Stock Market Wealth and the Real Economy:
A Local Labor Market Approach

Online Appendix

Gabriel Chodorow-Reich Plamen Nenov Alp Simsek

A Data Appendix

A.1 Details on the Capitalization Approach

A.1.1 Details on the IRS SOI

The IRS Statistics of Income (SOI) reports tax return variables aggregated to the zip code for 2004-2015 (and selected years before) and to the county for 1989-2015. Beginning in 2010 for the county files and in all available years for zip code files, the data aggregate all returns filed by the end of December of the filing year. Prior to 2010, the county files aggregate returns filed by the end of September of the filing year, corresponding to about 95% to 98% of all returns filed in that year. In particular, the county files before 2010 exclude some taxpayers who file form 4868, which allows a six month extension of the filing deadline to October 15 of the filing year.[1] To obtain a consistent panel, we first convert the zip code files to a county basis using the HUD USPS crosswalk file. We then implement the following algorithm: (i) for 2010 onward, use the county files; (ii) for 2004-2009, use the zip code files aggregated to the county level and adjusted by the ratio of 2010 dividends in the county file to 2010 dividends in the zip code aggregated file; (iii) for 1989-2003, use the county file adjusted by the ratio of 2004 dividends as just calculated to 2004 dividends in the county files. We implement the same adjustment for labor income. We exclude from the baseline sample 74 counties in which the ratio of dividend income from the zip code files to dividend income in the county files exceeds 2 between 2004 and 2009, as the importance of late filers in these counties makes the extrapolation procedure less reliable for the period before 2004.[2]

[1] See https://web.archive.org/web/20171019013107/https://www.irs.gov/statistics/soi-tax-stats-county-income-data-users-guide-and-record-layouts and https://web.archive.org/web/20190111012726/https://www.irs.gov/statistics/soi-tax-stats-individual-income-tax-statistics-zip-code-data-soi for data and documentation pertaining to the county and zip code files, respectively. For additional information on the timing of tax filings, see https://web.archive.org/web/20190211151353/https://www.irs.gov/newsroom/2019-and-prior-year-filing-season-statistics .

[2] Anecdotally, the filing extension option is primarily used by high-income taxpayers who may need to wait for additional information past the April 15 deadline (see e.g. Dale, Arden, "Late Tax Returns Common for the Wealthy," *Wall Street Journal*, March 29, 2013). Consistent with this, we find much less discrepancy in labor income than dividend income reported in the zip code and county files before 2010. Our results

Finally, since our benchmark analysis is at the quarterly frequency and the SOI income data is yearly data, we linearly interpolate the SOI data to obtain a quarterly series. Because the cross-sectional income distribution is persistent, measurement error arising from this procedure should be small.

A.1.2 Dividend yield adjustment

This section describes the county-specific dividend yield adjustment used in the capitalization of taxable county dividends. We start with the Barber and Odean (2000) data set, which contains a random sample of accounts at a discount brokerage, observed over the period 1991-96. The data contain monthly security-level information on financial assets held in the selected accounts. Graham and Kumar (2006) compare these data with the 1992 and 1995 waves of the SCF and show that the stock holdings of investors in the brokerage data are fairly representative of the overall population of retail investors.

We keep taxable individual and jointly owned accounts and remove margin accounts. We merge the monthly account positions data with the monthly CRSP stock price data and CRSP mutual funds data obtained from WRDS. Since our merge is based on CUSIP codes and mutual fund CUSIP codes are sometimes missing, we use a Fund Name-CUSIP crosswalk developed by Terry Odean and Lu Zheng. Additionally, we use an algorithm developed in Di Maggio et al. (forthcoming) based on minimizing the smallest aggregate price distance between mutual fund prices in household portfolios and in the CRSP fund-month data.[3] We drop household-month observations for which the value of total identified CRSP stocks and mutual funds is less than 95% of the value of the household's equity and mutual fund assets and also keep only identified CRSP stocks and mutual funds.[4] Finally, to be consistent with what we observe in the IRS-SOI data, we drop household-month observations with a zero dividend yield. Such households tend to be younger, hold few securities (around two on average), and hold only around 10% of total equity in the brokerage data.

We compute dividend yields by household and month using these data. Figure A.1 shows the average dividend yield by age of the household head (left panel) and by stock wealth percentile separately for different age bins (right panel), where household stock wealth is the total position equity in all accounts. As the figure shows, dividend yields increase with age. Moreover, within age bins, dividend yields have a weak relationship with wealth. These patterns motivate our focus on age.

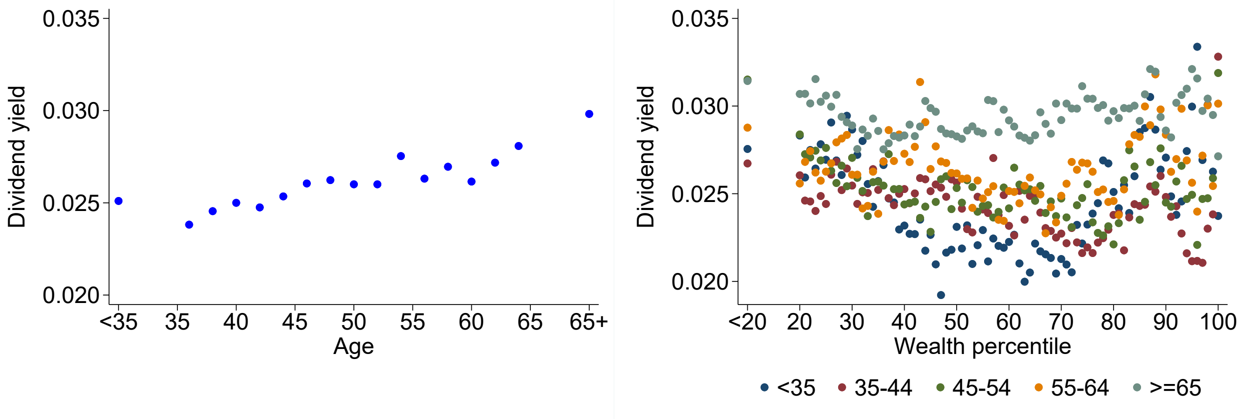
Table A.1 reports average dividend yields by age bin (weighted by wealth), separately for each Census Region. A few features merit mention. First, dividend yield increases with age, consistent

change little if we do not exclude the 74 counties from the analysis. For example, the coefficient for total payroll at the 7 quarter horizon changes from 2.18 to 2.27 (s.e.=0.67), and the coefficient for nontradable payroll changes from 3.23 to 2.67 (s.e.=0.83).

[3]We are grateful to Marco Di Maggio, Amir Kermani, and Kaveh Majlesi for sharing their codes.

[4]We are able to match more than 95% of equity and mutual fund position-months. The main type of equity assets that we cannot match are foreign stocks.

Figure A.1: Dividend Yield by Age and Wealth



Notes: The figures plot dividend yields by age and wealth quantile based on the Barber and Odean (2000) data from a discount brokerage firm merged with data on CRSP stocks and mutual funds. Wealth denotes the total position equity among all taxable accounts that a household has in the discount brokerage firm.

with the pattern shown in Figure A.1. Second, the age bin coefficients are precisely estimated and the R^2s are high. In column (5), which pools all geographic areas together, the five age bins explain 66% of the variation in dividend yield across households. Third, adding indicator variables for 10 wealth bins to the regression in column (6) has essentially no impact on the explanatory power of the regression or on the relative age bin coefficients.[5]

We combine the coefficients shown in columns (1)-(4) of Table A.1 with the county-year specific age structure from the Census Bureau and average wealth by age bin from the Survey of Consumer Finances (interpolated between SCF waves) to construct the wealth-weighted average of the age bin dividend yields in the county's Census region. The resulting county-year yields account for time series variation in a county's age structure and in relative wealth of different age groups, but not for changes in market dividend yields over time. Therefore, we scale these dividend yields so that the average dividend yield in each year is equal to the dividend yield on the value-weighted CRSP portfolio.[6]

We end this section with a discussion of (implied) dividend yields in the SCF and how those compare to the dividend yield distribution in the Barber and Odean (2000) data. The SCF contains information on taxable dividend income reported on tax returns together with self-reported information on directly held stocks (and stock mutual funds). Therefore, it is tempting to use the SCF data directly to compute dividend yields by demographic groups and use those for the dividend yield adjustment or, even more directly, use the relationship between taxable dividend income and total stock wealth in the SCF to impute total stock wealth directly from taxable dividends rather than doing the two-step procedure that we perform here. Unfortunately, there is one key difficulty in implementing this procedure with SCF data; in the SCF, stock wealth is reported for

[5]The age bin coefficients shift uniformly up by 0.37 to 0.38, reflecting the incorporation of average wealth.

[6]We also experimented with allowing the age-specific yields to vary with the CRSP yield, with almost no impact on our results.

Table A.1: Dividend Yields By Age

	Region 1	Region 2	Region 3	Region 4	Pooled	Pooled
	(1)	(2)	(3)	(4)	(5)	(6)
Right hand side variables:						
Age <35	2.81**	2.21**	2.28**	2.51**	2.45**	2.83**
	(0.16)	(0.19)	(0.25)	(0.18)	(0.11)	(0.15)
Age 35-44	2.48**	2.25**	2.43**	2.50**	2.43**	2.81**
	(0.11)	(0.16)	(0.18)	(0.14)	(0.08)	(0.12)
Age 45-54	2.65**	2.27**	2.51**	2.50**	2.49**	2.86**
	(0.16)	(0.09)	(0.30)	(0.08)	(0.08)	(0.13)
Age 55-64	3.00**	2.39**	2.40**	2.82**	2.69**	3.07**
	(0.11)	(0.14)	(0.20)	(0.10)	(0.08)	(0.13)
Age 65+	2.91**	2.73**	2.96**	3.27**	3.03**	3.40**
	(0.12)	(0.12)	(0.17)	(0.11)	(0.07)	(0.12)
Wealth bins	No	No	No	No	No	Yes
R^2	0.73	0.69	0.62	0.63	0.66	0.66
Individuals	1,965	1,586	2,192	3,556	9,299	9,299
Observations	73,486	60,987	83,112	133,149	350,734	350,734

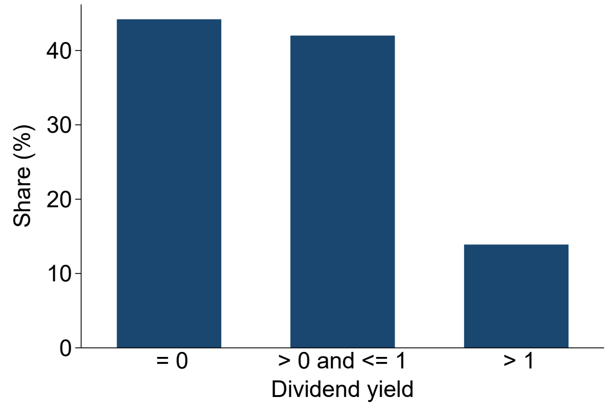
Notes: The table reports the coefficients from a regression of the account dividend yield on the variables indicated, at the account-month level. Standard errors in parentheses clustered by account. For readability, all coefficients multiplied by 100.

the survey year (more specifically, at the time of the interview), while taxable dividend income is based on the *previous* year's tax return. This creates biases in any dividend yields computed as the ratio of (previous year) dividend income to (current year) stock wealth. The bias is larger (in magnitude) for participants that (dis-)save more (either actively or passively through capital gains that the household does not respond to). Moreover, as we show in Figure A.2, a very large share of respondent-wave observations (more than 45%) report zero dividend income and positive stock wealth.[7] A large share of those are respondents that establish direct holdings of stocks (or mutual funds) some time between the end of the tax return year and the survey date. An analogous extensive margin adjustment may be taking place for respondents that report zero stock wealth and positive dividend income for the previous year. In that case the implied dividend yield is infinite.

Even if one disregards these two groups and only considers respondents for which the implied dividend yield is between zero and one, there is still substantial dispersion (and a possible bias) in the implied dividend yields. Figure A.3 shows the median implied dividend yields and interquartile ranges for 5 age groups for the 1992 and 1995 waves of the SCF and compares them against

[7]This is more than 2 times the account holders with zero dividend yield in the Barber and Odean (2000) data.

Figure A.2: SCF Implied Dividend Yield Categories



Notes: The figure shows the distribution of implied dividend yields in the SCF based on a comparison of the reported dividend income from tax returns against reported directly held stock market wealth.

the median dividend yields and inter-quartile ranges of (positive) dividend yields in the Barber and Odean (2000) data. Clearly the dividend yields in Barber and Odean (2000) are much more compressed around their median values compared to the SCF dividend yields. Moreover, the SCF dividend yields (conditional on being between zero and one) tend to be much higher than the Barber and Odean (2000) dividend yields.[8] Given these issues, we conclude that the SCF implied dividend yields cannot reliably be used for stock wealth imputation.

A.1.3 Non-taxable stock wealth adjustment

The SOI data exclude dividends held in non-taxable accounts (e.g. defined contribution retirement accounts). In this section, we describe how we adjust for non-taxable stock wealth to arrive at the stock market wealth variable we use in our empirical analysis.

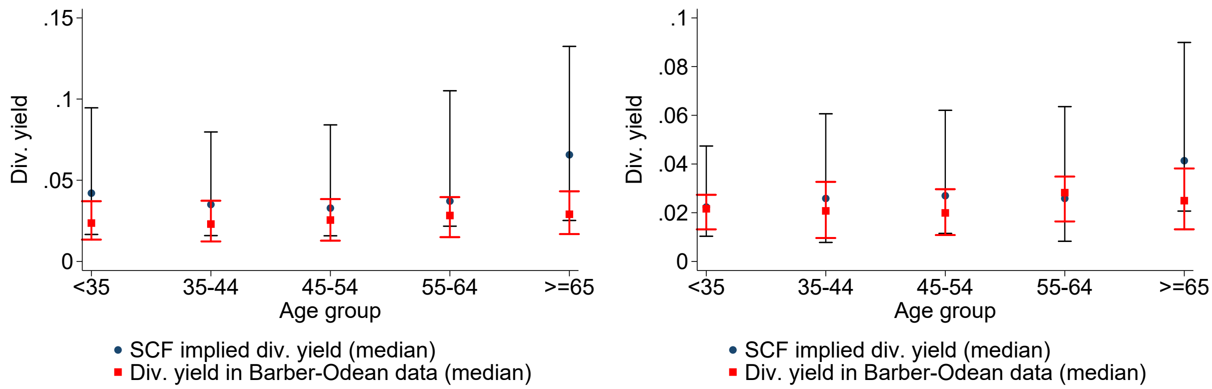
We begin by plotting in Figure A.4 the distribution of household holdings of corporate equity between taxable (directly held and non-IRA mutual fund) and non-taxable accounts using data from the Financial Accounts of the United States. Roughly 2/3 of corporate equity owned by households is held in taxable accounts.[9]

We next use data from the SCF to examine the relationship between total stock market wealth and stock market wealth held in taxable accounts in the cross-section of U.S. households. We pool all waves from 1992 to 2016, consistent with the sample period for our benchmark analysis. We

[8]This is also reflected in the mean dividend yields (not shown) in the SCF, which are substantially higher than the medians, while in Barber and Odean (2000) the two are comparable.

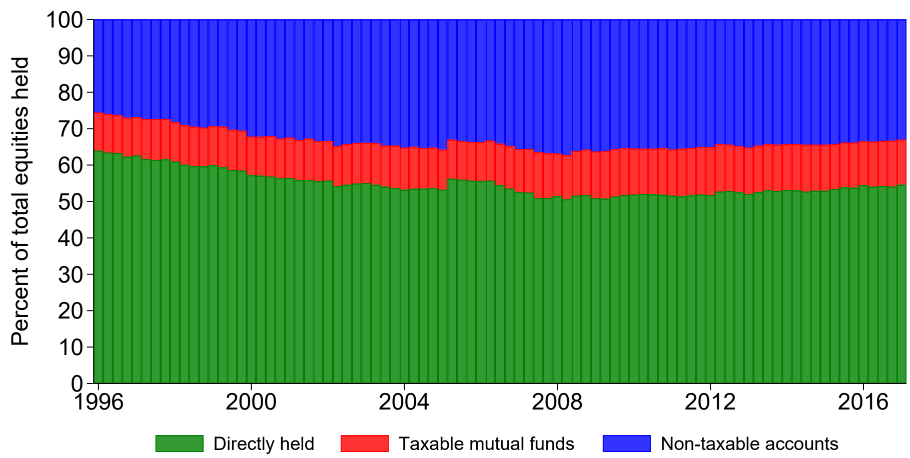
[9]Non-taxable retirement accounts here include only defined contribution accounts and exclude equity holdings of defined benefit plans. This definition accords with our empirical analysis since fluctuations in the market value of assets of defined benefit plans do not directly affect the future pension income of plan participants. The data plotted in Figure A.4 also include non-profit organizations, which hold about 10% of directly held equity and mutual fund shares.

Figure A.3: Dividend yield distributions by age group in the SCF and Barber and Odean (2000) data for 1992 (left) and 1995 (right)



Notes: Dots denote median values and bars show the inter-quartile range. The figures plot the distribution of implied dividend yields in the SCF (for dividend yields that are in (0, 1)) and dividend yields in the Barber and Odean (2000) data from a discount brokerage firm (for positive dividend yields) by age group for 1992 and 1995.

Figure A.4: Household Stock Market Wealth in the FAUS



Notes: The figure reports household equity wealth as reported in the Financial Accounts of the United States. We define stock market wealth as total equity wealth (table B.101.e line 14, code LM153064475Q) less the market value of S-corporations (table L.223 line 31, code LM883164133Q) and similarly define directly held stock market wealth as directly held equity wealth (table B.101.e line 15, code LM153064105Q) less the market value of S-corporations. Taxable mutual funds are total mutual fund holdings of equity shares (table B.101.e line 21, code LM653064155Q) less equity held in IRAs, where we compute the latter by assuming the same equity share of IRAs as of all mutual funds, IRA mutual fund equity = IRA mutual funds at market value (table L.227 line 16, code LM653131573Q) $\times$ total equities held in mutual funds /total value of mutual funds (table B.101.e line 21, code LM653064155Q + table B.101.e line 12, code LM654022055Q). Non-taxable accounts include equities held through life insurance companies (table B.101.e line 17, code LM543064153Q), in defined contribution accounts of private pension funds (table B.101.e line 18, code LM573064175Q), federal government retirement funds (table B.101.e line 19, code LM343064125Q), and state and local government retirement funds (table B.101.e line 20, code LM223064213Q), and through mutual funds in IRAs.

Table A.2: Summary Statistics (values are in 2016 dollars).

Variable	Mean	Std. Dev.	Min	Max
total stock wealth	119,402	1,144,358	0	9.87×10^8
taxable stock wealth	65,428	1,001,526	0	9.84×10^8

use the definition for stock-market wealth used in the Fed Bulletins.[10] Following the Fed Bulletin definition of stock-market wealth, we define taxable stock wealth as the sum of direct holdings of stocks, stock mutual funds and other mutual funds, and 1/2 of the value of combination mutual funds. All variables are expressed in constant 2016 dollars. Table A.2 reports summary statistics for total stock wealth and taxable stock wealth.

Table A.3 reports the coefficients from regressions of total stock wealth on taxable stock wealth. There is a positive constant term, indicating that nontaxable stock market wealth is more evenly distributed than taxable wealth. The coefficient on taxable stock wealth is between 1.08 and 1.09 and the R^2 is around 0.91. Therefore, total stock wealth and taxable stock wealth vary almost one-for-one.

The high R^2 from these regressions suggests that we can use the relationship between total stock wealth, taxable stock wealth, and demographics in the SCF to account for non-taxable stock wealth at the county level. Specifically, we again use all waves of the SCF from 1992 to 2016. For each survey wave, we use a specification as in Column (2) of Table A.3. We then interpolate these coefficient estimates for years in which no survey took place. Finally, we use the estimate of (real) taxable stock wealth from capitalizing taxable dividend income and county-level demographic information on population shares in different age bins and the college share (interpolated at yearly frequency from the decadal census and also extrapolated past 2010) to arrive at real total stock wealth for each county and year.

A.1.4 Non-public companies.

One remaining source of measurement error in our capitalization approach arises because dividend income reported on form 1040 includes dividends paid by private C-corporations. Such income accrues to owners of closely-held corporations and is highly concentrated at the top of the wealth distribution. Figure A.5 uses data from the Financial Accounts of the United States to plot the market value of equity issued by privately held C-corporations as a share of total equity issued by domestic C-corporations.[11] This share never exceeds 7% of total equity, indicating that as a

[10] The precise definition is available here: https://www.federalreserve.gov/econres/files/bulletin.macro.txt. Stock-market wealth appears as "financial assets invested in stock".

[11] Since 2015, table L.223 of the Financial Accounts of the United States has reported equity issued by domestic corporations separately by whether the corporation's equity is publicly traded, with the series extended back to 1996 using historical data. While obtaining market values of privately held corporations

Table A.3: Total stock wealth and taxable stock wealth

	(1)	(2)
Taxable stock wealth	1.09**	1.08**
	(0.01)	(0.01)
Age < 25		-12933.06**
		(1225.68)
Age 25-34		-22996.77**
		(1097.07)
Age 35-44		-2788.01*
		(1236.89)
Age 45-54		29412.54**
		(1790.46)
Age 55-64		64398.51**
		(2894.11)
Age 65+		34482.50**
		(2164.56)
College degree		102265.11**
		(2869.13)
Constant	48221.15**	
	(943.52)	
R^2	0.91	0.91
Observations	44,633	44,497

Notes: The table reports coefficient estimates from regressing (real) total stock wealth on (real) taxable stock wealth, and household head demographics in the SCF using the pooled 1992-2016 waves. Robust standard errors in parenthesis. * denotes significance at the 5% level, and ** denotes significance at the 1% level.

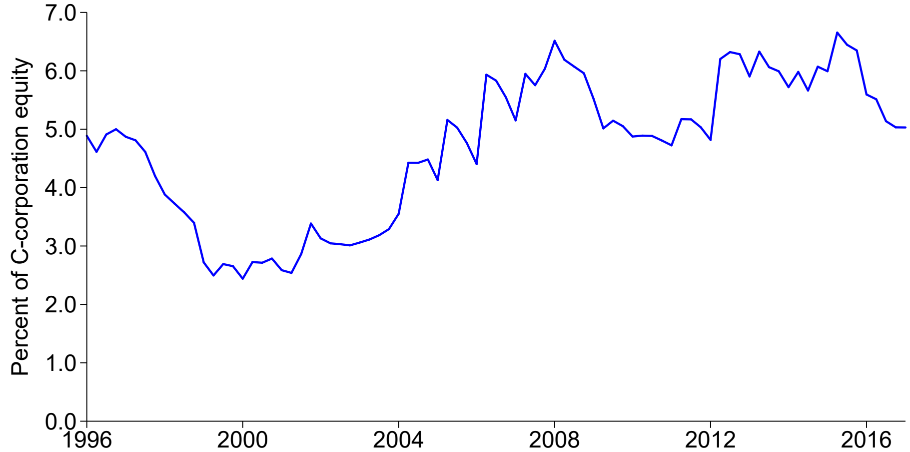
practical matter dividend income from non-public C-corporations is small. Moreover, as described in Appendix A.1 our baseline sample excludes a small number of counties with a substantial share of dividend income reported by late filers who disproportionately own closely-held corporations. Therefore, non-public C-corporation wealth does not appear to meaningfully affect our results.

A.1.5 Return heterogeneity

Similar to the dividend yield adjustment we also compute a county-specific stock market return. The systematic differences in dividend yields across households with different age that are the basis for our dividend yield adjustment in Appendix A.1.2 imply possible systematic differences in portfolio

necessarily requires some imputations (Ogden et al., 2016), we believe the results to be the best estimate of this split available and unlikely to be too far off.

Figure A.5: Equity of Privately Held C-Corporations



Notes: The figure reports the market value of equity of privately held C-corporations as a share of total (privately held plus publicly-traded) equity of domestic C-corporations as reported in the Financial Accounts of the United States table L.223 lines 29 and 32.

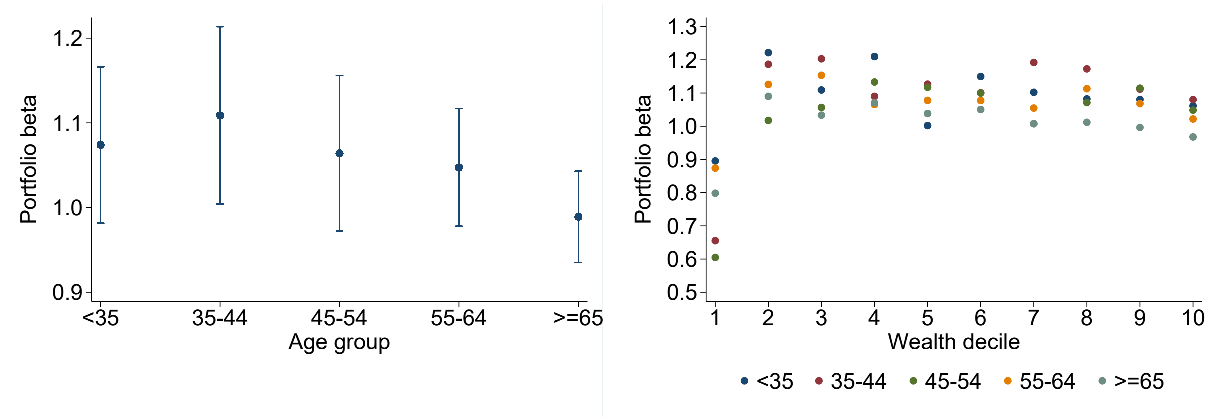
return characteristics across these same age groups. For example, it is well-known that stocks with higher dividend yields tend to be value stocks with a different return distribution than the stock market. Specifically, those stocks tend to have market betas below one. In that case the portfolio betas of households living in counties with predominantly older households will be lower than those of households living in counties with predominantly younger households. In this section we first present evidence using the Barber and Odean (2000) data set that there is indeed a systematic (although quite small) relation between portfolio betas and age. Second, as with the dividend yield adjustment from Appendix A.1.2 we use this relationship and county demographic information to construct a county-specific beta and compute a county-specific stock market return.

We use the household portfolio data described in Appendix A.1.2 and construct value-weighted portfolios by age group (for the same 5 age groups as in Appendix A.1.2).[12] We then construct monthly returns for these portfolios by computing the weighted one-month return on the underlying CRSP assets.[13] Using these monthly returns we estimate portfolio betas using the return on the CRSP value weighted index as the return on the market portfolio and the 3-month T-Bill yield as the risk free rate. Figure A.6 (left panel) plots the estimated portfolio betas together with a 95% confidence intervals. As the Figure shows there is a negative (albeit small in magnitude) relationship between beta and age with younger households having portfolios with higher beta (and beta above one) compared to older households.

[12]One difference relative to the sample we use in Appendix A.1.2 is that we also include household-month observations that have zero dividends. The reason for keeping these households in this case is that we want to construct a county-level stock market return that will be applied to county-level stock market wealth, which also includes the stock wealth of households that hold only non-dividend paying stocks in their portfolios.

[13]Household positions are recorded at the beginning of a month, so similar to Barber and Odean (2000) we implicitly assume that each household holds the assets in their portfolio for the duration of the month.

Figure A.6: Portfolio Beta by Age and Wealth



Notes: The figures plot the portfolio betas by age and wealth quantile based on the Barber and Odean (2000) data from a discount brokerage firm merged with data on CRSP stocks and mutual funds. Wealth denotes the total position equity among all taxable accounts that a household has in the discount brokerage firm.

We next use this relationship to construct a county-specific beta and from it a county-specific stock market return. Specifically, as with the dividend-yield adjustment, we combine the estimated betas shown in the left panel of Figure A.6 with the county-year specific age structure from the Census Bureau and average wealth by age bin from the Survey of Consumer Finances (interpolated between SCF waves) to construct the wealth-weighted average of the age bin portfolio betas for each county and year. Finally, we scale these betas so that the average beta in each year is equal to one (that is, we assume that on average counties hold the market portfolio). We then multiply CRSP total stock return by these county-year specific betas to arrive at a county-specific stock-market return.

A.2 Summary Statistics

Table A.4 reports the mean and standard deviation of the 8 quarter change in the labor market variables. It also reports the standard deviation after removing county-specific means and state-quarter means, with the latter being the variation used in the main analysis.

A.3 County demographic characteristics and stock wealth

To more clearly illustrate that our empirical strategy does not depend on stock wealth to labor income being randomly assigned across counties, we correlate the (time-averaged) county level value of stock wealth to labor income with a number of county level demographics. Specifically, we use time-averaged data from the 1990, 2000 and 2010 US Census to compute the county level shares of individuals 25 years and older with bachelor degree or higher, median age of the resident population, share of retired workers receiving social security benefits, share of females, and share

Table A.4: Summary Statistics

Variable	Source	Mean	SD	Within county SD	Within county and state-quarter SD	Obs.
Quarterly total return on market	CRSP	0.019	0.067			94
Capitalized dividends/labor income	IRS SOI	2.316	1.177	0.628	0.309	269 057
Log empl., 8Q change	QCEW	0.025	0.053	0.047	0.032	272 942
Log payroll, 8Q change	QCEW	0.084	0.077	0.072	0.048	272 942
Log nontradable empl., 8Q change	QCEW	0.031	0.069	0.064	0.054	269 774
Log nontradable payroll, 8Q change	QCEW	0.081	0.089	0.084	0.064	269 774
Log tradable empl., 8Q change	QCEW	−0.018	0.130	0.123	0.106	258 856
Log tradable payroll, 8Q change	QCEW	0.045	0.158	0.151	0.128	258 856

Notes: The table reports summary statistics. Within county standard deviation refers to the standard deviation after removing county-specific means. Within county and state-quarter standard deviation refers to the standard deviation after partialling out county and state-quarter fixed effects. All statistics weighted by 2010 population.

of the resident population identifying themselves as white.[14] Table A.5 reports the coefficient estimates from population weighted regressions of stock wealth to labor income on each demographic characteristics as well as a regression including all demographic characteristics (last column). All regressions include state fixed effects. Unsurprisingly, the share of retired workers and share with college degree are robustly positively related with the average stock wealth to labor income ratio in a county. The share of females and white is negatively related with stock wealth to labor income although the effects are smaller. Median age does not co-move with stock wealth to income after controlling for the other demographic characteristics.

A.4 Coefficients on control variables

This appendix reproduces the baseline results in Table 1 including the coefficients on the main control variables.

A.5 Responses by Category in the Consumer Expenditure Survey

This appendix describes our analysis of consumption responses by category using the interview module of the Consumer Expenditure Survey (CE). The CE interviews sampled households for up to four consecutive quarters about all expenditures over the prior three months on a detailed set

[14]For the college share we use the American Community Survey rather than the 2010 US Census.

Table A.5: County demographics regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Bachelor degree or higher (%)	0.06**					0.09**
	(0.01)					(0.01)
Median age		0.10*				−0.04*
		(0.04)				(0.02)
Retired (%)			0.12**			0.31**
			(0.04)			(0.03)
Female (%)				0.19**		−0.06*
				(0.04)		(0.03)
White (%)					−0.00	−0.02**
					(0.00)	(0.00)
Population weighted	Yes	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.31	0.21	0.22	0.18	0.15	0.54
Observations	3,141	3,141	3,141	3,141	3,141	3,141

Notes: The table reports coefficients and standard errors from regressing time-averaged total stock wealth by labor income on county demographics. Standard errors in parentheses are clustered by state. * denotes significance at the 5% level, and ** denotes significance at the 1% level.

of categories. While the survey does not ask directly about stock holdings, in the last interview it records information on security holdings. Dynan and Maki (2001) and Dynan (2010) use this information and the short panel structure of the survey to separately relate consumption growth of security holders and non-security holders to the change in the stock market. We follow the analysis in Dynan and Maki (2001) as closely as possible and extend it by measuring the response of retail and restaurant spending separately.[15]

The specification in Dynan and Maki (2001) is:

$$\Delta \ln C_{i,t} = \sum_{j=0}^{3} \beta_j \Delta \ln W_{t-j} + \Gamma' X_{i,t} + \epsilon_{i,t},$$

where $\Delta \ln C_{i,t}$ is the log change in consumption expenditure by household i between the second and fifth CE interviews,[16] $\Delta \ln W_{t-j}$ is the log change in the Wilshire 5000 between the recall periods

[15]The Dynan and Maki (2001) sample covers the period 1983-1998. Dynan (2010) finds negligible consumption responses when extending the sample through 2008, possibly reflecting the deterioration in the quality of the CE sample in the more recent years and the difficulty in recruiting high income and high net worth individuals to participate. Since our purpose is to compare the responses of different categories of consumption, we restrict to periods when the data can capture an overall response.

[16]The first CE interview introduces the household to the survey but does not collect consumption information. Therefore, the span between the second and fifth interviews is the longest span available.

Table A.6: Baseline Results

	All		Non-traded		Traded	
	Emp.	W&S	Emp.	W&S	Emp.	W&S
	(1)	(2)	(3)	(4)	(5)	(6)
Right hand side variables:						
$S_{a,t-1}R_{a,t-1,t}$	0.77*	2.18**	2.02*	3.24**	–0.11	0.71
	(0.36)	(0.63)	(0.80)	(1.01)	(0.64)	(0.74)
Bartik predicted employment	0.86**	1.46**	0.59**	0.84**	1.66**	2.11**
	(0.08)	(0.14)	(0.10)	(0.10)	(0.19)	(0.25)
Labor income interaction	–1.11+	–2.65**	0.96	–0.92	1.70	1.92
	(0.62)	(0.87)	(0.99)	(1.19)	(1.92)	(2.12)
Business income interaction	1.08+	2.53**	–1.26	0.58	–1.63	–1.90
	(0.61)	(0.83)	(0.99)	(1.17)	(1.89)	(2.05)
Bond return interaction	–0.07	–0.14	3.58+	2.80	0.20	–0.51
	(0.82)	(1.39)	(1.87)	(2.32)	(1.20)	(1.81)
House price interaction	–1.55	5.45	–8.33*	2.29	–9.91	–4.88
	(3.28)	(4.40)	(4.14)	(5.25)	(6.32)	(6.87)
Horizon h	Q7	Q7	Q7	Q7	Q7	Q7
Pop. weighted	Yes	Yes	Yes	Yes	Yes	Yes
County FE	Yes	Yes	Yes	Yes	Yes	Yes
State × time FE	Yes	Yes	Yes	Yes	Yes	Yes
Shock lags	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.66	0.64	0.39	0.48	0.35	0.36
Counties	2,901	2,901	2,896	2,896	2,877	2,877
Periods	92	92	92	92	92	92
Observations	265,837	265,837	263,210	263,210	252,928	252,928

Notes: The table reports coefficients and standard errors from estimating Eq. (1) for $h = 7$. Columns (1) and (2) include all covered employment and payroll; columns (3) and (4) include employment and payroll in NAICS 44-45 (retail trade) and 72 (accommodation and food services); columns (5) and (6) include employment and payroll in NAICS 11 (agriculture, forestry, fishing and hunting), NAICS 21 (mining, quarrying, and oil and gas extraction), and NAICS 31-33 (manufacturing). The shock occurs in period 0 and is an increase in stock market wealth equivalent to 1% of annual labor income. For readability, the table reports coefficients in basis points. Standard errors in parentheses and double-clustered by county and quarter. * denotes significance at the 5% level, and ** denotes significance at the 1% level.

covered by the second and fifth interviews ($j = 0$) or over consecutive, non-overlapping 9 month periods preceding the second interview ($j = 1, 2, 3$), and $X_{i,t}$ contains monthly categorical variables to absorb seasonal patterns in consumption, taste shifters (age, age^2, family size), socioeconomic variables (race, high school completion, college completion), labor earnings growth between the second and fifth interviews, and year categorical variables. Thus, this specification attempts to

Table A.7: Consumption Responses in the Consumer Expenditure Survey

	Non-durable goods and services		Retail and restaurants	
	SH	Other	SH	Other
	(1)	(2)	(3)	(4)
Right hand side variables:				
Stock return	0.369**	–0.015	0.198	–0.038
	(0.133)	(0.048)	(0.277)	(0.100)
Lag 1	0.385*	0.074	0.519+	0.121
	(0.151)	(0.053)	(0.312)	(0.109)
Lag 2	0.252+	0.050	0.447	0.065
	(0.134)	(0.047)	(0.278)	(0.097)
Lag 3	0.039	0.038	0.104	0.135+
	(0.103)	(0.037)	(0.220)	(0.077)
Sum of coefficients	1.044	0.146	1.268	0.283
R^2	0.02	0.01	0.02	0.01
Observations	4,086	28,329	4,026	28,376

Notes: The estimating equation is: $\Delta \ln C_{i,t} = \sum_{j=0}^{3} \beta_j \Delta \ln W_{t-j} + \Gamma' X_{i,t} + \epsilon_{i,t}$, where $\Delta \ln C_{i,t}$ is the log change in consumption expenditure by household i between the second and fifth CE interviews in the consumption category indicated in the table header and $\Delta \ln W_{t-j}$ is the log change in the Wilshire 5000 between the recall periods covered by the second and fifth interviews ($j = 0$) or over consecutive, non-overlapping 9 month periods preceding the second interview ($j = 1, 2, 3$). All regressions include controls for calendar month and year of the final interview, age, age^2, family size, race, high school completion, college completion, and labor earnings growth between the second and fifth interviews. The sample is 1983-1998. Columns marked SH include households with more than $10,000 of securities.

address the causal identification challenge by controlling directly for contemporaneous labor income growth and including year categorical variables, the latter which isolate variation in recent stock performance for households interviewed during different months of the same calendar year. Following Mankiw and Zeldes (1991), the specification is estimated separately for households above and below a cutoff value for total securities holdings.

Table A.7 reports the results. The left panel contains our replication of table 2 in Dynan and Maki (2001) and Dynan (2010). We find very similar results to those papers. Notably, expenditure on nondurable goods and services rises on impact for households categorized as stock holders and continues to rise over the next 18 months following a positive stock return. This sluggish response accords with the sluggish adjustment of labor market variables in our main analysis. Summing over the contemporaneous and lag coefficients, the total elasticity of expenditure to increases in stock market wealth is about 1. In contrast, total expenditure by non-stock holders does not increase.

The right panel replaces the consumption measure with purchases of non-durable and durable goods from retail stores and purchases at restaurants. These categories provide the closest correspondence to all purchases made at stores in the retail or restaurant sectors.[17] The cumulative consumption responses of purchases of goods from retail stores and at restaurants are very similar to the responses of total non-durable goods and services, albeit estimated with less precision.

Overall, these results provide support for our measure of local expenditure and of the homotheticity assumption we use to structural interpret our estimates. This conclusion holds even if one questions the causal identification of the Dynan and Maki (2001) framework, in which case one can interpret the relative responses across categories as reflective of general demand shocks rather than the stock market in particular.

B Model Details

In this appendix, we present the full model. In Section B.1, we describe the environment and define the equilibrium. For completeness, we repeat the key equations shown in the main text. In Section B.2, we provide a general characterization of equilibrium. In Section B.3, we provide a closed form solution for a benchmark case in which areas have the same stock wealth. In Section B.4, we log-linearize the equilibrium around the common-wealth benchmark and provide closed form solutions for the log-linearized equilibrium with heterogeneous stock wealth. In Section B.5, we use our results to characterize the cross-sectional effects of shocks to stock valuations. In Section B.6, we establish the robustness of the benchmark calibration of the model that we present in the main text. In Section B.7, we analyze the aggregate effects of shocks to stock valuations (when monetary policy is passive) and compare the results with our earlier results on the cross-sectional effects. Finally, we consider two extensions of the baseline model. In Section B.8, we extend the model to incorporate uncertainty, and we show that our results are robust to obtaining the stock price fluctuations from alternative sources such as changes in households' risk aversion or perceived risk. In Section B.9, we extend the model to consider more general levels for the EIS parameter and discuss how it would affect our analysis.

B.1 Environment and Definition of Equilibrium

Basic Setup and Interpretation. There are two factors of production: capital and labor. There is a continuum of measure one of areas (counties) denoted by subscript a. Areas are identical except for their initial ownership of capital.

There are two periods $t \in \{0, 1\}$. We view period 1 as "the long run" over which wages are flexible and all factors are mobile across the areas. In the long run, outcomes will be determined by productivity. In contrast, period 0 corresponds to "the short run" over which wages are somewhat

[17]Because we include durable goods, the categories in the right panel are not a strict subset of the categories in the left panel. We have experimented with excluding durable goods from the basket and obtain similar results.

sticky and labor is not mobile. In this case, outcomes will be determined by aggregate demand. Hence, we interpret a period in the model as corresponding to several years.

Our focus is to understand how fluctuations in stock wealth affect cross-sectional and aggregate outcomes in the short run. To this end, we will generate endogenous changes in the price of capital in period 0 from exogenous changes to the productivity of capital in period 1. We interpret these changes as capturing stock price fluctuations due to a "time-varying risk premium." We validate the risk premium interpretation in Section B.8, where we introduce uncertainty about capital productivity in period 1.

Goods and Production Technologies. In every period t, there is a composite tradable good that can be consumed everywhere. For each area a, there is also a corresponding nontradable good that can only be produced and consumed in area a. Labor and capital are perfectly mobile across the production technologies described below (but labor is not mobile across areas in period 0 as we will describe later). We assume production firms are competitive and not subject to nominal rigidities (we will assume nominal rigidities in the labor market).

The nontradable good in area a can be produced according to a standard Cobb-Douglas technology,

$$Y_{a,t}^N = \left(K_{a,t}^N/\alpha^N\right)^{\alpha^N} \left(L_{a,t}^N/\left(1-\alpha^N\right)\right)^{1-\alpha^N}. \tag{B.1}$$

Here, $L_{a,t}^N, K_{a,t}^N$ denotes the amount of area a labor and capital used to produce the nontradable good. The term $1-\alpha^N$ captures the share of labor in the nontradable sector.

The tradable good can be produced in two ways. First, it can be produced as a composite of tradable inputs across areas, where each input is produced according to a standard Cobb-Douglas technology:

$$\begin{aligned} Y_t^T &= \left(\int_a \left(Y_{a,t}^T\right)^{\frac{\varepsilon-1}{\varepsilon}} da\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ \text{where } Y_{a,t}^T &= \left(K_{a,t}^T/\alpha^T\right)^{\alpha^T} \left(L_{a,t}^T/\left(1-\alpha^T\right)\right)^{1-\alpha^T}. \end{aligned} \tag{B.2}$$

Here, $L_{a,t}^T, K_{a,t}^T$ denotes the amount of area a labor and capital used to produce the tradable good. The term $1-\alpha^T$ captures the share of labor in the tradable sector. The parameter, $\varepsilon > 0$, captures the elasticity of substitution across tradable inputs. When $\varepsilon > 1$ (resp. $\varepsilon < 1$), tradable inputs are gross substitutes (resp. gross complements).

Second, the tradable good can also be produced by another technology that uses only capital. This technology is linear,

$$\tilde{Y}_t^T = D_t^{1-\alpha^T} \tilde{K}_t^T. \tag{B.3}$$

Here, $\tilde{K}_t^T$ denotes the aggregate capital employed in the capital-only technology, and $\tilde{Y}_t^T$ denotes the tradables produced via this technology (we use the tilde notation to distinguish them from K_t^T and Y_t^T). The term, $D_t^{1-\alpha^T}$, captures the capital productivity in period t. The normalizing power

$1-\alpha^T$ ensures that we obtain relatively simple expressions. As we will verify below, the rental rate (and thus, the price) of capital will depend on the productivity in the capital-only sector, D_t.

Capital Supply. In each period t, capital supply is exogenous,

$$K_t = \overline{K} \equiv 1 \text{ for each } t \in \{0, 1\}. \tag{B.4}$$

To simplify the notation, we normalize the exogenous capital supply to one. Capital is perfectly mobile across areas in both periods (so its location is not important).

Financial Assets. There are two financial assets. First, there is a claim to capital (which we view as corresponding to the stock market). We let Q_0 denote the nominal cum-dividend price of capital in period 0. Recall that the supply of capital is normalized to one and its nominal rental rate is denoted by R_t. Thus, $Q_0 - R_0$ denotes the nominal ex-dividend price at the end of period 0.

Second, there is also a risk-free asset in zero net supply. We let R^f denote the nominal gross risk-free interest rate.

Heterogeneous Ownership of Capital. Households in different areas start with zero units of the risk-free asset but they can differ in their endowments of capital. Specifically, we let $1 + x_{a,t}$ denote the share of aggregate capital held by investors in area a in period t. The initial shares, $\{1 + x_{a,0}\}_a$, are exogenous and can be heterogeneous. The common-wealth benchmark corresponds to the special case with $x_{a,0} = 0$ for each a.

Nominal Prices. We let $W_{a,t}$ and $P^N_{a,t}$ denote, respectively, the nominal wage per unit of labor and the nominal price of the nontradable good in period t and area a. Likewise, we let R_t and P^T_t denote, respectively, the (nominal) rental rate of capital and the (nominal) price of the tradable good in period t.

Note that our assumption that labor is mobile across areas in period 1 implies that the nominal wage in period 1 is also the same across areas. We assume monetary policy stabilizes the nominal long-run wage at a constant level, that is:

$$W_{a,1} = \overline{W} \text{ for each } a. \tag{B.5}$$

Households' Optimization Decisions. The representative household in each area separates its consumption and labor choices as follows. At the beginning of period 0, the household splits into a consumer and a continuum of workers. The consumer makers consumption-saving decisions and the workers choose labor supply. At the end of the period the household recombines and makes a portfolio decision to allocate savings between capital (stock wealth) and the risk-free asset.[18]

[18]Without loss of generality, we allow the consumer to make the portfolio decision as well.

We choose to model consumption and labor decisions separately for two reasons. First, assuming workers choose labor according to Greenwood et al. (1988) (GHH) preferences allows us to ignore the wealth effects of labor supply. Second, we can endow consumers with standard time-separable preferences. In addition to simplifying the subsequent expressions, this setup accords with the fact that workers hold relatively little stock market wealth. At the same time, we sidestep some consequences of GHH preferences, such as leading to unplausibly large fiscal and monetary multipliers (Auclert and Rognlie, 2017)

Consumption-Saving and Portfolio Choice Problem. The household in area a divides its consumption $C_{a,t}$ between the tradable good, $C^T_{a,t}$, and the nontradable good, $C^N_{a,t}$, according to the intra-period preferences:

$$C_{a,t} = \left(C^N_{a,t}/\eta\right)^{\eta} \left(C^T_{a,t}/\left(1-\eta\right)\right)^{1-\eta}. \tag{B.6}$$

With this normalization, the ideal price index is given by,

$$P_{a,t} \equiv \left(P^N_{a,t}\right)^{\eta} \left(P^T_t\right)^{1-\eta}. \tag{B.7}$$

Households can be thought of as choosing the consumption aggregator $C_{a,t}$ at these prices. They then distribute their spending optimally across the two sectors. The optimal expenditure on each sector satisfies,

$$P^N_{a,t}C^N_{a,t} = \eta P_{a,t}C_{a,t} \text{ and } P^T_{a,t}C^T_{a,t} = \left(1-\eta\right)P_{a,t}C_{a,t}. \tag{B.8}$$

The household in area a chooses how much to consume and save and how to allocate savings across capital and the risk-free asset. The consumer's problem can then be written as,

$$\begin{aligned} &\max_{C_{a,0},1+x_{a,1}} \log C_{a,0} + \delta \log C_{a,1} \\ &P_{a,0}C_{a,0} + S_{a,0} = W_{a,0}L_{a,0} + \left(1+x_{a,0}\right)Q_0, \\ &S_{a,0} = S^f_{a,0} + \left(1+x_{a,1}\right)\left(Q_0 - R_0\right) \\ &P_{a,1}C_{a,1} = \overline{WL}_1 + \left(1+x_{a,1}\right)R_1 + S^f_{a,0}R^f. \end{aligned} \tag{B.9}$$

Here, $1+x_{a,1}$ denotes the units of capital that the household purchases. This purchase costs $\left(1+x_{a,1}\right)\left(Q_0-R_0\right)$ units of the consumption good in period 0. Households invest the rest of their savings, $S^f_{a,0} = S_{a,0} - \left(1+x_{a,1}\right)\left(Q_0-R_0\right)$, in the risk-free asset.

Labor Supply Problem. In period 1, the labor supply is exogenous (and constant across areas), that is:

$$L_{a,1} = \overline{L}_1 \text{ for each } a. \tag{B.10}$$

In period 0, the labor supply is endogenous. For the purpose of endogenizing the labor supply, we work with a GHH functional form for the intraperiod preferences between consumption and labor that eliminates the wealth effects on the labor supply. These effects seem counterfactual for business cycle analysis in general (Galí (2011)). In our context, they are likely to be insignificant also because stock wealth is a relatively small fraction of total household wealth (including human capital wealth).

Specifically, in each area the representative household consists of a continuum of workers denoted by $\nu \in [0,1]$. The workers provide specialized labor services. They set their individual wages and labor supply to maximize the intra-period utility function:

$$\log\left(C_{a,0} - \chi \int_0^1 \frac{(L_{a,0}(\nu))^{1+\varphi}}{1+\varphi} d\nu\right). \tag{B.11}$$

Here, $C_{a,0}$ denotes the composite of nontradable and tradable goods as in the main model and $L_{a,0}(\nu)$ denotes the labor supply by worker ν who specializes in providing a particular type of labor service. The parameter, φ, captures the inverse Frisch elasticity of the labor supply; and the parameter, χ, captures the disutility form labor. The intraperiod budget constraint is given by:

$$P_{a,0}C_{a,0} + S_{a,0} = \int_0^1 W_{a,0}(\nu) L_{a,0}(\nu) d\nu + (1 + x_{a,0}) Q_0. \tag{B.12}$$

Here, $P_{a,0}$ denotes the ideal price index over nontradable and tradable goods.

In each area a, there is also an intermediate firm that produces the labor services in the area by combining specific labor inputs from each worker type according to the aggregator:

$$L_{a,0} = \left(\int_0^1 L_{a,0}(\nu)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} d\nu\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}.$$

This leads to the labor demand equation:

$$L_{a,0}(\nu) = \left(\frac{W_{a,0}(\nu)}{W_{a,0}}\right)^{-\varepsilon_w} L_{a,0} \tag{B.13}$$

$$\text{where } W_{a,0} = \left(\int_0^1 W_{a,0}(\nu)^{1-\varepsilon_w} d\nu\right)^{1/(1-\varepsilon_w)}. \tag{B.14}$$

Here, $L_{a,0}$ denotes the aggregate equilibrium labor provided by the intermediate firm, which is the same as the labor supply in the main text.

In period 0, a fraction of the workers in an area, λ_w, reset their wages to maximize the intra-period utility function in $(B.11)$ subject to the labor demand equation in $(B.13)$ and the budget constraints in $(B.12)$. The remaining fraction, $1 - \lambda_w$, have preset wages given by $\overline{W}$ (which is the same as the long-run wage level for simplicity).

The wage level in an area is determined according to the ideal price index $(B.14)$. This index also ensures:

$$\int_0^1 W_{a,0}(\nu) L_{a,0}(\nu) d\nu = W_{a,0} L_{a,0}.$$

Substituting this into Eq. $(B.12)$, we obtain the budget constraint in problem $(B.9)$ stated earlier.

Optimal Wage Setting and the Wage Phillips Curve. First consider the flexible workers that reset their wages in period 0. These workers optimally choose $\left(W_{a,0}^{flex}, L_{a,0}^{flex}\right)$ that satisfy:

$$\begin{aligned} W_{a,0}^{flex} &\equiv P_{a,0} \frac{\varepsilon_w}{\varepsilon_w - 1} MRS_{a,0} \qquad \text{(B.15)} \\ \text{where } MRS_{a,0} &= \chi \left(L_{a,0}^{flex}\right)^{\varphi} \text{ and } L_{a,0}^{flex} = \left(\frac{W_{a,0}^{flex}}{W_{a,0}}\right)^{-\varepsilon_w} L_{a,0} \end{aligned}$$

In particular, workers set a real (inflation-adjusted) wage that is a constant markup over their marginal rate of substitution between labor and consumption (MRS). The functional form in $(B.11)$ ensures that the MRS depends on the level of labor supply but not on the level of consumption.

Note that $W_{a,0}^{flex}$ appears on both side of Eq. $(B.15)$. Solving for the fixed point, we further obtain:

$$\left(W_{a,0}^{flex}\right)^{1+\varphi\varepsilon_w} = \frac{\varepsilon_w}{\varepsilon_w - 1} \chi P_{a,0} W_{a,0}^{\varepsilon_w \varphi} L_{a,0}^{\varphi}. \qquad \text{(B.16)}$$

Next consider the sticky workers. These workers have preset wages, $\overline{W}$, and they provide the labor services demanded at these wages.

Next we use $(B.14)$ to obtain an expression for the aggregate wage level:

$$\begin{aligned} W_{a,0} &= \left(\lambda_w \left(W_{a,0}^{flex}\right)^{1-\varepsilon_w} + (1-\lambda_w) \overline{W}^{1-\varepsilon_w}\right)^{1/(1-\varepsilon_w)} \\ &= \left(\lambda_w \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \chi W_{a,0}^{\varepsilon_w \varphi} P_{a,0} L_{a,0}^{\varphi}\right)^{(1-\varepsilon_w)/(1+\varphi\varepsilon_w)} + (1-\lambda_w) \overline{W}^{1-\varepsilon_w}\right)^{1/(1-\varepsilon_w)}. \qquad \text{(B.17)} \end{aligned}$$

Here, the first line substitutes the wages of flexible and sticky workers. The second line substitutes the optimal wage for flexible workers from Eq. $(B.16)$. As we show in Section B.4 below, log-linearizing Eq. $(B.17)$ leads to Eq. (5) in the main text. Eq. $(B.17)$ illustrates that greater employment in an area, $L_{a,0}$, creates wage pressure. The amount of pressure depends positively on the fraction of flexible workers, λ_w, and negatively on the labor supply elasticity, $1/\varphi$, as well as on the elasticity of substitution across labor types, ε_w. An increase in the local price index, $P_{a,0}$, also creates wage pressure.

It is also instructive to consider the special case in which wages are fully flexible, $\lambda_w = 1$. In this case, all workers set the same wage, which implies $W_{a,0}^{flex} = W_{a,0}$. Using this observation Eq.

$(B.17)$ becomes:

$$\frac{W_{a,0}}{P_{a,0}} = \frac{\varepsilon_w}{\varepsilon_w - 1} \chi L_{a,0}^{\varphi}. \tag{B.18}$$

Hence, the frictionless labor supply in each area a is described by a neoclassical intratemporal optimality condition. In particular, real wage is a constant markup over the MRS between labor and consumption.

Market Clearing Conditions. The market clearing condition for nontradable goods and the tradable good can be written as, respectively,

$$Y_{a,t}^N = C_{a,t}^N \tag{B.19}$$

$$Y_t^T + \tilde{Y}_t^T = \int_a C_{a,1}^T da. \tag{B.20}$$

Here, $Y_{a,t}^N, Y_t^T, \tilde{Y}_t^T$ are given by Eqs. $(B.1 - B.3)$.

Labor and capital market clearing conditions for period 0 can be written as,

$$L_{a,0} = L_{a,0}^N + L_{a,0}^T \text{ for each } a \tag{B.21}$$

$$\overline{K} = 1 = \int_a \left(K_{a,0}^N + K_{a,0}^T + \tilde{K}_{a,0} \right) da. \tag{B.22}$$

The analogous conditions for period 1 can be written as,

$$\overline{L}_1 = \int_a \left(L_{a,1}^N + L_{a,1}^T \right) da \tag{B.23}$$

$$\overline{K} = 1 = \int_a \left(K_{a,1}^N + K_{a,1}^T + \tilde{K}_{a,1} \right) da. \tag{B.24}$$

Note that there is a single market clearing condition for capital because capital is mobile in either period. Likewise, there is a single market clearing condition for labor in period 1. In contrast, there is a separate market clearing condition in each area for labor in period 0.

Finally, the asset market clearing condition can be written as,

$$\int_a x_{a,1} da = 0. \tag{B.25}$$

Monetary Policy and Equilibrium. To close the model, it remains to specify how the monetary policy sets the nominal interest rate, R^f. For most of the analysis, we assume that the monetary policy sets R^f to ensure aggregate employment is "on average" equal to frictionless employment.

Specifically, we define $\overline{L}_0$ as the frictionless labor supply that would obtain when all areas have common wealth. It is the solution to the frictionless labor supply equation [cf. $(B.18)$]:

$$\frac{W_0}{P_0} = \frac{\varepsilon_w}{\varepsilon_w - 1}\chi \overline{L}_0^{\varphi}, \tag{B.26}$$

where $W_0 = W_{0,a}$ and $P_0 = P_{0,a}$ denote the common wage and price level across areas. Below, we characterize P_0 in terms of W_0 and the remaining parameters and provide a closed form solution for $\overline{L}_0$. We assume monetary policy sets R^f to ensure:

$$\int_a L_{a,0} da = \overline{L}_0. \tag{B.27}$$

We can then define the equilibrium as follows.

Definition 1. *Given a distribution of ownership of capital, $\{x_{a,0}\}_a$ (that sum to zero across areas), an equilibrium is a collection of cross-sectional and aggregate allocations together with (nominal) factor prices, $\left(\{W_{a,t}\}_a, R_t\right)$, goods prices, $\left(\left\{P_{a,t}^N\right\}_a, P_t^T\right)$, the asset price, Q_0, and the interest rate, R^f, such that:*

(i) Competitive firms maximize according to the production technologies described in $(B.1 - B.3)$.

(ii) Households choose their consumption and portfolios optimally [cf. problem $(B.9)$].

(ii) Capital supply is exogenous in both periods and given by $(B.4)$. Labor supply and nominal wages in period 1 are exogenous and given by Eqs. $(B.10)$ and $(B.5)$. Labor supply and nominal wages in period 0 are endogenous and satisfy Eq. $(B.17)$.

(iv) Monetary policy sets the interest rate R^f to ensure Eq. $(B.27)$ with $\overline{L}_0$ that solves Eq. $(B.26)$.

(v) Goods, factors, and asset markets clear [cf. Eqs. $(B.19 - B.25)$].

B.2 General Characterization of Equilibrium

We next provide a general characterization of equilibrium. We start by establishing the properties on the supply side that apply in both periods. We then use these properties to characterize the equilibrium in period 1. We then establish properties on the demand side and characterize the equilibrium in period 0. Throughout, we focus on an equilibrium in which the capital-only technology is used in equilibrium, $\tilde{K}_t \geq 0$. Later in the appendix, we will ensure this by making appropriate parametric assumptions on D_t.

Supply Side. The price of nontradable good in an area is equal to the unit production cost [cf. $(B.1)$]

$$P_{a,t}^N = \left(W_{a,t}\right)^{1-\alpha^N} R_t^{\alpha^N}. \tag{B.28}$$

Likewise, the price of the composite tradable good is equal to its unit production cost according to both the standard and the linear technology [cf. $(B.2)$ and $(B.3)$]

$$P_t^T = \left(\int_a \left(P_{a,t}^T\right)^{1-\varepsilon} da\right)^{1/(1-\varepsilon)} = R_t / D_t^{1-\alpha^T}, \tag{B.29}$$

$$\text{where } P_{a,t}^T = \left(W_{a,t}\right)^{1-\alpha^T} R_t^{\alpha^T}. \tag{B.30}$$

Here, we define $P_{a,t}^T$ as the unit cost of producing the tradable input in area a.

Using Eq. $(B.29)$, we also obtain an expression for the rental rate in terms of wages and the parameter D_t,

$$R_t^{1-\alpha^T} = D_t^{1-\alpha^T} \left(\int_a W_{a,t}^{\left(1-\alpha^T\right)(1-\varepsilon)} da\right)^{1/(1-\varepsilon)}. \tag{B.31}$$

Hence, the rental rate of capital is determined by the productivity of the linear technology together with wages in each area (that determine the price of the tradable good). This also implies that, given the wages in each area, we can uniquely calculate all other prices. The following lemma formalizes these results, and characterizes the prices when wages are equated across areas.

Lemma 1. *Given a collection of strictly positive nominal wages, $\{W_{a,t}\}_a$ and capital productivity, D_t, Eq. $(B.31)$ uniquely determines the rental rate of capital, R_t. Eqs. $(B.28 - B.30)$ determine the remaining prices, $P_{a,t}^N, P_{a,t}^T, P_t^T$, and Eq. $(B.7)$ determines the price of the final good in terms of these prices, $P_{a,t} = \left(P_{a,t}^N\right)^\eta \left(P_t^T\right)^{1-\eta}$. If $W_{a,t} \equiv W_t$ for each a, then*

$$\begin{aligned} R_t &= D_t W_t \\ P_{a,t}^N &= D_t^{\alpha^N} W_t \\ P_t^T &= P_{a,t}^T = D_t^{\alpha^T} W_t \\ P_{a,t} &= D_t^{\overline{\alpha}} W_t. \end{aligned}$$

Here, recall that $\overline{\alpha} = \eta\alpha^N + (1-\eta)\alpha^T$ denotes the weighted-average share of capital across the two sectors [cf. (7)].

We next characterize the demand for labor in the nontradable and tradable sectors. Note that the Cobb-Douglas production function in $(B.1)$ implies,

$$W_{a,t} L_{a,t}^N = \left(1-\alpha^N\right) P_{a,t}^N Y_{a,t}^N, \tag{B.32}$$

$$\text{where } P_{a,t}^N Y_{a,t}^N = P_{a,t}^N C_{a,t}^N.$$

Here, the second line substitutes the market clearing condition $(B.19)$. Hence, the demand for nontradable labor in an area is determined by the demand for nontradable goods in the area.

Likewise, the Cobb-Douglas production function in $(B.2)$ implies,

$$W_{a,t} L_{a,t}^T = \left(1-\alpha^T\right) P_{a,t}^T Y_{a,t}^T.$$

That is, the demand for tradable labor in an area is determined by the demand for tradable inputs from the area. To characterize this further, note that the CES production function in ($B.2$) implies,

$$P_{a,t}^{T} Y_{a,t}^{T} = \left(\frac{P_{a,t}^{T}}{P_{t}^{T}} \right)^{1-\varepsilon} P_{t}^{T} Y_{t}^{T}.$$

So the demand for tradable inputs in an area depends on the demand for the tradable good in the aggregate (that uses the standard technology) as well as the local unit cost. Combining these expressions, and using Eq. ($B.20$), we further obtain,

$$\begin{aligned} W_{a,t} L_{a,t}^{T} &= \left(1-\alpha^{T}\right) \left(\frac{P_{a,t}^{T}}{P_{t}^{T}} \right)^{1-\varepsilon} P_{t}^{T} Y_{t}^{T}, \\ \text{where } P_{t}^{T} Y_{t}^{T} &= \int_{a} P_{t}^{T} C_{a,t}^{T} da - P_{t}^{T} \tilde{Y}_{t}^{T} \text{ and } P_{t}^{T} \tilde{Y}_{t}^{T} = R_{t} \tilde{K}_{t}^{T}. \end{aligned} \tag{B.33}$$

Here, the second line captures that the demand for tradables that uses the standard technology is determined by the total demand for tradables net of the production via the capital-only technology. The following lemma summarizes this discussion. It also characterizes Eq. ($B.33$) further by solving for the amount of production in the tradable sector via the capital-only technology, $P_{t}^{T} \tilde{Y}_{t}^{T} = R_{t} \tilde{K}_{t}^{T}$.

Lemma 2. *The demand for nontradable labor is given by Eq.* ($B.32$). *The demand for tradable labor is given by Eq.* ($B.33$). *In equilibrium, the amount of capital employed in the capital-only technology satisfies,*

$$R_{t} \tilde{K}_{t}^{T} = \frac{1-\overline{\alpha}}{1-\alpha^{T}} R_{t} - \frac{\overline{\alpha}}{1-\alpha^{T}} \int_{a} W_{a,t} L_{a,t} da. \tag{B.34}$$

Therefore, Eq. ($B.33$) *can be further solved as,*

$$W_{a,t} L_{a,t}^{T} = \left(\frac{P_{a,t}^{T}}{P_{t}^{T}} \right)^{1-\varepsilon} \left[\left(1-\alpha^{T}\right) \int_{a} P_{t}^{T} C_{a,t}^{T} da - \left(1-\overline{\alpha}\right) R_{t} + \overline{\alpha} \int_{a} W_{a,t} L_{a,t} da \right]. \tag{B.35}$$

Proof. To establish Eq. ($B.35$), note that the analogue of Eqs. ($B.32$) and ($B.33$) also apply for capital. In particular, after aggregating across areas, we have,

$$\begin{aligned} R_{t} \int_{a} K_{a,t}^{N} da &= \alpha^{N} \int_{a} P_{a,t}^{N} C_{a,t}^{N} da \\ R_{t} \int_{a} K_{a,t}^{T} da &= \alpha^{T} \left(\int_{a} P_{t}^{T} C_{a,t}^{T} da - R_{t} \tilde{K}_{t}^{T} \right). \end{aligned}$$

Here, the second line uses $P_{t}^{T} = \left(\int_{a} \left(P_{a,t}^{T}\right)^{1-\varepsilon} da \right)^{1/(1-\varepsilon)}$. Adding these equations, and using the market clearing condition for capital in ($B.22$) and ($B.24$), we obtain,

$$R_{t} \left(1-\tilde{K}_{t}^{T}\right) = \alpha^{N} \int_{a} P_{a,t}^{N} C_{a,t}^{N} da + \alpha^{T} \left(\int_{a} P_{t}^{T} C_{a,t}^{T} da - R_{t} \tilde{K}_{t}^{T} \right).$$

Using Eq. $(B.8)$, we can express the right hand side in terms of aggregate consumption expenditure,

$$R_t\left(1-\tilde{K}_t^T\right)=\overline{\alpha}\int_a P_{a,t}C_{a,t}da-\alpha^T R_t\tilde{K}_t^T, \tag{B.36}$$

where recall that $\overline{\alpha}=\alpha^N\eta+\alpha^T(1-\eta)$ [cf. (7)].

Next note that, in equilibrium, aggregate consumption expenditure is equal to aggregate income,

$$\int_a P_{a,t}C_{a,t}da=\int_a W_{a,t}L_{a,t}da+R_t.$$

Substituting this into Eq. $(B.36)$, we solve for the production of tradables via capital-only technology as,

$$R_t\tilde{K}_t^T=\frac{1-\overline{\alpha}}{1-\alpha^T}R_t-\frac{\overline{\alpha}}{1-\alpha^T}\int_a W_{a,t}L_{a,t}da.$$

This establishes Eq. $(B.34)$. Substituting this expression into Eq. $(B.33)$, we obtain Eq. $(B.35)$, completing the proof. □

Equilibrium in Period 1 (Long Run). Our analysis so far enables us to characterize the equilibrium in period 1. Since labor is mobile across areas, the wages are equated across areas, $W_{a,1}\equiv\overline{W}$ for each a. Then, using Lemma 1, we obtain,

$$R_1=D_1\overline{W}. \tag{B.37}$$

Thus, the nominal rental rate of capital is determined by the productivity of capital, D_1, together with the the long-run nominal wage level, $\overline{W}$.

We can also explicitly solve for the aggregate consumption in nontradables and tradables, as well as the allocation of factors to these sectors. We skip these steps since they are not necessary for our analysis.

Average Labor Supply in Period 0 (Short Run). We can also utilize the analysis so far to solve Eq. $(B.26)$. Recall that this equation corresponds to the frictionless labor supply when all areas have common stock wealth. It describes the average labor supply $\overline{L}_0$ that monetary policy targets with an arbitrary distribution of stock wealth [cf. $(B.38)$].

When areas have common wealth, wages are equated across areas, $W_{a,0}=W_0$ for each a. Using Lemma 1, we also obtain $P_0=D_0^{\overline{\alpha}}W_0$. Substituting these expressions into $(B.26)$, we obtain:

$$\frac{W_0}{P_0}=\frac{1}{D_0^{\overline{\alpha}}}=\frac{\varepsilon_w}{\varepsilon_w-1}\chi\overline{L}_0^{\varphi}. \tag{B.38}$$

Note that, given the other parameters, there is a unique solution to Eq. $(B.38)$ that describes the frictionless labor supply $\overline{L}_0$. We next turn to the demand side and characterize the rest of the equilibrium in period 0.

Asset Prices in Period 0 (Short Run). Next consider households' portfolio decision in period 0. Since there is no risk in capital (for simplicity), problem ($B.9$) implies households take a non-zero position on capital if and only if its price satisfies,

$$\begin{aligned} Q_0 &= R_0 + \frac{R_1}{R^f} \\ &= R_0 + \frac{D_1 \overline{W}}{R^f}. \end{aligned} \tag{B.39}$$

Here, the second line substitutes for the future rental rate of capital from Eq. ($B.37$). Hence, a standard asset pricing condition applies to capital. In particular, households' stock wealth depends on (among other things) the productivity of capital and the interest rate, R^f.

Demand Side in Period 0 (Short Run). We next consider the households' consumption-savings decision in period 0. We define the households' human capital wealth in an area as,

$$H_{a,0} = W_{a,0} L_{a,0} + \frac{\overline{W L_1}}{R^f}. \tag{B.40}$$

We can then rewrite the households' budget constraints in ($B.9$) as a lifetime budget constraint,

$$P_{a,0} C_{a,0} + P_{a,1} C_{a,1} = H_{a,0} + (1 + x_{a,0}) Q_0.$$

Combining this with log utility, we obtain the optimality condition,

$$P_{a,0} C_{a,0} = \frac{1}{1+\delta} \left(H_{a,0} + (1 + x_{a,0}) Q_0 \right). \tag{B.41}$$

That is, households spend a constant fraction of their lifetime wealth, where the latter is a combination of their human capital and stock wealth. Combining this with Eq. ($B.8$), we further obtain,

$$P^N_{a,0} C^N_{a,0} = \frac{\eta}{1+\delta} \left(H_{a,0} + (1 + x_{a,0}) Q_0 \right), \tag{B.42}$$

$$P^T_0 C^T_{a,0} = \frac{1-\eta}{1+\delta} \left(H_{a,0} + (1 + x_{a,0}) Q_0 \right). \tag{B.43}$$

We next combine Eq. ($B.42$) with Eq. ($B.32$) from Lemma 2 to obtain,

$$W_{a,0} L^N_{a,0} = \frac{\left(1 - \alpha^N\right) \eta}{1+\delta} \left(H_{a,0} + (1 + x_{a,0}) Q_0 \right). \tag{B.44}$$

Thus, nontradable labor demand is determined by the local nontradable demand, which is equal to local wealth multiplied by the share of wealth spent ($1/(1+\delta)$) multiplied by the share of nontradables (η) multiplied by the share of labor in the nontradable sector ($1 - \alpha^N$).

Likewise, we combine Eq. ($B.43$) with Eq. ($B.35$) from Lemma 2 to obtain,

$$W_{a,0}L_{a,0}^{T}=\left(\frac{P_{a,0}^{T}}{P_{0}^{T}}\right)^{1-\varepsilon}\left\{\frac{\left(1-\alpha^{T}\right)\left(1-\eta\right)}{1+\delta}\left(H_{0}+Q_{0}\right)-\left(1-\overline{\alpha}\right)R_{0}+\overline{\alpha}\int_{a}W_{a,0}L_{a,0}da\right\}. \tag{B.45}$$

Here, we define the aggregate human capital wealth as, $H_0 = \int_a H_{a,0}da$. Hence, tradable labor demand is determined by aggregate demand for the tradable good, which depends on the aggregate wealth and similar coefficients as above.

After summing Eqs. ($B.44$) and ($B.45$), we obtain an expression for the total labor demand in an area as follows,

$$\begin{aligned}W_{a,0}L_{a,0}&=\frac{\left(1-\alpha^{N}\right)\eta}{1+\delta}\left(H_{a,0}+\left(1+x_{a,0}\right)Q_{0}\right) \qquad \text{(B.46)}\\&+\left(\frac{P_{a,0}^{T}}{P_{0}^{T}}\right)^{1-\varepsilon}\left\{\frac{\left(1-\alpha^{T}\right)\left(1-\eta\right)}{1+\delta}\left(H_{0}+Q_{0}\right)-\left(1-\overline{\alpha}\right)R_{0}+\overline{\alpha}\int_{a}W_{a,0}L_{a,0}da\right\}.\end{aligned}$$

After substituting $H_{a,0}$ from Eq. ($B.40$), we can also write the labor demand equation as follows,

$$\begin{aligned}W_{a,0}L_{a,0}&=\frac{\left(1-\alpha^{N}\right)\eta}{1+\delta}\left(W_{a,0}L_{a,0}+\frac{\overline{WL}_{1}}{R^{f}}+\left(1+x_{a,0}\right)Q_{0}\right) \qquad \text{(B.47)}\\&+\left(\frac{P_{a,0}^{T}}{P_{0}^{T}}\right)^{1-\varepsilon}\left\{\frac{\left(1-\alpha^{T}\right)\left(1-\eta\right)}{1+\delta}\left(\int_{a}W_{a,0}L_{a,0}da+\frac{\overline{WL}_{1}}{R^{f}}+Q_{0}\right)-\left(1-\overline{\alpha}\right)R_{0}+\overline{\alpha}\int_{a}W_{a,0}L_{a,0}da\right\}.\end{aligned}$$

The first line illustrates the local labor demand due to local spending on the nontradable good. The second line illustrates the local labor demand due to aggregate spending on the tradable good.

Next recall from Lemma 1 that the prices, $P_{a,0}^T, P_0^T, R_0$ are implicit functions of wages, $\{W_{a,0}\}_a$. Therefore, Eq. ($B.47$) is a collection of $|I|$ equations in $2|I|+1$ unknowns, $\{L_{a,0}, W_{a,0}\}_{a\in I}$ and R^f. Recall also that we have Eq. ($B.17$) that relates wages to the labor and the price level in each area. This provides $|I|$ additional equations in $\{L_{a,0}, W_{a,0}\}_{a\in I}$. The monetary policy rule in ($B.38$) provides the remaining equation, where $\overline{L}_0$ is given by Eq. ($B.38$). The equilibrium is characterized as the solution to these $2|I|+1$ equations.

B.3 Benchmark Equilibrium with Common Stock Wealth

We next characterize the equilibrium further in special cases of interest. In this section, we focus on a benchmark case in which areas have common wealth, $x_{a,0} = 0$ for each a, and provide a closed form solution. In the next section, we log-linearize the equilibrium around this benchmark and provide a closed-form solution for the log-linearized equilibrium. Throughout, we assume the productivity in the capital-only technology satisfies:

Assumption D. $D_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}_0$ and $D_1 \geq \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}_1$.

To understand the role of this assumption, note that the common-wealth benchmark features identical wages across areas as well as identical and frictionless employment (in either period), $W_{a,t} = W_t$ and $L_{a,t} = \overline{L}_t$. Using this observation, together with Lemmas 1 and 2, we obtain $D_t \tilde{K}_t^T = \frac{1-\overline{\alpha}}{1-\alpha^T} D_t - \frac{\overline{\alpha}}{1-\alpha^T} L_t$. Therefore, the inequality $D_t \geq \frac{\overline{\alpha}}{1-\overline{\alpha}} \overline{L}_t$ ensures that firms use the capital-only technology in equilibrium, $\tilde{K}_t^T \geq 0$. In period 0, we assume that the inequality holds as equality, which implies that firms are indifferent to use this technology and, moreover, $\tilde{K}_0^T = 0$. Thus, Assumption D ensures that the production in period 0 is homothetic across sectors despite the presence of the capital-only technology in the tradable sector—this homotheticity will be important for some of our results. This assumption also simplifies the expressions, e.g., it ensures that the share of labor is equal to its sector-weighted average share in the Cobb-Douglas technologies, $1 - \overline{\alpha}$.

Recall also that $\overline{L}_0$ is endogenous and corresponds to the solution to Eq. $(B.38)$. Given the other parameters, there is a unique level of $D_0, \overline{L}_0$ that satisfy Assumption D along with this equation.

To characterize the equilibrium in period 0 further, note that the areas are symmetric. Therefore, we drop the area subscript and denote the allocations with, W_0, P_0, L_0, R_0, H_0.

Substituting common wages, prices, and labor into Eq. $(B.17)$ and using Eq. $(B.26)$, we further obtain $W_0 = W_0^{flex} = \overline{W}$. Intuitively, since monetary policy targets the frictionless labor supply, the flexible-wage members of the household set the same wage level as the sticky-wage members. Therefore, the equilibrium wage level is the same as the sticky wage level, $\overline{W}$ (which we take as equal to the long-run wage level). Using Lemma 1, we also obtain, $R_0 = D_0 \overline{W}$ and $P_0 = D_0^{\overline{\alpha}} \overline{W}$.

Substituting these observations into the labor demand Eq. $(B.46)$, we obtain,

$$\overline{W} L_0 = \frac{1-\overline{\alpha}}{1+\delta} (H_0 + Q_0) - (1-\overline{\alpha}) D_0 \overline{W} + \overline{\alpha} \overline{W} L_0.$$

After rearranging terms, we obtain,

$$\overline{W} L_0 = \frac{1}{1+\delta} (H_0 + Q_0) - D_0 \overline{W}.$$

Rearranging further, we obtain,

$$(H_0 + Q_0) / \overline{W} = (1+\delta) \left(\overline{L}_0 + D_0 \right). \tag{B.48}$$

This expression says that the aggregate wealth (in real terms) must be a constant multiple of the supply-determined output level.

Next note that, after substituting the wages and the rental rate into Eqs. $(B.40)$ and $(B.39)$, human capital and stock wealth are given by, respectively,

$$H_0 / \overline{W} = \overline{L}_0 + \frac{\overline{L}_1}{R^{f,*}}, \tag{B.49}$$

$$Q_0 / \overline{W} = D_0 + \frac{D_1}{R^{f,*}}. \tag{B.50}$$

Combining the last three expressions, we can solve for "rstar" as,

$$R^{f,*} = \frac{1}{\delta} \frac{\overline{L}_1 + D_1}{\overline{L}_0 + D_0}. \tag{B.51}$$

Intuitively, monetary policy adjusts the interest rate ("rstar") so that aggregate wealth is at an appropriate level to ensure the implied amount of spending clears the goods market at the supply-determined output level. As expected, greater impatience (low δ) or greater expected growth of capital income (high D_1 relative to D_0) or expected growth of labor income (high $\overline{L}_1$ relative to $\overline{L}_0$) translates into a greater interest rate in equilibrium. We can also solve for the equilibrium levels of human capital and stock wealth as,

$$H_0/\overline{W} = \overline{L}_0 + \delta \left(\overline{L}_0 + D_0\right) \frac{\overline{L}_1}{\overline{L}_1 + D_1} \tag{B.52}$$

$$Q_0/\overline{W} = D_0 + \delta \left(\overline{L}_0 + D_0\right) \frac{D_1}{\overline{L}_1 + D_1} \tag{B.53}$$

These expressions are intuitive. For instance, an increase in D_1 increases stock prices as well as the risk-free rate, and it leaves total wealth unchanged. Intuitively, an increase in D_1 exerts upward pressure on aggregate wealth and increases aggregate demand. The interest rate increases to ensure output is at its supply determined level. This mitigates the rise in the stock price somewhat but it does not completely undo it, since some of the interest rate response is absorbed by human capital wealth. (The last point is the difference from Caballero and Simsek (forthcoming): here, "time-varying risk premium" translates into actual price movements because we have two different types of wealth and the "risk premium" varies only for one type of wealth.)

Next consider the determination of tradable and nontradable employment. Substituting $W_{a,0} = \overline{W}$ and $x_{a,0} = 0$ into Eqs. $(B.44)$ and $(B.45)$, we solve for aggregate nontradable and tradable employment as, respectively,

$$\begin{aligned} L_0^N &= \frac{\left(1-\alpha^N\right)\eta}{1+\delta} \left(H_0 + Q_0\right)/\overline{W} \\ L_0^T &= \frac{\left(1-\alpha^T\right)(1-\eta)}{1+\delta} \left(H_0 + Q_0\right)/\overline{W} - (1-\overline{\alpha}) D_0 + \overline{\alpha}\overline{L}_0. \end{aligned}$$

Combining this with Eq. $(B.48)$, we further obtain,

$$\begin{aligned} L_0^N &= \left(1-\alpha^N\right)\eta \left(\overline{L}_0 + D_0\right) \\ L_0^T &= \left(1-\alpha^T\right)(1-\eta)\left(\overline{L}_0 + D_0\right) - (1-\overline{\alpha}) D_0 + \overline{\alpha}\overline{L}_0 \end{aligned}$$

Finally, substituting $D_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}} \overline{L}_0$ from Assumption D, we can further simplify these expressions as follows,

$$
\begin{aligned}
L_0^N &= \frac{1-\alpha^N}{1-\overline{\alpha}}\eta\overline{L}_0, \\
L_0^T &= \frac{1-\alpha^T}{1-\overline{\alpha}}(1-\eta)\overline{L}_0.
\end{aligned}
\tag{B.54}
$$

Hence, the labor employed in the nontradable and tradable sectors is determined by the share of the corresponding good in household spending, with an adjustment for the differences in the share of labor across the two sectors.

Proposition 1. *Consider the model with Assumption D when areas have common stock wealth, $x_{a,0} = 0$ for each a. In equilibrium, all areas have identical allocations and prices. In period 0, labor is equal to its frictionless level, $L_0 = \overline{L}_0$, that solves Eq. $(B.38)$, and nominal wages and prices are given by $W_0 = \overline{W}$ and $P_0 = D_0^{\alpha}\overline{W}$. The nominal interest rate is given by Eq. $(B.51)$; human capital and stock wealth are given by Eqs. $(B.52)$ and $(B.53)$; the shares of labor employed in the nontradable and tradable sectors is given by Eq. $(B.54)$. In particular, an increase in D_1 decreases increases the interest rate and the price of capital but do not affect the labor market outcomes in period* 0.

B.4 Log-linearized Equilibrium with Heterogeneous Stock Wealth

We next consider the case with a more general distribution of stock wealth, $\{x_{a,0}\}_a$, that satisfies $\int_a x_{a,0}da = 0$. In this case, we log-linearize the equilibrium conditions around the common-wealth benchmark (for a fixed level of D_1), and we characterize the log-linearized equilibrium. To this end, we define the log-deviations of the local equilibrium variables around the common-wealth benchmark: $y = \log\left(Y/Y^b\right)$, where $Y \in \left\{L_{a,0}, L_{a,0}^N, L_{a,0}^T, W_{a,0}, P_{a,0}, P_{a,0}^T, H_{a,0}\right\}_a$. We also define the log-deviations of the endogenous aggregate variables: $y = \log\left(Y/Y^b\right)$, where $Y \in \left\{P_t^T, R_t, Q_t, R^f\right\}$. The following lemma simplifies the analysis (proof omitted).

Lemma 3. *Consider the log-linearized equilibrium conditions around the common-wealth benchmark. The solution to these equations satisfies $\int_a l_{a,0}da = \int_a w_{a,0}da = 0$ and $p_t^T = r_t = q_0 = r^f = 0$. In particular, the log-linearized equilibrium outcomes for the aggregate variables are the same as their counterparts in the common-wealth benchmark.*

We next log-linearize the equilibrium conditions and characterize the log-linearized equilibrium outcomes for each area a. We start by Eqs. $(B.28), (B.30)$ and $(B.7)$ that characterize the other prices in terms of nominal wages in an area. Log-linearizing Eqs. $(B.28)$ and $(B.30)$ we obtain,

$$
\begin{aligned}
p_{a,0}^N &= \left(1-\alpha^N\right) w_{a,0} \\
p_{a,0}^T &= \left(1-\alpha^T\right) w_{a,0}.
\end{aligned}
\tag{B.55}
$$

Log-linearizing Eq. $(B.7)$, we further obtain:

$$p_{a,0} = \eta p^N_{a,0} = \eta \left(1-\alpha^N\right) w_{a,0}. \tag{B.56}$$

Next, we log-linearize the labor supply equation $(B.17)$ to obtain:

$$w_{a,0} = \frac{\lambda_w}{1+\varphi\varepsilon_w}\left(p_{a,0} + \varphi\varepsilon_w w_{a,0} + \varphi l_{a,0}\right).$$

After rearranging terms and simplifying, we obtain Eq. (5) from the main text:

$$w_{a,0} = \lambda\left(p_{a,0} + \varphi l_{a,0}\right), \text{ where } \lambda = \frac{\lambda_w}{1+(1-\lambda_w)\varphi\varepsilon_w}. \tag{B.57}$$

Note that we derive the wage flexibility parameter, λ, in terms of the more structural parameters, $\lambda_w, \varphi, \varepsilon_w$. As expected, wage flexibility is greater when a greater fraction of members adjust wages (greater λ_w), labor supply is more inelastic (greater φ), labor types are less substitutable (smaller ε_w).

Note also that, combining Eqs. $(B.56)$ and $(B.57)$, we obtain the simplified labor supply equation:

$$w_{a,0} = \kappa l_{a,0}, \text{ where } \kappa = \frac{\lambda\varphi}{1-\lambda\eta\left(1-\alpha^N\right)}. \tag{B.58}$$

As expected, the wage adjustment parameter, κ, depends on the wage flexibility parameter, λ, and the inverse elasticity of the labor supply, φ. It also depends on the share of nontradable sector and the share of labor in the nontradable sector, $\eta, 1-\alpha^N$. These parameters capture the extent to which a change in local wages translate into local inflation, which creates further wage pressure.

Next, we log-linearize the labor demand equation $(B.47)$ to obtain,

$$\begin{aligned}(w_{a,0}+l_{a,0})\overline{WL}_0 &= \frac{\left(1-\alpha^N\right)\eta}{1+\delta}\left((w_{a,0}+l_{a,0})\overline{WL}_0 + x_{a,0}Q_0\right) \\ &\quad -(\varepsilon-1)\left(1-\alpha^T\right)w_{a,0}\overline{W}L^T_0.\end{aligned} \tag{B.59}$$

Here, the second line substitutes $p^T_{a,0} = \left(1-\alpha^T\right)w_{a,0}$ from Eq. $(B.55)$. It also observes that the term in the set brackets in $(B.47)$ is common across areas and is equal to $\overline{W}L^T_0$ at the commonwealth benchmark (the aggregate expendiure on tradable labor).

After rearranging terms, we further obtain the simplified labor demand equation:

$$\begin{aligned}(w_{a,0}+l_{a,0})\overline{WL}_0 &= \mathcal{M}\left(\frac{\left(1-\alpha^N\right)\eta}{1+\delta}x_{a,0}Q_0 - (\varepsilon-1)\left(1-\alpha^T\right)w_{a,0}\overline{W}L^T_0\right), \\ \text{where } \mathcal{M} &= \frac{1}{1-\left(1-\alpha^N\right)\eta/\left(1+\delta\right)}\end{aligned} \tag{B.60}$$

Here, we defined the parameter, $\mathcal{M}$, which captures the local Keynesian multiplier effects.

For each area a, Eqs. $(B.60)$ and $(B.58)$ represent 2 equations in 2 unknowns, $(w_{a,0}, l_{a,0})$. Hence, these equations characterize the local labor market outcomes in the log-linearized equilibrium.

Solving these equations, we also obtain the following closed-form characterization,

$$
\begin{aligned}
w_{a,0} + l_{a,0} &= \frac{1+\kappa}{1+\kappa\zeta}\mathcal{M}\frac{\left(1-\alpha^N\right)\eta}{1+\delta}\frac{x_{a,0}Q_0}{\overline{WL}_0} && \text{(B.61)}\\
l_{a,0} &= \frac{1}{1+\kappa}\left(w_{a,0}+l_{a,0}\right) && \text{(B.62)}\\
w_{a,0} &= \frac{\kappa}{1+\kappa}\left(w_{a,0}+l_{a,0}\right), && \text{(B.63)}\\
\text{where } \zeta &= 1+(\varepsilon-1)\left(1-\alpha^T\right)\frac{L_0^T}{\overline{L}_0}\mathcal{M}\\
&= 1+(\varepsilon-1)\frac{\left(1-\alpha^T\right)^2}{1-\overline{\alpha}}(1-\eta)\,\mathcal{M}.
\end{aligned}
$$

Here, the last line defines the parameter, ζ, and the last line substitutes $\frac{L_0^T}{\overline{L}_0} = 1-\eta$ [cf. Eq. $(B.54)$]. Eq. $(B.61)$ illustrates that the local spending on nontradables affects the local labor bill. Eqs. $(B.62)$ and $(B.63)$ illustrate that this also affects employment and wages according to the wage flexibility parameter, κ.

The term, $\frac{1+\kappa}{1+\kappa\zeta}$, in Eq. $(B.61)$ captures the effect that works through exports. In particular, an increase in local spending increases local wages, which generates an adjustment of local exports. As expected, this adjustment is stronger when wages are more flexible (higher κ). The adjustment is also stronger when tradable inputs are more substitutable across regions (higher ε, which leads to higher ζ). In fact, when tradable inputs are gross substitutes ($\varepsilon > 1$, which leads to $\zeta > 1$), the export adjustment dampens the direct spending effect on the labor bill. When tradable inputs are gross complements ($\varepsilon < 1$, which leads to $\zeta < 1$), the export adjustment amplifies the direct spending effect.

Finally, consider the effect on local labor employed in nontradable and tradable sectors. First consider the tradable sector. Log-linearizing Eq. $(B.45)$ and following the same steps that we use in deriving the second line in $(B.59)$, we obtain

$$
\begin{aligned}
w_{a,0} + l_{a,0}^T &= -(\varepsilon-1)\left(1-\alpha^T\right)w_{a,0}\\
&= -(\varepsilon-1)\left(1-\alpha^T\right)\frac{\kappa}{1+\kappa\zeta}\mathcal{M}\frac{\left(1-\alpha^N\right)\eta}{1+\delta}\frac{x_{a,0}Q_0}{\overline{WL}_0}. && \text{(B.64)}
\end{aligned}
$$

Here, the second line uses Eqs. $(B.63)$ and $(B.61)$. These expressions illustrate that the export adjustment described above affects the tradable labor bill. While the effect of stock wealth on the tradable labor bill is ambiguous (as it depends on whether $\varepsilon > 1$ or $\varepsilon < 1$), we show that the effect on tradable employment is always (weakly) negative, $dl_{a,0}^T/dx_{a,0} \leq 0$. Intuitively, the increase in local wages always generate some substitution of labor away from the area. On the other hand, labor bill can increase or decrease depending on the strength of the income effect relative to this substitution effect.

Next consider the nontradable sector. Log-linearizing Eq. $(B.47)$ (after substituting for $H_{a,0}$ from Eq. $(B.40)$), and simplifying the expression as before, we obtain an expression for the labor bill in the nontradable sector,

$$\begin{aligned}
w_{a,0}+l_{a,0}^{N} &= \frac{1}{\overline{WL}_0^N}\frac{\left(1-\alpha^N\right)\eta}{1+\delta}\left(\left(w_{a,0}+l_{a,0}\right)\overline{WL}_0+x_{a,0}Q_0\right) \\
&= \frac{1}{\overline{WL}_0^N}\frac{\left(1-\alpha^N\right)\eta}{1+\delta}\left(\left(w_{a,0}+l_{a,0}^N\right)\overline{WL}_0^N+\left(w_{a,0}+l_{a,0}^T\right)\overline{WL}_0^T+x_{a,0}Q_0\right) \\
&= \frac{1}{\overline{WL}_0^N}\mathcal{M}\frac{\left(1-\alpha^N\right)\eta}{1+\delta}\left(\left(w_{a,0}+l_{a,0}^T\right)\overline{WL}_0^T+x_{a,0}Q_0\right) \\
&= \frac{1}{\frac{1-\alpha^N}{1-\overline{\alpha}}\eta\overline{WL}_0}\mathcal{M}\frac{\left(1-\alpha^N\right)\eta}{1+\delta}\left(\left(w_{a,0}+l_{a,0}^T\right)\frac{1-\alpha^T}{1-\overline{\alpha}}\left(1-\eta\right)\overline{WL}_0+x_{a,0}Q_0\right) \\
&= \mathcal{M}\frac{1}{1+\delta}\left(\left(1-\overline{\alpha}\right)\frac{x_{a,0}Q_0}{\overline{WL}_0}+\left(1-\alpha^T\right)\left(1-\eta\right)\left(w_{a,0}+l_{a,0}^T\right)\right) \qquad \text{(B.65)}
\end{aligned}$$

Here, the second line separates the expression for the total labor bill into the labor bill for nontradable and tradable sectors. The third line accounts for the multiplier effects through the nontradable labor bill. The fourth line uses Eq. $(B.54)$ to substitute for $\overline{L}_0^N/\overline{L}_0$ and $\overline{L}_0^T/\overline{L}_0$. The last line simplifies and rearranges terms.

Eq. $(B.65)$ illustrates that greater stock wealth affects the nontradable labor bill due to a direct and an indirect effect. The direct effect is positive as it is driven by the impact of greater local wealth on local spending. There is also an indirect effect due to the impact of the stock wealth on the tradable labor bill (which in turn affects local labor income). The indirect effect has an ambiguous sign because stock wealth can decrease or increase the tradable labor bill depending on ε (cf. Eq. $(B.64)$). Nonetheless, we show that the direct effect always dominates. Specifically, regardless of ε, we have $d\left(w_{a,0}+l_{a,0}^N\right)/dx_{a,0}>0, dl_{a,0}^N/dx_{a,0}>0$: that is, greater stock wealth increases the nontradable labor bill as well as nontradable employment. The following result summarizes this discussion.

Proposition 2. *Consider the model with Assumption D when areas have an arbitrary distribution of stock wealth,* $\{x_{a,0}\}_a$*, that satisfies* $\int_a x_{a,0}da=0$*. In the log-linearized equilibrium, local labor and wages in a given area,*$(l_{a,0},w_{a,0})$*, are characterized as the solution to Eqs.* $(B.60)$ *and* $(B.58)$*. The solution is given by Eqs.* $(B.62)$ *and* $(B.63)$*. Local labor bill in nontradables and tradable sectors are given by Eqs.* $(B.64)$ *and* $(B.65)$*. In particular, local employment and wages satisfy the following comparative statics with respect to stock wealth:*

$$dl_{a,0}/dx_{a,0}>0, dw_{a,0}/dx_{a,0}\geq 0 \text{ and } d\left(l_{a,0}+w_{a,0}\right)/dx_{a,0}>0.$$

Moreover, regardless of ε*, employment and the labor bill in nontradable and tradable sectors satisfy* *the following comparative statics:*

$$d\left(l_{a,0}^{N}+w_{a,0}\right)/dx_{a,0}>0, dl_{a,0}^{N}/dx_{a,0}>0 \text{ and } dl_{a,0}^{T}/dx_{a,0}\leq 0.$$

Proof. Most of the proof is presented earlier. It remains to establish the comparative statics for the tradable employment, the nontradable employment and the nontradable labor bill.

First consider the tradable employment. Note that the first line of the expression in $(B.64)$ implies

$$l_{a,0}^{T}=-\left(1+(\varepsilon-1)\left(1-\alpha^{T}\right)\right)w_{a,0}. \tag{B.66}$$

Since $(\varepsilon-1)\left(1-\alpha^{T}\right)>-1$ (because $\varepsilon>0$) and $dw_{a,0}/dx_{a,0}\geq 0$ (cf. Eq. $(B.63)$), this implies the comparative statics for the tradable employment, $dl_{a,0}^{T}/dx_{a,0}\leq 0$.

Next consider the nontradable employment. Note that $L_{a,0}=L_{a,0}^{T}+L_{a,0}^{N}$. Log-linearizing this expression, we obtain,

$$l_{a,0}^{N}L_{a,0}^{N}=l_{a,0}\overline{L}_{0}-l_{a,0}^{T}L_{a,0}^{T}.$$

Differentiating this expression with respect to $x_{a,0}$ and using $dl_{a,0}/dx_{a,0}>0$ and $dl_{a,0}^{T}/dx_{a,0}\leq 0$, we obtain the comparative statics for the nontradable employment, $dl_{a,0}^{N}/dx_{a,0}>0$. Combining this with $dw_{a,0}/dx_{a,0}\geq 0$, we further obtain the comparative statics for the nontradable labor bill, $d\left(l_{a,0}^{N}+w_{a,0}\right)/dx_{a,0}>0$. □

B.5 Comparative Statics of Local Labor Market Outcomes

We next combine our results to investigate the impact of a change in aggregate stock wealth (over time) on local labor market outcomes. Specifically, consider the comparative statics of an increase in capital productivity from some D_{1}^{old} to $D_{1}^{new}>D_{1}^{old}$.

First consider the effect on the common-wealth benchmark. By Proposition 1, the equilibrium price of capital increases from Q_{1}^{old} to $Q_{1}^{new}>Q_{1}^{old}$. The labor market outcomes remain unchanged: in particular, $L_{0}=\overline{L}_{0}, W_{0}=\overline{W}, L_{0}^{N}/\overline{L}_{0}=\frac{1-\alpha^{N}}{1-\overline{\alpha}}\eta$ and $L_{0}^{T}/\overline{L}_{0}=\frac{1-\alpha^{T}}{1-\overline{\alpha}}(1-\eta)$.

Next consider the effect when areas have heterogeneous wealth. We use the notation $\Delta X=X^{new}-X^{old}$ for the comparative statics on variable X. Consider the effect on labor market outcomes, for instance, the (log of the) local labor bill $\log\left(W_{a,0}L_{a,0}\right)$. Note that we have:

$$\log\left(W_{a,0}L_{a,0}\right)\simeq\log\left(\overline{W}L_{0}\right)+w_{a,0}+l_{a,0}.$$

Here, $w_{a,0}, l_{a,0}$ are characterized by Proposition 2 as linear functions of capital ownership, $x_{a,0}$; and the approximation holds up to second-order terms in capital ownership, $\left\{x_{a,0}\right\}_{a}$. Note also that the change of D_{1} does not affect $\log\left(\overline{W}L_{0}\right)$. Therefore, the comparative statics in this case can be written as,

$$\begin{aligned}\Delta \log \left(W_{a,0} L_{a,0}\right) &\simeq \Delta \left(w_{a,0} + l_{a,0}\right) \\ &= \left(w_{a,0}^{new} + l_{a,0}^{new}\right) - \left(w_{a,0}^{old} + l_{a,0}^{old}\right),\end{aligned}$$

where the approximation holds up to second-order terms in $\{x_{a,0}\}_a$. Put differently, up to second-order terms, the change of D_1 affects the (log of the) local labor bill through its effect on the log-linearized equilibrium variables.

Recall that the log-linearized equilibrium is characterized by Proposition 2. In particular, considering Eq. $(B.61)$ for D_1^{old} and D_1^{new}, we obtain:

$$\begin{aligned}w_{a,0}^{old} + l_{a,0}^{old} &= \frac{1+\kappa}{1+\kappa\zeta} \mathcal{M} \frac{\left(1-\alpha^N\right)\eta}{1+\delta} \frac{x_{a,0} Q_0^{old}}{\overline{W} L_0}, \\ w_{a,0}^{new} + l_{a,0}^{new} &= \frac{1+\kappa}{1+\kappa\zeta} \mathcal{M} \frac{\left(1-\alpha^N\right)\eta}{1+\delta} \frac{x_{a,0} Q_0^{new}}{\overline{W} L_0}.\end{aligned}$$

These equations illustrate that the change of D_1 affects the log-linearized equilibrium only through its effect on the price of capital, Q_0. Taking their difference, we obtain Eq. (11) in the main text that describes $\Delta \left(w_{a,0} + l_{a,0}\right)$.

Applying the same argument to Eqs. $(B.62), (B.65), (B.64)$, we also obtain Eqs. $(12), (13), (14)$ in the main text that describe, respectively, $\Delta l_{a,0}, \Delta \left(w_{a,0} + l_{a,0}^N\right), \Delta \left(w_{a,0} + l_{a,0}^T\right)$. These equations illustrate that an increase in local stock wealth due to a change in aggregate stock wealth has the same impact on local labor market outcomes as an increase of stock wealth in the cross section that we characterized earlier.

Comparative Statics of Local Consumption. We next derive the comparative statics of local consumption that we use in Section 6 (see Eq. (19)). For simplicity, we focus on the case $\varepsilon = 1$. Combining Eqs. $(B.8)$ and $(B.47)$, we obtain

$$P_{a,0} C_{a,0} = \frac{W_{a,0} L_{a,0}^N}{\left(1-\alpha^N\right)\eta}.$$

Log-linearizing this expression around the common-wealth benchmark, we obtain

$$\begin{aligned}\left(p_{a,0} + c_{a,0}\right) P_0 C_0 &= \left(w_{a,0} + l_{a,0}^N\right) \frac{\overline{W} L_0^N}{\left(1-\alpha^N\right)\eta} \\ &= \mathcal{M} \frac{1}{1+\delta} x_{a,0} Q_0\end{aligned}$$

Here, the second line uses the third line of Eq. $(B.65)$ and observes that $w_{a,0}+l_{a,0}^{T}=0$ when $\varepsilon=1$. After rearraning terms, and considering the change from D_1^{old} to $D_1^{new}>D_1^{old}$, we obtain

$$\Delta\left(p_{a,0}+c_{a,0}\right)=\mathcal{M}\frac{1}{1+\delta}\frac{x_{a,0}\Delta Q_0}{P_0C_0}. \tag{B.67}$$

After an appropriate change of variables, this equation gives Eq. (19) in the main text.

B.6 Details of the Calibration Exercise

This appendix provides the details of the calibration exercise in Section 6. We start by summarizing the solution for the local labor market outcomes that we derived earlier. In particular, we use the change of variables, $\frac{1}{1+\delta}=\rho\mathbb{T}$ and write the differenced versions of Eqs. $(B.61-B.65)$ as follows:

$$\begin{aligned}
\frac{\Delta\left(w_{a,0}+l_{a,0}\right)}{SR} &= \frac{1+\kappa}{1+\kappa\zeta}\mathcal{M}\left(1-\alpha^{N}\right)\eta\rho, \\
\frac{\Delta l_{a,0}}{SR} &= \frac{1}{1+\kappa}\frac{\Delta\left(w_{a,0}+l_{a,0}\right)}{SR} \qquad \text{(B.68)}\\
\frac{\Delta w_{a,0}}{SR} &= \frac{\kappa}{1+\kappa}\frac{\Delta\left(w_{a,0}+l_{a,0}\right)}{SR}\\
\frac{\Delta\left(w_{a,0}+l_{a,0}^{T}\right)}{SR} &= -\left(\varepsilon-1\right)\left(1-\alpha^{T}\right)\frac{\Delta w_{a,0}}{SR}\\
\frac{\Delta\left(w_{a,0}+l_{a,0}^{N}\right)}{SR} &= \mathcal{M}\rho\left(1-\overline{\alpha}\right)\left(1-\left(\varepsilon-1\right)\frac{\left(1-\alpha^{T}\right)^{2}}{1-\overline{\alpha}}\left(1-\eta\right)\mathbb{T}\frac{\Delta w_{a,0}}{SR}\right) \qquad \text{(B.69)}\\
\text{where } S &= \frac{x_{a,0}Q_{a,0}}{\overline{WL}_0/\mathbb{T}}, R=\frac{\Delta Q_0}{Q_0}\\
\text{and } \mathcal{M} &= \frac{1}{1-\left(1-\alpha^{N}\right)\eta\rho\mathbb{T}}\\
\text{and } \zeta &= 1+\left(\varepsilon-1\right)\frac{\left(1-\alpha^{T}\right)^{2}}{1-\overline{\alpha}}\left(1-\eta\right)\mathcal{M}.
\end{aligned}$$

Our calibration relies on two model equations that determine the key parameters κ and ρ. Specifically, we calibrate κ by using Eq. $(B.68)$, which replicates Eq. (20) from the main text. We calibrate ρ by using Eq. $(B.69)$ which generalizes Eq. (16) from the main text. For reasons we describe in the main text, we do not use the response of the tradable sector for calibration purposes (see Footnote 38).

Note that combining Eq. $(B.68)$ with the empirical coefficients for employment and the total labor bill from Table 1 (for quarter 7), we obtain:

$$0.77\% \leq \frac{1}{1+\kappa}2.18\%$$

As we discuss in the main text, while the model makes predictions for total labor supply including changes in hours per worker, in the data we only observe employment. A long literature dating to

Okun (1962) finds an elasticity of total hours to employment of 1.5. Applying this adjustment and using the coefficients for total employment and the total labor bill from Table 1 yields:

$$\frac{\Delta l_{a,0}}{S_{a,0}R_0} = 1.5 \times 0.77\%$$
$$\frac{\Delta\left(w_{a,0} + l_{a,0}\right)}{S_{a,0}R_0} = 2.18\%.$$

Combining these with Eq. (20), we obtain:

$$\kappa = 0.9. \tag{B.70}$$

Thus, a one percent change in labor is associated with a 0.9% change in wages at a horizon of two years.

That leaves us with Eq. (B.69) to determine the stock wealth effect parameter, ρ. In the main text, we focus on a baseline calibration that assumes unit elasticity for tradables, $\varepsilon = 1$, which leads to a particularly straightforward analysis. In this appendix, we first provide the details of the baseline calibration. We then show that this calibration is robust to considering a wider range for the tradable elasticity parameter, $\varepsilon \in [0.5, 1.5]$.

Throughout, we set the labor share parameters in the two sectors so that the weighted-average share of labor is equal to the standard empirical estimates [cf. (7)]:

$$1 - \overline{\alpha} = \frac{2}{3}.$$

To keep the calibration simple, we set the same labor share for the two sectors:

$$1 - \alpha^L = 1 - \alpha^N = \frac{2}{3}.$$

Eq. (B.69) (when $\varepsilon = 1$) shows that our analysis is robust to allowing for heterogeneous labor share across the two sectors.

B.6.1 Details of the Baseline Calibration

Setting $\varepsilon = 1$ in Eq. (B.69) reduces to Eq. (16) in the main text,

$$\frac{\Delta\left(w_{a,0} + l^N_{a,0}\right)}{SR} = \mathcal{M}\left(1 - \overline{\alpha}\right)\rho.$$

Combining this expression with the empirical coefficient for the nontradable labor bill from Table 1 (for quarter 7), we obtain:

$$\mathcal{M}\left(1 - \overline{\alpha}\right)\rho = 3.23\%. \tag{B.71}$$

We also require the local income multiplier to be consistent with empirical estimates from the literature, which implies:

$$\mathcal{M}=\frac{1}{1-\left(1-\alpha^{N}\right)\rho\eta\mathbb{T}}=1.5 \tag{B.72}$$

With these assumptions, as we discussed in the main text, Eq. $(B.71)$ determines the stock wealth effect parameter independently of the other parameters such as $\kappa, \eta, \mathbb{T}$. In particular, we have:

$$\rho = 3.23\%.$$

Combining this with Eq. $(B.72)$ to match the multiplier, we also obtain:

$$\eta\mathbb{T} = 15.48.$$

Hence, our calibration of the multiplier determines the product of η and $\mathbb{T}$.

The parameter, η, is difficult to calibrate precisely because there is no good measure of the trade bill at the county level. Therefore, we allow for a wide range of possibilities:

$$\eta \in \left[\underline{\eta}, \overline{\eta}\right], \text{ where } \underline{\eta} = 0.5 \text{ and } \underline{\eta} = 0.8. \tag{B.73}$$

Then, our calibration of the multiplier implies:

$$\mathbb{T} = \mathbb{T}\left(\eta\right) \equiv \frac{15.48}{\eta}, \text{ where } \mathbb{T}\left(\overline{\eta}\right) = 19.35 \text{ and } \mathbb{T}\left(\underline{\eta}\right) = 30.96.$$

In particular, for every choice of η, there exists a horizon parameter $\mathbb{T}$ that supports the calibration of the multiplier in our model. Since our model is stylized in the time dimension (it has only two periods), we do not interpret $\mathbb{T}$ literally but view it as a modeling device to calibrate the multiplier $\mathcal{M}$. In particular, we view the implied high levels of $\mathbb{T}$ as capturing reasons outside our model (such as borrowing constraints) that would increase the income multiplier in practice.[19]

B.6.2 Robustness of the Baseline Calibration

Next consider the case with general ε. In this case, Eq. $(B.69)$ is more complicated and given by:

$$\frac{\Delta\left(w_{a,0}+l_{a,0}^{N}\right)}{SR} = \mathcal{M}\rho\left(1-\overline{\alpha}\right)\left(1-\left(\varepsilon-1\right)\frac{\left(1-\alpha^{T}\right)^{2}}{1-\overline{\alpha}}\left(1-\eta\right)\mathbb{T}\frac{\Delta w_{a,0}}{SR}\right).$$

In particular, the nontradable labor bill in this case also depends on the effect on local wages. The intuition is that the change in local wages affects the tradable labor bill, which affects local

[19]The dependence of $\mathcal{M}$ on $\mathbb{T}$ in our model can be understood by considering the intertemporal Keynesian cross (see Auclert et al. (2018) for an exposition). When output is determined by aggregate demand, an increase in future spending increases not only future income but also current income through a wealth effect. In our environment, increasing $\mathbb{T}$ increases the time-length of period 0 over which output is determined by aggregate demand. This leads to stronger multiplier effects.

households' income. This in turn affects local households' spending and the nontradable labor bill. Consistent with this intuition, the magnitude of this effect depends on the parameters $\varepsilon, \alpha^T, \eta$.[20]

Recall also that we have Eq. $(B.68)$ that describes the change in wages as a function of the change in the total labor bill:

$$\frac{\Delta w_{a,0}}{SR} = \frac{\kappa}{1+\kappa} \frac{\Delta\left(w_{a,0} + l_{a,0}\right)}{SR}.$$

Substituting this expression into Eq. $(B.69)$, and using the empirical coefficients for the nontradable and the total labor bill from Table 1 (for quarter 7), we obtain the following generalization of Eq. $(B.71)$:

$$\mathcal{M}\left(1-\overline{\alpha}\right)\rho\left(1-\left(\varepsilon-1\right)\frac{\left(1-\alpha^T\right)^2}{1-\overline{\alpha}}\left(1-\eta\right)\mathbb{T}\frac{\kappa}{1+\kappa}2.18\%\right) = 3.23\%. \tag{B.74}$$

As this expression illustrates, the stock wealth effect parameter in this case is not determined independently of the remaining parameters, $\kappa, \eta, \mathbb{T}$. We have already established that $\kappa = 0.9$ [cf. Eq. $(B.70)$]. We also assume $1-\overline{\alpha} = 1-\alpha^T = 2/3$. We also assume η lies in the range $(B.73)$ that we described earlier. Recall also that we choose $\mathbb{T}$ to ensure Eq. $(B.72)$ given all other parameters. Hence, for any fixed ε, Eq. $(B.74)$ describes ρ as a function of η, where η is required to lie in the range $(B.73)$.

Figure B.1 illustrates the possible values of ρ for $\varepsilon = 0.5$ (the left panel) and $\varepsilon = 1.5$ (the right panel). As the figure illustrates the implied values for ρ remain close to their corresponding levels from the baseline calibration with $\varepsilon = 1$. As expected, the largest deviations from the benchmark obtain when the share of nontradables is small—as trade has the largest impact on households' incomes in this case. However, ρ lies within 5% of its corresponding level from the baseline calibration even if we set $\eta = 0.5$.

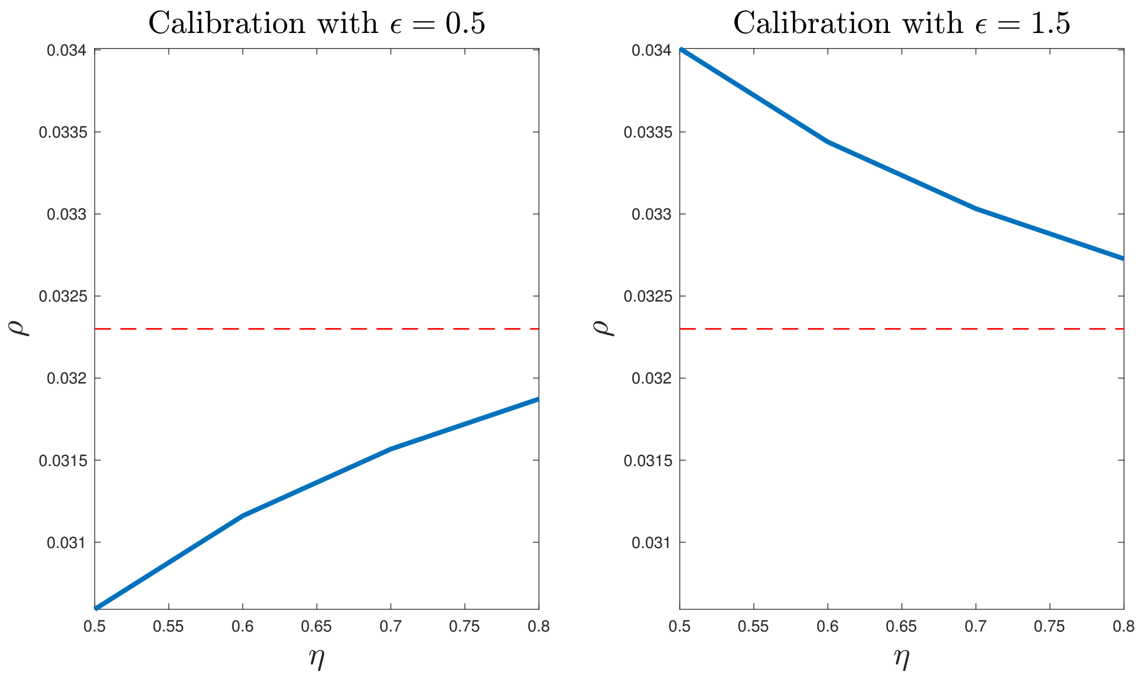
The intuition for robustness can be understood as follows. As we described earlier, the additional effects emerge from the adjustment of the tradable labor bill due to a change in local wages. As long as wages do not change by much, the effect has a negligible effect on our baseline calibration. As it turns out, the value of κ that we find given our calibration is such that the deviations from the benchmark are relatively small. Put differently, our analysis suggests that wages in an area do not change by much in response to stock wealth changes. Consequently, the tradable labor bill of the area also does not change by much either even if ε is somewhat different than 1.

B.7 Aggregation When Monetary Policy is Passive

So far, we assumed the monetary policy changes the interest rate to neutralize the impact of stock wealth changes on aggregate employment. In this appendix, we characterize the equilibrium under

[20]Less obviously, the magnitude also depends on the horizon parameter, $\mathbb{T}$. This parameter enters the equation for the same reason it enters the equation for the multiplier, $\mathcal{M}$ (see Footnote 19). As before, the dependence of the equation on $\mathbb{T}$ can be thought of as capturing reasons outside our model (such as households' borrowing constraints) that would amplify the spending effect of any change in households' incomes due to trade considerations.

Figure B.1:



Notes: Notes: The left panel (resp. the right panel) illustrates the implied ρ as a function of η given $\varepsilon = 0.5$ (resp. $\varepsilon = 1.5$), as we vary η over the range in $(B.73)$. The red dashed lines illustrate the implied ρ for the baseline calibration with $\varepsilon = 1$.

the alternative assumption that monetary policy leaves the interest rate unchanged in response to stock price fluctuations. In Section 7 of the main text, we use this characterization together with our calibration to describe how stock price fluctuations would affect aggregate labor market outcomes if they were not countered by monetary policy.

The model is the same as in Section B.1 with the only difference that the monetary policy keeps the nominal interest rate at a constant level, $R^f = \overline{R}^f$. In particular, we continue to assume monetary policy stabilizes the long-run nominal wage at the constant level, $\overline{W}$. For simplicity, we also focus attention on the common-wealth benchmark, $x_{a,0} = 0$. Consequently, the areas have symmetric allocations that we denote by dropping the subscript a.

First note that the rental rate of capital is given by $R_0 = D_0 W_0$ [cf. Lemma 1]. Consequently, the analogues of Eqs. $(B.49)$ and $(B.50)$ also apply in this setting. In particular, human capital wealth is given by,

$$H_0 = W_0 L_0 + \frac{\overline{W L_1}}{\overline{R}^f} \quad \text{(B.75)}$$

and the stock wealth is given by,

$$Q_0 = W_0 D_0 + \frac{\overline{W} D_1}{\overline{R}^f}. \quad \text{(B.76)}$$

Next note that the labor demand Eq. $(B.46)$ applies also in this case. Using $x_{a,0} = 0$ along with the definition of $\overline{\alpha}$ [cf. (7)], we obtain

$$W_0L_0=\frac{1-\overline{\alpha}}{1+\delta}\left(H_0+Q_0\right)-\left(1-\overline{\alpha}\right)R_0+\overline{\alpha}W_0L_0.$$

Using $R_0=W_0D_0$ and the expressions for H_0 from Eq. $(B.75)$, we further obtain,

$$W_0L_0=\frac{1-\overline{\alpha}}{1+\delta}\left(W_0L_0+\frac{\overline{WL_1}}{\overline{R}^f}+Q_0\right)-\left(1-\overline{\alpha}\right)W_0D_0+\overline{\alpha}W_0L_0.$$

Simplifying further, we obtain,

$$W_0L_0+W_0D_0=\frac{1}{1+\delta}\left(W_0L_0+\frac{\overline{WL_1}}{\overline{R}^f}+Q_0\right). \tag{B.77}$$

This equation says that the total amount of spending in the aggregate (on capital and labor) depends on the lifetime wealth multiplied by the propensity to spend out of wealth.

Next note that the labor supply equation $(B.17)$ applies also in this case. Since areas have common wealth, we can rewrite this equation as:

$$W_0^{1-\varepsilon_w}=\lambda_w\left(\frac{\varepsilon_w}{\varepsilon_w-1}\chi P_0W_0^{\varepsilon_w\varphi}L_0^{\varphi}\right)^{(1-\varepsilon_w)/(1+\varphi\varepsilon_w)}+\left(1-\lambda_w\right)\overline{W}^{1-\varepsilon_w}. \tag{B.78}$$

Using Lemma 1, we also have:

$$P_0=W_0D_0^{\overline{\alpha}}. \tag{B.79}$$

The equilibrium is characterized by Eqs. $(B.76),(B.77),(B.78),(B.79)$ in four variables, (Q_0,W_0,L_0,P_0).

Next note that there exists a level of D_1, denoted by $\overline{D}_1$, that ensures these equations are satisfied with $L_0=\overline{L}_0$ and $W_0=\overline{W}$, along with $\overline{Q}_0\equiv\overline{W}D_0+\frac{\overline{WD_1}}{\overline{R}^f}$. To simplify the expressions further, we next log-linearize the equations around the equilibrium with $D_1=\overline{D}_1$.

Log-linearized Aggregate Equilibrium. Log-linearizing the stock pricing Eq. $(B.76)$, we obtain,

$$q_0\overline{Q}_0=w_0\overline{W}D_0+d_1\frac{\overline{WD_1}}{\overline{R}^f}. \tag{B.80}$$

Log-linearizing the labor demand Eq. $(B.77)$, we obtain,

$$\left(w_0+l_0\right)\overline{WL}_0+w_0\overline{W}D_0=\frac{1}{1+\delta}\left(\left(w_0+l_0\right)\overline{WL}_0+q_0\overline{Q}_0\right).$$

After substituting Eq. $(B.80)$, and rearranging terms to account for the multiplier effects, we further obtain,

$$\left(w_0+l_0\right)\overline{WL}_0+w_0\overline{W}D_0 \;=\; \tilde{\mathcal{M}}^A\frac{1}{1+\delta}d_1\frac{\overline{WD_1}}{\overline{R}^f}, \tag{B.81}$$

$$\text{where } \tilde{\mathcal{M}}^A = \frac{1}{1 - 1/(1+\delta)}.$$

Log-linearizing the labor supply equation $(B.78)$, we obtain:

$$w_0 = \lambda(p_0 + \varphi l_0), \text{ where } \lambda = \frac{\lambda_w}{1 + (1-\lambda_w)\varphi\varepsilon_w}. \tag{B.82}$$

Log-linearizing Eq. $(B.79)$, we obtain:

$$p_0 = w_0. \tag{B.83}$$

Combining the last two equations, we further obtain:

$$w_0 = \kappa^A l_0, \text{ where } \kappa^A \equiv \frac{\lambda\varphi}{1-\lambda} > \kappa = \frac{\lambda\varphi}{1 - \lambda\eta(1-\alpha^N)}. \tag{B.84}$$

The log-linearized equilibrium is characterized by Eqs. $(B.80), (B.81), (B.84)$ in three variables (q_0, w_0, l_0). Given these variables, we also characterize the price level as $p_0 = w_0$ [cf. $(B.83)$]. The equations for (q_0, w_0, l_0) can also be solved in closed form. We conjecture a linear solution:

$$\begin{aligned} q_0\overline{Q}_0 &= A_Q Q^A \\ w_0\overline{WL}_0 &= A_W Q^A \\ l_0\overline{WL}_0 &= A_L Q^A, \\ \text{where } Q^A &= \frac{\overline{WD}_1 d_1}{\overline{R}^f} \end{aligned} \tag{B.85}$$

Here, Q^A denotes the log-linear approximation to the exogenous component of stock wealth $(\frac{\overline{WD}_1}{\overline{R}^f})$. Hence, the coefficients A_Q, A_W, A_L describe the effect of a one dollar increase in the exogenous component of stock wealth on endogenous equilibrium outcomes.

To solve for these coefficients, we substitute the linear functional form in $(B.85)$ into Eqs. $(B.80), (B.81), (B.84)$. We also use Assumption D to substitute $D_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}_0$ and simplify the expressions, to obtain the system of equations,

$$\begin{aligned} A_Q &= \frac{\overline{\alpha}}{1-\overline{\alpha}}A_W + 1 \\ A_W &= \kappa^A A_L \\ A_W + A_L + \frac{\overline{\alpha}}{1-\overline{\alpha}}A_W &= \tilde{\mathcal{M}}^A \frac{1}{1+\delta}. \end{aligned}$$

Using these equations, we obtain the closed-form solution for the effect on the aggregate labor bill,

$$\begin{aligned} A_W + A_L &= \mathcal{M}^A \frac{1-\overline{\alpha}}{1+\delta}, \\ \text{where } \mathcal{M}^A &= \mathcal{F}^A \tilde{\mathcal{M}}^A \text{ and } \mathcal{F}^A = \frac{1+\kappa^A}{1-\overline{\alpha}+\kappa^A} \end{aligned} \tag{B.86}$$

The effect on the aggregate employment and wages are given by

$$A_L = \frac{1}{1+\kappa^A}\left(A_W + A_L\right), \tag{B.87}$$

$$A_W = \frac{\kappa^A}{1+\kappa^A}\left(A_W + A_L\right). \tag{B.88}$$

Substituting the solutions in $(B.86 - B.88)$ into Eqs. $(B.85)$, we obtain

$$w_0 + l_0 = \mathcal{M}^A \frac{1-\overline{\alpha}}{1+\delta}\frac{Q_0^A}{\overline{WL_0}}$$
$$l_0 = \frac{1}{1+\kappa^A}\left(w_0 + l_0\right).$$

Considering the equation for two different levels of future dividends, d_1^{old} and d_1^{new}, and taking the difference, we obtain Eqs. (22) and (23) in the main text.

Comparison with the Log-linearized Local Equilibrium. It is instructive to compare the log-linearized labor supply equations $(B.82)$ and $(B.84)$ with their counterparts in the local analysis. Note that Eq. $(B.82)$ is the same as its local counterpart, Eq. $(B.57)$. Hence, controlling for prices as well as labor, the aggregate labor supply curve is the same as the local one. However, Eq. $(B.84)$ is different than its local counterpart, Eq. $(B.58)$. This is because the impact of aggregate nominal wages on the aggregate price level is greater than the impact of local wages on the local price level: specifically, we have $p_0 = w_0$ as opposed to $p_{0,a} = w_{0,a}\eta\left(1-\alpha^N\right)$ [cf. Eqs. $(B.83)$ and $(B.56)$]. Therefore, the real wage $w - p$ responds locally but not in the aggregate. The real wage response generates a neoclassical local labor supply response, with strength determined by the magnitude of the Frish elasticity $1/\phi$, that does not extend to the aggregate level. Rewriting the expressions for κ and κ^A to eliminate the wage stickiness parameter, λ, we obtain:

$$\frac{1}{\kappa} = \frac{1}{\varphi}\left(1 - \eta\left(1-\alpha^N\right)\right) + \frac{1}{\kappa^A}.$$

This expression illustrates that the local labor response, $\frac{1}{\kappa}$, combines a neoclassical response to higher real wages, $\frac{1}{\varphi}\left(1-\eta\left(1-\alpha^N\right)\right)$, that only occurs locally, and a term due to wage stickiness that extends to the aggregate, $\frac{1}{\kappa^A}$.

It is also instructive to consider the intuition for the labor bill characterized in $(B.86)$. Note that $1/\left(1+\delta\right)$ describes the effect of stock wealth on total spending. Multiplying this with $1-\overline{\alpha}$ gives the direct effect on the aggregate labor bill. This direct effect is amplified by two types of multipliers. First, there is a standard aggregate spending multiplier captured by, $\tilde{\mathcal{M}}^A = \frac{1}{1-1/(1+\delta)} > 1$. Second, there is also a second multiplier, which we refer to as the *factor-share multiplier*, denoted by $\mathcal{F}^A = \frac{1+\kappa^A}{1-\overline{\alpha}+\kappa^A} > 1$. The multiplier we use in the main text, $\mathcal{M}^A = \mathcal{F}^A\tilde{\mathcal{M}}^A$, is a composite of the two multipliers. The factor-share multiplier is somewhat specific to our model. In particular, it emerges from the assumption that labor is somewhat flexible but capital is not. These features (combined with the production technologies we work with) implies that labor absorbs a greater fraction of demand-driven fluctuations in aggregate spending compared to capital. Consistent with this intuition, the factor-share multiplier is decreasing in the degree of wage flexibility, κ^A, and it approaches one in the limit with perfectly flexible wages, $\kappa^A \to \infty$.

Finally, we compare the aggregate effect in ($B.86$) with its local counterpart characterized earlier. Specifically, recall that Eqs. ($B.62$) and ($B.63$) imply the effect of stock wealth on the *local* labor bill is given by,

$$\frac{(l_{a,0} + w_{a,0})\, \overline{WL}_0}{x_{a,0} Q_0} = \mathcal{M} \frac{1+\kappa}{1+\kappa\zeta} \frac{\left(1-\alpha^N\right)\eta}{1+\delta}. \tag{B.89}$$

Comparing this expression with Eq. ($B.86$) illustrates that the aggregate effect differs from the local effect for three reasons. First, the direct spending effect is greater in the aggregate than at the local level, $\frac{1-\overline{\alpha}}{1+\delta} > \frac{\eta\left(1-\alpha^N\right)}{1+\delta}$. Here, the inequality follows since $1-\overline{\alpha} = \eta\left(1-\alpha^N\right) + (1-\eta)\left(1-\alpha^T\right)$. Intuitively, spending on tradables increases the labor bill in the aggregate but not locally. Second, the aggregate labor bill does not feature the export adjustment term, $\frac{1+\kappa}{1+\kappa\zeta}$, because this adjustment is across areas. Third, the multiplier is greater in the aggregate than at the local level, $\mathcal{M}^A > \mathcal{M}$. In particular, the standard spending multiplier is greater at the aggregate level, $\tilde{\mathcal{M}}^A > \mathcal{M}$, because spending on tradables (as well as the mobile factor, capital) generates a multiplier effect in the aggregate but not locally. The factor-share multiplier increases the aggregate multiplier further, $\mathcal{F}^A > 1$.

Note also that, as long as $\varepsilon \geq 1$, the aggregate effect is greater than the local effect. In this case, $\zeta \geq 1$ and thus the export adjustment also dampens the local effect relative to the aggregate effect. When $\varepsilon < 1$, the export adjustment tends to make the local effect greater than the aggregate effect. However, all other effects (captured by $\eta < 1$ and $\mathcal{M}^A > \mathcal{M}$) tend to make the aggregate effect greater than the local effect.

B.8 Extending the Model to Incorporate Uncertainty

In this appendix, we generalize the baseline model to introduce uncertainty about capital productivity in period 1. We show that changes in households' risk aversion or perceived risk generate the same qualitative effects on the price of capital (as well as on "rstar") as in our baseline model. Moreover, conditional on a fixed amount of change in the price of capital, the model with uncertainty features the same *quantitative* effects on local labor market outcomes. Therefore, this exercise illustrates that our baseline analysis is robust to generating stock price fluctuations from alternative channels than the change in expected stock payoffs that we consider in our baseline analysis.

The model is the same as in Section B.1 with two differences. First, an aggregate state $s \in S$ is realized at the beginning of period 1 with probability $\pi(s)$ (with $\sum_{s\in S} \pi(s) = 1$). States determine the productivity of the capital-only technology. We adopt the normalization,

$$D_1(s) = s, \tag{B.90}$$

so that the state is equal to the productivity of capital, and we assume that S is a finite subset of $\mathbb{R}_+$. The baseline model is the special case in which S has a single element. We denote the equilibrium allocations in period 1 as functions of s, e.g., $C_{a,1}(s)$ denotes the consumption in area a and period 1 conditional on the aggregate state s.

Second, to analyze the effect of risk aversion, we also consider Epstein-Zin preferences that are more general than time-separable log utility. Specifically, we replace the preferences in (3) with,

$$\begin{aligned} & \log C_{a,0} + \delta \log U_{a,1}, \qquad \text{(B.91)} \\ \text{where } U_{a,1} &= \left(E\left[C_{a,1}(s)^{1-\gamma} \right] \right)^{1/(1-\gamma)}. \end{aligned}$$

Here, $U_{a,1}$ captures the household's (and particularly, the consumer's) certainty-equivalent consumption. The parameter γ captures her risk aversion. The baseline model is the special case with $\gamma = 1$. Note that we still assume the elasticity of intertemporal substitution is equal to one. The consumer chooses $C_{a,0}, S_{a,0}, 1 + x_{a,1}$ to maximize $(B.91)$ subject to the budget constraints:

$$\begin{aligned} P_{a,0}C_{a,0} + S_{a,0} &= W_{a,0}L_{a,0} + (1 + x_{a,0}) Q_0 \qquad \text{(B.92)} \\ S_{a,0} &= S^f_{a,0} + (1 + x_{a,1})(Q_0 - R_0) \\ P_{a,1}(s) C_{a,1}(s) &= \overline{W} L_{a,1}(s) + (1 + x_{a,1}) R_1(s) + S^f_{a,0} R^f. \end{aligned}$$

In period 0, the budget constraint is the same as before. In period 1, there is a separate budget constraint for each state. The rest of the equilibrium is unchanged.

General Characterization of Equilibrium with Uncertainty. Most of our analysis from the baseline case applies also in this case. First consider the equilibrium in period 1. As before, we have $W_{a,1}(s) \equiv \overline{W}$ and $L_{a,1}(s) = \overline{L}_1$ for each a and s. Using Lemma 1, we also obtain the following analogue of Eq. $(B.37)$

$$R_1(s) = D_1(s)\overline{W}. \qquad \text{(B.93)}$$

Note also that aggregating the budget constraint across all areas, we obtain the aggregate budget constraint:

$$\int_a P_{a,1}(s) C_{a,1}(s)\, da = R_1(s) + \overline{W}\overline{L}_1.$$

By Lemma 1, the price of the consumption good is the same across areas,

$$P_{a,1}(s) = P_1(s) \equiv D_1(s)^{\overline{\alpha}}\, \overline{W}.$$

After substituting this expression and using $(B.93)$, the aggregate budget constraint implies,

$$\int_a C_{a,1}(s)\, da = \frac{D_1(s) + \overline{L}_1}{(D_1(s))^{\overline{\alpha}}}. \qquad \text{(B.94)}$$

In the common-wealth benchmark, the areas are identical so Eq. $(B.94)$ provides a closed-form solution for consumption.

Next consider the equilibrium in period 0. The following lemma characterizes the consumers' optimal consumption and portfolio choice. To state the result let $H_{a,0} = W_{a,0}L_{a,0} + \frac{\overline{WL}_1}{R^f}$ denote the human capital wealth in area a as in the baseline model.

Lemma 4. *The optimal consumption for area a satisfies,*

$$P_{a,0}C_{a,0} = \frac{1}{1+\delta}\left[H_{a,0} + (1+x_{a,0})\, Q_0\right]. \tag{B.95}$$

Optimal portfolios in area a are such that the risk-free interest rate satisfies,

$$1/R^f = E\left[M_{a,1}(s)\right] \tag{B.96}$$

and the price of capital satisfies,

$$Q_0 = R_0 + E\left[M_{a,1}(s)\, R_1(s)\right], \tag{B.97}$$

where $M_{a,1}(s)$ denotes the (nominal) stochastic discount factor *for area a and is given by*

$$M_{a,1}(s) = \delta \frac{P_{a,0}C_{a,0}}{P_{a,1}(s)\, C_{a,1}(s)} \frac{C_{a,1}(s)^{1-\gamma}}{E\left[C_{a,1}(s)^{1-\gamma}\right]}. \tag{B.98}$$

Eq. $(B.41)$ illustrates that the consumption wealth effect remains unchanged in this case [cf. Eq. $(B.41)$]. This is because we use Epstein-Zin preferences with an intertemporal elasticity of substitution equal to one. Eqs. $(B.96)$ and $(B.97)$ illustrate that standard asset pricing conditions apply in this setting. Specifically, the risk-free asset as well as capital are priced according to a stochastic discount factor (SDF) that might be specific to the area. Eq. $(B.98)$ characterizes the SDF. When $\gamma = 1$, the SDF has a familiar form corresponding to time-separable log utility. We relegate the proof of Lemma 4 to the end of this section.

Since the optimal consumption Eq. $(B.95)$ remains unchanged (and the remaining features of the model are also unchanged), the rest of the general characterization in Section B.2 also applies in this case. We next characterize the equilibrium further in the common-wealth benchmark.

Common-wealth Benchmark with Uncertainty. Consider the benchmark case with $x_{a,0} = 0$ for each a. We generalize Assumption D as follows.

Assumption D$^{\text{U}}$. $D_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}_0$ and $D_1(s) \geq \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}_1$ for each $s \in S$.

As before, this assumption ensures that $\tilde{K}_0^T = 0$ and $\tilde{K}_1^T(s) \geq 0$ for each s.

Note also that we still have $L_{a,0} = \overline{L}_0$ where $\overline{L}_0$ corresponds to the solution to $(B.38)$.

Next note that, since areas are identical, we have $C_{a,1}(s) = C_1(s)$. We also have $W_{a,1}(s) = \overline{W}$. By Lemma 1, this implies,

$$P_{a,1}(s) = (D_1(s))^{\overline{\alpha}}\,\overline{W}. \tag{B.99}$$

Combining these observations with Eq. $(B.94)$, we obtain a closed-form solution for consumption,

$$C_1(s) = \frac{D_1(s) + \overline{L}_1}{(D_1(s))^{\overline{\alpha}}}. \tag{B.100}$$

Next note that we also have $W_{a,0} = \overline{W}$, and

$$P_{a,0} = D_0^{\overline{\alpha}}\,\overline{W}. \tag{B.101}$$

Therefore, the analogous equation also applies in period 0,

$$C_0 = \frac{D_0 + \overline{L}_0}{D_0^{\overline{\alpha}}}. \tag{B.102}$$

Substituting this into Eq. $(B.95)$, and using $(B.101)$, we obtain,

$$\left(D_0 + \overline{L}_0\right)\overline{W} = \frac{1}{1+\delta}\left[H_{a,0} + Q_0\right].$$

After rearranging the expression, we find that Eq. $(B.48)$ also applies in this setting:

$$\left(H_0 + Q_0\right)/\overline{W} = (1+\delta)\left(\overline{L}_0 + D_0\right). \tag{B.103}$$

As before, the sum of capital and human capital wealth must be equal to a multiple of the frictionless output level. This is necessary so that the implied wealth effect is sufficiently large to clear the goods market.

Next note that, after substituting Eqs. $(B.100)$ and $(B.102)$ for consumption and Eqs. $(B.99)$ and $(B.101)$ for goods prices, we obtain a closed-form solution for the stochastic discount factor in $(B.98)$,

$$M_1(s) = \delta \frac{D_0 + \overline{L}_0}{D_1(s) + \overline{L}_1} \frac{\left(\frac{D_1(s)+\overline{L}_1}{(D_1(s))^{\overline{\alpha}}}\right)^{1-\gamma}}{E\left[\left(\frac{D_1(s)+\overline{L}_1}{(D_1(s))^{\overline{\alpha}}}\right)^{1-\gamma}\right]}. \tag{B.104}$$

Combining this expression with Eqs. $(B.96)$ and $(B.97)$, we also obtain closed-form solutions for $R^{f,*}$ ("rstar") and Q_0:

$$1/R^{f,*} = E\left[M_1(s)\right] \tag{B.105}$$

$$Q_0/\overline{W} = D_0 + E\left[M_1(s)\,D_1(s)\right]. \tag{B.106}$$

Here, the second line substitutes $R_0 = D_0\overline{W}$ and $R_1(s) = D_1(s)\,\overline{W}$. We can also calculate the human capital wealth as,

$$H_0/\overline{W} = \overline{L}_0 + \frac{\overline{L}_1}{R^f} = \overline{L}_0 + \overline{L}_1 E\left[M_1(s)\right]. \tag{B.107}$$

Note also that, when $\gamma = 1$, we have time-separable log utility and Eq. $(B.104)$ reduces to the more familiar form, $M_1(s) = \frac{D_0+\overline{L}_0}{D_1(s)+\overline{L}_1}$. Using this expression, note that, when there is a single state, Eqs. $(B.105)$ $(B.106)$, and $(B.107)$ become identical to their counterparts in the earlier analysis [cf. Eqs. $(B.51)$, $(B.53)$, and $(B.52)$].

Since the aggregate wealth $H_0 + Q_0$ remains unchanged [cf. $(B.103)$], the rest of the characterization in Section B.3 remains unchanged. In particular, labor shares in nontradable and tradable sectors are given by $L_0^N/\overline{L}_0 = \frac{1-\alpha^N}{1-\overline{\alpha}}\eta$ and $L_0^T/\overline{L}_0 = \frac{1-\alpha^T}{1-\overline{\alpha}}(1-\eta)$ [cf. Eq. $(B.54)$].

Recall that, in the baseline model without uncertainty, we generate fluctuations in Q_0 as well as R_f^* from changes in D_1. We next show that this aspect of the model also generalizes. In particular, after summarizing the above discussion, the following proposition establishes that changes in risk or risk aversion generate the same effects on asset prices as changes in future productivity in the baseline model. To state the result, recall that we normalize $D_1(s) = s$ so that the probability distribution for states, $\pi(s)$, is the same as the distribution for capital productivity.

Proposition 3. *Consider the model with uncertainty with Assumption D^U and the normalization in $(B.90)$ Suppose areas have common stock wealth, $x_{a,0} = 0$ for each a. In equilibrium, all areas have identical allocations and prices. In period 0, labor is at its frictionless level, $L_0 = \overline{L}_0$, and nominal wages are at their expected level, $W_0 = \overline{W}$; the stochastic discount factor is given by Eq. $(B.104)$; the nominal interest rate is given by Eq. $(B.105)$; the human capital and stock wealth are given by Eqs. $(B.107)$ and $(B.106)$; the shares of labor employed in the nontradable and tradable sectors are given by Eq. $(B.54)$.*

Consider any one of the following changes:

(i) Suppose $\gamma = 1$ and the probability distribution, $\left(\pi^{old}(s)\right)_{s\in S}$, changes such that $\left(\pi^{new}(s)\right)_{s\in S}$ first-order stochastically dominates $\left(\pi^{old}(s)\right)_{s\in S}$.

(ii) Suppose $\gamma = 1$ and the probability distribution, $\left(\pi^{old}(s)\right)_{s\in S}$, changes such that $\left(\pi^{old}(s)\right)_{s\in S}$ is a mean-preserving spread of $\left(\pi^{new}(s)\right)_{s\in S}$.

(iii) Suppose $(\pi(s))_s$ remains unchanged but risk-aversion decreases, $\gamma^{new} < \gamma^{old}$.

These changes increase Q_0 and reduce R_f^* in equilibrium but do not affect the labor market outcomes in period 0.

The first part is a generalization of the comparative statics exercise that we consider in the baseline model. It shows that the price of capital increases also if households perceive greater capital productivity in the first-order stochastic dominance sense. The second part shows that a similar result obtains if households' expected belief for capital productivity remains unchanged but they perceive less risk in capital productivity. For analytical tractability, these two parts focus on the case, $\gamma = 1$, which corresponds to time-separable log utility as in the baseline model. The last part considers the case with general γ, and shows that a similar result obtains also if households' belief distribution remains unchanged but their risk aversion declines. We relegate the proof of Proposition 3 to the end of this section.

Comparative Statics of Local Labor Market Outcomes with Uncertainty. Recall that since the optimal consumption Eq. $(B.95)$ remains unchanged, all equilibrium conditions for period 0 derived in Section B.2 continue to apply conditional on Q_0 and R^f. Therefore, the log-linearized equilibrium conditions derived in Section B.4 also continue to apply conditional on Q_0. Moreover, as we show in Section B.5, the comparative statics in Proposition 3 affect these conditions only through their effect on Q_0. It follows that, conditional on generating the same change in the price of capital, ΔQ_0, the model with uncertainty features the same *quantitative* effects on local labor market outcomes as in our our baseline model. Combining this result with the comparative static results in Proposition 3, we conclude that our baseline analysis is robust to generating stock price fluctuations from alternative sources such as changes in households' risk aversion or perceived risk about stock payoffs.

Proof of Lemma 4. To analyze the households' problem, we consider the change of variables,

$$\tilde{S}_{a,0} = S_{a,0} + \frac{\overline{WL_1}}{R^f}.$$

Note that $L_{a,1}(s) \equiv \overline{L}_1$. Hence, $\tilde{S}_{a,0}$ can be thought of as the households' "effective savings" that incorporates the present discounted value of her lifetime wealth. We also consider the change of variables

$$\omega_{a,1} \equiv \frac{(1 + x_{a,1})(Q_0 - R_0)}{\tilde{S}_{a,0}}.$$

Here, $\omega_{a,1}$ captures the fraction of households' effective savings that she invests in capital (recall that $Q_0 - R_0$ denotes the ex-dividend price of capital). The remaining fraction, $1 - \omega_{a,1}$, is invested in the risk-free asset. After substituting this notation into the budget constraints, the households' problem can be equivalently written as,

$$\begin{aligned} \max_{\tilde{S}_{a,0},\omega_{a,1}} \quad & \log C_{a,0} + \delta \log U_{a,1}, \\ \text{where } U_{a,1} &= \left(E\left[C_{a,1}(s)^{1-\gamma}\right]\right)^{1/(1-\gamma)} \\ P_{a,0}C_{a,0} + \tilde{S}_{a,0} &= W_{a,0}L_{a,0} + \frac{\overline{WL_1}}{R^f} + (1 + x_{a,0})\,Q_0 \\ P_{a,1}(s)\,C_{a,1}(s) &= \tilde{S}_{a,0}\left(R^f + \omega_{a,1}\left(\frac{R_1(s)}{Q_0 - R_0} - R^f\right)\right) \end{aligned} \tag{B.108}$$

Here, $\frac{R_1(s)}{Q_0 - R_0}$ denotes the gross return on capital. When $\omega_{a,1} = 0$, the household does not invest in capital so her portfolio return is the gross risk-free rate, R^f. When $\omega_{a,1} = 1$, the household invests all of her savings in capital so her portfolio return is the gross return to capital, $\frac{R_1(s)}{Q_0-R_0}$.

Next consider the optimality condition for $\tilde{S}_{a,0}$ in problem $(B.108)$. This gives:

$$\begin{aligned}
\frac{1}{P_{a,0}C_{a,0}} &= \delta E\left[\frac{U_{a,1}^{\gamma}C_{a,1}(s)^{-\gamma}}{U_{a,1}}\frac{1}{P_{a,1}(s)}\left(R^f+\omega_{a,1}\left(\frac{R_1(s)}{Q_0-R_0}-R^f\right)\right)\right] \\
&= \delta E\left[U_{a,1}^{\gamma-1}C_{a,1}(s)^{-\gamma}\frac{C_{a,1}(s)}{\tilde{S}_{a,0}}\right] \\
&= \delta E\left[U_{a,1}^{\gamma-1}U_{a,1}^{1-\gamma}\frac{1}{\tilde{S}_{a,0}}\right] \\
&= \delta\frac{1}{\tilde{S}_{a,0}}.
\end{aligned}$$

Here, the second line uses the budget constraint in period 1 to substitute for the return in terms of $C_{a,1}(s)$; the third line uses $U_{a,1}^{1-\gamma}=E\left[C_{a,1}(s)^{1-\gamma}\right]$ (from the definition of the certainty-equivalent return), and the last line simplifies the expression. Combining the resulting expression with the budget constraint in period 1, we obtain,

$$P_{a,0}C_{a,0}=\frac{1}{1+\delta}\left[W_{a,0}L_{a,0}+\frac{\overline{WL}_1}{R^f}+(1+x_{a,0})\,Q_0\right].$$

This establishes $(B.95)$.

Next, to establish the asset pricing condition for the risk-free asset, consider the optimality condition for $S_{a,0}^f$ in the original version of the problem (as this corresponds to saving in the risk-free asset). This gives:

$$\frac{1}{P_{a,0}C_{a,0}}=E\left[\frac{\delta}{P_{a,1}(s)\,C_{a,1}(s)^{\gamma}\,E\left[C_{a,1}(s)^{1-\gamma}\right]}R^f\right]. \tag{B.109}$$

Rearranging terms and substituting $M_{a,1}(s)$ from Eq. $(B.98)$, we obtain Eq. $(B.96)$. Finally, to establish the asset pricing condition for capital, consider the optimality condition for $\omega_{a,1}$ in problem $(B.108)$. This gives:

$$E\left[\frac{C_{a,1}(s)^{-\gamma}}{P_{a,1}(s)}\left(\frac{R_1(s)}{Q_0-R_0}-R^f\right)\right]=0.$$

Rearranging terms, we obtain,

$$\begin{aligned}
Q_0 &= R_0+\frac{1}{R^f E\left[\frac{1}{P_{a,1}(s)C_{a,1}(s)^{\gamma}}\right]}E\left[\frac{1}{P_{a,1}(s)\,C_{a,1}(s)^{\gamma}}R_1(s)\right] \\
&= R_0+\delta E\left[\frac{P_{a,0}C_{a,0}}{P_{a,1}(s)\,C_{a,1}(s)^{\gamma}\,E\left[C_{a,1}(s)^{1-\gamma}\right]}R_1(s)\right]
\end{aligned}$$

$$= \quad R_0 + E\left[M_1(s)\, R_1(s)\right].$$

Here, the second line uses Eq. (B.109) to simplify the expression and the last line substitutes for $M_1(s)$ from Eq. (B.98). This establishes (B.97) and completes the proof of the lemma. □

Proof of Proposition 3. It remains to establish the comparative statics exercises. Recall that the aggregate wealth and human capital wealth satisfy [cf. Eqs. (B.48) and (B.107)],

$$\begin{aligned}
(H_0 + Q_0)/\overline{W} &= (1+\delta)\left(\overline{L}_0 + D_0\right) \\
H_0/\overline{W} &= \overline{L}_0 + \frac{\overline{L}_1}{R^{f,*}}.
\end{aligned}$$

Note that the probability distribution, $(\pi(s))_{s \in S}$, or the risk aversion, γ, affect these equations only through their effect on Q_0 and R^f. These equations imply that if Q_0 increases in equilibrium, then $R^{f,*}$ must also increase. Specifically, the first equation implies that if Q_0 increases then H_0 decreases. The second equation implies that if H_0 decreases then $R^{f,*}$ increases. Therefore, it suffices to establish the comparative statics exercises for the price of capital, Q_0.

First consider the comparative statics exercises in parts (i) and (ii). After substituting $\gamma = 1$ and $D_1(s) = s$ into Eqs. (B.106) and (B.104), we obtain the following expression for the price of capital:

$$\begin{aligned}
Q_0 &= D_0 + \delta\left(D_0 + \overline{L}_0\right) E\left[f(s)\right], \qquad \text{(B.110)} \\
\text{where } f(s) &= \frac{s}{s + \overline{L}_1}.
\end{aligned}$$

Here, the second line defines the function $f : \mathbb{R}_+ \to \mathbb{R}_+$. Note that this function is strictly increasing and strictly concave: that is, $f'(s) > 0$ and $f''(s) < 0$ for $s > 0$. Combining this observation with Eq. (B.110) proves the desired comparative statics. To establish (i), note that $E^{new}[f(s)] \geq E^{old}[f(s)]$ because $f(s)$ is increasing in s, and $\pi^{new}(s)$ first-order stochastically dominates $\pi^{old}(s)$. To establish (ii), note that $E^{new}[f(s)] \geq E^{old}[f(s)]$ because $f(s)$ is increasing and concave in s, and $\pi^{new}(s)$ second-order stochastically dominates $\pi^{old}(s)$ (which in turn follows because $\pi^{old}(s)$ is a mean-preserving spread of $\pi^{new}(s)$).

Finally, consider the comparative statics exercise in part (iii). In this case, Eqs. (B.106) and (B.104) imply,

$$Q_0 = D_0 + \delta\left(D_0 + \overline{L}_0\right) \frac{E\left[f(s)\, g(s)^{1-\gamma}\right]}{E\left[g(s)^{1-\gamma}\right]}, \qquad \text{(B.111)}$$

$$\text{where } g(s) = \frac{s + \overline{L}_1}{s^{\overline{\alpha}}}.$$

Here, the second line defines the function $g : \mathbb{R}_+ \to \mathbb{R}_+$. We first claim that this function is increasing in s over the relevant range. To see this, note that,

$$g'(s) = s^{-\overline{\alpha}-1}\left((1-\overline{\alpha})s - \overline{\alpha}\overline{L}_1\right).$$

Assumption D^U implies that $s \geq \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}_1$, which in turn implies $g'(s) \geq 0$. Therefore, $g(s)$ is increasing in s over the range implied by Assumption D^U.

Next note that Eq. $(B.111)$ can be rewritten as

$$Q_0 = D_0 + \delta\left(D_0 + \overline{L}_0\right)E^*\left[f(s)\right],$$

where $E^*[\cdot]$ denotes the expectations under the endogenous probability distribution $\{\pi_s^*\}_{s\in S}$, defined by,

$$\pi_s^* = \frac{\pi_s g(s)^{1-\gamma}}{\sum_{\tilde{s}\in S}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}} \text{ for each } s \in S. \tag{B.112}$$

Hence, using our result from part (i), it suffices to show that $\pi_s^{*,new}$ (which corresponds to $\gamma^{new} < \gamma^{old}$) first-order stochastically dominates $\pi_s^{*,old}$.

To establish the last claim, define the cumulative distribution function corresponding to the endogenous probability distribution,

$$\Pi_s^*(\gamma) = \sum_{\tilde{s}\leq s}\pi_{\tilde{s}}^* = \frac{\sum_{\tilde{s}\leq s}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}}{\sum_{\tilde{s}\in S}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}} \text{ for each } s \in S. \tag{B.113}$$

We made the dependence of the distribution function on γ explicit. To prove the claim, it suffices to show that $\frac{d\Pi_s^*(\gamma)}{d\gamma} \geq 0$ for each $s \in S$ (so that a decrease in γ decreases $\Pi_s^*(\gamma)$ for each s and thus increases the distribution in the first-order stochastic dominance order). We have:

$$\begin{aligned}
\frac{d\Pi_s^*(\gamma)}{d\gamma} &= \frac{\sum_{\tilde{s}\leq s}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}}{\sum_{\tilde{s}\in S}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}}\left(-\frac{\sum_{\tilde{s}\leq s}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}\log g(\tilde{s})}{\sum_{\tilde{s}\leq s}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}} + \frac{\sum_{\tilde{s}\in S}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}\log g(\tilde{s})}{\sum_{\tilde{s}\in S}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}}\right) \\
&= \frac{\sum_{\tilde{s}\leq s}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}}{\sum_{\tilde{s}\in S}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}}\left(-\sum_{\tilde{s}\leq s}\frac{\pi_{\tilde{s}}^*}{\Pi_s^*(\gamma)}\log g(\tilde{s}) + \sum_{\tilde{s}\in S}\pi_{\tilde{s}}^*\log g(\tilde{s})\right) \\
&= \frac{\sum_{\tilde{s}\leq s}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}}{\sum_{\tilde{s}\in S}\pi_{\tilde{s}} g(\tilde{s})^{1-\gamma}}\left(-E^*\left[\log g(\tilde{s}) \quad | \tilde{s} \leq s\right] + E^*\left[\log g(\tilde{s})\right]\right).
\end{aligned}$$

Here, the second line substitutes the definition of the endogenous distribution and its cumulative distribution from Eqs. $(B.112)$ and $(B.113)$. The last line substitutes the unconditional and conditional expectations. It follows that $\frac{d\Pi_s^*(\gamma)}{d\gamma} \geq 0$ for some $s \in S$ if and only if the unconditional expectation exceeds the conditional expectation, $E^*\left[\log g(\tilde{s})\right] \geq E^*\left[\log g(\tilde{s}) \mid \quad \tilde{s} \leq s\right]$. This is true because $\log g(s)$ is increasing in s (since $g(s)$ is increasing), which implies that the conditional expectation is increasing in s. This proves the claim and completes the proof of part (iii). □

B.9 Extending the Model for More General EIS

We next generalize the model to consider more general levels of EIS. For simplicity, suppose all areas except for one have time-separable log utility (3) as in the baseline model. The remaining area, denoted by a, has the following more general utility function,

$$u(C_{a,0}) + \delta u(C_{a,1}) \text{ where } u(C) = \frac{\varepsilon}{\varepsilon - 1}\left(C^{\frac{\varepsilon-1}{\varepsilon}} - 1\right). \tag{B.114}$$

We characterize the equilibrium in area a and illustrate how it depends on the EIS parameter, ε. To simplify the analysis, we assume all other areas have equal wealth, $x_{\tilde{a},0} = 0$ for each $\tilde{a} \neq a$. Since area a has zero mass, this ensures that the aggregate allocations and prices, as well as the allocations and prices in each area $\tilde{a} \neq a$, are described by the common-wealth benchmark characterized in Section B.7.

To characterize the equilibrium in area a, first note that (after substituting the equilibrium price for Q_0) households' budget constraints can be combined into a lifetime budget constraint,

$$P_{a,0}C_{a,0} + \frac{P_{a,1}C_{a,1}}{R^f} = H_{a,0} + (1 + x_{a,0}) Q_0.$$

Households in area a maximize $(B.114)$ subject to this constraint. The optimality condition gives the Euler equation,

$$\begin{aligned} P_{a,1}C_{a,1} &= \delta^{\varepsilon} R^f \left(R^{fr}\right)^{\varepsilon-1} P_{a,0}C_{a,0} \\ \text{where } R_a^{fr} &= R^f \frac{P_{a,0}}{P_{a,1}} \end{aligned} \tag{B.115}$$

Here, R_a^{fr} denotes the real interest rate in area a. Substituting this into the budget constraint, we obtain the following analogue of Eq. (6),

$$P_{a,0}C_{a,0} = \frac{1}{1 + \delta^{\varepsilon}\left(R_a^{fr}\right)^{\varepsilon-1}} \left(H_{a,0} + (1 + x_{a,0}) Q_0\right),. \tag{B.116}$$

This expression illustrates that a similar relationship between wealth and consumption exists once we replace the exogenous parameter, δ, with its counterpart, $\delta^{\varepsilon}\left(R_a^{fr}\right)^{\varepsilon-1}$. When $\varepsilon = 1$, the wealth-effect coefficient, $\frac{1}{1+\delta^{\varepsilon}\left(R_a^{fr}\right)^{\varepsilon-1}}$, does not depend on the real interest rate. In this case, which we analyze in the main text, the income and substitution effects are exactly balanced so that we have a pure wealth effect. When $\varepsilon > 1$, the wealth-effect coefficient is decreasing in the interest rate. In this case, there is a net substitution effect so that greater interest rate increases savings and reduces consumption. Conversely, when $\varepsilon < 1$, the wealth-effect coefficient is increasing in the interest rate due to a net-income effect.

To characterize the rest of the equilibrium, note that much of the analysis in Section B.2 applies also in this case. In particular, after using $x_{\tilde{a},0} = 0$ for each $\tilde{a}$, the labor demand equation in area a is given by the following analogue of Eq. $(B.47)$:

$$W_{a,0}L_{a,0} = \frac{\left(1-\alpha^N\right)\eta}{1+\delta^{\varepsilon}\left(R_a^{fr}\right)^{\varepsilon-1}}\left(W_{a,0}L_{a,0} + \frac{\overline{WL}_1}{R^f} + (1+x_{a,0})\,Q_0\right) + \left(\frac{P_{a,0}^T}{P_0^T}\right)^{1-\varepsilon}\overline{W}L_0^T$$

Here, recall that R_a^{fr} is given by Eq. $(B.115)$ where $P_{a,t} = \left(P_{a,t}^N\right)^{\eta}\left(P_{a,t}^T\right)^{1-\eta}$ and $P_{a,t}^N, P_{a,t}^T$ as well as $P_{a,t}$ are characterized by Lemma 1. Using $x_{\tilde{a},0} = 0$, we also have,

$$P_{a,t} = \left(\frac{W_{a,0}}{\overline{W}}\right)^{\eta\left(1-\alpha^N\right)} D_t^{\overline{\alpha}}\overline{W} \text{ and } \frac{P_{a,0}^T}{P_0^T} = \left(\frac{W_{a,0}}{\overline{W}}\right)^{1-\alpha^T}.$$

After substituting these expressions, we simplify the labor demand equation as follows,

$$\begin{aligned} W_{a,0}L_{a,0} &= \frac{\left(1-\alpha^N\right)\eta}{1+\delta^{\varepsilon}\left(R_a^{fr}\right)^{\varepsilon-1}}\left(W_{a,0}L_{a,0} + \frac{\overline{WL}_1}{R^f} + (1+x_{a,0})\,Q_0\right) + \left(\frac{W_{a,0}}{\overline{W}}\right)^{\left(1-\alpha^T\right)(1-\varepsilon)}\overline{W}L_0^T, \\ \text{where } R_a^{fr} &= R^f\frac{D_0^{\overline{\alpha}}}{D_1^{\overline{\alpha}}}\left(\frac{W_{a,0}}{\overline{W}}\right)^{\eta\left(1-\alpha^N\right)}. \end{aligned}$$

The equilibrium in area a is characterized by solving this equation together with the labor supply equation $(B.17)$.

To make progress, consider the special case in which wages are perfectly sticky, $\lambda_w = 0$ (which also leads to $\lambda = 0$). In this case, $W_{a,0} = \overline{W}$ and the labor demand equation can be further simplified as,

$$\begin{aligned} \overline{W}L_{a,0} &= \frac{\left(1-\alpha^N\right)\eta}{1+\delta^{\varepsilon}\left(R^{fr}\right)^{\varepsilon-1}}\left(\overline{W}L_{a,0} + \frac{\overline{WL}_1}{R^f} + (1+x_{a,0})\,Q_0\right) + \overline{W}L_0^T, \\ \text{where } R^{fr} &= R^f\frac{D_0^{\overline{\alpha}}}{D_1^{\overline{\alpha}}}. \end{aligned} \quad \text{(B.117)}$$

Here, R^{fr} denotes the aggregate real interest rate. This expression illustrates that the labor market equilibrium in area a is characterized in similar fashion to the equilibrium in other areas. The main difference concerns the wealth-effect coefficient, $\frac{\left(1-\alpha^N\right)\eta}{1+\delta^{\varepsilon}\left(R^{fr}\right)^{\varepsilon-1}}$. The new coefficient illustrates that the level of the real interest rate affects the labor bill.

Next note that the aggregate equilibrium is unchanged and characterized as in Appendix B.7. In particular, the nominal interest rate is characterized by,

$$R^f = \frac{1}{\delta}\frac{\overline{L}_1 + D_1}{\overline{L}_0 + D_0}.$$

Thus, the real interest rate is characterized by,

$$R^{fr} = \frac{1}{\delta}\frac{\overline{L}_1 + D_1}{\overline{L}_0 + D_0}\frac{D_0^{\overline{\alpha}}}{D_1^{\overline{\alpha}}}.$$

Note that, we have:

$$\frac{dR^{fr}}{dD_1} = \frac{1}{\delta}\frac{D_0^{\overline{\alpha}}}{\overline{L}_0 + D_0} D_1^{-\overline{\alpha}-1}\left(-\overline{\alpha}\overline{L}_1 + (1-\overline{\alpha}) D_1\right) \geq 0,$$

where the inequality follows from Assumption D. Therefore, an increase in D_1 increases not only the nominal interest rate but also the real interest rate. Combining this observation with Eq. $(B.117)$ illustrates that a shock to D_1 that changes the price of capital has two effects on the labor markets in area a with high stock wealth, $x_{a,0}$. First, it creates a wealth effect as in the earlier analysis. Second, since it increases R^{fr}, it also creates a net substitution or income effect depending on whether $\varepsilon > 1$ or $\varepsilon < 1$.