Appendix to "Beyond Competitive Devaluations:
The Monetary Dimensions of Comparative Advantage"
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1. Demand equations not listed in text

The composition of expenditure on adjustment costs, both for prices and bond holding, follows the same preferences as for consumption, and the associated demands mirror equations (4)-(9). Adjustment costs for bond holding are as follows:

$$AC_{B,D,t} = \theta P_t AC_{B,t} / P_{D,t} \qquad AC_{B,N,t} = (1-\theta) P_t AC_{B,t} / P_{N,t}$$

$$d_{AC,B,t}(h) = \left(p_t(h)/P_{D,t}\right)^{-\phi} AC_{B,D,t} \qquad d_{AC,B,t}(f) = \left(p_t(f)/P_{D,t}\right)^{-\phi} AC_{B,D,t}$$

$$AC_{B,H,t} = \nu\left(P_{H,t}/P_{N,t}\right)^{-\eta} AC_{B,N,t} \qquad AC_{B,F,t} = (1-\nu)\left(P_{F,t}/P_{N,t}\right)^{-\eta} AC_{B,N,t}$$

The economy-wide demand for goods arising from price adjustment costs sums across the demand arising among *n* home firms: $AC_{P,t} = n_t AC_{P,t}(h)$. This is allocated as follows:

$$AC_{P,D,t} = \theta P_t AC_{P,t} / P_{D,t} \qquad AC_{P,N,t} = (1-\theta) P_t AC_{P,t} / P_{N,t}$$

$$d_{AC,P,t}(h) = \left(p_t(h)/P_{D,t}\right)^{-\phi} AC_{P,D,t} \qquad d_{AC,P,t}(f) = \left(p_t(f)/P_{D,t}\right)^{-\phi} AC_{P,D,t}$$

$$AC_{P,H,t} = \nu\left(P_{H,t}/P_{N,t}\right)^{-\eta} AC_{P,N,t} \qquad AC_{P,F,t} = (1-\nu)\left(P_{F,t}/P_{N,t}\right)^{-\eta} AC_{P,N,t}$$

2. Entry condition

The single-period version of the entry condition (25) is:

$$W_t K = E_t\left[\beta \frac{\mu_t}{\mu_{t+1}} \pi_{t+1}(h)\right].$$

Combine with the single-period version of the profit function (24), in which the dynamic adjustment cost ($AC_{p,t}(h)$) is set to zero, and simplify:

$$W_t K = E_t\left[\beta \frac{\mu_t}{\mu_{t+1}}\left(\left(p_{t+1}(h) - \frac{W_{t+1}}{\alpha_{t+1}}\right)c_{t+1}(h) + \left(e_{t+1}p^*_{t+1}(h) - (1+\tau)\frac{W_{t+1}}{\alpha_{t+1}}\right)c^*_t(h)\right)\right]$$

Under producer currency pricing of exports:

$$W_t K = E_t\left[\beta \frac{\mu_t}{\mu_{t+1}}\left(\left(p_{t+1}(h) - \frac{W_{t+1}}{\alpha_{t+1}}\right)c_{t+1}(h) + \left((1+\tau)p_{t+1}(h) - (1+\tau)\frac{W_{t+1}}{\alpha_{t+1}}\right)c^*_{t+1}(h)\right)\right]$$

$$W_t K = E_t\left[\beta \frac{\mu_t}{\mu_{t+1}}\left(\left(p_{t+1}(h) - \frac{W_{t+1}}{\alpha_{t+1}}\right)\left(c_{t+1}(h) + (1+\tau)c^*_{t+1}(h)\right)\right)\right]$$

Using demand equations for $C_{M,t}$ and $c_t(h)$, as well as definition of $P_{M,t}$:

$$W_t K = E_t\left[\beta \frac{\mu_t}{\mu_{t+1}}\left(\left(p_{t+1}(h) - \frac{W_{t+1}}{\alpha_{t+1}}\right)\left(\left(\frac{p_{t+1}(h)}{P_{M,t+1}}\right)^{-\phi}\theta\left(\frac{P_{t+1}}{P_{M,t+1}}\right)C_{t+1} + (1+\tau)^{1-\phi}\left(\frac{p_{t+1}(h)/e_{t+1}}{P^*_{M,t+1}}\right)^{-\phi}\theta\left(\frac{P^*_{t+1}}{P^*_{M,t+1}}\right)C^*_t\right)\right)\right]$$

$$W_t K = E_t\left[\beta \frac{\mu_t}{\mu_{t+1}}\left(\left(p_{t+1}(h) - \frac{W_{t+1}}{\alpha_{t+1}}\right)p_{t+1}(h)^{-\phi}\theta\left(\begin{array}{l}\left(n_{t+1}p_{t+1}(h)^{1-\phi} + n^*_{t+1}p_{t+1}(f)^{1-\phi}\right)^{-1}P_{t+1}C_{t+1} \\ +(1+\tau)^{1-\phi}e_{t+1}{}^{\phi}\left(n_{t+1}p^*_{t+1}(h)^{1-\phi} + n^*_{t+1}p^*_{t+1}(f)^{1-\phi}\right)^{-1}P^*_{t+1}C^*_t\end{array}\right)\right)\right]$$

Under log utility, where $W_t = \mu_t$ and $P_t C_t = \mu_t$, this becomes equation (46).

3. Entry under full stabilization

Substitute prices, $p_{t+1}(h) = p^*_{t+1}(f)\left(\phi/(\phi-1)\right)$, and policy rules ($\mu_t = \alpha_t, \quad \mu^*_t = \alpha^*_t$) into (46) and simplify:

$$\frac{K\phi}{\beta\theta} = E_t\left[\left(n_{t+1} + n^*_{t+1}\left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{1-\phi}(1+\tau)^{1-\phi}\right)^{-1} + (1+\tau)^{1-\phi}\left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{\phi-1}\left(n_{t+1}\left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{\phi-1}(1+\tau)^{1-\phi} + n^*_{t+1}\right)^{-1}\right]$$

Impose symmetry across countries:

$$n_{t+1} = \frac{\beta\theta}{K\phi}E_t\left[\left(1 + \left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{1-\phi}(1+\tau)^{1-\phi}\right)^{-1} + (1+\tau)^{1-\phi}\left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{\phi-1}\left(\left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{\phi-1}(1+\tau)^{1-\phi} + 1\right)^{-1}\right]$$

$$n_{t+1} = \frac{\beta\theta}{K\phi}E_t\left[\frac{2 + \left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{1-\phi}\left((1+\tau)^{\phi-1} + (1+\tau)^{1-\phi}\right)}{1 + \left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{1-\phi}\left((1+\tau)^{\phi-1} + (1+\tau)^{1-\phi}\right) + \left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{2(1-\phi)}}\right]$$

Which is the same as for the flexible price case.

To compare to the no stabilization case, write this as

$$n^{stab}_{t+1} = n^{no\,stab}_{t+1}E_t\Gamma_{t+1}$$

$$\text{where } \Gamma = \frac{2 + \left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{1-\phi}\left((1+\tau)^{\phi-1} + (1+\tau)^{1-\phi}\right)}{1 + \left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{1-\phi}\left((1+\tau)^{\phi-1} + (1+\tau)^{1-\phi}\right) + \left(\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right)^{2(1-\phi)}}$$

Note that $n^{stab}_{t+1} > n^{no\,stab}_{t+1}$ if $E_t\Gamma_{t+1} > 1$. However Γ_{t+1} switches from a concave function of $\alpha_{t+1}/\alpha^*_{t+1}$ to a convex function near the symmetric steady state value of $\alpha_{t+1}/\alpha^*_{t+1} = 1$. Hence we cannot apply Jensen's inequality to determine whether $E_t\Gamma_{t+1} > 1$. This finding reflects the fact that the effects of symmetric stabilization are small. Our analysis, nonetheless, will show that the effects of asymmetric stabilization can be large.

4. Case of fixed exchange rate rule

Substitute prices and policy rules ($\mu_t = \alpha_t$, $\mu_t^* = \mu_t = \alpha_t$) into (46):

$$\frac{K}{\beta\theta} = E_t\left[\left[\left(\left(\frac{\phi}{\phi-1}-1\right)\left(\frac{\phi}{\phi-1}\right)^{-\phi}\left(\begin{array}{l}\left(n_{t+1}\left(\frac{\phi}{\phi-1}\right)^{1-\phi}+n_{t+1}^*\left(\frac{\phi}{\phi-1}E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}(1+\tau)^{1-\phi}\right)^{-1}\\ +(1+\tau)^{1-\phi}\left(n_{t+1}\left(\frac{\phi}{\phi-1}\right)^{1-\phi}(1+\tau)^{1-\phi}+n_{t+1}^*\left(\frac{\phi}{\phi-1}E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}\right)^{-1}\end{array}\right)\right)\right]\right]$$

Pass through expectations and simplify

$$\frac{K\phi}{\beta\theta} = \left(\left(n_{t+1}+n_{t+1}^*\left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}(1+\tau)^{1-\phi}\right)^{-1}+\left(n_{t+1}+n_{t+1}^*\left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}(1+\tau)^{1-\phi}\right)^{-1}\right)$$

Do the same for the foreign entry condition:

$$\frac{K\phi}{\beta\theta} = \left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}\left(\left(n^*{}_{t+1}\left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}+n_{t+1}(1+\tau)^{1-\phi}\right)^{-1}+\left(n^*{}_{t+1}\left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}n_{t+1}(1+\tau)^{\phi-1}\right)^{-1}\right)$$

Rewrite the home and foreign conditions as fractions:

Home: $\frac{K\phi}{\beta\theta} = \frac{1}{n_{t+1}+An_{t+1}^*}+\frac{1}{n_{t+1}+Bn_{t+1}^*}$

Foreign: $\frac{K\phi}{\beta\theta} = \frac{A}{n_{t+1}+An^*{}_{t+1}}+\frac{B}{n_{t+1}+Bn^*{}_{t+1}}$

Where we define:

$$A \equiv \left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}(1+\tau)^{1-\phi}, \quad B \equiv \left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*{}_{t+1}}\right]\right)^{1-\phi}(1+\tau)^{\phi-1}$$

Equating across countries:

$$\frac{2n_{t+1}+(A+B)n_{t+1}^*}{(n_{t+1}+An_{t+1}^*)(n_{t+1}+Bn_{t+1}^*)} = \frac{(A+B)n_{t+1}+2ABn^*{}_{t+1}}{(n_{t+1}+An^*{}_{t+1})(n_{t+1}+Bn^*{}_{t+1})}$$

$$\frac{n_{t+1}}{n^*{}_{t+1}} = \frac{2AB-A-B}{2-A-B}$$

so $\frac{n_{t+1}}{n^*{}_{t+1}} > 1$ if $\frac{2AB-A-B}{2-A-B} > 1$

Note that the denominator will be negative provided the standard deviation of shocks is small relative to the iceberg costs, which will be true for all our cases:

$$\sigma < \left(\ln\left(2/\left((1+\tau)^{1-\phi}+(1+\tau)^{\phi-1}\right)\right)/\frac{1-\phi}{2}\right)^{0.5}$$

For shocks independently log normally distributed with standard deviation σ so that $E_t\left[\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right]=e^{\frac{1}{2}\sigma^2}$. For example, with τ=0.1 and ϕ =6, σ must be less than 0.209. Our calibration of σ is 0.017.

So $\frac{n_{t+1}}{n^*_{t+1}}>1$ if $2AB-A-B<2-A-B$ or $AB<1$

$$AB=\left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right]\right)^{1-\phi}(1+\tau)^{1-\phi}\left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right]\right)^{1-\phi}(1+\tau)^{\phi-1}=\left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right]\right)^{2(1-\phi)}$$

For independent log normal distributions of productivity:

$$\left(E_t\left[\frac{\alpha_{t+1}}{\alpha^*_{t+1}}\right]\right)^{2(1-\phi)}=e^{(1-\phi)\sigma^2}<1 \text{ since } \phi>1$$

We can conclude that $n_t>n_t$*.

5. Local currency pricing (LCP) model specification

Under the specification that prices for domestic sales, $p_t(h)$, and exports, $p^*_t(h)$, are set separately in the currencies of the buyers, the Rotemberg price setting equations for our model become:

$$p_t(h)=\frac{\phi}{\phi-1}\frac{W_t}{\alpha_t}+\frac{\kappa}{2}\left(\frac{p_t(h)}{p_{t-1}(h)}-1\right)^2 p_t(h)-\kappa\frac{1}{\phi-1}\left(\frac{p_t(h)}{p_{t-1}(h)}-1\right)\frac{p_t(h)^2}{p_{t-1}(h)}$$
$$+\frac{\beta\kappa}{\phi-1}E_t\left[\frac{\mu_t}{\mu_{t+1}}\frac{\Omega_{H,t+1}}{\Omega_{H,t}}\left(\frac{p_{t+1}(h)}{p_t(h)}-1\right)\frac{p_{t+1}(h)^{2-\phi}}{p_t(h)^{1-\phi}}\right]$$

and

$$p^*_t(h)=\frac{\phi}{\phi-1}\frac{W_t(1+\tau_t)}{\alpha_t e_t}+\frac{\kappa(1+\tau_t)}{2}\left(\frac{p^*_t(h)}{p^*_{t-1}(h)}-1\right)^2 p^*_t(h)-\frac{1}{\phi-1}\kappa(1+\tau_t)\left(\frac{p^*_t(h)}{p^*_{t-1}(h)}-1\right)\frac{p^*_t(h)^2}{p^*_{t-1}(h)}$$
$$+\beta\frac{\kappa}{\phi-1}E_t\left[\frac{\mu_t}{\mu_{t+1}}\frac{\Omega^*_{H,t+1}}{\Omega^*_{H,t}}\left((1+\tau_{t+1})\left(\frac{p^*_{t+1}(h)}{p^*_t(h)}-1\right)\frac{e_{t+1}}{e_t}\frac{p^*_{t+1}(h)^{2-\phi}}{p^*_t(h)^{1-\phi}}\right)\right]$$

where $\Omega_{H,s}=\left(\frac{p_s(h)}{P_{D,s}}\right)^{-\phi}\left(C_{D,s}+G_s+ne_s(1-\theta_K)K_s+AC_{P,D,s}+AC_{B,D,s}\right)\frac{1}{\mu_s}$, and

$$\Omega^*_{H,s}=\left(\frac{(1+\tau_D)p_s(h)}{e_s P^*_{D,s}}\right)^{-\phi}(1+\tau_D)\left(C^*_{D,s}+G^*_s+ne^*_s(1-\theta_K)K^*_s+AC^*_{P,D,s}+AC^*_{B,D,s}\right)\frac{1}{\mu_s}.$$

6 . Additional sensitivity analysis

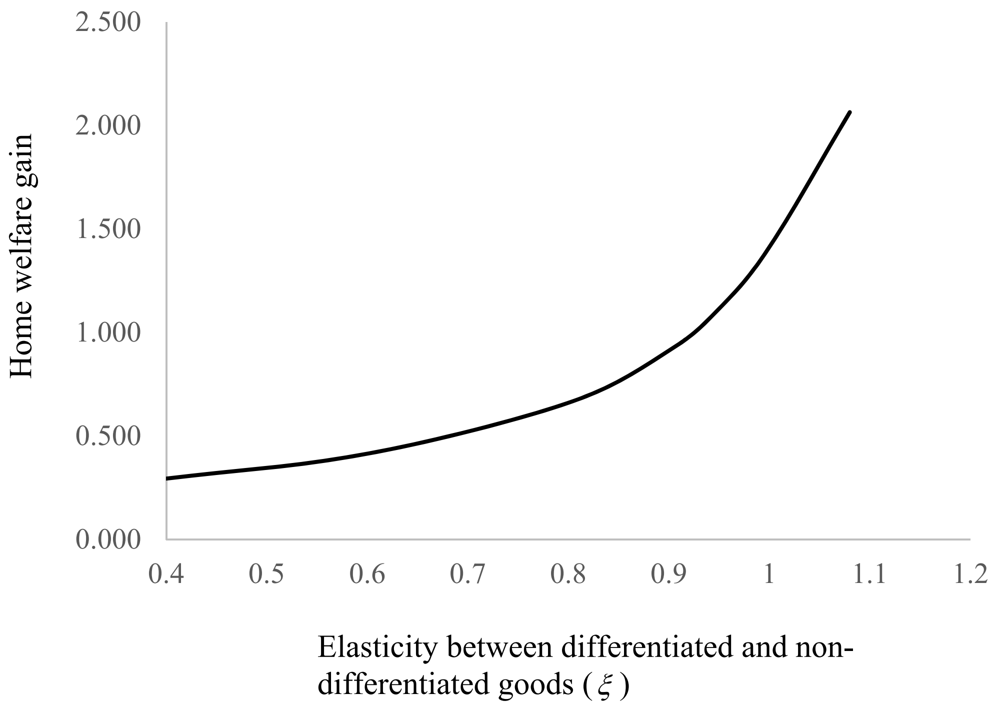
6.1. Elasticity between differentiated and non-differentiated goods

The benchmark model implies a unitary elasticity between differentiated and non-differentiated goods. We can generalize the aggregator to a CES specification, with elasticity ξ :

$$C_t \equiv \left(\theta^{\frac{1}{\xi}} C_{D,t}^{\frac{\xi-1}{\xi}} + \left(1-\theta\right)^{\frac{1}{\xi}} C_{N,t}^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}} .$$

Figure A1 below shows the effect of alternative assumptions about the elasticity ξ on home welfare gain when the foreign country pegs and home targets inflation, relative to the Ramsey solution. The home welfare gain is reduced as the two goods become more complementary, and it rises as they become more substitutable, although the range is limited where Ramsey can be solved numerically in the latter case.

Figure A1: Effect of elasticity of substitution between sectors on the home welfare from foreign peg



Home welfare gain is percentage difference from Ramsey policy welfare, in consumption units.

6.2. Endogenous tradedness of goods

The benchmark model makes the standard assumption in the trade literature on production relocation, that all differentiated goods are traded, and the relevant entry decision is whether a potential entrant should pay the sunk cost of firm creation. We consider here an alternative model where the entry decision instead is whether to export, where those firms that do not export continue to produce for just the domestic market as nontraded varieties.

The new model assumes a fixed unit mass of differentiated goods producers in each country, and n_t becomes the fraction of domestic firms that choose to become exporters. For those firms that choose to be nonexporters, the sales abroad for their varieties are set to zero ($d_t^*(h)$, defined from the counterpart of equation (22) in the text). Firm profits and firm valuations are defined accordingly. For exporters, the specifications of demand for their exports, profits, and firm valuations are the same as in the benchmark model. Firms choose to be an exporter when the firm value of being an exporter minus that of being a nonexporter equals the sunk export entry cost. The sunk cost is calibrated to imply the same ratio of exports to GDP as in the benchmark model (implying $\bar{K} = 0.126$). This implies that 29% of domestic firms choose to become exporters, which is a standard value in the literature.

Simulations in Appendix Table A1 indicate that the production relocation effect is very small, and there is only a small welfare gain for the home country that stabilizes inflation when the foreign country pegs. The main effect of the foreign peg is that both countries lose firms and welfare compared to the Ramsey policy. The reason is that if tradability is endogenous but not the location of production, then the production relocation effect cannot have its full effect. The scope for comparative advantage to shape domestic production is very limited if domestic firms are not forced to leave the market. It is possible that the effects of production relocation might be restored if there were also a sunk cost of domestic entry as well as exporting. However, two simultaneous sunk costs would greatly multiply the complexity of solution, as firms might pay the sunk cost of domestic firm creation in order to secure the option of future export entry under particular realizations of shocks. This option value problem would require different solution methods.

Table A1. Models with nontraded goods

	(1) Endogenous traded margin	(2) Nontraded sector
Welfare:		
Home	-0.290	0.856
Foreign	-0.591	-1.179
Total	-0.440	-0.165
Diff. goods export share:		
Home	-7.678	4.478
Foreign	-7.822	-4.643

6.3. Exogenously nontraded goods

Even if tradedness is not endogenous, the presence of nontraded goods could limit the relocation mechanism driving our result by reducing the scope for comparative advantage. We propose another variant of the model where half of the differentiated goods varieties are defined as nontradable. In this model, the nontradable and tradable sectors both consist of differentiated goods producers, but each subsector is handled independently. There is a mass of n_x differentiated goods firms that both export and sell domestically, and there is mass n_d domestic firms that sell only to the domestic market. The tradable firms face a sunk cost entry decision identical to that in the benchmark model. The nontraded firms are assumed to be of a constant mass and do not face an entry decision, but their number is calibrated as half of the number of firms in the benchmark model (n_d = 0.2). This restriction was required by the fact that both sectors face the same demands for their varieties in the home market, since they face the same marginal costs and price stickiness. If they were subject to the same sunk entry cost, then there is no solution that supports both an endogenous number of domestic firms and export firms, where the firm value of the latter is necessarily greater than the former. We adopt the local currency pricing specification of price stickiness discussed in the text, as this allows us to model a single set of prices for both sets of firms when selling domestically.

This model is calibrated with the same sunk entry cost as in the benchmark model. The steady state shows that approximately half the differentiated goods varieties are not traded, and half of domestic consumption of differentiated goods is of nontraded varieties. But the smaller number of differentiated goods varieties export a proportionately larger quantity of output, so that the share of exports in overall GDP is the same as in the benchmark model.

Results in appendix Table A1 indicate that the magnitude of production relocation is reduced compared to the benchmark model, but it still remains substantial. The foreign peg still shifts production of differentiated goods from foreign to home, raising home welfare and lowering foreign welfare relative to the symmetric Ramsey solution. The magnitude of these asymmetric effects on welfare are slightly more than half of the magnitudes under the benchmark model. This lower magnitude reflects the smaller share of tradable differentiated goods in the consumption bundle in this version of the model.

6.4. Investment in physical capital

In this version of the model, we introduce investment in physical capital, to investigate whether standard capital accumulation can replace the sunk entry cost of firm entry in generating the production relocation effect. In this version of the model firm entry is suspended and the number of firms in each country is fixed.

Consumers invest in new capital subject to quadratic adjustment costs. They earn a competitive rate of return, r_t, while capital depreciates at rate δ. The household budget constraint becomes:

$$P_t C_t + (M_t - M_{t-1}) + (B_{H,t} - B_{H,t-1}) + e_t(B_{F,t} - B_{F,t-1}) =$$
$$W_t l_t + \Pi_t + i_{t-1}B_{H,t-1} + i^{\star}_{t-1}B_{F,t-1} - P_t AC_{B,t} - T_t + r_t K_{t-1} - I_t - AC_{K,t}.$$

Adjustment costs, $AC_{K,t}$, are quadratic while investment follows the standard definition:

$$AC_{K,t} = \frac{\psi_k}{2}\frac{(K_t - K_{t-1})^2}{K_{t-1}},$$
$$I_t = K_t - K_{t-1}(1-\delta).$$

The consumer's first order condition for capital is:

$$\beta E_t\left(\frac{\mu_t}{\mu_{t+1}}\left[r_{t+1} + 1 - \delta + \psi_k\left(\frac{(\Delta K_{t+1})^2}{2} + \Delta K_{t+1}\right)\right]\right) = 1 + \psi_k \Delta K_t,$$

where $\Delta K_t = (K_t - K_{t-1})/K_{t-1}$ and μ_t is the inverse of the nominal marginal utility.

The firm problem is different in two ways. First, the firm minimizes cost with capital as a new input. Second, we drop the entry condition when the firm chooses prices. Output becomes a function of capital, and marginal costs are similar to before but now incorporate payments to capital:

$$y_t(h) = \left[(G_t(h))^{1-\zeta}(l_t(h))^{\zeta}\right]^{1-\gamma}[K_t(h)]^{\gamma},$$
$$mc_t = \frac{(r_t)^{\gamma}(W_t)^{(1-\gamma)(\zeta)}(P_{d,t})^{(1-\gamma)(1-\zeta)}}{\alpha_{d,t}(\gamma)^{\gamma}((1-\gamma)(\zeta))^{(1-\gamma)(\zeta)}((1-\gamma)(1-\zeta))^{(1-\gamma)(1-\zeta)}},$$
$$r_t K_{t-1}(h) = W_t l_t(h)\frac{\gamma}{(1-\zeta)(1-\gamma)},$$

where the last equation comes from cost minimization. Investment is funded from differentiated goods so that the new market clearing condition in the home country for the individual firm is:

$$d_t(h) = c_t(h) + d_{G,t}(h) + d_{AC,P,t}(h) + d_{AC,B,t}(h) + d_{K,t}(h) + d_{AC,K,t}(h).$$

The difference here are the last two terms, $d_{K,t}(h)$ and $d_{AC,K,t}(h)$, which are demand for new investment goods and demand for the differentiated goods to cover adjustment costs. These are respectively:

$$d_{K,t}(h) = \left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi} I_t,$$

$$d_{AC,K,t}(h) = \left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi} AC_{K,t}.$$

From the firm's optimization problem, we can now update the expression for Ω_t from the text so that the stochastic discount factor for the firm becomes

$$\Omega_t = \left[\left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi} \left(C_{D,t} + G_t + ne_t(1-\theta_k)K_t + AC_{P,D,t} + AC_{B,D,t} + AC_{K,t} + I_t\right)\right]/\mu_t$$
$$+ \left[\left(\frac{p_t(h)(1+\tau_D)}{e_t P^\star_{D,t}}\right)^{-\phi} \left(C^\star_{D,t} + G^\star_t + ne^\star_t(1-\theta_k)K^\star_t + AC^\star_{P,D,t} + AC^\star_{B,D,t} + AC^\star_{K,t} + I^\star_t\right)\right]/\mu_t.$$

The number of firms, n_t, is now fixed so that $n_t = n^\star_t = 0.4$. We then set new entry to zero. Simulations use standard values for the new parameters: $\psi_k = 0.05$, $\delta = 0.06$, $\gamma = 0.3$.

Simulation results indicate that this model does not generate a large production relocation effect. Assuming policies where the foreign country pegs the exchange rate while the home country fully stabilizes differentiated goods producer price inflation, the home share of differentiated goods in exports rises only 0.039 percentage points, and the foreign share falls just 0.005 percentage points, relative to a case where both countries fully target differentiated goods inflation. These values work in the same direction as the results from the benchmark model simulation, but they are two orders of magnitude smaller. This result serves simply to reiterate the claim in the main text that the large production reallocation effect in the benchmark model depends crucially upon endogenous firm entry in the differentiated goods sector, in order to facilitate a large production reallocation of sectors between countries.

6.5. Calvo price stickiness

Under Calvo pricing, demand for the differentiated goods, $d_t(h)$, must satisfy:

$$d_t(h) = c_t(h) + d_{G,t}(h) + d_{AC,B,t}(h) + d_{K,t}(h).$$

Using the definitions for each of the components, we arrive at

$$d_t(h) = \left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi} \Delta_t,$$

where $\Delta_t = C_{D,t} + G_t + AC_{B,D,t} + AC_{K,t} + ne_t(1-\theta_k)K_t$. The foreign country has $\Delta^\star_t = C^\star_{D,t} + G^\star_t + AC^\star_{B,D,t} + AC^\star_{K,t} + ne^\star_t(1-\theta_k)K^\star_t$. Total output of variety h is then $y_t(h) = d_h + d^\star_t(h)(1+\tau_D)$ so that we can write this as:

$$y_t(h) = \left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi} \left(\Delta_t + \Delta^\star_t(1+\tau_D)^{1-\psi}\left(\frac{P_{D,t}}{e_t P^\star_{D,t}}\right)^{-\phi}\right).$$

From here onward, we let $\overline{\Delta}_t$ be the second term on the right in parenthesis, so that

$$\overline{\Delta}_t = \left(\Delta_t + \Delta_t^\star (1+\tau_D)^{1-\phi} \left(\frac{P_{D,t}}{e_t P_{D,t}^\star}\right)^{-\phi}\right).$$

Using this demand function in the optimization problem for the firm, allowing share $1-\rho$ of firms to adjust price each period, we arrive at the price chosen by any firm in time t:

$$p_t^\# = \frac{\phi}{\phi-1} \frac{E_t\left\{\sum_{s=0}^{\infty} (\rho\beta)^s mc_{t+s} \widetilde{\Omega}_{t+s} P_{D,t+s}^{\phi}\right\}}{E_t\left\{\sum_{s=0}^{\infty} (\rho\beta)^s \widetilde{\Omega}_{t+s} P_{D,t+s}^{\phi}\right\}},$$

and the term $\widetilde{\Omega}_{t+1}$ is defined as

$$\widetilde{\Omega}_{t+s} = \frac{\mu_t}{\mu_{t+s}} \overline{\Delta}_{t+1}.$$

Because share ρ of firms are locked into the price they set today, and share $1-\rho$ is able to readjust and set prices at $p_t^\#$, aggregating across all firms we arrive at the average price for domestically sold differentiated goods, $\tilde{p}_t^h$:

$$\left(\tilde{p}_t^h\right)^{1-\phi} = (1-\rho)(p_t^\#)^{1-\phi} + \rho\left(\tilde{p}_{t-1}^h\right)^{1-\phi}.$$

Abroad, the foreign country has a similar condition:

$$\left(\tilde{p}_t^{f,\star}\right)^{1-\phi} = (1-\rho)\left(p_t^{\#,\star}\right)^{1-\phi} + \rho\left(\tilde{p}_{t-1}^{f,\star}\right)^{1-\phi}.$$

Using the definition for the domestic price of the foreign differentiated good,

$$\tilde{p}_t^f = e_t(1+\tau_D)\tilde{p}_t^{f,\star}.$$

Using the price together with the domestic price, we arrive at the price index for domestic and foreign differentiated goods:

$$P_{D,t} = \left(n_t\left(\tilde{p}_t^h\right)^{1-\phi} + n_t^\star\left(\tilde{p}_t^f\right)^{1-\phi}\right)^{\frac{1}{1-\phi}}.$$

To compute the price dispersion, v_p, we set demand equal to supply and integrate across all varieties:

$$\alpha_{D,t} \int_0^{n_t} (G_t(h))^{1-\zeta} (l_t(h))^{\zeta} dh = \overline{\Delta}_t \int_0^{n_t} \left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi} dh.$$

Since technology is identical across firms and returns to scale are constant, this yields:

$$\alpha_{D,t}\left(G_t^{\zeta}\right)\left(l_{D,t}^{1-\zeta}\right) = n_{t-1} v_{p,t} \overline{\Delta}_t,$$

where $v_{p,t}$ is the degree of price dispersion and is equal to: $v_{p,t} = \int_0^1 \left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi} dh.$

Integrating, we can write this in terms of $\pi_{D,t}$ and $\pi_{D,t}^\#$, which are defined respectively as $\pi_{D,t} = P_{D,t}/P_{D,t-1}$ and $\pi_{D,t}^\# = p_t^\#/P_{D,t-1}$. The price dispersion is

$$v_{p,t} = (1-\rho)\left(\frac{\pi_{D,t}}{\pi_{D,t}^\#}\right)^{\phi} + \rho \pi_{D,t}^{\phi} v_{p,t-1}.$$

Using this expression, we now replace the variety-specific demands (differentiated by h) with average demands across varieties. To arrive at the average demand across varieties for the various uses of the differentiated good, we simply integrate with respect to h and divide by the number of firms. For example, defining the average consumption of differentiated goods as $\tilde{c}_t$,

$$\tilde{c}_t = \frac{1}{n_t}\int_0^{n_t} c_t(h)dh = \frac{1}{n_t}\int_0^{n_t}\left(\frac{p_t(h)}{P_{D,t}}\right)^{-\phi} C_{D,t}dh = v_{p,t}C_{D,t}.$$

Doing the same to demand across all uses for differentiated goods, i.e. $d_{G,t}(h)$, $d_{AC,B,t}(h)$, and $d_{K,t}(h)$, the average demands are,

$$\tilde{d}_{G,t} = v_{p,t}G_{K,t}$$
$$\tilde{d}_{AC,B,t} = v_{p,t}AC_{B,t}$$
$$\tilde{d}_{K,t} = v_{p,t}ne_t(1-\theta_k)K_t.$$

We use these expressions to replace demand for variety h with average demand across all varieties. This change has no material impact on the steady state or even the entry condition for firms into the differentiated goods sector, as we assume that firms choose to enter or not before they learn if they are able to set prices for that period. In experiments we set parameter $\rho = 0.5$.

Simulation results indicate that this model produces results very similar to the benchmark model with Rotemberg pricing, if we retain the feature of free entry of firms into the differentiated goods sector. Assuming policies where the foreign country pegs the exchange rate while the home country fully stabilizes differentiated goods producer price inflation, the home share of differentiated goods in exports rises by 3.33 percentage points, and the foreign share falls a similar 3.41 percentage points, relative to a case where both countries fully target differentiated goods inflation. This production relocation is facilitated by a shift in the location of firms, with a rise in the number of home firms by 6.26 percent, and fall in the number of foreign firms by 5.12 percent.

When firm entry is eliminated from the model and the number of firms is exogenously fixed, the production relocation effects becomes very small. A foreign peg raises the home share of differentiated goods by just 0.018 percentage points and lowers foreign share by 0.038 percentage points. These values have the same sign as the benchmark model, but the values are two orders of magnitude smaller. Again, this reiterates the point that the production relocation effect depends crucially upon endogenous firm entry.