B Online Appendix: Empirical Application

B.1 Representative household utility in model of Section 5.1

Labor Supply. Assume that the representative household preferences have the following nested structure, if $\sum_{kr} L_{kr} = 1$,

$$U_c = \left[\sum_d \nu_d \left(\sum_s (\nu_{sd} C_{sd}) (L_{sd})^{-\frac{1}{\phi_e}} \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1}} \right]^{\frac{\phi_m}{\phi_m - 1}}$$

and $U_c = -\infty$ whenever $\sum_{kr} L_{kr} \neq 1$.

In sector s of region d, the real consumption is $C_{sd} = \omega_{sd} L_{sd}$. Thus, the representative household solves the following second-stage problem:

$$\max_{\{L_{sd}\}_{sd}} \left[\sum_d \nu_d \left((L_{Hd})^{\frac{\phi_e - 1}{\phi_e}} + \sum_s (\nu_{sd} \omega_{sd}) (L_{sd})^{\frac{\phi_e - 1}{\phi_e}} \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1}} \right]^{\frac{\phi_m}{\phi_m - 1}} \quad \text{s.t.} \quad \sum_{kr} L_{kr} = 1.$$

Let μ be Lagrange multiplier of the constraint. The first-order condition for L_{sd} is

$$\kappa_c \tilde{w}_d (L_{Hd})^{-\frac{1}{\phi_e}} = \mu$$

$$\kappa_c \tilde{w}_d (\nu_{sd} \omega_{sd}) (L_{sd})^{-\frac{1}{\phi_e}} = \mu$$

where

$$\kappa_c \equiv \left[\sum_d \nu_d \left((L_{Hd})^{\frac{\phi_e - 1}{\phi_e}} + \sum_s (\nu_{sd} \omega_{sd}) (L_{sd})^{\frac{\phi_e - 1}{\phi_e}} \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1}} \right]^{\frac{\phi_m}{\phi_m - 1} - 1}$$

$$\tilde{w}_d \equiv \nu_d \left((L_{Hd})^{\frac{\phi_e - 1}{\phi_e}} + \sum_s (\nu_{sd} \omega_{sd}) (L_{sd})^{\frac{\phi_e - 1}{\phi_e}} \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1} - 1}.$$

Thus,

$$\frac{\bar{L}_{sd}}{\bar{L}_{Hd}} = (\nu_{sd} \omega_{sd})^{\phi_e}$$

and

$$\bar{L}_{Hd} = \frac{L_{Hd}}{L_{Hd} + \sum_s L_{sd}} = \frac{1}{1 + \sum_s (\nu_{sd} \omega_{sd})^{\phi_e}}.$$

The first-order condition for L_{sd} also implies that

$$\frac{\tilde{w}_d}{\tilde{w}_0} = \left(\frac{L_{Hd}}{L_{H0}} \right)^{\frac{1}{\phi_e}}$$

such that

$$\tilde{w}_d = \nu_d \left((L_{Hd})^{-\frac{1}{\phi_e}} \left(L_{Hd} + \sum_s L_{sd} \right) \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1} - 1}.$$

Defining $\tilde{L}_d \equiv L_{Hd} + \sum_s L_{sd}$,

$$\frac{\nu_d}{\nu_0} \left(\left(\frac{L_{Hd}}{L_{H0}} \right)^{-\frac{1}{\phi_e}} \left(\frac{\tilde{L}_d}{\tilde{L}_o} \right) \right)^{-\frac{\phi_e - \phi_m}{\phi_m(\phi_e - 1)}} = \left(\frac{L_{Hd}}{L_{H0}} \right)^{\frac{1}{\phi_e}}$$

and finally

$$\frac{\tilde{L}_d}{\tilde{L}_o} = \left(\frac{\nu_d}{\nu_0} \right)^{\phi_m} \left(\frac{\bar{L}_{Hd}}{\bar{L}_{H0}} \right)^{-\frac{\phi_m - 1}{\phi_e - 1}}. \tag{58}$$

B.2 Shift-Share representation of regional shock exposure in model of Section 5.1

Consider our parametric model of Section 5.1. To obtain a shift-share representation, we simplify the model by imposing Cobb-Douglas preferences between manufacturing and non-manufacturing. That is, for simplicity, we assume that $\chi = 0$. In this case, the revenue exposure is the impact of the shock on a market's revenue holding constant wages and employment everywhere:

$$\log \hat{\eta}^R_{kr}(\hat{\boldsymbol{\tau}}) \equiv \sum_{n,o,sd} \frac{\partial \log Y^0_{kr}}{\partial \log \tau^t_{n,ko,sd}} d \log \tau^t_{n,ko,sd}$$

$$= \sum_{n,o,sd} \frac{x^0_{kr,sd} \left(w^0_{sd} L^0_{sd} \right)}{Y^0_{kr}} \frac{\partial \log x^0_{kr,sd}}{\partial \log \tau^t_{n,ko,sd}} d \log \tau^t_{n,ko,sd}$$

By combining this expression with equation (35),

$$\frac{\partial \log x^0_{kr,sd}}{\partial \log \tau^t_{n,ko,sd}} = \frac{x^0_{n,kr,sd}}{x^0_{kr,sd}} \frac{\partial \log x^0_{n,kr,sd}}{\partial \log \tau^t_{n,ko,sd}} = -\bar{\chi}_n \frac{x^0_{n,kr,sd}}{x^0_{kr,sd}} \left(1[r = o] - \bar{x}_{n,ko,sd} \right)$$

where $\bar{x}_{n,ko,sd}$ is the share of spending on goods from region r in industry n by sd.

Let$X^t_{n,kr,sd} \equiv x^t_{n,kr,sd} \left(w^t_{sd} L^t_{sd} \right)$ be the total sales of industry n of market kr to region sd. So,

$$\log \hat{\eta}^R_{kr}(\hat{\boldsymbol{\tau}}) \equiv - \sum_{n,o,sd} \frac{X^0_{n,kr,sd}}{Y^0_{kr}} \left(1[r = o] - \bar{x}_{n,ko,sd} \right) \bar{\chi}_n d \log \tau^t_{n,ko,sd}.$$

As in Section 5.3, we consider a shock to the productivity of a foreign country $\bar{o}$ such that, for all destination sd, $d\log\tau^t_{n,kr,sd} = 0$ if $r \neq \bar{o}$ and $d\log\tau^t_{n,k\bar{o},sd} = d\log\tau^t_n$. Thus,

$$\log\hat{\eta}^R_{kr}(\hat{\boldsymbol{\tau}}) \equiv \sum_n \left(\sum_{sd} \frac{X^0_{n,kr,sd}}{Y^0_{kr}} \bar{x}_{n,k\bar{o},sd}\right) \bar{\chi}_n d\log\tau^t_n.$$

This expression clearly outlines that, in our empirical application, the revenue exposure has a shift-share structure where the industry shock is $\bar{\chi}_n d\log\tau^t_n$ and industry-market exposure is $\sum_{sd}(X^0_{n,kr,sd}\bar{x}_{n,k\bar{o},sd})/Y^0_{kr}$. This shift-share expression entails two adjustments. First, our model implies that the magnitude of the industry-level shock must be adjusted by the the industry's trade elasticity. Intuitively, conditional on the same exogenous productivity change, the demand response is larger in industries with a higher demand elasticity. Second, the industry-region exposure adjusts the share of industry n in market kr revenue by the importance of country $\bar{o}$ across destination markets sd. Because of the gravity-trade structure, the demand response in market sd is proportional to the initial spending share of that market on goods from $\bar{o}$.

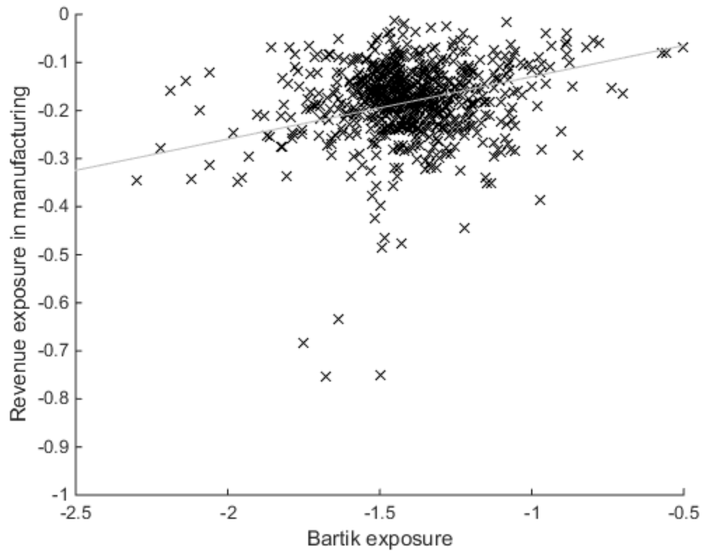
Whenever these two sources of heterogeneity are shut down, the revenue exposure is proportional to a shift-share specification based on industry-region employment shares and the industry shocks. To see this, assume that all destination markets are have identical industry-level spending share on country $\bar{o}$ (i.e., $\bar{x}_{n,k\bar{o},sd} = \bar{x}_{k\bar{o}}$ for all sd and n), and the trade elasticity is identical in all industries ($\bar{\chi}_n = \bar{\chi}$). In this special case,

$$\log\hat{\eta}^R_{kr}(\hat{\boldsymbol{\tau}}) = (\bar{\chi}\bar{x}_{k\bar{o}}) \sum_n l^0_{n,kr}\left(d\log\tau^t_n\right),$$

which $l^0_{n,kr}$ is the share of industry n in the total employment of sector k of region r in the initial equilibrium.

To evaluate the importance of these adjustments in practice, Figure 4 reports the relation between the revenue exposure in manufacturing in our baseline empirical model and the shift-share exposure measure, $\sum_n l^{1997}_{n,kr}\hat{\delta}_{n,China}$. The two measures have a correlation of 0.3. This indicates that they rely on different sources of cross-regional variation.

Figure 4: Manufacturing revenue exposure and shift-share exposure



Notes: Scatter plot of the revenue exposure in manufacturing in the baseline empirical model against the shift-share exposure measure. A least-square best fit line is reported.

B.3 Additional Results: Empirical Specification

B.3.1 Chinese export growth shock

In this section, we present our measure of the shock to Chinese exports between 1997 and 2007. Table 5 presents the list of industries in our sample, along with the calibrated trade elasticity and the various sources of industry-level Chinese cost shock. As explained in the main text, we obtain the estimates of the trade elasticity from Caliendo and Parro (2014). The adjusted export shock is our baseline shock divided by the trade elasticity, $\hat{\delta}_{n,China}/\bar{\chi}_n$. To obtain the inverted bilateral trade cost, we implement the procedure in Head and Ries (2001) for China and each CZ r (that is, $\hat{\tau}_{n,rC} = \hat{\tau}_{n,Cr} = (\hat{x}_{n,rC}\hat{x}_{n,Cr}/\hat{x}_{n,rr}\hat{x}_{n,CC})^{-1/\bar{\chi}_n}$. For the NTR gap, we use the data in Pierce and Schott (2016a) to compute the change in the trade cost between each CZ and China by taking the simple average of the NTR Gaps among the HS6 goods in the corresponding SCTG. Finally, we computed the firm-level productivity growth in 1997-2007 using the unadjusted annual measured productivity growth in column (3) of Table 6 in Hsieh and Ossa (2016).

There are two striking features in the table. First, there is great cross-industry variation in the magnitude of the cost shock, which we exploit in estimation. Second, the different measures of industry-level shocks are only imperfectly correlated, providing us with different sources of variation for estimation.

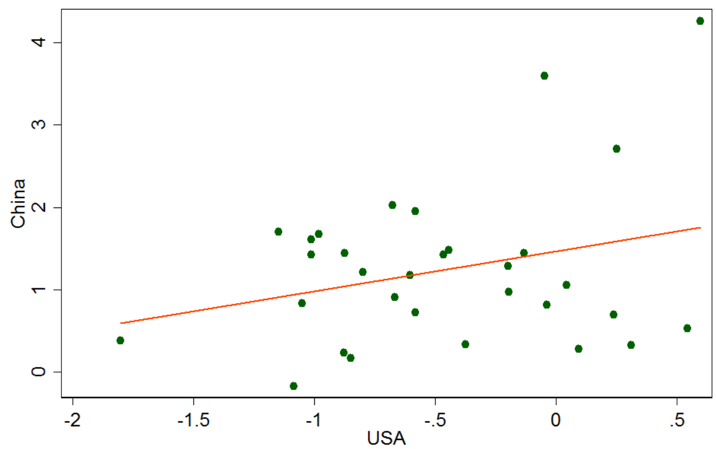
Before proceeding, Figure 5 investigates the cross-industry correlation between the exporter fixed-effects of China and the US. To this end, we obtain $\hat{\delta}_{n,US}$ by estimating equation (45) with the US in the sample. The figure presents a scatter plot of $\hat{\delta}_{n,China}$ and $\hat{\delta}_{n,US}$ for the 31 manufacturing industries in our sample. We can see that they have a weak positive correlation.

Table 5: Industries: Parameters and Shocks

Industry	SCTG	Trade Elast.	Chinese cost shock: Export (baseline)	Export (adj)	Inverted cost	NTR Gap	Prod. Hsieh-Ossa
Animals, cereals	1-2	8.59	0.33	0.04	-0.18	0.04	1.06
Other agriculture	3	8.59	0.23	0.03	-0.04	0.11	1.06
Animal origin goods	4	8.59	0.17	0.02	-0.09	0.07	1.06
Meat, fish, seafood	5	8.59	0.90	0.11	-0.13	0.09	1.06
Grain products	6	2.83	0.83	0.29	-0.31	0.10	1.16
Other prepared food	7	2.83	0.72	0.26	-0.17	0.13	1.16
Alcoholic beverages	8	2.83	-0.17	-0.06	-0.10	0.34	1.16
Tobacco products	9	8.59	0.38	0.04	-0.03	0.22	1.16
Mining	10	14.83	0.28	0.02	-0.02	0.12	1.06
Oil products	15-19	69.31	0.52	0.01	0.02	0.04	0.72
Basic chemicals	20	3.64	1.05	0.29	-0.16	0.14	1.29
Pharmaceutical	21	3.64	0.70	0.19	-0.18	0.17	1.29
Fertilizers	22	3.64	4.26	1.17	-0.28	0.00	1.29
Chemical products	23	3.64	1.28	0.35	-0.29	0.21	1.29
Plastics and rubber	24	0.88	1.17	1.33	-0.95	0.29	0.92
Logs and other wood	25	10.19	0.33	0.03	-0.08	0.00	1.02
Wood products	26	10.19	1.61	0.16	-0.13	0.21	1.02
Pulp, paper	27	8.32	3.60	0.43	-0.21	0.20	0.89
Paper articles	28	8.32	1.70	0.20	-0.10	0.29	0.89
Printed products	29	8.32	1.48	0.18	-0.18	0.14	0.89
Textiles and leather	30	5.99	1.22	0.20	-0.16	0.42	0.65
Nonmetallic mineral	31	3.38	1.45	0.43	-0.16	0.32	1.13
Base metals	32	6.58	2.71	0.41	-0.14	0.17	1.17
Articles of base metal	33	5.03	1.43	0.28	-0.18	0.32	1.17
Machinery	34	2.87	1.95	0.68	-0.39	0.31	1.18
Electronic equip.	35	11.02	1.68	0.15	-0.09	0.32	1.23
Vehicles	36	0.49	1.44	2.94	-2.12	0.18	1.06
Transportation equip.	37	0.9	2.03	2.25	-0.88	0.25	1.06
Precision instruments	38	4.95	0.97	0.20	0.00	0.36	0.70
Furniture	39	4.95	1.42	0.29	-0.12	0.40	0.70
Miscellaneous	40-43	4.95	0.82	0.17	0.01	0.38	0.70
Services	NA	5	-	-	-	-	-
Median		5.02	1.11	0.20	-0.15	0.19	1.06
Average		7.89	1.20	0.41	-0.25	0.20	1.01
St. Dev.		11.51	0.96	0.64	0.40	0.12	0.26
Correl. w/ baseline		-0.17	1.00	0.42	-0.21	0.11	0.22

Notes: The inverted trade shocks are, for each industry, the average change across US CZs in the cost of imports from China.

Figure 5: Change in Exporter Fixed-Effect of China and US: 31 manufacturing industries, 1997-2007



Notes: Scatter plot of the estimated industry-level exporter fixed effect for China against the corresponding fixed effect for US. A least-square best fit line is reported.

B.3.2 Estimation of Structural Parameters: Robustness

This section investigates the robustness of the results reported in Table 1. In every specification, we compute the predicted changes in CZ-level outcomes using the first-step estimates of the structural parameters reported in Panel A of Table 1.

We start by reporting, in Table 6, the results obtained with alternative sets of controls. We can see that the additional controls do not affect significantly the estimates of ϕ_e and ψ reported in Panels B and C. However, the estimate of the migration elasticity is sensitive to the control set: as we sequentially include controls, the first-stage becomes weaker and the estimate more imprecise.

Table 7 reports the estimates of the structural parameters using the model's predicted response of labor market outcomes with alternative parameter estimates and shock sources. Column (2) shows that we obtain similar estimates when MOIV is computed with the second-step estimates reported in Panel B of Table 1. This suggests that there are small efficiency gains of moving beyond the two-step feasible implementation of the MOIV estimator, as indeed suggested by Proposition 5.

Columns (3)–(7) report estimates obtained with the alternative configuration of the industry-level shock described above. Relative to the baseline estimates, the estimated elasticities of labor supply remain similar but the agglomeration elasticity is can be larger. In fact, column (8) reports the results of estimation of the structural parameters with the predicted responses with all sources of cost shocks. The p-value of the over-identification test is low, which suggests that either the model is not well specified or the exogeneity restriction is not valid for one of the shocks.

Finally, Table 8 reports the estimated agglomeration elasticity under different parametrization of the function controlling how local productivity depends on employment of other regions. Specifically, column (2) reports the estimation of our model under the assumption of $\pi_{rr} = 1$

and $\pi_{rd} = 0$ for all $r \neq d$. In this case, productivity only depends on the own-market employment level, as in Krugman (1980) and Allen and Arkolakis (2014). The estimated parameter of 0.60 indicates strong local agglomeration forces. Alternatively, in column (3), we estimate the model under the assumption that the decay of π_{ij} on distance is 0.35 – the estimate reported in columns (3) of Table 5 of Ahlfeldt et al. (2015). In this case, the estimated parameter suggests even stronger productivity spillovers across markets.

Table 6: Structural Parameter Estimates: Alternative Control Set

	(1)	(2)	(3)	(4)
Panel A: $-\phi_m/\phi_e$				
	-0.395	0.320	-1.576	-0.262
S.E.	(0.467)	(0.462)	(2.311)	(1.724)
F Stat.	5.94	3.67	1.14	0.61
Panel B: $1/\phi_e$				
	0.917***	0.876***	0.823***	0.796***
S.E.	(0.121)	(0.098)	(0.107)	(0.089)
F Stat.	16.46	16.85	17.79	15.65
Panel C: ψ				
	0.635***	0.588***	0.506***	0.479**
S.E.	(0.177)	(0.161)	(0.195)	(0.204)
F Stat.	9.22	9.57	8.94	9.23
Sector composition controls:	No	Yes	No	Yes
Demographic controls:	No	No	Yes	Yes

Notes: Sample of 722 Commuting Zones and 2 Sectors in 1997-2007. Models are weighted by start of period CZ share of national population. Control sets defined in Table 2. Instrumental variable computed with First-Step estimates of Table 1A. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 7: Structural Parameter Estimates: Alternative Instrumental Variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: $-\phi_m/\phi_e$							
	-0.390	-0.300	-1.230	0.390	-0.82	-1.83	-0.33
S.E.	(0.467)	(0.374)	(0.936)	(0.35)	(0.91)	(1.63)	(0.45)
F Stat.	5.94	8.32	2.96	7.30	2.38	1.88	2.99
J-test (p-value)							0.003
Panel B: $1/\phi_e$							
	0.92***	0.90***	0.71***	1.18***	1.25***	1.41***	1.19***
S.E.	(0.121)	(0.122)	(0.117)	(0.09)	(0.19)	(0.24)	(0.11)
F Stat.	16.46	16.21	7.90	120.23	40.84	9.77	51.88
J-test (p-value)							0.052
Panel C: ψ							
	0.64***	0.63***	0.260	0.55***	1.12***	1.08***	0.70***
S.E.	(0.177)	(0.195)	(0.188)	(0.15)	(0.20)	(0.23)	(0.15)
F Stat.	9.22	8.81	5.02	76.75	30.12	15.37	37.95
J-test (p-value)							0.016
Cost Shock:							
Export (baseline)	Yes	Yes	No	No	No	No	Yes
Export (adjsuted)	No	No	Yes	No	No	No	Yes
Firm productivity	No	No	No	Yes	No	No	Yes
NTR Gap	No	No	No	No	Yes	No	Yes
Inverted trade cost	No	No	No	No	No	Yes	Yes
MOIV parameters:	1st	2nd	2nd	2nd	2nd	2nd	2nd

Notes: Sample of 722 Commuting Zones and 2 Sectors in 1997-2007. Models are weighted by start of period CZ share of national population. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 8: Structural Parameter Estimates: Alternative Agglomeration Specification

	(1)	(2)	(3)
	0.635***	0.604***	0.871***
S.E.	(0.177)	(0.169)	(0.270)
F Stat.	9.22	9.35	8.07
Distance decay:	$\delta = 1$	$\delta = \infty$	$\delta = 0.35$

Notes: Sample of 722 Commuting Zones and 2 Sectors in 1997-2007. Models are weighted by start of period CZ share of national population. Instrumental variable computed with estimates Table 1A. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

B.3.3 Model Fit: Robustness

This section investigates the differential responses in manufacturing employment to alternative measures of the CZ's exposure to Chinese import competition. Specifically, We present the estimation of equation (46) using alternative measures of the Chinese export shock and alternative shift-share exposure measures. All specifications include the full set of controls in column (4) of Table 2.

Table 9 investigates the cross-regional employment effects obtained with shift-share exposure measures. Column (1) replicates the results in Autor, Dorn, and Hanson (2013). To this end, the industry-level "shift" is the change in Chinese imports of other developed countries normalized by the 1990 employment in the US, $(X^{2007}_{n,China,j} - X^{1997}_{n,China,j})/L^{1990}_{n,US}$, and the "share" is the share of industry n in the CZ's total employment. Notice that Autor, Dorn, and Hanson (2013) multiply the log-change in manufacturing employment by 100. So, in order to compare our estimates to theirs, we need to multiply the estimated coefficient in Table 10 by 100. In this case, our estimated cross-regional effect is 6.5, which is similar to the estimated effect of 4.2 in Table 5 of Autor, Dorn, and Hanson (2013).

Column (2) reports the differential employment effect of a similar shift-share exposure where the "shift" is Chinese export shock $\{\hat{\delta}_{n,Chna}\}_n$ (as described in Section 5). The estimated coefficient indicates that CZs more exposed to the Chinese import competition experienced a statistically significant lower relative growth in manufacturing employment. Finally, column (3) reports the cross-regional impact of the shift-share measure where the "share" is the share of industry n in manufacturing employment. In this case, the point estimate is negative, but it is not statistically significant.

Column (1) replicates the baseline results of Table 2. In column (2), we adjust the Chinese export shock by the industry's trade elasticity: the cost shock is $z_n = \hat{\delta}_{n,China}/\bar{\chi}_n$ using the $\bar{\chi}_n$ reported in Table 5. Despite the fact that the average magnitude of this adjusted shock measure is 30% of the average baseline shock, the estimated coefficient in column (2) is only 50% higher than the coefficient in column (1). This indicates that the cross-regional variation in predicted changes in manufacturing employment is mainly driven by industries with a low trade elasticity – in fact, the cross-industry correlation between $\hat{\delta}_{n,China}$ and $\bar{\chi}_n$ is -0.2.

Column (3) shows that the estimated coefficient is higher when the cost shock measure is the firm-level productivity growth of Hsieh and Ossa (2016). This is partially driven by the lower cross-regional variation in exposure to the measured productivity shock due to its lower cross-industry variation.

Columns (4) and (5) show that the estimated coefficients are much higher when we consider the impact of removing NTR gaps and changes in bilateral trade costs. In all these cases, the cross-regional correlation between predicted employment responses to the baseline and the alternative cost shocks is above 0.4. So, the smaller effects of the trade cost shocks in columns (4) and (5) are partially capturing the larger impact of changes in Chinese productivity.

Table 9: Model Fit: Manufacturing Employment - Alternative Specifications

Dependent variable: Log-change in manufacturing employment, 1997-2007

	(1)	(2)	(3)
Shift-share exposure	-0.065***	-0.768***	-0.084
	(0.017)	(0.237)	(0.088)
R^2	0.270	0.234	0.268
Industry-level shock			
Baseline:	No	Yes	Yes
Normalized import change (ADH):	Yes	No	No
CZ's industry employment share in			
Total employment:	Yes	Yes	No
Manufacturing employment:	No	No	Yes

Notes: Sample of 722 Commuting Zones in 1997-2007. Models are weighted by start of period CZ share of national population. All specifications include the set of baseline controls in column (4) of Table 2. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 10: Model Fit: Manufacturing Employment - Alternative Industry-level Shocks

Dependent variable: Log-change in manufacturing employment, 1997-2007

	(1)	(2)	(3)	(4)	(5)
Predicted manuf. log-change in employment	6.84***	9.53**	12.99**	43.25***	50.69**
	(2.079)	(3.883)	(5.232)	(7.87)	(24.51)
R^2	0.27	0.23	0.24	0.34	0.27
Cost Shock:					
Export (baseline)	Yes	No	No	No	No
Export (adjsuted)	No	Yes	No	No	No
Firm productivity	No	No	Yes	No	No
NTR Gap	No	No	No	Yes	No
Inverted trade cost	No	No	No	No	Yes

Notes: Sample of 722 Commuting Zones in 1997-2007. Models are weighted by start of period CZ share of national population. All specifications include the set of baseline controls in column (4) of Table 2. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

B.4 Additional Results: Counterfactual Analysis

B.4.1 Reduced-form elasticities for real wages

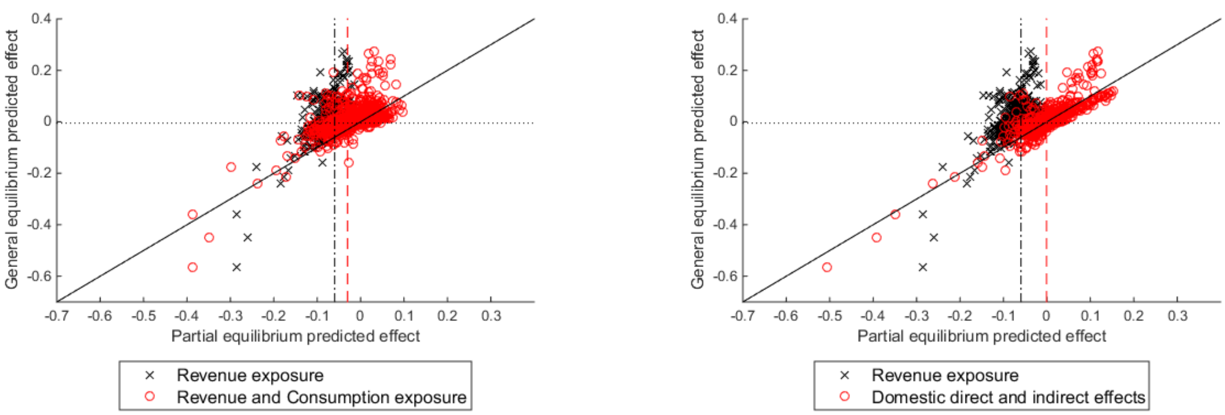
Table 11: Reduced-Form Elasticities of Employment to Local Manufacturing Shock Exposure

	Own region		Other regions	
	$\gamma_{Mr,Mr}$	$\gamma_{Nr,Mr}$	$\sum_{d\neq r} l_{Md}\gamma_{Md,Mr}$	$\sum_{d\neq r} \gamma_{Mr,Md}$
	(1)	(2)	(3)	(4)
Panel A: Employment elasticity to revenue exposure ($\gamma^R_{Mr,Md}$)				
Avg.	0.1741	-0.0082	0.0004	-0.1435
Panel B: Employment elasticity to consumption exposure ($\gamma^C_{Mr,Md}$)				
Avg.	1.2096	0.2313	0.0008	-0.3625

Notes: Average reduced-form elasticity computed using the estimates in Panel B of Table 1 and the observed equilibrium in 1997. M denotes the manufacturing sector and N denotes the non-manufacturing sector.

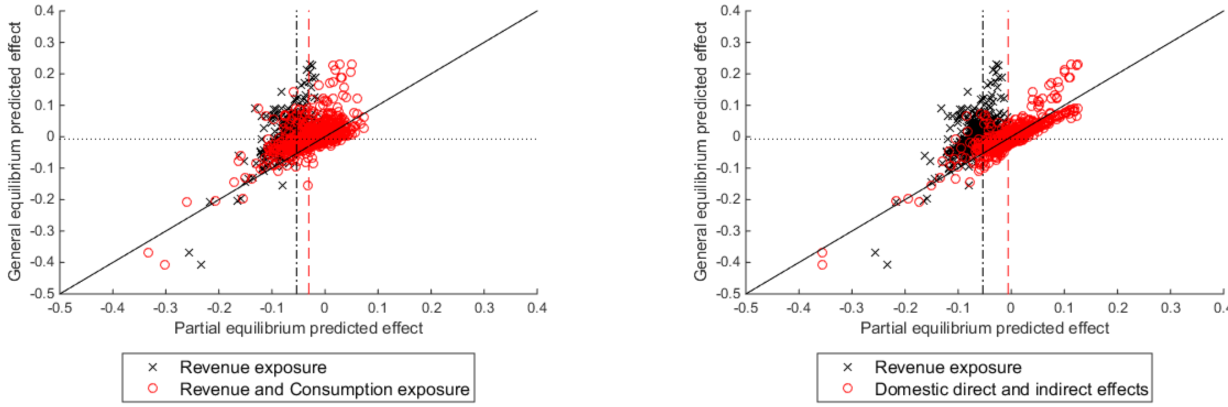
B.4.2 Robustness

Figure 6: Predicted Change in Manufacturing Real Wage, China shock



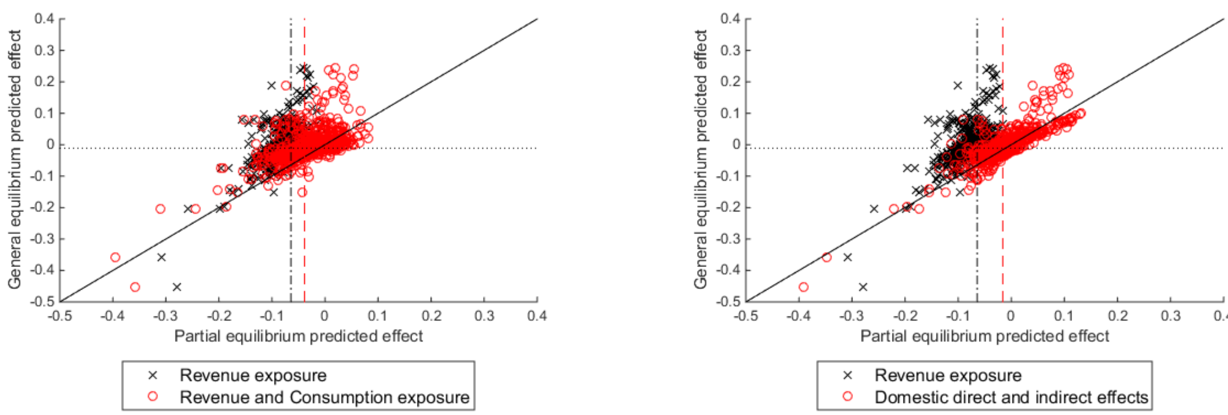
Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing real wage against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

Figure 7: Predicted Change in Manufacturing Employment, no labor links



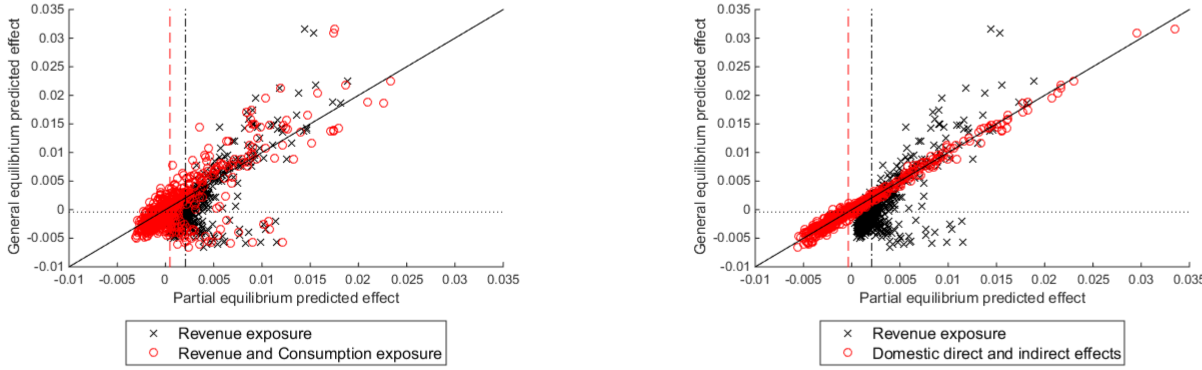
Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing employment in a model without agglomeration and migration across regions, against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

Figure 8: Predicted Change in Manufacturing Employment, no home sector



Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing employment in a model without the home sector, against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

Figure 9: Predicted Change in Manufacturing Employment, NAFTA shock



Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing employment after a NAFTA shock, against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

C Online Appendix: Data Construction

Our sample consists of 722 US commuting zones, 58 foreign countries plus Alaska and Hawaii. Table 12 lists the foreign countries. We divide the manufacturing sector in 31 industries, listed in Table 5.

Table 12: Sample of Countries

Argentina	Kazakhistan
Australia	South Korea
Austria	Mexico
Baltic Republics	Malaysia
Bulgaria	North Africa
Belarus	Netherlands
Benelux	Norway
Brazil	New Zealand
Canada	Pakistan
Switzerland	Philippines
Chile	Poland
China	Portugal
Colombia	Rest of Africa
Czech Republic	Rest of Asia
Germany	Rest of Central America
Denmark	Rest of Europe
Spain	Rest of South America
Finland	Romania
France	Russia
United Kingdom	Saudi Arabia
Greece	Singapore
Croatia	Slovakia
Hungary	Sweden
Indonesia	Thailand
India	Taiwan
Ireland	Ukraine
Iran	Uruguay
Italy	Venezuela
Japan	South Africa

Notes: Baltic Republics includes Estonia, Lithuania and Latvia; North Africa includes Algeria, Egypt, Ethiopia, Morocco, Tunisia; countries named "Rest of X" include all the remaining countries of continent X not included in the table.

C.1 World Trade Matrix

We construct a matrix of bilateral industry-level trade flows among 722 US Commuting Zones, Alaska, Hawaii and 58 foreign countries for 1997 and 2007.

1. We create country-to-country matrix of trade flows at the 2-digit SCTG classification used in the CFS. To this end, we use the BACI trade dataset from UN Comtrade at the HS6 level. We use it to construct trade flows for the 31 industries in Table 5 between the USA and the 58 countries in Table 12. We merge this data with the Eora MRIO dataset to obtain the domestic spending share in each industry. Since the EORA dataset uses a more aggregated industry classification, we assign identical spending shares to all SCTG industries in the EORA sectors. We obtain trade flows in non-manufacturing directly from EORA.

2. We then create a trade matrix between US states and foreign countries at the SCTG-level. We use state-to-state shipments data at the SCTG level from the Commodity Flow Survey

released by the US Census in 1997 and 2007. One issue is that in the CFS dataset some shipment values are suppressed or missing. We use a gravity-based approach to impute these suppressed values, that we describe in the sub-section C.1.1 below. Finally, we convert shipment flows into trade flows as follows.

(a) Let $\left(Z_{dj}^{kt}, Z_{jd}^{kt}\right)$ denote the trade flows between each of the 40 US custom districts, d, and foreign country, j, by sector k and year t. We obtain $\left(Z_{dj}^{kt}, Z_{jd}^{kt}\right)$ from the US Merchandise Trade Files released annually by the US Census between 1990 and 2016. The exports and imports of state i to foreign country j are

$$X_{ij}^{kt} = \sum_{d} a_i^{dj,kt} \cdot Z_{dj}^{kt}$$

$$X_{ji}^{kt} = \sum_{d} b_i^{dj,kt} \cdot Z_{jd}^{kt}$$

where $a_i^{dj,kt}$ and $b_i^{dj,kt}$ correspond to the share of total exports and imports in district d whose respective origin and destination are state i.

i. We construct bilateral trade flows between US states for each sector and year. Let $\tilde{X}_{ir}^{kt}$ denote the value of shipments from state i to state r of goods in sector k at year t. The trade flows between state i to state r are services:

$$X_{ir}^{kt} = \tilde{X}_{ir}^{kt} - \sum_{d,j} \left(\tilde{a}_{ir}^{dj,kt} \cdot Z_{dj}^{kt} + \tilde{b}_{ir}^{dj,kt} \cdot Z_{jd}^{kt}\right)$$

where $\tilde{a}_{ir}^{dj,kt}$ and $\tilde{b}_{ir}^{dj,kt}$ correspond respectively to the share of total exports and imports in district d transiting between states i and r. To compute the variables above, we assume that the transit route is the same for all export and import of all sectors with identical state of origin/destination, port of exit/entry, and foreign country of origin/destination. Using the US Census data on state of origin exports by port and destination, we compute the following variables:

$$a_i^{dj,kt} = b_i^{dj,kt} = \frac{\text{exports}_i^{dj,t}}{\sum_l \text{exports}_l^{dj,t}} \quad \text{and} \quad \tilde{a}_{ir}^{dj,kt} = \tilde{b}_{ir}^{dj,kt} = \frac{\text{exports}_{ij}^{dj,t}}{\sum_{r,l} \text{exports}_{rl}^{dj,t}}$$

ii. We adjust domestic sales of the residual sector to include local spending in

$$X_{ii}^{NT,t} = \left(\sum_{k \neq NT} \sum_{r} X_{ri}^{kt}\right) e_i^t$$

where e_i^t is the expenditure ratio between non-tradeable and tradeable goods of state i at year t obtained from the BEA state-level accounts.

(b) We merge the trade bilateral trade flows of US states with the bilateral trade flows of the US and other countries in the BACI database. To this end, we use US domestic sales in BACI to normalize total expenditures of US states on goods produced from other US states. We also distribute the bilateral trade flows of the US in the BACI among US states using each state share in total trade flows to/from other foreign countries obtained in the previous step. The final output consists of $X_{ij,k}$: trade flow from i to j in SCTG sector k, where i, j are US states or foreign countries.

3. The final step is to use the trade matrix with US states and foreign countries to construct trade flows for US Commuting Zones at the SCTG-level. To this end, we construct the participation of each CZ r in its state $i(r)$ production and consumption. The production share is the CZ's share in the state's total employment in industry n, $R^t_{n,r}(j) \equiv L^t_{n,r}/(\sum_{r' \in i(r)} L^t_{n,r'})$, and the spending share is the CZ's total employment share in the state, $E^t_{n,r}(i) \equiv \bar{L}^t_r/(\sum_{r' \in i(r)} \bar{L}^t_{r'})$ with $\bar{L}^t_r \equiv \sum_{n'} L^t_{r,n'}$. For each CZ, export value to country j is $X^t_{cj,n} = R^t_{n,r} X_{i(c)j,n}$ and import value from country j is $X^t_{ir,n} = E^t_{n,r} X_{ij(c),n}$. Finally, we impute trade flows between CZs using a gravity procedure. We first use state-to-state trade flows computed in the previous step to estimate a gravity regression for each industry n:

$$\log X^t_{ij,n} = \beta^t_0 + \beta^t_1 1[i = j] + \beta^t_2 \ln d_{ij} + \beta_3 \ln Y^t_{i,n} + \beta_4 \ln E^t_{j,n} + e^t_{ij,k}$$

where d_{ij} is the bilateral distance between state i and j, $Y^t_{i,n}$ is the total production in state i, $E^t_{j,n}$ is the total expenditure in state j. We then use the estimated coefficients to compute the predicted flows between CZ r and d

$$\log \hat{x}^t_{rd,k} = \hat{\beta}^t_0 + \hat{\beta}^t_1 1[i = j] + \hat{\beta}^t_2 \ln d_{ij} + \hat{\beta}_3 \ln R^t_{i,n} + \hat{\beta}_4 \ln E^t_{j,n}$$

where $R^t_{n,r}$ are employment shares and $E^t_{n,k}$ are expenditure shares computed in the previous step. We rescale these predicted values by the corresponding share of national flows coming from US domestic sales from the BACI dataset. In the non-manufacturing sector, we use a similar procedure using a higher value for the distance elasticities, $\hat{\beta}_1$ and $\hat{\beta}_2$. In particular, we follow Eckert (2018) by adjusting these parameters to be 50% higher than the estimates for the manufacturing sector.

C.1.1 Methodology to replace suppressed values in the CFS

We implement the imputation procedure separately for each of the 31 industries in Table 5. To simplify the notation, we drop the industry subscript. Using observed data on bilateral shipments between US states in the tradeable sector, we estimate the following gravity equation, for every year t:

$$\log \tilde{X}_{ij} = \beta_0 + \beta_1 \ln d_{ij} + \beta_2 \ln Y_i + \beta_2 \ln E_j + e_{ij}$$

where d_{ij} is the bilateral distance between state i and j, Y_i is the total production in state i, E_j is the total expenditure in state j, and e_{ij} is the econometric error. Then we obtain the predicted values

$$\log \hat{X}_{ij} = \hat{\beta}_0 + \hat{\beta}_1 \ln d_{ij} + \hat{\beta}_2 \ln X_i + \hat{\beta}_2 \ln X_j.$$

We compute the residual outflows for each state as $\bar{Y}_i = Y_i - \sum_j \tilde{X}_{ij}$, and the residual inflows as $\bar{E}_j = E_j - \sum_i \tilde{X}_{ij}$. For suppressed values, we assume that the true trade flow equals:

$$\tilde{X}_{ij} = \hat{X}_{ij} \xi_i \gamma_j.$$

We must have that the summation of predicted flows across destinations for each origin has to be equal to total production: $\sum_j \tilde{X}_{ij} = \bar{Y}_i$. Also, the summation of predicted flows across origins for each destination has to be equal to total expenditures: $\sum_i \tilde{X}_{ij} = \bar{E}_j$. To compute ξ_i and γ_j, we use the following algorithm. For state i, consider the vector of exports to all states $\tilde{X}_{ij}$ and the imports $\tilde{X}_{ji}$. Then, we compute the following ratios: $\xi_i = \sum_j \tilde{X}_{ij}/\bar{Y}_i$ and $\gamma_i = \sum_j \tilde{X}_{ji}/\bar{E}_j$. We then adjust $\hat{X}_{ij} = \tilde{X}_{ij}/\xi_i$ and $\hat{X}_{ji} = \tilde{X}_{ji}/\gamma_i$. For state $j+1$, repeat the same procedure, but keeping constant the exports and imports of the previous adjusted states 1 to j, and adjusting the total expenditures and production. Finally, we use these predicted (and consistent with the aggregates) values to fill the suppressed shipments.

D Online Appendix: Equivalences and Extensions

This online appendix has three parts. First, we establish the general solution of our model in the non-linear system of equilibrium conditions. Second, we formally establish the equivalence of our model's counterfactual predictions to those implied by a number of existing trade and geography models. Finally, we extend our model to account for other sources of cross-market links.

D.1 Non-Linear DEK Expressions

Consider the solution of the non-linear system of equilibrium equations following changes in economic fundamentals. Consider an equilibrium with positive production in all markets. The labor market module in (14)–(16) imply written in changes:

$$L_i^0 \hat{L}_i = \Phi_i\left(\left\{\omega_j^0 \hat{\omega}_j\right\}_j\right) \tag{59}$$

$$\hat{w}_i = \hat{p}_i \frac{\Psi_i\left(\left\{L_j^0 \hat{L}_j\right\}_j\right)}{\Psi_i\left(\left\{L_j^0\right\}_j\right)}. \tag{60}$$

In addition, the market clearing condition in (17) yields

$$w_i^0 L_i^0 \left(\hat{w}_i \hat{L}_i\right) = \sum_j x_{ij}^0 \hat{x}_{ij} w_j^0 L_j^0 \left(\hat{w}_j \hat{L}_j\right), \tag{61}$$

where the changes in spending shares and price indices in (12)–(13) are given by

$$x_{ij}^0 \hat{x}_{ij} = X_{ij}\left(\left\{\frac{\tau_{oj}^0 p_o^0}{P_j^0}\hat{\tau}_{oj}\hat{p}_o\right\}_o\right) \quad \text{and} \quad \hat{P}_j = P_j\left(\left\{\frac{\tau_{oj}^0 p_o^0}{P_j^0}\hat{\tau}_{oj}\hat{p}_o\right\}_o\right). \tag{62}$$

The system (59)–(62) determines the changes in endogenous variables, $\{\hat{p}_i, \hat{P}_i, \hat{L}_i, \hat{\omega}_i\}_i$, implied by any combination of shocks, $\{\hat{\tau}_{ij}\}_{i,j}$. It depends on the aggregate mappings $\{\{X_{ij}(\cdot)\}_j, \Phi_i(\cdot), \Psi_i(\cdot)\}_i$ as well as initial outcomes, $\{\{x_{ij}^0\}_j, w_i^0, L_i^0\}_i$, and initial prices and shifters, $\{\{\tau_{ij}^0\}_j, p_i^0, P_i^0\}_i$. Notice however that our model – and thus a large number of spatial models – is over-identified: there are multiple degrees of freedom to match observed labor and trade outcomes in the initial equilibrium. We show that it is always possible to choose the location of the preference and productivity shifters in $\{\{X_{ij}(\cdot)\}_j, \Phi_i(\cdot), \Psi_i(\cdot)\}_i$ to replicate the initial levels of trade flows and labor market outcomes across markets, while normalizing shifters of trade costs, and productivity in the initial equilibrium. The normalization of bilateral effective prices in the initial equilibrium is analogous to that imposed in neoclassical economies by Adao, Costinot, and Donaldson (2017).

Thus, in equations (59)–(62), we choose initial shifters such that

$$\tau_{ij}^0 p_i^0 \equiv 1, \quad P_j^0 \equiv 1, \quad \Psi_i\left(\left\{L_j^0\right\}_j\right) \equiv 1 \quad \forall i, j. \tag{63}$$

Given the normalization in (63), we can use the system in (59)–(62) to characterize the counterfactual predictions of our model.

Proposition 6. *Consider the Generalized Spatial Economy satisfying Assumptions 1 and 2. Conditional on initial levels of endogenous variables* $\{\{x_{ij}^0\}_j, w_i^0, L_i^0\}_i$*, the mappings* $\{\{X_{ij}(\cdot)\}_j, \Phi_i(\cdot), \Psi_i(\cdot)\}_i$

are sufficient to uniquely characterize counterfactual changes in endogenous outcomes, $\{\hat{p}_i, \hat{P}_i, \hat{L}_i, \hat{\omega}_i\}_i$, implied by any combination of shocks, $\{\hat{\tau}_{ij}\}_{i,j}$, as a solution of (59)–(62).

Proof. Proposition 2 immediately guarantees that there is a unique equilibrium for the initial and the final set of shifters. So, we only need to show that, by specifying preferences and technology, we obtain an equilibrium with identical trade and labor outcomes as the initial equilibrium under the normalization in (63).

Initial Equilibrium. Consider an initial equilibrium such that

$$L_j^0 = \Phi_j\left(\left\{\frac{w_i^0}{P_i^0}\right\}_i\right)$$

$$p_i^0 = \frac{w_i^0}{P_i^0}\frac{1}{\Psi_i\left(\left\{L_j^0\right\}_j\right)}$$

$$w_i^0 L_i^0 = \sum_j x_{ij}^0 w_j^0 L_j^0$$

$$x_{ij}^0 = X_{ij}\left(\left\{\tau_{oj}^0 p_o^0\right\}_o\right) \quad \text{and} \quad P_j^0 = P_j\left(\left\{\tau_{oj}^0 p_o^0\right\}_o\right).$$

Alternative Economy. Denote $\Psi_i^0 \equiv \Psi_i\left(\left\{L_j^0\right\}_j\right)$. Let us construct an alternative economy without trade costs ($\tilde{\tau}_{ij} \equiv 1$), where technology is given by

$$\tilde{\Psi}_i\left(\{L_j\}_j\right) \equiv \frac{1}{\Psi_i^0}\Psi_i\left(\{L_j\}_j\right),$$

and preferences are given by

$$\tilde{U}_c\left(\{C_j\}_j, \{L_j\}_j\right) \equiv U_c\left(\left\{\frac{1}{P_j^0}C_j\right\}_j, \{L_j\}_j\right)$$

$$\tilde{V}_j\left(\{c_{ij}\}_i\right) \equiv V_j\left(\left\{c_{ij}\frac{\Psi_i^0 P_j^0}{\tau_{ij}^0}\right\}_i\right).$$

In this case, we immediately get that

$$\tilde{\Phi}_j\left(\{\omega_i\}_i\right) = \Phi_j\left(\left\{\frac{1}{P_i^0}\omega_i\right\}_i\right)$$

and

$$\tilde{X_{ij}}\left(\{p_{oj}\}_o\right) = X_{ij}\left(\left\{\frac{\tau_{oj}^0}{P_j^0\Psi_i^0}p_{oj}\right\}_o\right) \quad \text{and} \quad \tilde{P}_j\left(\{p_{oj}\}_o\right) = P_j\left(\left\{\frac{\tau_{oj}^0}{P_j^0\Psi_i^0}p_{oj}\right\}_o\right).$$

Equilibrium of Alternative Economy. In this economy, the equilibrium entails $\tilde{w}_i = w_i^0$, $\tilde{L}_i = L_i^0$,$\tilde{x}_{ij} = x_{ij}^0$, and $\tilde{p}_{ij} = \tilde{P}_i = 1$. To see this, notice that

$$\tilde{p}_i = \frac{\tilde{w}_i}{\tilde{\Psi}_i\left(\left\{L_j^0\right\}_j\right)} = w_i^0,$$

$$\tilde{\Phi}_j\left(\left\{\frac{\tilde{w}_i}{\tilde{P}_i}\right\}_i\right) = \Phi_j\left(\left\{\frac{1}{P_i^0}w_i^0\right\}_i\right) = L_j^0,$$

$$\tilde{x}_{ij} = \tilde{X}_{ij}\left(\{\tilde{\tau}_{oj}\tilde{p}_o\}_o\right) = X_{ij}\left(\left\{\frac{\tau_{oj}^0 p_o^0}{P_j^0}\right\}_o\right) = x_{ij}^0,$$

$$\tilde{P}_j = \tilde{P}_j\left(\{\tilde{\tau}_{oj}\tilde{p}_o\}_o\right) = P_j\left(\left\{\frac{\tau_{oj}^0 p_o^0}{P_j^0}\right\}_o\right) = 1$$

Finally, the labor market clearing condition holds:

$$\tilde{w}_i\tilde{L}_i = w_i^0 L_i^0 = \sum_j x_{ij}^0 w_j^0 L_j^0 = \sum_j \tilde{x}_{ij}\tilde{w}_j\tilde{L}_j. \quad \blacksquare$$

D.2 Equivalences

We now discuss how our theoretical environment unifies a number of existing frameworks in spatial economics. We show that the shape of the mappings $\{\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\}_j$ encompasses the central forces in a wide range of spatial and trade models. We start by introducing a formal definition of the models for which the equilibrium outcomes of the Generalized Spatial Model of Section 3 are observationally equivalent to.

Definition 2. The Generalized Spatial Model of Section 3 is observationally equivalent to Economy N with respect to the shifters $\{\tau_{ij}\}_{i,j}$ if

1. There exist *unique* mappings $\{\{X_{ij}^N(\cdot)\}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot)\}_j$ such that the equilibrium of Economy N is characterized by conditions (12)–(17) for *any* levels of $\{\tau_{ij}\}_{i,j}$;

2. There exist preferences, (1) and (10), and technology, (4), that imply $\{\{X_{ij}^N(\cdot)\}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot)\}_j$.

This definition requires that, independent of the levels of the exogenous shifters, Economy N must satisfy the equilibrium conditions (12)–(17) for unique mappings $\{\{X_{ij}^N(\cdot)\}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot)\}_j$. This implies that any combination of shocks to the shifters $\{\tau_{ij}\}_{i,j}$ yields identical counterfactual outcomes in labor markets. We use Definition 2 to establish that our model is observationally equivalent to several existing frameworks under specific parametric restrictions on the shape of $\{\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\}_j$. In particular, we show the equivalence with, respectively: (i) Neoclassical models with economies of scale, (ii) New trade theory models, (iii) New economic geography models, (iv) Spatial assignment models, and (v) Spatial assignment models with other factors of production.

D.2.1 Neoclassical Economy

Environment. Consider a neoclassical economy with a single factor of production. We denote all the variables of this economy that are potentially different from the Generalized Spatial Economy with a superscript N. The proofs follows the logic of the proof of Adao, Costinot, and Donaldson

(2017) but extending to the case of labor mobility and agglomeration spillovers. We assume that the agglomeration function, the labor supply function, and the exogenous shifters are the same as for the Generalized Economy, so that we do not use superscripts for those objects.

As in the Generalized Spatial Model, each country has a representative agent with preferences for consumption and labor supply in different markets, with utility function given by

$$U_c\left(\left\{C_j^N\right\}_j, \left\{L_j^N\right\}_j\right).$$

The main difference is that we explicitly allow for preferences over goods, z:

$$C_j^N \equiv V^N\left(\left\{c_{z,ij}^N\right\}_{z,i}\right),$$

where $V^N(\cdot)$ is twice differentiable, quasi-concave, homothetic, and increasing in all arguments. Notice that $V^N(\cdot)$ allows for the possibility that goods from different origins are imperfect substitutes.

The representative household's budget constraint is

$$\sum_i \sum_z p_{z,ij}^N c_{z,ij}^N = w_j^N L_j^N.$$

There are many perfectly competitive firms supplying each good in any market. The production technology uses only labor and entails external economies of scale at the market level. In particular, the technology of producing good z in i and delivering to j is given by

$$Y_{z,ij}^N = \Psi_i\left(\left\{L_j^N\right\}_j\right)\frac{L_{z,ij}^N}{\tau_{ij}\alpha_{z,ir}^N},$$

where $\alpha_{z,ij}^N$ is good-specific productivity shifter of producing in i and delivering in j.

Equilibrium. We use the fact that $V^N(\cdot)$ is homothetic to derive the price index in market j:

$$P_j^N = P_j^N\left(\left\{p_{k,oj}^N\right\}_{k,o}\right) \equiv \min_{\left\{c_{k,oj}\right\}_{k,o}} \sum_{k,o} p_{k,oj}^N c_{k,oj}^N \quad \text{s.t.} \quad V^N\left(\left\{c_{k,oj}\right\}_{k,o}\right) \geq 1 \tag{64}$$

where the associated spending share on good z from i is

$$x_{ij,z}^N \in X_{ij,z}^N\left(\left\{p_{k,oj}^N\right\}_{k,o}\right). \tag{65}$$

Conditional on prices, the representative household solves the utility maximization problem that yields the labor supply in market j:

$$L_j^N = \Phi_j\left(\left\{\omega_i^N\right\}_i\right). \tag{66}$$

Profit maximization implies that

$$p_{z,ij}^N = \tau_{ij} p_i^N \alpha_{z,ij}^N \tag{67}$$

where

$$p_i^N = \frac{w_i^N}{\Psi_i\left(\left\{L_j^N\right\}_j\right)} \tag{68}$$

Finally, the labor market clearing condition is

$$w_i^N L_i^N = \sum_j \sum_z x_{z,ij}^N \cdot \left(w_j^N L_j^N\right). \tag{69}$$

The competitive equilibrium corresponds to $\left\{\left\{p_{z,ij}^N\right\}_{z,i}, w_j^N, L_j^N, P_j^N\right\}_j$ such that equations (64)–(69) hold. Thus, the equilibrium can be written as $\left\{p_i^N, \omega_i^N, L_i^N, P_i^N\right\}_i$ solving (12)–(17) with $\Phi_j(\cdot)$, $\Psi_j(\cdot)$, and

$$X_{ij}^N\left(\left\{\tau_{oj}p_o^N\right\}_o\right) \equiv \left\{x_{ij}^N = \sum_z x_{ij,z}^N : \ x_{ij,z}^N \in X_{ij,z}^N\left(\left\{\tau_{oj}p_o^N\alpha_{k,oj}^N\right\}_{o,k}\right)\right\}$$

such that

$$P_i^N\left(\left\{\tau_{oj}p_o^N\right\}_o\right) = P_i^N\left(\left\{\tau_{oj}p_o^N\alpha_{k,oj}^N\right\}_{o,k}\right).$$

Equivalence. We now construct an equivalent Generalized Spatial Economy. We only need to show that there exist preferences and technology that are consistent with $\left\{\left\{X_{ij}^N(.)\right\}_i, \Phi_j(.), \Psi_j(.)\right\}_j$. We also assume that the production function of the market-specific composite good in the Generalized Economy is

$$Y_{ij} = \Psi_i\left(\left\{L_j\right\}_j\right)\frac{L_{ij}}{\tau_{ij}}.$$

In addition, consider the preferences in Section 3 with

$$V_j\left(\left\{c_{ij}\right\}_i\right) \equiv \max_{\left\{c_{z,ij}\right\}_{z,i}} V^N\left(\left\{c_{z,ij}\right\}_{z,i}\right) \quad \text{s.t.} \quad \textstyle\sum_z \alpha_{z,ij}^N c_{z,ij} = c_{ij} \ . \tag{70}$$

Intuitively, the preference structure in (70) implies that, if the representative household acquires c_{ij} units of $i's$ composite good for j's consumption, then it optimally allocates the composite good into the production of different goods, given the exogenous weights $\alpha_{z,ij}^N$ that are now embedded into the representative agent's preferences. Since the relative price of goods in market i only depends on $\alpha_{z,ij}^N$, this decision yields allocations that are identical to those in the competitive equilibrium of the decentralized economy.

To see this, denote the spending shares associated with the cost minimization problem with $V_j(\cdot)$ by $x_{ij} \in X_{ij}\left(\left\{\tau_{ij}p_i\right\}_i\right)$. Thus, the equivalence follows from showing that

$$X_{ij}\left(\left\{\tau_{oj}p_o\right\}_o\right) = X_{ij}^N\left(\left\{\tau_{oj}p_o\right\}_o\right) \quad \forall \left\{\tau_{oj}p_o\right\}_o. \tag{71}$$

First, we show that $x_{ij} \in X_{ij}\left(\left\{\tau_{oj}p_o\right\}_o\right) \implies \exists x_{ij,z}^N \in X_{ij,z}^N\left(\left\{\tau_{oj}p_o\alpha_{z,oj}^N\right\}_{k,o}\right)$ with $x_{ij} = \sum_z x_{ij,z}^N$. Let $\left\{c_{z,ij}\right\}_{z,i}$ be the solution of the good allocation problem in the definition of $V_j\left(\left\{c_{ij}\right\}\right)$ in (70). We proceed by contradiction to show that $\left\{c_{z,ij}\right\}_{z,i}$ implies spending shares, $\left\{x_{z,ij}\right\}_{z,i} =$

$\left\{\tau_{oj}p_o\alpha^N_{z,ij}c_{z,ij}\right\}_{z,i}$, such that $x_{z,ij} \in X^N_{ij,z}\left(\left\{\tau_{oj}p_o\alpha^N_{z,oj}\right\}_{k,o}\right)$. Suppose there exists a feasible allocation $\left\{c^N_{z,ij}\right\}_{z,i}$ such that

$$V^N\left(\left\{c^N_{z,ij}\right\}_{z,i}\right) > V^N\left(\left\{c_{z,ij}\right\}_{z,i}\right) \quad \text{and} \quad \sum_i\sum_z \tau_{ij}p_i\alpha^N_{z,ij}c^N_{z,ij} \leq 1. \tag{72}$$

Notice that $\sum_i\sum_z \tau_{ij}p_i\alpha^N_{z,ij}c^N_{z,ij} \leq 1$, which implies that the allocation $c^N_{ij} \equiv \sum_z \alpha^N_{z,ij}c^N_{z,ij}$ is feasible in the Generalized Spatial Competitive Economy. Thus,

$$V^N\left(\left\{c_{z,ij}\right\}_{z,i}\right) = V_j\left(\left\{c_{ij}\right\}\right) \geq V_j\left(\left\{c^N_{ij}\right\}\right) \geq V^N\left(\left\{c^N_{z,ij}\right\}_{z,i}\right),$$

which is a contradiction of inequality (72).

Second, we show that $x_{ij} = \sum_z x^N_{ij,z}$ with $x^N_{ij,z} \in X^N_{ij,z}\left(\left\{\tau_{oj}p_o\alpha^N_{z,oj}\right\}_{k,o}\right)$, and $c^N_{ij} = \sum_z \alpha^N_{z,ij}c^N_{z,ij} \implies x_{ij} \in X_{ij}\left(\left\{\tau_{oj}p_o\right\}_o\right)$. We start with $c^N_{ij} = \sum_z \alpha^N_{z,ij}c^N_{z,ij}$ implied by the solution of the consumer's problem in the Neoclassical Economy. We proceed by contradiction to show that $\left\{c^N_{ij}\right\}_i$ is optimal in the Generalized Spatial Competitive Economy given prices $\left\{\tau_{ij}p_i\right\}_i$. Suppose there exists a feasible allocation $\left\{c_{ij}\right\}_i$ in the Generalized Spatial Competitive Economy such that

$$V_j\left(\left\{c_{ij}\right\}\right) > V_j\left(\left\{c^N_{ij}\right\}\right) \quad \text{and} \quad \sum_i p_{ij}c_{ij} \leq \sum_i p_{ij}c^N_{ij} = 1.$$

Let $\left\{c_{z,ij}\right\}_{z,i}$ be the be the solution of the good allocation problem in the definition of $V_j\left(\left\{c_{ij}\right\}\right)$ in (70). Thus,

$$\sum_i \tau_{ij}p_i\sum_z \alpha^N_{z,ij}c_{z,ij} = \sum_i \tau_{ij}p_ic_{ij} \leq 1$$

and, by revealed preference,

$$V_j\left(\left\{c^N_{ij}\right\}\right) \geq V^N\left(\left\{c^N_{z,ij}\right\}_{z,i}\right) \geq V^N\left(\left\{c_{z,ij}\right\}_{z,i}\right) = V_j\left(\left\{c_{ij}\right\}\right).$$

This establishes the contradiction. Since we have found preferences and technology that imply the mappings $\left(\Phi_j(.), \Psi_j(.), X^N_{ij}(.)\right)$, we have proven the equivalence.

D.2.2 New Trade Theory

Environment. The utility function is as in the Generalized Spatial Model. We assume that C_j has a nested preference structure across sectors, $C^N_j = V^N_j\left(\left\{C_{k,j}\right\}_k\right)$ with $V^N_j(\cdot)$ strictly quasi-concave and homogeneous of degree one. Sectors are divided into two groups: competitive sectors, $k \in K^{N_C}$, and monopolistic competitive sectors, $k \in K^{N_M}$.

In any competitive sector $k \in K^{N_C}$, firms in each country produce one homogeneous good with the production technology in (4). In particular, assume that technology is subject to external economies of scale with the marginal production cost given by $\zeta_{kr}\Psi^{N_C}_{kr}\left(\left\{L_j\right\}_j\right)$. Let $C^{N_C}_{k,j}$ be an aggregator of goods from different origins r, $C^{N_C}_{k,j} \equiv V^{N_C}_{k,j}\left(\left\{c_{kr,j}\right\}_r\right)$, where $V^{N_C}_{k,j}(\cdot)$ is twice differentiable, increasing, quasi-concave, and homogeneous of degree one. Notice that the utility function allows the goods produced in different regions to be perfect substitutes and, therefore, it covers homogeneous goods.

In any sector $k \in K^{N_M}$, there is large mass of potential entrants in each region that produce a differentiated good, indexed by z, and operate in monopolistic competition. We assume that all potential entrants in sector-region (k, r) have access to the same increasing returns technology where, in terms of labor, the fixed entry cost is $\mu_{kr}\Psi_{kr}^{N_E}\left(\{L_j\}_j\right)$ and the marginal production cost is $\zeta_{kr} \cdot \Psi_{kr}^{N_P}\left(\{L_j\}_j\right)$. We explicitly allow $\Psi_{kr}^{N_P}(\cdot)$ and $\Psi_{kr}^{N_E}(\cdot)$ to depend on employment, but we assume that firms perceive them as given. So, these function incorporate external agglomeration and congestion forces at the market level.

We also assume that, for $k \in K^{N_M}$, preferences are CES across the available differentiated goods with elasticity $\sigma > 1$:

$$C_{k,j}^N = \left[\int_{z \in Z_{k,j}} (c(z))^{\frac{\sigma}{\sigma-1}} \, dz\right]^{\frac{\sigma-1}{\sigma}},$$

where $Z_{k,j}$ is the set of goods in sector $k \in K^{N_M}$ available in market j.

Equilibrium. As in the Generalized Spatial Model, the representative household's problem yields the labor supply in region-sector j,

$$L_j^N \in \Phi_j\left(\{\omega_i^N\}_i\right). \tag{73}$$

Consider now a competitive sector $k \in K^{N_C}$. Cost minimization implies that

$$p_{kr,j}^N = \tau_{kr,j}\frac{w_{kr}^N}{\Psi_{kr}^{N_C}\left(\left\{L_j^N\right\}_j\right)} \tag{74}$$

For the monopolistic competitive sector $k \in K^{N_M}$, all firms in region r choose the same price:

$$p_{kr,j}^N = \tau_{kr,j}\frac{\sigma}{\sigma-1}\frac{w_{kr}^N}{\Psi_{kr}^{N_P}\left(\left\{L_j^N\right\}_j\right)}. \tag{75}$$

We now characterize the mass of operating firms, M_{kr}. The labor market clearing and the free entry conditions in (k, r) imply

$$M_{kr} = \frac{1}{\sigma\mu_{kr}} \cdot \frac{L_{kr}^N}{\Psi_{kr}^{N_E}\left(\left\{L_j^N\right\}_j\right)}.$$

Thus, in the monopolistic competitive sector $k \in K^{N_M}$, we can express prices as

$$p_{kr,j}^N = \tau_{kr,j}\frac{w_{kr}^N}{\Psi_{kr}^{N_M}\left(\left\{L_j^N\right\}_j\right)} \tag{76}$$

with

$$\Psi_{kr}^{N_M}\left\{L_j^N\right\}_j \equiv \frac{\sigma-1}{\sigma}\Psi_{kr}^{N_P}\left(\left\{L_j^N\right\}_j\right)\left(\frac{1}{\sigma\mu_{kr}} \cdot \frac{L_{kr}^N}{\Psi_{kr}^{N_E}\left(\left\{L_j^N\right\}_j\right)}\right)^{\frac{1}{1-\sigma}}. \tag{77}$$

Using these expressions, it is straightforward to show that the labor market clearing condition in sector k of region r, $i = (k, r)$, is

$$w_{kr}^N L_{kr}^N = \sum_j x_{kr,j}^N w_j^N L_j^N. \tag{78}$$

Equivalence. We now construct an equivalent Generalized Spatial Model. To establish the equivalence, we need to set $\Psi_{kr}(\cdot) = \Psi_{kr}^{N_M}(\cdot)$ for $k \in K^{N_M}$ and $\Psi_{kr}(\cdot) = \Psi_{kr}^{N_C}(\cdot)$ for $k \in K^{N_C}$. We also need to specify sector-level preferences such that $V_{k,j}\left(\{c_{kr,j}\}_r\right) = V_{k,j}^{N_C}\left(\{c_{kr,j}\}_r\right)$ for $k \in K^{N_C}$ and $V_{k,j}\left(\{c_{kr,j}\}_r\right) = \left[\sum_r (c_{kr,j})^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}}$ for $k \in K^{N_M}$. In addition, we must specify the same upper-level consumption aggregator across sectors: $V_j\left(\{c_{kr,j}\}_{k,r}\right) \equiv V_j^N\left(\left\{V_{k,j}\left(\{c_{kr,j}\}_r\right)\right\}_k\right)$.

D.2.3 New Economic Geography

Environment. For the next equivalence result we consider an economy with production structure and preference for goods identical to those in the New Trade Theory Economy of Section D.2.2. We assume that each country c is populated by a continuum of individuals with identical preferences for goods. These individuals differ in terms of mobility across markets. As in Krugman (1991), there are two groups of markets in each country, $J_c^{N_I}$ and $J_c^{N_M}$. Market $j \in J_c^{N_I}$ is populated by a subset of completely immobile individuals such that

$$L_j^N = \bar{L}_j \quad \forall j \in J_c^{N_I}, \tag{79}$$

In addition, there is a mass $\bar{L}_c$ of individuals that is completely mobile across markets $j \in J_c^{N_M}$ such that

$$\sum_{j \in J_c^{N_M}} L_j^N = \bar{L}_c. \tag{80}$$

Mobile individuals have identical preferences for being employed in any $j \in J_c^{N_M}$:

$$U_j\left(\omega_j^N, L_j^N\right) = \omega_j^N \left(L_j^N\right)^\beta \cdot$$

where ω_j^N is the real wage in market j.

Equilibrium. We restrict attention to equilibria with positive employment in every $j \in J_c^{N_M}$, and analyze separately the cases of $\beta \neq 0$ and $\beta = 0$.

If $\beta = 0$, any employment allocation is feasible as long as $\nu_i \omega_i^N = \bar{u}$. Thus, the labor supply is

$$\left\{L_i^N\right\}_i = \Phi_c^N\left(\left\{\nu_i \omega_i^N\right\}_i\right) = \begin{cases} L_j^N = \bar{L}_c,\ L_i^N = 0 & \text{if} \quad \omega_i^N > \omega_i^N\ \forall i \in J_c^{N_M} \\ \left\{L_j^N : \sum_j L_j^N = \bar{L}_c\right\} & \text{if} \quad \omega_i^N = \bar{u}\ \forall i \in J_c^{N_M} \end{cases}. \tag{81}$$

If $\beta \neq 0$, in this case, any $j \in J_c^{N_M}$ with positive employment must have

$$\omega_j^N \left(L_j^N\right)^\beta = \bar{u} \quad \implies L_j^N = \left(\frac{\bar{u}}{\omega_i^N}\right)^{\frac{1}{\beta}}$$

From equation (80),

$$L_j^N = \Phi_j^N\left(\left\{\omega_i^N\right\}_i\right) \equiv \bar{L}_c \frac{\left(\omega_j^N\right)^{-\frac{1}{\beta}}}{\sum_{i \in J_c^{N_M}} \left(\omega_i^N\right)^{-\frac{1}{\beta}}} \tag{82}$$

The equilibrium of this economy is $\left\{p_i^N, P_i^N, L_i^N, \omega_i^N\right\}$ solving (12)–(17) with $\Phi_j\left(\left\{\omega_i^N\right\}_i\right) = \bar{L}_j$ if $j \in J^{N_I}$ and $\Phi_j\left(\left\{\omega_i^N\right\}_i\right) = \Phi_j^N\left(\left\{\omega_i^N\right\}_i\right)$ if $j \in J^{N_M}$.

Equivalence. To establish equivalence, we construct preferences for the the representative household in the Generalized Spatial Model that yield the labor supply function $\Phi_j(\cdot) = \Phi_j^N(\cdot)$. Specifically, consider the following preferences:

$$U_c\left(\left\{C_j, L_j\right\}_j\right) = \begin{cases} \left[\sum_{j \in J_c^M} \left(C_j\right)\left(L_j\right)^{\beta}\right]^{\frac{1}{1+\beta}} & \text{if } \sum_{j \in J_c^M} L_j = \bar{L}_c \text{ and } L_i = \bar{L}_i \ \forall i \in J^I \\ -\infty & \text{otherwise} \end{cases}$$

Since the budget constraint implies that $C_j = \omega_j L_j$, the labor supply function is the solution of

$$\left\{\Phi_j\left(\left\{\omega_i\right\}_i\right)\right\}_j = \arg\max_{\{L_j\}} \left[\sum_{j \in J_c^M} \left(\omega_j\right)\left(L_j\right)^{1+\beta}\right]^{\frac{1}{1+\beta}} \quad \text{s.t.} \sum_{j \in J_c^M} L_j = \bar{L}_c.$$

If $\beta = 0$, it is straightforward to see that the solution of the utility maximization problem yields equation (81). If $\beta \neq 0$, the solution of the maximization problem is the same as equation (82). Since we have assumed a production structure and preferences for goods identical to those in the New Trade Theory Economy, the assumptions on technology and consumption aggregator imposed in the previous section imply that the functions $X_{ij}(\cdot)$ and $\Psi_i(\cdot)$ deliver the equivalence.

D.2.4 Spatial Assignment Models

Environment. Suppose that countries are populated by a continuum of individuals, $\iota \in I_c$, that are heterogeneous in terms of preferences and efficiency across markets (i.e, sector-region pairs). We assume individual ι has market specific preferences, $a_j(\iota)$, and market specific efficiency, $e_j(\iota)$. In particular, if employed in market j, we assume that individual ι has homothetic preferences given by

$$U_j(\iota) = a_j(\iota) + V_j^N\left(\left\{c_{ij}^N\right\}_j\right),$$

with a budget constraint given by

$$\sum_i p_{ij}^N c_{ij}^N = w_j^N e_j(\iota).$$

We further assume that individuals take independent draws of $(a_j(\iota), e_j(\iota))$ from a common distribution:

$$\left\{a_j(\iota), e_j(\iota)\right\}_j \sim F^N(\boldsymbol{a}, \boldsymbol{e}).$$

On the production side, we maintain the same structure of the Generalized Spatial Model. That is, there is a representative competitive firm in each market with the production technology in (4).

Equilibrium. We start by characterizing spending shares across markets. Conditional on choosing j, individuals choose spending shares that minimize total cost:

$$P_j^N\left(\left\{p_{oj}^N\right\}_j\right) \equiv \sum_o p_{oj}^N c_{oj}^N \text{ s.t. } V_j^N\left(\left\{c_{oj}^N\right\}_o\right) = 1, \tag{83}$$

with associated spending shares given by

$$x_{ij} \in X_{ij}^N\left(\left\{p_{oj}^N\right\}_o\right). \tag{84}$$

The solution of this problem implies that, for individual ι, the utility of being employed in j is $U_j(\iota) = a_j(\iota) + \omega_j^N e_j(\iota)$. Thus, the set of individuals choosing j is

$$I_j\left(\left\{\omega_i^N\right\}_i\right) \equiv \left\{(\boldsymbol{a}, \boldsymbol{e}):\ a_j + e_j\omega_j \geq a_i + e_i\omega_i\ \forall i\right\},$$

with the associated labor supply given by

$$L_j = \Phi_j^N\left(\left\{\omega_i^N\right\}_i\right) \equiv \int_{I_j\left(\left\{\omega_i^N\right\}_i\right)} e_j\ dF_c(\boldsymbol{a}, \boldsymbol{e}). \tag{85}$$

Notice that the function $\Phi_j^N(\cdot)$ is homogeneous of degree zero with $\frac{\partial \Phi_j^N}{\partial \omega_j} \geq 0$ and $\frac{\partial \Phi_j^N}{\partial \omega_i} \leq 0$.[46]

Profit maximization and labor market clearing are still given by (15)–(17). Thus, the equilibrium can be written as $\left\{p_i^N, P_i^N, L_i^N, \omega_i^N\right\}$ solving (12)–(17) with $\Psi_j(\cdot)$, $X_{ij}(\cdot) = X_{ij}^N(\cdot)$, and $\Phi_j(\cdot) = \Phi_j^N(\cdot)$.

Equivalence. To establish the equivalence, it is sufficient to show that there are preferences for the representative household of the Generalized Spatial Model that yield $\Phi_j(\cdot) = \Phi_j^N(\cdot)$ and $X_{ij}(\cdot) = X_{ij}^N(\cdot)$. Specifically, consider the following preferences:

$$C_j = V_j^N\left(\left\{c_{ij}\right\}_j\right),$$

and

$$U\left(\left\{C_j\right\}_j \left\{L_j\right\}_j\right) \equiv \max_{\left\{\left\{I_j(a,e)\right\}_j\right\}_{(a,e)}} \sum_j C_j + \int \sum_j a_j I_j(a, e)\ dF^N(a, e)$$

subject to

$$L_j = \int e_j I_j(a, e)\ dF^N(a, e)\ \forall j$$

$$\sum_j I_j(a, e) = 1\ \forall (a, e),$$

$$I_j(a, e) \geq 0\ \forall j,\ \forall (a, e).$$

It is straight forward to see that the first-stage problem in the Generalized Spatial Model yields

$$X_{ij}\left(\left\{p_{oj}\right\}_o\right) = X_{ij}^N\left(\left\{p_{oj}\right\}_o\right) \text{ and } P_j\left(\left\{p_{oj}\right\}_o\right) = P_j^N\left(\left\{p_{oj}\right\}_o\right).$$

[46]The homogeneity of $\Phi_j^N(\cdot)$ follows immediately from the definition of I_j. To see that $\frac{\partial \Phi_j^N}{\partial \omega_j} \geq 0$ and $\frac{\partial \Phi_j^N}{\partial \omega_i} \leq 0$, notice that $I_i\left(\tilde{\boldsymbol{\omega}}_c\right) \subset I_i\left(\boldsymbol{\omega}_c\right)$ and $I_j\left(\boldsymbol{\omega}_c\right) \subset I_j\left(\tilde{\boldsymbol{\omega}}_c\right)$ whenever $\tilde{\omega}_j > \omega_j$ and $\tilde{\omega}_i = \omega_i$.

Also, the second-stage problem in the Generalized Spatial Model yields a labor supply function that solves

$$\{\Phi_j(\{\omega_i\}_i)\}_j = \arg\max_{\{L_j\}_j} U\left(\{\omega_j L_j\}_j \{L_j\}_j\right).$$

Using the definition above, the solution of this problem is

$$\Phi_j(\{\omega_i\}_i) = \int e_j I_j^*(a,e)\ dF^N(a,e)\ \forall j$$

where

$$\left\{\{I_j^*(a,e)\}_j\right\}_{(a,e)} \equiv \arg \max_{\left\{\{I_j(a,e)\}_j\right\}_{(a,e)}} \int \sum_j (a_j + \omega_j e_j)\, I_j(a,e)\ dF^N(a,e)$$

subject to

$$\forall (a,e):\ \sum_j I_j(a,e) = 1,\ \text{and}\ I_j(a,e) \geq 0.$$

To solve this problem, we substitute the first constraint into the objective function to eliminate $I_o(a,e)$ for an arbitrary o. Then, we consider the problem's Lagrangian:

$$\max_{\{I_j(a,e)\geq 0\}_{j\neq o}} \int (a_o + \omega_o e_o)\ dF(a,e) + \int \sum_j (a_j + \omega_j e_j - a_o - \omega_o e_o)\, I_j(a,e)\ dF^N(a,e).$$

The first-order condition of this problem implies that, for all $j \neq o$, $I_j(a,e) = 0$ if $a_o + \omega_o e_o > a_j + \omega_j e_j$. Thus, $I_o(a,e) = 1$ if, and only if, $a_o + \omega_o e_o \geq a_j + \omega_j e_j$. Since o was arbitrarily chosen, we can write

$$\forall i:\ I_j^*(a,e) = 1 \ \Leftrightarrow\ (a,e) \in I_j(\{\omega_i\}_i) \equiv \{(\boldsymbol{a},\boldsymbol{e}):\ a_j + \omega_j e_j \geq a_o + \omega_o e_o\ \forall o\}.$$

Thus, the system of labor supply constraints implies that

$$\Phi_j(\{\omega_i\}_i) = \int_{I_j(\{\omega_i\}_i)} e_j\ dF^N(a,e),$$

and, therefore,

$$\Phi_j(\{\omega_i\}_i) = \Phi_j^N(\{\omega_i\}_i).$$

D.2.5 Spatial Assignment Models with Other Factors in Production

Environment. Consider an economy with a representative household with the preferences in (1)–(10) subject to the budget constraint in (11). We denote an origin sector-region pair as $i \equiv (k,r)$ and a destination sector-region pair as $j \equiv (s,d)$. We impose additional restrictions on preferences to obtain the equivalence result. First, assume that individuals employed in all sectors of region r have identical preferences, $V_{sd}(\cdot) = V_d(\cdot)$, and face identical prices, $p_{kr,sd} = p_{kr,d}$. Second, assume that preferences are such that the labor supply function is invertible (up to a scalar).

In sector-region pair, there is a representative competitive firm that uses labor, L_{kr}^N, and another factor, T_{kr}^N, in production, with the following Cobb-Douglas production function:

$$Y_{kr}^N = \tilde{\zeta}_{kr} \tilde{\Psi}_{kr}^N\left(\left\{L_{sd}^N\right\}_{sd}\right) \left(L_{kr}^N\right)^{\alpha_{kr}^N} \left(T_{kr}^N\right)^{1-\alpha_{kr}^N}. \tag{86}$$

Each region r has an endowment of the other factor, $\bar{T}_r^N$. We assume that the other factor is mobile across sectors within a region, but that it is immobile across regions – like land in spatial models. Similar to Caliendo et al. (2018b), there is a national mutual fund that owns the other factor in all regions. We assume that the local government in region r owns a share κ_r of the national fund, and it transfers all dividends to local residents. In particular, we impose that the per-capita transfer rate to individuals employed in sector k of region r, ρ_{kr}^N, is inversely proportional to the share of labor in the total cost of the sector,

$$\rho_{kr}^N = \rho_r^N / \alpha_{kr}^N. \tag{87}$$

Equilibrium. To characterize the equilibrium, it is useful to work with the adjusted wage rate, $\tilde{w}_{kr}^N \equiv w_{kr}^N / \alpha_{kr}^N$. The representative household's cost minimization problem yields spending share and price indices that are given by, for all s,

$$x_{kr,sd}^N \in X_{kr,sd}\left(\left\{p_{kr,sd}^N\right\}_{kr}\right) = X_{kr,d}\left(\left\{p_{kr,d}^N\right\}_{kr}\right) \quad \text{and} \quad P_{sd}^N = P_{sd}\left(\left\{p_{kr,sd}^N\right\}_{kr}\right) = P_d\left(\left\{p_{kr,dd}^N\right\}_{kr}\right). \tag{88}$$

As in Section (3), the utility maximization problem of the representative household yields the labor supply function. Using the transfer rule in (87), the labor supply in j is

$$L_{sd} \in \Phi_{sd}\left(\left\{\rho_r^N \tilde{\omega}_{kr}^N\right\}_{kr}\right). \tag{89}$$

Thus, the optimization of consumption and labor choice is corresponds directly to the one of the Generalized Economy with transfers such that the budget constraint in market j is $\sum_{kr} c_{kr,sd} p_{kr,sd} = \rho_d^N w_{sd} L_{sd}$.

In addition, the profit maximization problem of firms implies that

$$p_{kr,sd}^N = \tau_{kr,sd} p_{kr}^N$$

where

$$p_{kr}^N = \frac{\tilde{w}_{kr}^N}{\zeta_{kr} \tilde{\Psi}_{kr}^N\left(\left\{L_{sd}^N\right\}_{sd}\right)} \cdot \left(\frac{R_{kr}^N}{\tilde{w}_{kr}^N}\right)^{1-\alpha_{kr}^N}$$

where R_{kr}^N is the price of other factor faced by the producer in sector k of region r, and, abusing notation, $\zeta_{kr} \equiv \tilde{\zeta}_{kr}(1-\alpha_{kr}^N)^{(1-\alpha_{kr}^N)}$.

To obtain the equilibrium level of R_{kr}^N, consider the market clearing condition for the other factor in region r: $\bar{T}_r^N = \sum_k T_{kr}^N = \sum_k \left(1-\alpha_{kr}^N\right) \tilde{w}_{kr}^N L_{kr}^N / R_{kr}^N$. Since the other factor is perfectly mobile across sectors, $R_{kr}^N = R_r^N$ for all k and, therefore,

$$R_r^N = \frac{\sum_k \left(1-\alpha_{kr}^N\right) \tilde{w}_{kr}^N L_{kr}^N}{\bar{T}_r^N}$$

We use this expression to eliminate R_{kr}^N in the expression of $p_{kr,kr}^N$ for sector k in region r. After some manipulation, we obtain

$$p_{kr,kr}^N = \frac{\tilde{w}_{kr}^N}{\tilde{\Psi}_{kr}^N\left(\left\{L_{sd}^N\right\}_{sd}\right)} \left(\frac{1}{\bar{T}_r^N} \sum_s \left(1-\alpha_{sr}^N\right) \frac{\rho_r^N \tilde{\omega}_{sr}^N}{\rho_r^N \tilde{\omega}_{kr}^N} L_{sr}^N\right)^{1-\alpha_{kr}^N}$$

Thus, the invertibility of the labor supply function yields

$$p^N_{kr} = \frac{\tilde{w}^N_{kr}}{\Psi^N_{kr}\left(\left\{L^N_{sd}\right\}_{sd}\right)} \tag{90}$$

with

$$\Psi^N_{kr}\left(\left\{L^N_{sd}\right\}_{sd}\right) \equiv \tilde{\Psi}^N_{kr}\left(\left\{L^N_{sd}\right\}_{sd}\right)\left(\frac{1}{\bar{T}^N_r}\sum_s \left(1-\alpha_{sr}\right)\Phi^{-1}_{kr,sr}\left(\left\{L^N_{sd}\right\}_{sd}\right)L^N_{sr}\right)^{\alpha_i-1}. \tag{91}$$

where we used invertibility of labor supply up to a scalar to write

$$\frac{\rho^N_r\tilde{\omega}^N_{sr}}{\rho^N_r\tilde{\omega}^N_{kr}} = \Phi^{-1}_{kr,sr}\left(\left\{L^N_{sd}\right\}_{sd}\right).$$

To close the equilibrium, we consider the labor market clearing condition that can be written in terms of the revenue share accruing to labor in every sector-region pair:

$$\tilde{w}^N_{kr}L^N_{kr} = \sum_{sd} x^N_{kr,sd}\rho^N_d\tilde{w}^N_{sd}L^N_{sd}. \tag{92}$$

Finally, the transfer rate in region r is determined by its share in the dividend paid by the mutual fund:

$$\kappa_r\sum_{sd}(1-\alpha^N_{sd})\tilde{w}^N_{sd}L^N_{sd} = \sum_k(\rho^N_{kr}-1)\alpha_{kr}\tilde{w}^N_{kr}L^N_{kr} = \sum_k(\rho^N_r-\alpha_{kr})\tilde{w}^N_{kr}L^N_{kr}$$

$$\rho^N_r = \frac{\kappa_r\sum_{sd}(1-\alpha^N_{sd})\tilde{w}^N_{sd}L^N_{sd} + \sum_k\alpha_{kr}\tilde{w}^N_{kr}L^N_{kr}}{\sum_k\tilde{w}^N_{kr}L^N_{kr}} \tag{93}$$

where the left hand side is region r's total transfer payments, and the right hand side is region r's share in the total land revenue in the country.

The equilibrium of this economy is characterized by $\left\{p^N_i, P^N_i, L^N_i, \omega^N_i\right\}$ that solve equations (88)–(92), with $\left(\Phi_j(\cdot), \Psi^N_j(\cdot), X_{ij}(\cdot)\right)$, conditional on the transfer rule $\{\rho_r\}$ in (93).

Equivalence. To establish the equivalence, we consider the Generalized Spatial Model of Section 3, with $\Psi_{kr}(\cdot) = \Psi^N_{kr}(\cdot)$ in (91) and the transfer rule in (93). This establishes that the Generalized Spatial Model is equivalent to spatial assignment models with other factors of production that are mobile across sectors but not across regions – e.g., land and other natural resources. A similar argument yields the equivalence with models with other factors of production that are mobile across both regions and sectors. The only restriction is that the invertibility step to obtain (91) requires the same transfer rate across markets in the country, as in Caliendo et al. (2018b).

D.2.6 Special Case with Mobile Capital

Environment. Consider the simplified economy of Section 2. Assume that assume that preferences are such that the labor supply function is invertible (up to a scalar), so that we can write

$$\frac{w_j}{w_i} = \Phi^{-1}_{i,j}(\boldsymbol{L}). \tag{94}$$

We introduce capital by assuming that the production function takes the following Cobb-Douglas form:

$$Y_i = \frac{1}{\kappa_i}\tau_i \left(L_i\right)^{\alpha_i}\left(K_i\right)^{1-\alpha_i},$$

where $\kappa_i \equiv \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}$.

Assume that capital is fully mobile across regions, so that rent is identical in all regions: $R_i = R$ for all i. There is an exogenous capital endowment in the economy given by $\bar{K}$.

Equilibrium. The cost minimization problem of the firm and the zero profit conditions imply that, in every region i,

$$p_i = \frac{w_i}{\tau_i}\left(\frac{R}{w_i}\right)^{1-\alpha_i}. \tag{95}$$

In this economy, capital market clearing condition requires $R\bar{K} = \sum_i RK_i$. Using the fact that firms spend a share $1-\alpha_i$ of their revenue on capital, we get the following expression for the rent in equilibrium:

$$R = \frac{1}{\bar{K}}\sum_j \frac{1-\alpha_j}{\alpha_j} w_j L_j.$$

Substituting this expression into (95),

$$p_i = \frac{w_i}{\tau_i}\left(\frac{1}{\bar{K}}\sum_j \frac{1-\alpha_j}{\alpha_j}\frac{w_j}{w_i}L_j\right)^{1-\alpha_i},$$

which combined with the inverse labor supply in (94) yields

$$p_i = \frac{w_i}{\tau_i}\left(\frac{1}{\bar{K}}\sum_j \frac{1-\alpha_j}{\alpha_j}\Phi_{i,j}^{-1}\left(\boldsymbol{L}\right)L_j\right)^{1-\alpha_i}.$$

Equivalence. We establish the equivalence with the model of Section 2 by specifying

$$\Psi_i\left(\boldsymbol{L}\right) \equiv \left(\frac{1}{\bar{K}}\sum_j \frac{1-\alpha_j}{\alpha_j}\Phi_{i,j}^{-1}\left(\boldsymbol{L}\right)L_j\right)^{\alpha_i-1}. \tag{96}$$

An illustrative example. To gain intuition for the labor productivity spillovers implied by factor mobility, consider the special case of a gravity labor supply structure: $\Phi_i\left(\boldsymbol{\omega}\right) = \frac{\omega_i^{\phi}}{\sum_j \omega_j^{\phi}}$ such that $\frac{w_j}{w_i} = \left(\frac{L_j}{L_i}\right)^{\frac{1}{\phi}}$. In this case,

$$\Psi_i\left(\boldsymbol{L}\right) = \left[\frac{\bar{K}\left(L_i\right)^{\frac{1}{\phi}}}{\sum_j \frac{1-\alpha_j}{\alpha_j}\left(L_j\right)^{1+\frac{1}{\phi}}}\right]^{1-\alpha_i}$$

Thus, for $i \neq j$, the elasticity of labor productivity in market i to employment in market j is

$$\psi_{ij} \equiv \frac{\partial \log \Psi_i(\boldsymbol{L})}{\partial \log L_j} = -(1-\alpha_i)\frac{\frac{1-\alpha_j}{\alpha_j}(L_j)^{1+\frac{1}{\phi}}}{\sum_{j'}\frac{1-\alpha_{j'}}{\alpha_{j'}}(L_{j'})^{1+\frac{1}{\phi}}}\left(1+\frac{1}{\phi}\right) < 0.$$

Intuitively, since the labor-to-capital spending ration is constant, higher employment in market j triggers an increase in the capital demand in market j, which causes rent prices to increase in the entire economy. The higher capital cost increases the production cost everywhere and, therefore, acts as a congestion on other markets.

D.3 Extensions

D.3.1 Generalized Spatial Model with Multiple Labor Types

Multiple Worker Types. Consider an extension of our model with multiple worker groups – groups are indexed by g and g'. We write the equilibrium in terms of factor-content of trade as in Adao, Costinot, and Donaldson (2017). Each market now is defined as a triple of sector-region-group. We denote origin markets as $i \equiv (k, r, g)$, and destination markets as $j \equiv (s, d, g')$. As before, the representative consumer has preferences over consumption and labor across markets (i.e., sector-region-group markers):

$$U_c\left(\{C_j\}, \{L_j\}_j\right).$$

We assume that the consumption index depends on the factor content of trade from different sectors and regions. That is, the consumption index depends directly on a composite good produced by each sector-region-group triple:

$$C_j = V_j\left(\{c_{ij}\}_i\right)$$

Finally, assume that there is a competitive firm producing the market-level composite good with production function given by

$$Y_i = \Psi_i\left(\{L_o\}_o\right) L_i.$$

All our results remain valid in this environment with spending shares in terms of factor content of trade. That is, x_{ij} is the spending share on the composite good produced in the sector-region-group triple.

Equivalent Armington Economy Multiple Worker Types. To gain intuition for this economy, we now derive preferences in terms of factor content of trade in the case of an Armington economy with multiple labor types. Assume that the representative household has preferences over the allocation of the multiple worker groups across sector-region pairs j,

$$U_c\left(\left\{C_j^g\right\}_{j,g}, \left\{L_j^g\right\}_{j,g}\right).$$

The consumption index is a function of the quantities consumed of goods produced in different origin sector-region pairs i:

$$C_j^g = \tilde{V}_j\left(\left\{c_{ij}^g\right\}_i\right).$$

Assume that each sector-region pair i has a representative firm that combines labor from different worker types with a constant returns to scale technology:

$$Y_i = F_i\left(\{\Psi_i^g(\boldsymbol{L})\, L_i^g\}_g\right)$$

where $F_{ij}(.)$ is homogeneous of degree one.

Thus, as in the equivalence with the Ricardian economy above, we must define preferences of the representative agent that incorporate the technology to produce final goods,

$$C_j = V_j\left(\left\{c_{ij}^g\right\}_{i,g}\right) \equiv \tilde{V}_j\left(\left\{F_i\left(\left\{c_{ij}^g\right\}_g\right)\right\}_i\right),$$

where c_{ij}^g is the amount of "effective" labor of group g in market i used in the production of goods shipped to market j.

let the production technology of "effective" labor of group g in market i be

$$c_i^g = \Psi_i^g(\boldsymbol{L})\, L_i^g.$$

In equilibrium, the production cost of "effective" labor of group g in market i

$$p_i^g = \frac{w_i^g}{\Psi_i^g(\boldsymbol{L})}$$

In this case, the spending share on factor g in sector-region pair i is simply

$$x_{ij}^g = \alpha_i^g\left(\{p_i^g\}_g\right) x_{ij}$$

where $\alpha_i^g\left(\{p_i^g\}_g\right)$ is the share of factor g in the production cost of sector-region pair i, and x_{ij} is the spending share on goods from sector-region pair i.

D.3.2 Generalized Spatial Model with Intermediate Goods in Production

We now derive the decomposition between direct and indirect effects in a model with input-output linkages.

Preferences. On the consumption side, we maintain the same structure of Section 3, in which the representative household preferences yield a market-level price index of

$$P_j^C = P_j^C\left(\boldsymbol{p}_j\right) \equiv \min_{\boldsymbol{c}_j} \sum_o p_{oj} c_{oj} \quad \text{s.t.} \quad V_j\left(\boldsymbol{c}_j\right) = 1, \tag{97}$$

with the associated final spending share on goods from origin i given by

$$x_{ij}^C = X_{ij}^C\left(\boldsymbol{p}_j\right). \tag{98}$$

Also, the utility maximization problem of the representative agent yields the labor supply in any market j:

$$L_j = \Phi_j(\boldsymbol{\omega}). \tag{99}$$

We also maintain the assumption of iceberg trade costs such that

$$p_{ij} = \tau_{ij} p_i \tag{100}$$

Production. The main change is on the production function, which we assume to take the following Cobb-Douglas form between labor and an intermediate input aggregator:

$$Y_i = \frac{1}{\kappa_i} \Psi_i (\boldsymbol{L}) (L_i)^{\varpi_i} (M_i)^{1-\varpi_i},$$

where $\kappa_i = \varpi_i^{\varpi_i} (1-\varpi_i)^{1-\varpi_i}$, and M_i is an index of intermediate inputs used in production:

$$M_i = F_i \left(\{M_{ji}\}_j \right).$$

In this case, the cost minimization problem of the representative firm implies that the zero profit condition is

$$p_i = \frac{(w_i)^{\varpi_i} \left(P_i^M\right)^{1-\varpi_i}}{\Psi_i (\boldsymbol{L})} \tag{101}$$

where

$$P_i^M = P_i^M (\boldsymbol{p}_i) \equiv \min_{\boldsymbol{M}_i} \sum_o p_{ji} M_{ji} \quad \text{s.t.} \quad F_i \left(\{M_{ji}\}_j \right) = 1 \tag{102}$$

with associated input spending shares given by

$$x_{ji}^M = X_{ji}^M \left(\{p_{ji}\}_j \right) \equiv \frac{\partial \ln P_i^M}{\partial \ln p_{ji}}. \tag{103}$$

Market clearing. To close the model, consider the market clearing condition for labor in each market. The total revenue of market i from sales in market j is

$$X_{ij} = x_{ij}^C w_j L_j + x_{ij}^M (1-\varpi_j) \sum_d X_{jd}$$

$$X_{ij} = x_{ij}^C w_j L_j + x_{ij}^M \frac{1-\varpi_j}{\varpi_j} w_j L_j$$

$$X_{ij} = \left(x_{ij}^C + x_{ij}^M \frac{1-\varpi_j}{\varpi_j} \right) w_j L_j$$

Thus,

$$\frac{1}{\varpi_i} w_i L_i = \sum_j \left(\varpi_j x_{ij}^C + x_{ij}^M (1-\varpi_j) \right) \frac{1}{\varpi_j} w_j L_j. \tag{104}$$

Equilibrium. The equilibrium entails $\{w_i, P_i, L_i, p_i\}$ that satisfy (97)–(104) given $p_m \equiv 1$ for a reference market.

There are two points that are worth mentioning. The equilibrium requires knowledge of the labor share, ϖ_i, and the cost function, $F_i(.)$ (which determines the producer price index $P_i^M(\cdot)$ and the intermediate spending shares $\pi_{ij}(\cdot)$). Second, this environment is a generalization of the model in Caliendo and Parro (2014) that imposes a gravity structure on the demand for final products, $X_{ij}(\cdot)$, and intermediate products, $X_{ij}^M(\cdot)$. In particular, their model assumes that final and intermediate consumption is identical within each sector, but have different sector-level spending shares.

To write the labor and the trade modules, we combine first equations (99) and (101):

$$\log \omega_i = \frac{1}{\varpi_i} \log Q_i + \frac{1}{\varpi_i} \log \Psi_i (\boldsymbol{\Phi} (\boldsymbol{\omega})), \tag{105}$$

where

$$Q_i = \frac{p_i}{\left(P_i^C\right)^{\varpi_i}\left(P_i^M\right)^{1-\varpi_i}} \tag{106}$$

with P_i^C given by (97) and P_i^M given by (102).

The trade module follows from the combination of (101) and (104):

$$\left[\frac{p_i\Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\omega}\right)\right)}{\left(P_i^M\right)^{1-\varpi_i}}\right]^{\frac{1}{\varpi_i}} \frac{\Phi_i(\boldsymbol{\omega})}{\varpi_i} = \sum_j X_{ij}\left(\{p_{oj}\}_o\right)\left[\frac{p_j\Psi_{ij}\left(\boldsymbol{\Phi}\left(\boldsymbol{\omega}\right)\right)}{\left(P_j^M\right)^{1-\varpi_j}}\right]^{\frac{1}{\varpi_i}} \frac{\Phi_j(\boldsymbol{\omega})}{\varpi_j} \tag{107}$$

with P_i^C given by (97), P_i^M given by (102), and

$$X_{ij}\left(\{p_{oj}\}_o\right) \equiv X_{ij}^C\left(\{p_{oj}\}_o\right)\varpi_j + X_{ij}^M\left(\{p_{oj}\}_o\right)(1-\varpi_j)$$

Decomposition of direct and indirect effects. In terms of the modified competitiveness measure, we have the same labor market module equation:

$$\log\hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}}\log\hat{\boldsymbol{Q}} \tag{108}$$

with $\bar{\boldsymbol{\beta}} = \left(\bar{\boldsymbol{\varpi}} - \bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}\right)^{-1}$ and $\bar{\boldsymbol{\varpi}}$ is a diagonal matrix with entries ϖ_i.

We also have that

$$\log\hat{\boldsymbol{Q}} = \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\bar{\boldsymbol{x}}^C - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\bar{\boldsymbol{x}}^M\right)\log\hat{\boldsymbol{p}} - \bar{\boldsymbol{\varpi}}\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}}) - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}}) \tag{109}$$

where the consumption and the production cost exposure are given by

$$\log\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) \equiv \sum_j x_{ji}^C\log\hat{\tau}_{ji}, \tag{110}$$

$$\log\hat{\eta}_i^M(\hat{\boldsymbol{\tau}}) \equiv \sum_j x_{ji}^M\log\hat{\tau}_{ji}. \tag{111}$$

Notice that, if the production and the consumption shares are the same $x_{ji} = x_{ji}^M = x_{ji}^C$, then $\log\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) = \log\hat{\eta}_i^M(\hat{\boldsymbol{\tau}})$.

Define $\chi_{oij} \equiv \frac{\partial\log X_{ij}(\{p_{oj}\}_o)}{\partial\log p_{oj}}$, with the associate matrix $\bar{\boldsymbol{\chi}} \equiv [\sum_d y_{id}\chi_{jid}]_{i,j}$. As before, we define the revenue exposure as

$$\log\hat{\eta}_i^R(\hat{\boldsymbol{\tau}}) \equiv \sum_j\sum_o y_{ij}^0\chi_{oij}\log\hat{\tau}_{oj}. \tag{112}$$

Thus, the trade module yields

$$\left[\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}} - \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\varpi}}\right]\bar{\boldsymbol{\varpi}}^{-1}\log\hat{\boldsymbol{p}} + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\varpi}}^{-1}\bar{\boldsymbol{\psi}}\right)\bar{\boldsymbol{\phi}}\log\hat{\boldsymbol{\omega}} = \log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\bar{\boldsymbol{\varpi}}^{-1}\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\hat{\boldsymbol{P}}^M$$

Let us define

$$\bar{\boldsymbol{x}} \equiv \bar{\boldsymbol{\varpi}}\bar{\boldsymbol{x}}^C + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\bar{\boldsymbol{x}}^M$$

$$\bar{\boldsymbol{\mu}}^M \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\bar{\boldsymbol{\varpi}}^{-1}$$

$$\bar{\boldsymbol{\mu}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\varpi}}^{-1}\bar{\boldsymbol{\psi}}\right)\bar{\boldsymbol{\phi}}$$

$$\bar{\boldsymbol{\gamma}} \equiv \left[\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}} - \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\varpi}} + \bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right)\bar{\boldsymbol{\varpi}} - \bar{\boldsymbol{\mu}}^M\bar{\boldsymbol{x}}^M\right]\bar{\boldsymbol{\varpi}}^{-1}$$

By substituting (108) and (109) into the expression above, we obtain

$$\bar{\boldsymbol{\gamma}}\log\hat{\boldsymbol{p}} = \log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\varpi}}\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}}) + \left(\bar{\boldsymbol{\mu}}^M + \bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}\right)\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}}).$$

Applying this expression into (109),

$$\log\hat{\boldsymbol{Q}} = \bar{\boldsymbol{\alpha}}^R\log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^C\bar{\boldsymbol{\varpi}}\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^M\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}}) \tag{113}$$

where $\bar{\boldsymbol{\alpha}} \equiv \bar{\boldsymbol{M}}'\left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'\right)^{-1}\bar{\boldsymbol{M}}$, $\bar{\boldsymbol{\alpha}}^R \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right)\bar{\boldsymbol{\alpha}}$, $\bar{\boldsymbol{\alpha}}^C \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{\alpha}}^R\bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}$, and $\bar{\boldsymbol{\alpha}}^M \equiv \bar{\boldsymbol{\alpha}}^C - \bar{\boldsymbol{\alpha}}^R\bar{\boldsymbol{\mu}}^M$.

Thus, equations (108) and (113) yield

$$\log\hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}}\left[\bar{\boldsymbol{\alpha}}^R\log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^C\bar{\boldsymbol{\varpi}}\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}},\hat{\boldsymbol{\zeta}}) - \bar{\boldsymbol{\alpha}}^M\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}})\right] \tag{114}$$

Notice that if $x_{ji} = x_{ji}^M = x_{ji}^C$ then $\log\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) = \log\hat{\eta}_i^M(\hat{\boldsymbol{\tau}})$, as discussed above, so that then the relationship can be written as

$$\log\hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}}\left[\bar{\boldsymbol{\alpha}}^R\log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) - \left[\bar{\boldsymbol{\alpha}}^C\bar{\boldsymbol{\varpi}} + \bar{\boldsymbol{\alpha}}^M\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\right]\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}})\right].$$

Thus, under the assumption of $x_{ji} = x_{ji}^M = x_{ji}^C$, the model with intermediate inputs can generate the same counterfactuals as a model without intermediate inputs, as long as the elasticities of the models with and without the intermediates are set to be the same.

An illustrative example. To see this point more clearly, we consider a simple example that draws on the model of Section 2. In particular, we assume the presence of a single homogeneous good as in Section 2 such that the production function with intermediate goods is

$$Y_i = \tau_i\Psi_i\left(\boldsymbol{L}\right)\left(L_i\right)^{\varpi}\left(M_i\right)^{1-\varpi}.$$

The derivations above yield the following expression for the labor market module:

$$\log\boldsymbol{\omega} - \varpi^{-1}\log\boldsymbol{\Psi}\left(\boldsymbol{\Phi}\left(\boldsymbol{\omega}\right)\right) = \varpi^{-1}\log\boldsymbol{\tau}.$$

Consider the case of log-linear functions of agglomeration and labor supply: $\Phi_i\left(\boldsymbol{\omega}\right) = \omega_i^{\phi}$ and $\Psi_i(\boldsymbol{L}) = L_i^{\psi}$. Thus,

$$\log\omega_i = \frac{\varpi^{-1}}{1 - \varpi^{-1}\psi\phi}\log\tau_i = \frac{1}{\varpi - \psi\phi}\log\tau_i.$$

The interpretation of this condition is that, for given elasticities ϕ and ψ, a lower value of ϖ (higher share of intermediates) means a stronger response of labor outcomes to economic shocks. However, the response of real wages to shocks in τ_i is going to be the same if the aggregate elasticity $(\varpi - \psi\phi)^{-1}$ is set to be the same across models. This is a similar point to the one made by Allen, Arkolakis, and Takahashi (2018) in that, for certain counterfactuals, the predictions of a spatial model with respect to fundamentals may be the same with intermediate inputs or not as long as some aggregate elasticities are set to be invariant across models.

D.3.3 Generalized Spatial Model with Commuting

We now define a Generalized Spatial Competitive Economy with Commuting between markets.

Preferences. We assume that the representative household has preferences over consumption and labor for individuals residing in market j and commuting to market d:

$$U\left(\{C_{jd}\}_{j,d}, \{L_{jd}\}_{j,d}\right)$$

where L_{jd} is the mass of workers residing in market j and working on market d, and C_{jd} denoting the associated consumption index of these workers.

We assume that individuals consume in their market of residence. For labor in market j commuting to d, the homothetic consumption index is

$$C_{jd} = V_j\left(\{c_{ijd}\}_i\right)$$

and the budget constraint is

$$\sum_i p_{ij} c_{ijd} = w_d L_{jd}.$$

As in the baseline model, the first-stage problem yields the price index and the spending shares,

$$P_j\left(\{p_{ij}\}_i\right) \quad \text{and} \quad X_{ij}\left(\{p_{oj}\}_o\right). \tag{115}$$

Notice that, because $V_j(.)$ does not vary with the commuting destination, the price index and the spending shares do not vary with the commuting destination. This implies that

$$\sum_i p_{ij} c_{ijd} = P_j C_{jd} \quad \Rightarrow \quad C_{jd} = \frac{w_d}{P_j} L_{jd} = \omega_{jd} L_{jd}$$

where $\omega_{jd} = w_d / P_j$ is the real wage of working in market d and residing in market j.

Thus, the second-stage problem is

$$\max_{\{L_{jd}\}_{jd}} U\left(\{(\omega_{jd}) L_{jd}\}_{j,d}, \{L_{jd}\}_{j,d}\right)$$

which yields the labor supply mapping,

$$L_{jd} \in \Phi_{jd}(\boldsymbol{\omega}, \boldsymbol{P}) \equiv \Phi_{jd}\left(\left\{\omega_i \frac{P_i}{P_o}\right\}_{oi}\right). \tag{116}$$

Production. As in the baseline model, we consider the profit maximization problem of firms in market i yields the same equilibrium conditions

$$p_{ij} = \tau_{ij} p_i, \tag{117}$$

$$p_i = \frac{w_i}{\Psi_i\left(\{L_{jd}\}_{j,d}\right)}. \tag{118}$$

Notice that we all agglomeration to depend on the entire vector of commuting flows, $\{L_{ij}\}_{i,j}$. This general formulation covers two possible specifications of agglomeration forces. When agglomeration depends only on employment in each market, $\Psi_i\left(\{L_{jd}\}_{j,d}\right) = \Psi_i\left(\left\{\sum_j L_{jd}\right\}_d\right)$. Alternatively, when agglomeration depends only on residence population in each market, we have that $\Psi_i\left(\{L_{jd}\}_{j,d}\right) = \Psi_i\left(\{\sum_d L_{jd}\}_j\right)$.

Market clearing. To close the model, we consider the labor market clearing condition: total labor payments to labor in market i equals total revenue of market i from selling to all other markets in the world economy. That is,

$$\sum_o w_i L_{oi} = \sum_j x_{ij}\left(\sum_d w_d L_{jd}\right). \tag{119}$$

Equilibrium. The competitive equilibrium in this economy corresponds to $\{p_i, w_i, P_j, L_{ij}\}$ such that conditions (115)–(119) hold. In this case, we need to extend the notion of labor supply to capture commuting flows across markets. In other words, counterfactual predictions require knowledge of the extended labor supply mapping with between-market worker commuting flows, $L_{jd} \in \Phi_{jd}\left(\{\omega_{oi}\}_{oi}\right)$.

Let bold variable with a tilde denote the $N^2 \times 1$ vector of stacked market-to-market vector, with $\tilde{\boldsymbol{L}} \equiv \{L_{jd}\}_{jd}$ and $\tilde{\boldsymbol{\omega}} \equiv \{\omega_{jd}\}_{jd}$.

Using this notation, the combination of equations (116) and (118) yields the labor market module

$$\omega_i = \frac{p_i}{P_i}\Psi_i\left(\{\Phi_{jd}\left(\boldsymbol{\omega}, \boldsymbol{P}\right)\}_{jd}\right) \tag{120}$$

The combination of (118) and (119) yields the trade module:

$$p_i \Psi_i\left(\tilde{\boldsymbol{\Phi}}\left(\tilde{\boldsymbol{\omega}}\right)\right)\sum_o \Phi_{oi}\left(\tilde{\boldsymbol{\omega}}\right) = \sum_j x_{ij}\left(\sum_d p_d \Psi_d\left(\tilde{\boldsymbol{\Phi}}\left(\tilde{\boldsymbol{\omega}}\right)\right)\Phi_{jd}\left(\tilde{\boldsymbol{\omega}}\right)\right), \tag{121}$$

where the price index and the spending shares are given by (115).

Decomposition of direct and indirect effects. We now log-linearize the system to obtain the decomposition into direct and indirect spillover effects. The labor market module in (116) implies that

$$\log\hat{\omega}_i = \log\hat{p}_i - \log\hat{P}_i + \sum_{jd}\psi_{i,jd}\sum_l\sum_o \phi_{jd,ol}\left(\log\hat{\omega}_l + \log\hat{P}_l - \log\hat{P}_o\right)$$

$$\log\hat{\omega}_i - \sum_{jd}\psi_{i,jd}\sum_l\left(\sum_o \phi_{jd,ol}\right)\log\hat{\omega}_l = \log\hat{p}_i - \log\hat{P}_i + \sum_{jd}\psi_{i,jd}\sum_l\left(\sum_o\left(\phi_{jd,ol} - \phi_{jd,lo}\right)\right)\log\hat{P}_l$$

In matrix form, we write

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^{\omega}\right)\log\hat{\boldsymbol{\omega}} = \log\hat{\boldsymbol{p}} - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^{P}\right)\log\hat{\boldsymbol{P}}$$

where

$$\bar{\boldsymbol{\psi}} = [\psi_{i,jd}]_{i,jd} \quad \bar{\boldsymbol{\phi}}^{\omega} \equiv [\sum_o \phi_{jd,ol}]_{jd,l} \quad \bar{\boldsymbol{\phi}}^{P} \equiv [\sum_o(\phi_{jd,ol} - \phi_{jd,lo})]_{jd,l}.$$

We now have two elasticity matrices of commuting flows: $\bar{\boldsymbol{\phi}}^{\omega}$ and $\bar{\boldsymbol{\phi}}^{P}$. First, a change in the real wage of market l affects the payoff of all commuting flows with destination l and, therefore, has a total effect on the flow in jd of $\phi^{\omega}_{jd,l} \equiv \sum_o \phi_{jd,ol}$. Second, conditional on real wages, a change in the price index of market l has an effect on the payoff of all pairs with an origin effect in l, generating a total response in the jd flow of $\phi^{P}_{jd,l} \equiv \sum_o (\phi_{jd,ol} - \phi_{jd,lo})$.

Recalling that $\log \hat{\boldsymbol{P}} = \log \hat{\boldsymbol{\eta}}^C + \bar{\boldsymbol{x}} \log \hat{\boldsymbol{p}}$, we get that

$$\log \hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\pi}} \bar{\boldsymbol{x}} \right) \log \hat{\boldsymbol{p}} - \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} \log \hat{\boldsymbol{\eta}}^C \tag{122}$$

where we define

$$\bar{\boldsymbol{\beta}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^{\omega} \right)^{-1} \quad \text{and} \quad \bar{\boldsymbol{\pi}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^{P} \right).$$

From the trade module in

$$\begin{gathered}
\log \hat{p}_i + \sum_{jd} \psi_{i,jd} \phi^{\omega}_{jd,l} \log \hat{\omega}_l + \sum_{jd} \psi_{i,jd} \phi^{P}_{jd,l} \log \hat{P}_l + \sum_o \tfrac{L_{oi}}{\sum_l L_{li}} \left(\phi^{\omega}_{oi,l} \log \hat{\omega}_l + \phi^{P}_{oi,l} \log \hat{P}_l \right) = \\
\log \eta^R_i + \sum_o \left(\sum_j y_{ij} \chi_{oij} \right) \log \hat{p}_o + \log \hat{p}_d + \sum_j y_{ij} \sum_d \tfrac{L_{jd}}{\sum_o L_{jo}} \left(\sum_{ko} \psi_{d,ko} \phi^{\omega}_{ko,l} \log \hat{\omega}_l + \sum_{ko} \psi_{d,ko} \phi^{P}_{ko,l} \log \hat{P}_l \right) \\
+ \sum_j y_{ij} \sum_d \tfrac{L_{jd}}{\sum_o L_{jo}} \left(\phi^{\omega}_{jd,l} \log \hat{\omega}_l + \phi^{P}_{jd,l} \log \hat{P}_l \right)
\end{gathered}$$

Thus,

$$\begin{gathered}
\log \hat{\boldsymbol{p}} + \left(\bar{\boldsymbol{\psi}} + \bar{\boldsymbol{L}}^E \right) \left(\bar{\boldsymbol{\phi}}^{\omega} \log \hat{\boldsymbol{\omega}} + \bar{\boldsymbol{\phi}}^{P} \log \hat{\boldsymbol{P}} \right) = \log \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\chi}} \log \hat{\boldsymbol{p}} \\
+ \bar{\boldsymbol{y}} \bar{\boldsymbol{L}} \left(\log \hat{\boldsymbol{p}} + \bar{\boldsymbol{\psi}} \left(\bar{\boldsymbol{\phi}}^{\omega} \log \hat{\boldsymbol{\omega}} + \bar{\boldsymbol{\phi}}^{P} \log \hat{\boldsymbol{P}} \right) \right) + \bar{\boldsymbol{y}} \bar{\boldsymbol{L}}^R \left(\bar{\boldsymbol{\phi}}^{\omega} \log \hat{\boldsymbol{\omega}} + \bar{\boldsymbol{\phi}}^{P} \log \hat{\boldsymbol{P}} \right)
\end{gathered}$$

where $\bar{\boldsymbol{L}} = [L_{ij} / \sum_o L_{io}]_{i,j}$, $\bar{\boldsymbol{L}}^R = [L^R_{i,jd}]_{i,jd}$ with $L^R_{j,od} = L_{od} / \sum_i L_{ji} 1[j = o]$, and $\bar{\boldsymbol{L}}^E = [L^E_{i,jd}]_{i,jd}$ with $L^E_{i,jd} = L_{jd} / \sum_o L_{oi} 1[i = d]$.

Rearranging the expression above,

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}} \bar{\boldsymbol{L}} \right) \log \hat{\boldsymbol{p}} = \log \hat{\boldsymbol{\eta}}^R - \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \log \hat{\boldsymbol{\omega}} + \bar{\boldsymbol{\phi}}^{P} \log \hat{\boldsymbol{P}} \right)$$

with $\bar{\boldsymbol{\mu}} \equiv \bar{\boldsymbol{\psi}} + \bar{\boldsymbol{L}}^E - \bar{\boldsymbol{y}} \left(\bar{\boldsymbol{L}} \bar{\boldsymbol{\psi}} + \bar{\boldsymbol{L}}^R \right)$.

Using (122),

$$\bar{\boldsymbol{\gamma}} \log \hat{\boldsymbol{p}} = \log \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} - \bar{\boldsymbol{\phi}}^{P} \right) \log \hat{\boldsymbol{\eta}}^C$$

with $\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}} \bar{\boldsymbol{L}} + \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\pi}} \bar{\boldsymbol{x}} \right) + \bar{\boldsymbol{\phi}}^{P} \bar{\boldsymbol{x}} \right)$.

By combining this expression and (122),

$$\log \hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\pi}} \bar{\boldsymbol{x}} \right) \left(\bar{\boldsymbol{\alpha}} \log \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\alpha}} \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} - \bar{\boldsymbol{\phi}}^{P} \right) \log \hat{\boldsymbol{\eta}}^C \right) - \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} \log \hat{\boldsymbol{\eta}}^C,$$

which implies that

$$\log \hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{\alpha}}^R \log \hat{\boldsymbol{\eta}}^R - \bar{\boldsymbol{\alpha}}^C \log \hat{\boldsymbol{\eta}}^C \right) \tag{123}$$

where $\bar{\boldsymbol{\alpha}} \equiv \bar{\boldsymbol{M}}' \left(\bar{\boldsymbol{M}} \bar{\boldsymbol{\gamma}} \bar{\boldsymbol{M}}' \right)^{-1} \bar{\boldsymbol{M}}$, $\bar{\boldsymbol{\alpha}}^R \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\pi}} \bar{\boldsymbol{x}} \right) \bar{\boldsymbol{\alpha}}$, $\bar{\boldsymbol{\alpha}}^C \equiv \bar{\boldsymbol{\pi}} - \bar{\boldsymbol{\alpha}}^R \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} - \bar{\boldsymbol{\phi}}^{P} \right)$.