ONLINE APPENDIX

Does Parental Quality Matter?

Evidence on the Transmission of Human Capital Using Variation in Parental Influence from Death, Divorce, and Family Size

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Appendix A

This appendix presents a simple model to organize our empirical findings. We highlight the challenges in disentangling the mechanisms by which parental education affects their children's human capital and how each layer of our analysis can shed light on potential mechanisms.

Model Structure

The model characterizes parental investments in their children's human capital, k, during a single period of childhood, which determines the child's test performance at the end of childhood (age 18).

Parents choose their own consumption, c_j, time with the child, l_j, which are both parent-specific, $j \in \{m, f\}$; and an amount to invest financially in the child, e. We assume that household utility at time t is a Cobb-Douglas function of each parent's consumption, time with the child, and the continuation value of the child's human capital, k.[1] Denoting the parameters of the household utility function by α's yields:

$$u(l_m, l_f, c_m, c_f, k) = \alpha_{lm} \ln l_m + \alpha_{lf} \ln l_f + \alpha_{cm} \ln c_m + \alpha_{cf} \ln c_f + \alpha_k \ln k.$$

Human capital at the end of childhood, k, depends on human capital at birth, k_0, and is a Cobb-Douglas function of the time spent by each parent with the child (provided that both parents are alive), the financial investments in the child, e, and a total factor productivity (TFP) term $R(s_m, s_f)$, which depends on each parent's schooling, s_m and s_f. In our context, TFP represents the quality of parenting conditional on the amount of time and money invested in the child. We assume TFP to be a non-decreasing function of each parent's schooling. Denoting the parameters of the production function by δ's yields:

$$lnk = R(s_m, s_f) + s_m^{\theta} \delta_m lnl_m + s_f^{\theta} \delta_f lnl_f + \delta_e lne + \delta_k lnk_0.$$

[1] Our model is inspired by that in Del Boca et. al. (2014), but we modify their model in a number of ways. We include separate consumption variables for each parent (Del Boca et. al. (2014) includes a single, joint consumption variable). We also allow time with the child to affect utility (as well as the child's human capital) and for one parent to potentially derive more utility from time with children than the other (the formulation in Del Boca et. al. (2014) is the case where $\alpha_{lm} = \alpha_{lf} = 0$).

This formulation allows for a parent's schooling to affect both TFP (through $R(s_m, s_f)$) as well as the marginal returns to time investments (through s_j^{θ}).[2] Genetics are a confounding factor in many studies of the effect of parental schooling on children's human capital. In this setup, genetics are captured by the initial level of human capital, k_0, which is allowed to be correlated with s_m and s_f.

Model Solution

Parents maximize their utility subject to the law of motion for human capital and the time and budget constraints. The time constraint is:

$$T = l_j + h_j, \quad for\ j \in \{m, f\},$$

where h_j denotes hours worked by parent *j*. The budget constraint is given by:

$$c_m + c_f + e = w_m h_m + w_f h_f + I,$$

where I denotes unearned income and w_j denotes the wage of parent *j*. Although we do not write it explicitly, we assume that w_j is an increasing function of s_j.

Assuming that both parents are at interior solutions of their work decision, the optimal investments will be characterized by:

$$l_m = \varphi(\alpha_{lm} + \alpha_k s_m^{\theta} \delta_m)\left(\frac{w_m + w_f}{w_m} T + \frac{I}{w_m}\right)$$

$$l_f = \varphi(\alpha_{lf} + \alpha_k s_f^{\theta} \delta_f)\left(\frac{w_m + w_f}{w_f} T + \frac{I}{w_f}\right)$$

$$e = \varphi \alpha_k \delta_e [(w_m + w_f) T + I]$$

where[3] $\varphi \equiv [\alpha_{lm} + \alpha_{lf} + \alpha_{cm} + \alpha_{cf} + \alpha_k(s_m^{\theta} \delta_m + s_f^{\theta} \delta_f + \delta_e)]^{-1}$.

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[2] This formulation allows for, e.g. mother's schooling to have a greater effect on TFP than father's schooling as would arise if mothers are more likely to make decisions that affect children.
[3] Appendix B provides a range of additional results including corner solutions for parental time at work.

These expressions follow a standard Cobb-Douglas structure and are quite intuitive. Since the demand for financial investments in children is derived and the demand for time with children has direct and derived components, the Cobb-Douglas function that determines parental utility from investing in their children contains composite parameters. The sum of the underlying and composite parameters, including those for consumption, is given by φ^{-1}.

Consider the choice of the financial investment in the child, *e*. The share of total potential income spent on financial investments is given by $\varphi\alpha_k\delta_e$, where α_k affects the utility from the child's human capital, δ_e represents the productivity of financial investments, and φ^{-1} is the sum of the underlying and composite parameters. The optimal time for parent *j* to invest in the child is similar. It reflects total potential income; utility obtained directly from time with the child (α_{lm} or α_{lf}); the value of the child's human capital, α_k; and the productivity of time with the child, $s_j^{\theta}\delta_j$. The optimal time investment is decreasing in parent *j*'s own wage rate (holding own schooling constant), and increasing in the spouse's wage rate and unearned income. Additionally, if $\theta > 0$, when one parent's schooling increases, that parent's productivity and time with the child increase, φ decreases, and other investments (e.g. the other parent's time with the child and, holding income constant, expenditures on the child) decline. (By contrast, if schooling does not increase the productivity of parental time with children ($\theta = 0$), the model implies that parental time with children decreases with schooling because of the indirect effect of schooling on the wage rate.)

The optimized value of the child's human capital at the end of childhood is:

$$lnk = R(s_m, s_f) + s_m^{\theta}\delta_m ln\left[\varphi\left(\alpha_{lm} + \alpha_k s_m^{\theta}\delta_m\right)\left(\frac{w_m+w_f}{w_m}T + \frac{I}{w_m}\right)\right] + s_f^{\theta}\delta_f ln\left[\varphi\left(\alpha_{lf} + \alpha_k s_f^{\theta}\delta_f\right)\left(\frac{w_m+w_f}{w_f}T + \frac{I}{w_f}\right)\right] + \delta_e ln\left[\varphi\alpha_k\delta_e\left[\left(w_m + w_f\right)T + I\right]\right] + \delta_k lnk_0. \quad (1)$$

Implication 1: The child's human capital is increasing in the schooling level of each parent, but the effect of parental schooling cannot in general be separated from the effect of genetics represented by k_0 using observational data on children's human capital and parental schooling.

Implication 1 states one of the primary challenges to the identification of the causal effect of parental schooling in the literature, which can be seen from equation (1). Specifically, if parental schooling is related to k_0, then a regression of children's human capital on parental schooling will be biased upward.

Parental Death

To estimate the causal effect of parental schooling on a child's human capital, and to shed light on potential mechanisms, we examine the occurrence of a parental death during childhood. In particular, we compare a child's human capital when no parent dies to the case when one parent dies during their formative years. (The very rare cases where both parents die during childhood are not considered.) We will also leverage variations in the timing of parental death during childhood. To do this with our simple single-period model, we consider a child's human capital to be a weighted average of their human capital under two scenarios: (1) their human capital with two living parents; and (2) the case where one parent dies and one parent survives. The weights are determined by the age of the child when one parent dies. In the limit case, a child who loses a parent very late in childhood is approximated by a child who always lived with both parents.[4]

In order to incorporate parental death into the structure of the model above, we eliminate time with children, time at work, and the consumption of the deceased parent from the utility function, budget constraint, and the child's human capital production function. We also drop the deceased parent's schooling from TFP. Without loss of generality, we let "d" denote the parent who dies and "a" the parent who remains alive. We assume that:

$$\tilde{u}(\tilde{l}_a, \tilde{c}_a, \tilde{k}_a) = \alpha_{la} \ln \tilde{l}_a + \alpha_{ca} \ln \tilde{c}_a + \alpha_k \ln \tilde{k}_a$$

and

[4] A 2-period model is sketched in Appendix B.

$$ln\tilde{k}_a = Q(s_a) + s_a^\theta \delta_a ln\tilde{l}_a + \delta_e ln\tilde{e}_a + \delta_k ln\tilde{k}_{a0}$$

Here, we denote variables for children with only one parent by a tilde and a subscript for the surviving parent. We replace $R(\cdot,\cdot)$ by $Q(\cdot)$ to account for unobserved changes to the human capital production as a result of a parent's death, as well as any effects of grief ($R(s_m, s_f) - Q(s_a)$). Because children who lose a parent may differ from those who do not in terms of their socioeconomic background and genetics, we allow the initial human capital of a child who loses parent d, $\tilde{k}_{a0}$, to differ from those who do not, k_0. The time constraint for the surviving parent remains the same, while the budget constraint becomes:

$$\tilde{c}_a + \tilde{e} = w_a\tilde{h}_a + \tilde{I}.$$

$\tilde{I}$ reflects both unearned income and survivor benefits, which may be generous, but generally only partially compensate for the income loss of the deceased parent.[5] The optimized investments are given by:

$$\tilde{l}_a = \tilde{\varphi}(\alpha_{la} + \alpha_k s_a^\theta \delta_a)\left(T + \frac{\tilde{I}}{w_a}\right)$$

$$\tilde{e} = \tilde{\varphi}\alpha_k\delta_e(w_a T + \tilde{I})$$

where $\tilde{\varphi} \equiv [\alpha_{la} + \alpha_{ca} + \alpha_k(s_a^\theta \delta_a + \delta_e)]^{-1}$. Substituting these values into the production function yields:

$$ln\tilde{k}_a = Q(s_a) + s_a^\theta \delta_a \ln\left[\tilde{\varphi}(\alpha_{la} + \alpha_k s_a^\theta \delta_a)\left(T + \frac{\tilde{I}}{w_a}\right)\right] + \delta_e \ln[\tilde{\varphi}\alpha_k\delta_e(w_a T + \tilde{I})] + \delta_k ln\tilde{k}_{a0} .$$

[5] There are two sources of survivor benefits in Israel. In the period covered by our data, people had the option to purchase insurance to cover most of their salary (overtime and other supplemental pay were not insurable). These benefits provided 60% of the insured amount plus 20% for each child up to two children. Due to the progressivity of the tax code and because the insurance premium is tax deductible, high income people tend to insure more than low income people. The cap for the tax credit on the insurance is around the 90th percentile of personal income. The state also pays an allowance that depends on the number of children, but not on the deceased parents' earnings. These benefits are on the order of 20% of average earnings and are decreasing in the amount of insurance received.

The difference in human capital between children with both parents and children who lose a parent is given by the difference between equation (1) and the preceding expression for $ln\tilde{k}_a$. As indicated, we use this difference to approximate losing a parent late in childhood relative to earlier. Formally,

$$lnk - ln\tilde{k}_a = R(s_m, s_f) - Q(s_a) + s_a^\theta \delta_a ln\left[\varphi\left(\frac{w_a + w_d}{w_a} T + \frac{I}{w_a}\right)\right] + s_d^\theta \delta_d ln\left[\varphi\left(\alpha_{ld} + \alpha_k s_d^\theta \delta_d\right)\left(\frac{w_a + w_d}{w_d} T + \frac{I}{w_d}\right)\right] + \delta_e ln\left[\varphi[(w_a + w_d)T + I]\right] - s_a^\theta \delta_a ln\left[\tilde{\varphi}\left(T + \frac{\tilde{I}}{w_a}\right)\right] - \delta_e ln[\tilde{\varphi}(w_a T + \tilde{I})] + \delta_k\left(lnk_0 - ln\tilde{k}_{a0}\right) \quad (2)$$

Parental Schooling and Death

We now focus on how parental death affects the relationship between each parent's schooling and the child's human capital. We perform two comparisons. We look first at how the relationship between a child's human capital and a parent's schooling depends on whether that parent survives versus dies. Afterwards, we study how the relationship between a child's human capital and a parent's schooling depends on whether the *other* parent dies or survives. For the first analysis, we differentiate (2) with respect to the schooling of the deceased parent:

$$\frac{d\left(lnk - ln\tilde{k}_a\right)}{ds_d} = \frac{\partial R}{\partial s_d} + \theta s_d^{\theta-1}\delta_d lnl_d + s_d^\theta \delta_d \frac{\alpha_k \theta s_d^{\theta-1}\delta_d}{\alpha_{ld} + \alpha_k s_d^\theta \delta_d} - \alpha_k \theta s_d^{\theta-1}\delta_d \varphi\left(s_a^\theta \delta_a + s_d^\theta \delta_d + \delta_e\right) + \left[\left(s_a^\theta \delta_a + s_d^\theta \delta_d + \delta_e\right)\frac{T}{(w_a + w_d)T + I} - \frac{s_d^\theta \delta_d}{w_d}\right]\frac{\partial w_d}{\partial s_d} \quad (3)$$

At an empirical level, under the assumption that the relationship between a parent's schooling and the child's initial human capital does not depend on whether the parent dies, we can difference out initial human capital (genetics) by comparing the relationship between parental schooling and children's human capital for children who have two surviving parents to those who lose a parent. Thus, the difference in the relationship between a child's outcome and a parent's schooling for a parent who himself (or herself) dies versus survives can provide a causal estimate of the effect of parental schooling on children's outcomes. It does not allow us to identify underlying mechanisms because of the presence of both a "direct effect" and an "indirect effect" through income (the term in the square bracket).

As indicated, we also study the effect of losing a parent on the relationship between children's human capital and the schooling of the *surviving* parent. Specifically,

$$\frac{d(lnk-ln\tilde{k}_a)}{ds_a} = \frac{\partial R}{\partial s_a} - \frac{\partial Q}{\partial s_a} + \theta s_a^{\theta-1}\delta_a\left(lnl_a - ln\tilde{l}_a\right) + \alpha_k \theta s_a^{\theta-1}\delta_a\tilde{\varphi}\left(s_a^{\theta}\delta_a + \delta_e\right) - \alpha_k\theta s_a^{\theta-1}\delta_a\varphi\left(s_a^{\theta}\delta_a + s_d^{\theta}\delta_d + \delta_e\right) + \left[\left(s_a^{\theta}\delta_a + s_d^{\theta}\delta_d + \delta_e\right)\frac{T}{(w_a+w_d)T+I} - \frac{(s_a^{\theta}\delta_a+\delta_e)T}{w_aT+\tilde{I}}\right]\frac{\partial w_a}{\partial s_a} \quad (4)$$

Again, empirically, under the assumption that the relationship between the schooling of a parent and the child's initial human capital does not depend on whether the other parent dies, we can difference out initial human capital (genetics) by comparing the relationship between parental schooling and children's human capital for children who have two surviving parents to those who lose a parent. Thus, looking at the effect of the surviving parent's schooling when the other parent dies versus both parents survive can provide a causal estimate of the effect of schooling on children's outcomes, although it does not allow us to distinguish between the productivity of time from the indirect effect of parental income.

Although we do not incorporate the timing of parental death in the model explicitly, as noted above, we can analyze the timing by modeling a child's human capital as a weighted average of their human capital under the case where the child lives with both parents versus the case where only one parent survives. A death later into childhood would shift the weights toward the former case. For children who lose a parent, it is reasonable to assume that the child's initial human capital does not depend on when the parent dies, so that lnk_0 and $ln\tilde{k}_{a0}$ difference out of (2). As such, a child's genetics can be controlled for by comparing the human capital of children who lose a parent at different ages. Taken together, examining the effect of each parent's schooling on children who lose a parent, and according to the age of parental death, is a powerful mechanism to control for (difference out) genetics.

Implication 2: Genetics can be eliminated from the relationship between children's human capital and parent's schooling by comparing the human capital of children who lose a parent at different ages or by estimating the relationship between parent's schooling and children's human capital for parents who survive versus die, but additional variation is necessary to disentangle underlying mechanisms.

Substitution Between Parents

We now formally examine how the death of a parent alters the effect of the dying parent's human capital relative to that of the surviving parent's human capital, which we refer to as "substitution." Assuming, for the moment, that the two parents are identical (in terms of preferences, productivity, wages, and schooling), substitution is given by:

$$
\begin{aligned}
\frac{dln\tilde{k}_a}{ds_d} - \frac{dln\tilde{k}_a}{ds_a} &= \frac{d\left(\ln k - ln\tilde{k}_a\right)}{ds_a} - \frac{d\left(\ln k - ln\tilde{k}_a\right)}{ds_d} \\
&= -\frac{\partial Q}{\partial s_a} - \theta s_a^{\theta-1}\delta_a ln\tilde{l}_a \\
&\quad - \alpha_k \theta s_a^{\theta-1}\delta_a \frac{\alpha_{ca} s_a^{\theta}\delta_a}{\left[\alpha_{la} + \alpha_{ca} + \alpha_k\left(s_a^{\theta}\delta_a + \delta_e\right)\right]\left(\alpha_{la} + \alpha_k s_a^{\theta}\delta_a\right)} \\
&\quad + \alpha_k \theta s_a^{\theta-1}\delta_a \frac{\delta_e \alpha_{la}}{\left[\alpha_{la} + \alpha_{ca} + \alpha_k\left(s_a^{\theta}\delta_a + \delta_e\right)\right]\left(\alpha_{la} + \alpha_k s_a^{\theta}\delta_a\right)} \\
&\quad + \frac{s_a^{\theta}\delta_a \tilde{I}_a - w_a \delta_e T}{(w_a T + \tilde{I}_a) w_a}\frac{\partial w_a}{\partial s_a}
\end{aligned}
$$

The first term reflects the productivity of investments (both time and financial) in children and is negative. The second term gives changes in the productivity of time with children and is also negative. An increase in parental schooling leads the surviving parent to spend more time with the child and less time working if $\theta > 0$. The third term represents the increased time with children while the fourth represents the induced reduction in financial investments in children as a parent works less. Together, these terms will be negative unless financial investments are sufficiently important relative to time. The last term represents the indirect effect due to changes in wages and can only be positive if unearned income is large. Thus, unless expenditures are the dominant effect, there is a presumption that the dying parent's schooling will become less important relative to the surviving parent's schooling. At an empirical level, under the assumption that the relationship between parent's schooling and the child's initial human capital (through genetics) is the same for both parents, we can difference out genetics by comparing the relationship between a parent's education and a child's human capital for a parent who dies to that for a parent who survives.

Death of a Mother Versus a Father

Beyond identifying the causal effect of parental schooling, we seek to distinguish between the direct effect of parental education on the productivity of time spent with children versus the indirect effect of parental education on family income. To disentangle these mechanisms, we exploit variation in the time allocation and the wage rate across parents. Specifically, mothers typically work and earn less (overall and on an hourly basis) and spend more time with children than fathers. Thus, the loss of a mother will have a larger impact on time spent with the child and smaller effect on family income than the loss of a father.

Our first analysis compares the effect of a mother surviving (as opposed to dying) on the effect of her schooling to the effect of a father surviving (as opposed to dying) on the effect of his schooling. We assume that the effect of schooling on log wages does not differ by gender, so that $\frac{\partial \ln w_m}{\partial s_m} = \frac{\partial \ln w_f}{\partial s_f} = \frac{\partial \ln w}{\partial s}$.[6] We also assume that $s_m^\theta \delta_m = s_f^\theta \delta_f = s^\theta \delta$, which holds when both parents have the same amount of schooling (our empirical analysis will control for differences in schooling) and the parameters on both parent's (schooling-adjusted) time are the same ($\delta_m = \delta_f = \delta$). Differencing equation (3) when the mother dies from when the father dies yields:

[6] Under the assumption that $\frac{\partial w_m}{\partial s_m} = \frac{\partial w_f}{\partial s_f} = \frac{\partial w}{\partial s}$, the effect of parent's schooling on children's human capital differences out in the sense that δ_e does not appear in the expression for the effect of losing a mother on the effect of her schooling relative to that of losing a father on the effect of his schooling.

$$\frac{d(\ln k - ln\tilde{k}_f)}{ds_m} - \frac{d(\ln k - ln\tilde{k}_m)}{ds_f}$$

$$= \frac{\partial \ln R}{\partial s_m} - \frac{\partial \ln R}{\partial s_f}$$

$$+ \ \theta s^{\theta-1}\delta \left[ln\left(\frac{w_f}{w_m}\frac{\alpha_{lm} + \alpha_k s^\theta \delta}{\alpha_{lf} + \alpha_k s^\theta \delta}\right) - \frac{(\alpha_{lm} - \alpha_{lf})\alpha_k \theta s^\theta \delta}{(\alpha_{lm} + \alpha_k s^\theta \delta)(\alpha_{lf} + \alpha_k s^\theta \delta)}\right]$$

$$+ (2s^\theta \delta + \delta_e)\frac{T}{(w_m + w_f)T + I}(w_m - w_f)\frac{\partial \ln w}{\partial s}$$

To produce testable implications that allow us to identify underlying mechanisms, we shut down all mechanisms related to the productivity of time by assuming: $\delta_m = \delta_f = 0$ and $\frac{\partial \ln R}{\partial s_m} = \frac{\partial \ln R}{\partial s_f} = 0$. Under these assumptions:

$$\frac{d(\ln k - ln\tilde{k}_f)}{ds_m} - \frac{d(\ln k - ln\tilde{k}_m)}{ds_f} = \frac{\delta_e T}{(w_m + w_f)T + I}(w_m - w_f)\frac{\partial \ln w}{\partial s} < 0$$

if $w_m < w_f$. This yields the following:

Implication 3: Since mothers typically earn less and spend more time with children and less time working than fathers, a finding that the survival of mothers increases the effect (on children's human capital) of mother's schooling more than the survival of fathers increases the effect of father's schooling implies that parent's schooling affects children's human capital through time investments (and that the time channel dominates the expenditure channel).

Intuitively, because mothers earn lower wages and work less, a mother's schooling should be "less important" than father's schooling if time investments are not important.

We can also learn about mechanisms from how a death alters the importance of the dying parent's schooling relative to the surviving parent's schooling, the substitution process discussed above. Again, we assume that $s_m^\theta \delta_m = s_f^\theta \delta_f = s^\theta \delta$ and $\frac{\partial \ln w_m}{\partial s_m} = \frac{\partial \ln w_f}{\partial s_f}$. From equations 3 and 4, the amount of substitution from the father to the mother (as measured by the effect of the father's schooling relative to the mother's schooling) when the father dies versus survives is given by:

$$\frac{d\left(ln\tilde{k}_m - lnk\right)}{ds_f} - \frac{d\left(ln\tilde{k}_m - lnk\right)}{ds_m} = \frac{d\left(lnk - ln\tilde{k}_m\right)}{ds_m} - \frac{d\left(lnk - ln\tilde{k}_m\right)}{ds_f}$$

$$= \frac{\partial R}{\partial s_m} - \frac{\partial R}{\partial s_f} - \frac{\partial Q}{\partial s_m} + \theta s^{\theta-1}\delta\left(lnl_m - ln\tilde{l}_m - lnl_f\right)$$

$$+ \alpha_k \theta s^{\theta-1}\delta\left(\tilde{\varphi}\left(s^{\theta}\delta + \delta_e\right) - \frac{s^{\theta}\delta}{\alpha_{lf} + \alpha_k s^{\theta}\delta}\right)$$

$$+ \left[\left(2s^{\theta}\delta + \delta_e\right)\frac{T\left(w_m - w_f\right)}{\left(w_m + w_f\right)T + I} + s^{\theta}\delta - \frac{\left(s^{\theta}\delta + \delta_e\right)w_m T}{w_m T + \tilde{I}}\right]\frac{\partial \ln w}{\partial s}$$

$$= \frac{\partial R}{\partial s_m} - \frac{\partial R}{\partial s_f} - \frac{\partial Q}{\partial s_m} + \theta s^{\theta-1}\delta ln\frac{w_f\left[\alpha_{lm} + \alpha_{cm} + \alpha_k\left(s^{\theta}\delta + \delta_e\right)\right]}{\left(\alpha_{lf} + \alpha_k s^{\theta}\delta\right)\left(w_m T + \tilde{I}\right)}$$

$$+ \alpha_k \theta s^{\theta-1}\delta\left(\frac{s^{\theta}\delta + \delta_e}{\alpha_{lm} + \alpha_{cm} + \alpha_k(s^{\theta}\delta + \delta_e)} - \frac{s^{\theta}\delta}{\alpha_{lf} + \alpha_k s^{\theta}\delta}\right)$$

$$+ \left[\left(2s^{\theta}\delta + \delta_e\right)\frac{T\left(w_m - w_f\right)}{\left(w_m + w_f\right)T + I} + s^{\theta}\delta - \frac{\left(s^{\theta}\delta + \delta_e\right)w_1 T}{w_m T + \tilde{I}}\right]\frac{\partial \ln w}{\partial s}$$

Again, our goal is to show how the difference in substitution can be used to generate testable implications for whether time or only financial investments affect children's human capital, so again we assume that only financial investments are important (that $\delta_m = \delta_f = 0$ and that $R(\cdot,\cdot) = Q(\cdot) = 0$). Under these assumptions, the case where mothers earn less than fathers ($w_m < w_f$) and where survivor benefits are positive ($\tilde{I} \geq I$), but not too large ($Tw_m + I > \tilde{I}$) yields:

$$\frac{d\left(lnk - ln\tilde{k}_m\right)}{ds_m} - \frac{d\left(lnk - ln\tilde{k}_m\right)}{ds_f} - \left(\frac{d\left(lnk - ln\tilde{k}_f\right)}{ds_f} - \frac{d\left(lnk - ln\tilde{k}_f\right)}{ds_m}\right)$$

$$= \left\{\delta_e T\left[w_m\left(\frac{2}{\left(w_m + w_f\right)T + I} - \frac{1}{w_m T + \tilde{I}}\right)\right.\right.$$

$$\left.\left. - w_f\left(\frac{2}{\left(w_m + w_f\right)T + I} - \frac{1}{w_f T + \tilde{I}}\right)\right]\frac{\partial \ln w}{\partial s}\right\} < 0$$

A detailed analysis is presented in the Appendix B. Intuitively, and as shown by Fadlon, Itzik and Nielsen [2015], for most households, mothers will increase time working when a father dies but, when a mother dies, fathers will, if anything, spend less time working. If time investments are not important, the increase in mother's work when the father dies will lead her schooling to become more important, but the death of a mother will not induce the same increase in the importance of father's schooling and may cause father's schooling to become less important. Thus, if the death of a mother generates more substitution toward fathers than does the death of a father does toward mothers, it would indicate that time investments are important.

Implication 4: Since mothers typically earn less and spend more time with children and less time working in the labor market relative to fathers, a finding that the death of a mother leads to more substitution away from mothers and toward fathers (as measured by the relative importance of their human capital for children's human capital) than the death of a father generates away from fathers and toward mothers, implies that parental schooling affects children's human capital through time investments.

As discussed above, while both of these results are derived for whether a parent dies versus survives, we analyze the timing of parental death using a weighted average of parental death and survival, where the weights depend on the timing of death. A finding that later death reduces substitution more when a mother dies relative to when a father dies indicates the importance of time investments.

Divorce

In our empirical analysis, we study parental divorce as well as parental death. Of course, the incidence and timing of parental divorce is less likely to be exogenous than parental death. At the same time, divorce is far more common and has similarities to paternal death in that mothers typically retain custody in most societies. In Israel, alimony and child support are also quite limited, so the effects on income are similar to cases of parental death. Given this institutional setup, our model's predictions and results regarding paternal death are also applicable to families experiencing parental divorce. As such, our empirical analysis of parental divorce serves as a robustness check for our findings regarding paternal death.

Family Size

Information about the mechanisms that connect parental schooling to children's human capital can also be obtained from variation across households of different sizes. In particular, parents in larger families typically display a stronger degree of specialization in terms of their time allocation to child rearing, and therefore, this variation produces testable implications for how a child's human capital depends on the schooling level of each parent.

We adapt the production function to allow for differences in the number of children. In this case, lnk is taken to be an index of all the children's human capital that enters the parental utility function:

$$lnk = R(s_m, s_f) + s_m^{\theta}\delta_m(n)lnl_m + s_f^{\theta}\delta_f(n)lnl_f + \delta_{\mathrm{e}}(n)lne + \delta_k lnk_0,$$

where n is the number of children in the household and $\delta'_i(n) \leq 0$ for $i \in \{m, f, e\}$.[7] This model is sufficiently general to allow for a range of crowding-out/dilution of time and financial investments - no dilution is represented by $\delta_i(n) = \delta_i$; full dilution occurs when $\delta_i(n) = \delta_i/n$.

As is common in models of investments in multiple children, we allow α_k, the weight put on the children's human capital in the parental utility function, to vary with the number of children. In our case, we expect $\alpha'_k(n) \geq 0$, indicating that the total utility from the index of children's human capital is non-decreasing in the number of children (although utility derived per child may be decreasing).

Differentiating human capital with respect to number of children yields:

[7] For instance, we can assume that all children have the same human capital or assume that parent's utility depends on the average log human capital of children, which is measured by lnk.

$$\frac{dlnk}{dn} = s_m^\theta \delta_m' lnl_m + s_m^\theta \delta_m \frac{dlnl_m}{dn} + s_f^\theta \delta_f' lnl_f + s_f^\theta \delta_f \frac{dlnl_f}{dn} + \delta_e' lne + \delta_e \frac{dlne}{dn}$$

Differentiating this expression with respect to mother's schooling (the results for father's schooling are analogous) yields:

$$\frac{d^2lnk}{dnds_m} = \theta s_m^{\theta-1} \delta_m' lnl_m + s_m^\theta \delta_m' \frac{dlnl_m}{ds_m} + \theta s_m^{\theta-1} \delta_m \frac{dlnl_m}{dn} + s_m^\theta \delta_m \frac{dlnl_m}{dnds_m} + s_f^\theta \delta_f' \frac{dlnl_f}{ds_m} + s_f^\theta \delta_f \frac{dlnl_f}{dnds_m} + \delta_e' \frac{dlne}{ds_m} + \delta_e \frac{dlne}{dnds_m}$$

As above, we produce testable implications that allow us to identify underlying mechanisms by assuming that time investments are not important. Specifically, when $\delta_m(n) = \delta_f(n) = 0$, the previous equation becomes:

$$\frac{d^2lnk}{dnds_m} = \delta_e' \frac{dlne}{ds_m} + \delta_e \frac{d^2lne}{dnds_m} = \delta_e' \frac{T}{(w_m + w_f)T + I} \frac{\partial w_m}{\partial s_m} - \delta_e \frac{TI'(n)}{\left[(w_m + w_f)T + I\right]^2} \frac{\partial w_m}{\partial s_m} \leq 0$$

Intuitively, in the absence of an effect of time on children's human capital, increases in family size dilute the effects of parental schooling on children's human capital because financial investments are spread across more children.[8] For the effects of parental education on lnk to be increasing in family size, there must be a positive interaction between family size and parental education in resource allocation. The importance of parental time can be increasing in family size if $\theta > 0$ because the share of potential income that parents allocate to time with children (as opposed to consumption) increases with education, whereas the share of potential income allocated to financial investments is independent of parental education.

[8] Additionally, financial transfers from the government may increase in family size, which reduces a mother's (and a father's) labor supply, and hence, the effect of a mother's education on child outcomes declines.

Implication 5: A finding that family size increases the effect of parental schooling on children's human capital implies that parent's schooling affects children's human capital through time investments.

If time investments are important, the relationship between a mother's schooling and her children's human capital is increasing in family size if time investments in children by the mother increase as well. (As indicated, the model is analogous for fathers.) Moreover, the mother's schooling will become more important for the child's human capital relative to the father's schooling if mothers increase their time with children relative to fathers in larger families. Formally, $\frac{d^2 lnk}{dnd\partial s_m} - \frac{d^2 lnk}{dnds_f}$ is increasing in $\frac{d \ln l_m}{dn} - \frac{d \ln l_f}{dn}$ under the assumption of symmetry between mothers and fathers (used above) that $\frac{\partial \ln w_m}{\partial s_m} = \frac{\partial \ln w_f}{\partial s_f}$ and $s_m^\theta \delta_m = s_f^\theta \delta_f = s^\theta \delta$, and that the effect of family size on the productivity of mother's and father's time is the same, $\delta_m'(n) = \delta_f'(n) = \delta'(n)$. Thus, a finding that family size increases the importance of the mother's schooling relative to father's schooling on a child's human capital is consistent with time investments in children being important and with mothers increasing their time with children more than fathers in larger families.

Appendix B

A Corner Solution

A corner solution occurs when $l_j \geq T$. For sake of concreteness, assume that the mother doesn't work and the father does, which implies: $l_m \geq T \; and \; l_f < T$ (the calculations are analogous for the father). That is:

$$l_m = \varphi\left(\alpha_{lm} + \alpha_k s_m^{\theta}\delta_m\right)\left(\frac{w_m+w_f}{w_m}T + \frac{I}{w_m}\right) \geq T.$$

In that case the optimal allocation is:

$$l_m = T$$

$$l_f = \varphi\left(\alpha_{lf} + \alpha_k s_f^{\theta}\delta_f\right)\left(T + \frac{I}{w_f}\right)$$

$$e = \varphi\alpha_k\delta_e\left(w_f T + I\right).$$

Hence, equation (1) in the text becomes:

$$lnk =$$

$$R\left(s_m, s_f\right) + s_m^{\theta}\delta_m lnT + s_f^{\theta}\delta_f ln\left[\varphi\left(\alpha_{lf} + \alpha_k s_f^{\theta}\delta_f\right)\left(T + \frac{I}{w_f}\right)\right] +$$

$$\delta_e ln\left[\varphi\alpha_k\delta_e\left(w_f T + I\right)\right] + \delta_k lnk_0.$$

Two period model

Parents maximize:

$$\begin{aligned} u\left(l_{m1}, l_{m2}, l_{f1}, l_{f2}, c_{m1},, c_{m2},, c_{f1}, c_{f2}, k_1, k\right) \\ = \alpha_{lm}\ln l_{m1} + \alpha_{lf}\ln l_{f1} + \alpha_{cm}\ln c_{m1} \\ + \alpha_{cf}\ln c_{f1} + \alpha_k \ln k_1 + \beta\left[\alpha_{lm}\ln l_{m2} + \alpha_{lf}\ln l_{f2} + \alpha_{cm}\ln c_{m2}\right. \\ \left. + \alpha_{cf}\ln c_{f2} + \alpha_k\, lnk_2\right] + \psi\alpha_k lnk_2 \end{aligned}$$

Subject to:

$T = l_{jt} + h_{jt}, \quad for \quad j \in \{m, f\}, \quad t \in \{1,2\}.$

$c_{mt} + c_{ft} + e_t = w_m h_{mt} + w_f h_{ft}, \quad for\ t \in \{1,2\}$

$lnk_t = R(s_m, s_f) + s_m^{\theta}\delta_m lnl_{mt} + s_f^{\theta}\delta_f lnl_{ft} + \delta_e lne_t + \delta_k lnk_{t-1}, \ \ t \in \{1,2\}, k_0\ given$

where $\psi\alpha_k lnk_2$ represents the terminal value to the household of the child's human capital after the parents have completed their investment (in our case, after the child takes the matriculation exams).

The optimal allocation of time and resources is:

$$l_{jt} = \frac{\varphi_{jt} - \alpha_{cj}}{\varphi_{m1} + \varphi_{f2} + \varphi_{et}}\left(\frac{w_m + w_f}{w_j}T + \frac{I_t}{w_j}\right)$$

$$e_t = \frac{\varphi_{et}}{\varphi_{mt} + \varphi_{ft} + \varphi_{et}}\left((w_m + w_f)T + I_t\right)$$

where:

$\varphi_{j1} \equiv \alpha_{lj} + \alpha_{cj} + \alpha_k s_j^{\theta}\delta_j(1 + \delta_k\psi); \quad \varphi_{j2} \equiv \alpha_{lj} + \alpha_{cj} + \alpha_k s_j^{\theta}\delta_j\psi;$

$\varphi_{e1} \equiv \alpha_k\delta_e(1 + \delta_k\psi); \quad \varphi_{et} \equiv \alpha_k\delta_e\psi$

<u>Proof of implication 4</u>

Claim: If $Tw_m + I > \tilde{I}, \ I \leq \tilde{I}$ and $w_f > w_f$ then:

$$\frac{d(lnk - ln\tilde{k}_m)}{ds_m} - \frac{d(lnk - ln\tilde{k}_m)}{ds_f} - \left(\frac{d(lnk - ln\tilde{k}_f)}{ds_f} - \frac{d(lnk - ln\tilde{k}_f)}{ds_m}\right) < 0$$

Proof:

$$\frac{d(lnk - ln\tilde{k}_m)}{ds_m} - \frac{d(lnk - ln\tilde{k}_m)}{ds_f} - \left(\frac{d(lnk - ln\tilde{k}_f)}{ds_f} - \frac{d(lnk - ln\tilde{k}_f)}{ds_m}\right)$$
$$= \frac{d(ln\tilde{k}_m + ln\tilde{k}_f - 2\ln k)}{ds_f} - \frac{d(ln\tilde{k}_m + ln\tilde{k}_f - 2\ln k)}{ds_m}$$

Assuming that $s_m^{\theta}\delta_m = s_f^{\theta}\delta_f = 0$, $\frac{\partial \ln w_m}{\partial s_m} = \frac{\partial \ln w_f}{\partial s_f}$, and $R(\cdot,\cdot) = Q(\cdot) = 0$, then:

$$ln\tilde{k}_m + ln\tilde{k}_f - 2\ln k = Q(s_m) + Q(s_f) - 2R(s_m, s_f) + \delta_e(ln\tilde{e}_m + ln\tilde{e}_f - 2lne).$$

Hence,

$$\frac{d(ln\tilde{k}_m + ln\tilde{k}_f - 2\ln k)}{ds_f} - \frac{d(ln\tilde{k}_m + ln\tilde{k}_f - 2\ln k)}{ds_m}$$
$$= \delta_e T\left[w_m\left(\frac{2}{(w_m + w_f)T + I} - \frac{1}{w_m T + \tilde{I}}\right)\right.$$
$$\left. - w_f\left(\frac{2}{(w_m + w_f)T + I} - \frac{1}{w_f T + \tilde{I}}\right)\right]\frac{d\ln w}{ds}.$$

Note that if $w_f = w_m$, the above equation equals zero. Because $w_f > w_m$, it is sufficient to show that its derivative with respect to w_f is negative;

$$\frac{d\left[w_m\left(\frac{2}{(w_m + w_f)T + I} - \frac{1}{w_m T + \tilde{I}}\right) - w_f\left(\frac{2}{(w_m + w_f)T + I} - \frac{1}{w_f T + \tilde{I}}\right)\right]}{dw_f}$$
$$= -\frac{2Tw_m}{[(w_m + w_f)T + I]^2} - \left(\frac{2}{(w_m + w_f)T + I} - \frac{1}{w_f T + \tilde{I}}\right)$$
$$+ \frac{2Tw_f}{[(w_m + w_f)T + I]^2} - \frac{Tw_f}{[w_f T + \tilde{I}]^2}$$
$$= \frac{2}{(w_m + w_f)T + I}\left[\frac{T(w_f - w_m)}{(w_m + w_f)T + I} - 1\right] + \frac{1}{w_f T + \tilde{I}}\left[1 - \frac{Tw_f}{w_f T + \tilde{I}}\right]$$
$$= -\frac{2(2Tw_m + I)}{[(w_m + w_f)T + I]^2} + \frac{\tilde{I}}{[w_f T + \tilde{I}]^2}$$

Under the assumptions that $Tw_m + I > \tilde{I}$, $I \leq \tilde{I}$, and $w_f \geq w_f$, it is possible to show that:

$$-\frac{2(2Tw_m+I)}{[(w_m+w_f)T+I]^2}+\frac{\tilde{I}}{[w_fT+\tilde{I}]^2}<2\left(-\frac{2Tw_m+I}{[(w_m+w_f)T+I]^2}+\frac{Tw_m+I}{[(w_m+w_f)T+I]^2}\right)<0$$

This completes the proof.

Appendix Table A1: Incidence of Parental Death by Age of Child

Age of Child	Maternal Death			Paternal Death		
	Frequency	Column Percent	Cumulative Column Percent	Frequency	Column Percent	Cumulative Column Percent
0	60	0.3	0.3	247	0.5	0.5
1	76	0.38	0.68	327	0.66	1.16
2	100	0.5	1.19	367	0.74	1.9
3	129	0.65	1.83	440	0.89	2.78
4	150	0.75	2.59	445	0.9	3.68
5	169	0.85	3.44	530	1.07	4.75
6	203	1.02	4.46	563	1.14	5.89
7	234	1.18	5.63	602	1.21	7.1
8	263	1.32	6.95	685	1.38	8.48
9	296	1.49	8.44	758	1.53	10.01
10	309	1.55	9.99	812	1.64	11.65
11	323	1.62	11.62	898	1.81	13.46
12	344	1.73	13.34	1,028	2.07	15.53
13	418	2.1	15.44	1,050	2.12	17.65
14	470	2.36	17.8	1,164	2.35	20
15	519	2.61	20.41	1,386	2.79	22.79
16	553	2.78	23.19	1,487	3	25.79
17	699	3.51	26.7	1,607	3.24	29.03
18	739	3.71	30.41	1,705	3.44	32.47
19	738	3.71	34.12	1,804	3.64	36.11
20	800	4.02	38.14	2,060	4.15	40.26
21	894	4.49	42.63	2,095	4.22	44.48
22	836	4.2	46.83	2,165	4.37	48.85
23	913	4.59	51.42	2,203	4.44	53.29
24	940	4.72	56.14	2,252	4.54	57.83
25	847	4.26	60.4	2,126	4.29	62.12
26	873	4.39	64.78	2,192	4.42	66.54
27	891	4.48	69.26	2,094	4.22	70.76
28	845	4.25	73.5	2,074	4.18	74.95
29	834	4.19	77.69	1,979	3.99	78.94
30	797	4	81.7	1,939	3.91	82.85
31	735	3.69	85.39	1,760	3.55	86.4
32	711	3.57	88.96	1,583	3.19	89.59
33	632	3.18	92.14	1,541	3.11	92.7
34	561	2.82	94.96	1,270	2.56	95.26
35	447	2.25	97.2	1,062	2.14	97.4
36	334	1.68	98.88	782	1.58	98.98
37	179	0.9	99.78	426	0.86	99.83
38	44	0.22	100	82	0.17	100
Total	19,905	100		49,590	100	

Notes: The sample includes native born Israeli Jews who were not in the religious school system that were born between 1974 and 1991 (i.e. in the 1992 to 2009 12th grade cohorts). Paternal deaths are reported up to 2012, so many cohorts did not reach all the ages reported in the table.

Appendix Table A2: Causes of Parental Deaths (1992-2004 Cohorts)

	Mother Loss before Age 18		Father Loss before Age 18	
	Frequency	Percent of Non-Missing Cause of Deaths	Frequency	Percent of Non-Missing Cause of Deaths
Infections	66	1.23	53	0.39
Neoplasms	3,574	66.42	4,007	29.71
Endocrine	68	1.26	321	2.38
Blood Disease	23	0.43	35	0.26
Mental	70	1.30	317	2.35
Nervous System	29	0.54	74	0.55
Circulatory	496	9.22	3,719	27.58
Respiratory	85	1.58	354	2.62
Digestive	76	1.41	435	3.23
Urinary	37	0.69	156	1.16
Pregnancy	40	0.74		
Skin	3	0.06	7	0.05
Musculatory-Skeletal	22	0.41	19	0.14
Congenital	12	0.22	28	0.21
Unknown Illness	178	3.31	1,146	8.50
Traffic Accident	208	3.87	925	6.86
Self-harm	148	2.75	579	4.29
Assault	41	0.76	162	1.20
War	13	0.24	94	0.70
Other Unnatural Cause	192	3.57	1,055	7.82
Frequency of Missing Cause of Death	844		2900	
Fraction of Deaths with Missing Cause of Death	0.14		0.18	

The table shows causes of death. The sample in the left panel is restricted to families where the father did not die before the child was 18. The sample in the left panel is restricted to families where the mother did not die before the child was 18.

Appendix Table A3: Mother Loss Analysis with Parental Income

	Dependent Variable: Pass Matriculation Exam													
	All	Excludes Parental Deaths before 1991												
		Sample Includes Families with a Parent's Income = 0						Sample Excludes Families where either Parent's Income = 0						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Mother's Education	0.0182***	0.0182***		0.0150***	0.0182***		0.0150***	0.0182***	0.0176***		0.0134***	0.0176***		0.0134***
	(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)	(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)
Father's Education	0.0171***	0.0171***		0.0139***	0.0171***		0.0139***	0.0171***	0.0168***		0.0124***	0.0168***		0.0124***
	(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)	(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)
Mother's Log Income Fixed-Effect			0.0523***	0.0315***		0.0523***	0.0315***			0.0607***	0.0381***		0.0608***	0.0381***
			(0.0008)	(0.0008)		(0.0008)	(0.0008)			(0.0010)	(0.0010)		(0.0010)	(0.0010)
Father's Log Income Fixed-Effect			0.0548***	0.0352***		0.0548***	0.0352***			0.0686***	0.0452***		0.0686***	0.0452***
			(0.0007)	(0.0008)		(0.0007)	(0.0008)			(0.0010)	(0.0010)		(0.0010)	(0.0010)
Mother Died when Child < 18 interacted with:														
Mother's Education	-0.0073***	-0.0036		-0.0014	-0.0208**		-0.0182*	-0.0073***	-0.0039		0.001	-0.0510***		-0.0439***
	(0.0023)	(0.0028)		(0.0029)	(0.0099)		(0.0100)	(0.0023)	(0.0041)		(0.0042)	(0.0150)		(0.0151)
Father's Education	0.002	0.0001		-0.0005	0.0128		0.0071	0.002	0.0015		0.0002	0.0162		0.0033
	(0.0022)	(0.0027)		(0.0028)	(0.0099)		(0.0106)	(0.0022)	(0.0039)		(0.0042)	(0.0155)		(0.0174)
Mother's Educ*Age of Child when Mother Died					0.0013*		0.0013*					0.0034***		0.0033***
					(0.0007)		(0.0007)					(0.0010)		(0.0010)
Father's Educ*Age of Child when Mother Died					-0.001		-0.0006					-0.0012		-0.0004
					(0.0007)		(0.0007)					(0.0011)		(0.0012)
Mother's Log Income Fixed-Effect			-0.0372***	-0.0237***		-0.0531	-0.0287			-0.0508***	-0.0410***		-0.0533	-0.0149
			(0.0075)	(0.0075)		(0.0333)	(0.0334)			(0.0108)	(0.0110)		(0.0440)	(0.0445)
Father's Log Income Fixed-Effect			0.0123	0.0074		0.0458	0.0457			0.0168	0.014		0.0323	0.0714
			(0.0083)	(0.0090)		(0.0341)	(0.0380)			(0.0121)	(0.0133)		(0.0510)	(0.0569)
Mother's Log Income Fixed-Effect*Age of Child when Mother Died						0.0010	0.0001						0.0000	-0.0022
						(0.0023)	(0.0023)						(0.0030)	(0.0030)
Father's Log Income Fixed-Effect*Age of Child when Mother Died						-0.0024	-0.0027						-0.0011	-0.0041
						(0.0023)	(0.0026)						(0.0035)	(0.0039)
Number of School Fixed Effects	849	848	849	848	848	849	848	849	841	841	841	841	841	841
Observations	636748	634944	645467	634944	634944	645467	634944	636748	434003	434003	434003	434003	434003	434003
Mother Died when Child < 18 interacted with:														
Mother's Educ - Father's Educ	-0.0046**	-0.0016		-0.0001	-0.0171*		-0.0143	-0.0046**	-0.0025		0.0012	-0.0349***		-0.0267**
	(0.0020)	(0.0025)		(0.0025)	(0.0089)		(0.0089)	(0.0020)	(0.0036)		(0.0037)	(0.0133)		(0.0135)
(Mother's Educ - Father's Educ)*Age of Child when Mother Died					0.0012*		0.0011*					0.0024***		0.0021**
					(0.0006)		(0.0006)					(0.0009)		(0.0009)
Mother's Income Fixed-Effect - Father's Income Fixed-Effect			-0.0323***	-0.0218***		-0.0294	-0.0196			-0.0361***	-0.0300***		-0.0414	-0.0263
			(0.0069)	(0.0069)		(0.0322)	(0.0322)			(0.0087)	(0.0089)		(0.0370)	(0.0373)
(Mother's Income Fixed-Effect - Father's Income Fixed-Effect)*Age of Child when Mother Died						-0.0003	-0.0002						0.0003	-0.0004
						(0.0022)	(0.0022)						(0.0025)	(0.0025)
Number of School Fixed Effects	849	848	849	848	848	849	848	849	841	841	841	841	841	841
Observations	636748	634944	645467	634944	634944	645467	634944	636748	434003	434003	434003	434003	434003	434003
Controls for Each Parent's Income being Zero Directly and Interacted with Losing Parent < Age 18		Yes	Yes	Yes	Yes	Yes	Yes							
Controls for Each Parent's Income being Zero interacted with Age of Child when Parent Died					Yes	Yes	Yes							

Notes: For every column, the upper and lower panels represent separate regressions. The sample is restricted to individuals who lost a mother. Standard errors appear in parentheses. Significance levels are indicated by one, two, or three stars which represent 10 percent, 5 percent, and 1 percent levels, respectively. All regressions are performed using OLS and include controls for mother's age at the time of the child's birth, father's age at the time of the child's birth, gender, number of siblings, birth order, and dummy variables for each cohort year. All specifications also control for socioeconomic status of the locality, a dummy for whether the mother died before age 18, age mother died, education of the mother, and education of the father.

Appendix Table A4: Mother Loss Analysis - Results for Different Subjects

	Pass Matriculation	Math > 80	Math > 70	English > 80	English > 70	Bible > 80	Bible > 70	Hebrew > 80	Hebrew > 70
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Mother Died when Child < 18 interacted with:									
Mother's Education	-0.0233***	-0.0028	-0.0120**	-0.006	-0.0053	-0.0071	-0.0064	-0.0148***	-0.0223***
	(0.0060)	(0.0060)	(0.0061)	(0.0056)	(0.0060)	(0.0054)	(0.0060)	(0.0053)	(0.0058)
Father's Education	0.0129**	0.0025	0.0017	0.0081	0.0032	0.0146***	0.0063	0.0107*	0.0132**
	(0.0062)	(0.0061)	(0.0062)	(0.0058)	(0.0062)	(0.0055)	(0.0062)	(0.0055)	(0.0060)
Mother's Educ*Age of Child when Mother Died	0.0015***	0.0003	0.0006	0.0001	-0.0001	0.0003	0.0002	0.0008*	0.0013***
	(0.0005)	(0.0005)	(0.0005)	(0.0004)	(0.0005)	(0.0004)	(0.0005)	(0.0004)	(0.0004)
Father's Educ*Age of Child when Mother Died	-0.0009*	-0.0006	-0.0003	-0.0004	0.0001	-0.0005	-0.0002	-0.0003	-0.0008*
	(0.0005)	(0.0005)	(0.0005)	(0.0004)	(0.0005)	(0.0004)	(0.0005)	(0.0004)	(0.0004)
Number of School Fixed-Effects	651	651	651	651	651	651	651	651	651
Observations	19,905	19,905	19,905	19,905	19,905	19,905	19,905	19,905	19,905

Notes: The sample includes only those that lost a mother at any age. Standard errors appear in parentheses. Significance levels are indicated by one, two, or three stars which represent 10 percent, 5 percent, and 1 percent levels, respectively. All regressions are performed using OLS and include controls for mother's age at the time of the child's birth, father's age at the time of the child's birth, gender, number of siblings, birth order, and dummy variables for each cohort year. All specifications also control for socioeconomic status of the locality, a dummy for whether the parent died before age 18, age parent died, the interaction between age the parent died and losing the parent before age 18, education of the mother, and education of the father.

Appendix Table A5: Father Loss Analysis with Parental Income

	Includes Parental Death in all Years	Excludes Parental Deaths before 1991											
		Sample Includes Families with a Parent's Income = 0						Sample Excludes Families where either Parent's Income = 0					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(15)
Mother's Education	0.0182***	0.0182***		0.0150***	0.0182***		0.0150***	0.0175***		0.0134***	0.0176***		0.0134***
	(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)
Father's Education	0.0170***	0.0170***		0.0138***	0.0170***		0.0138***	0.0167***		0.0123***	0.0168***		0.0123***
	(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)	(0.0003)		(0.0003)
Mother's Log Income Fixed-Effect			0.0523***	0.0315***		0.0523***	0.0315***		0.0607***	0.0381***		0.0607***	0.0381***
			(0.0008)	(0.0008)		(0.0008)	(0.0008)		(0.0010)	(0.0010)		(0.0010)	(0.0010)
Father's Log Income Fixed-Effect			0.0545***	0.0350***		0.0545***	0.0350***		0.0683***	0.0450***		0.0683***	0.0450***
			(0.0007)	(0.0008)		(0.0007)	(0.0008)		(0.0010)	(0.0010)		(0.0010)	(0.0010)
<u>Father Died when Child < 18 interacted with</u>:													
Mother's Education	0.0032**	0.0037**		0.0036*	0.0180***		0.0213***	0.0011		-0.0001	0.0024		0.006
	(0.0015)	(0.0018)		(0.0019)	(0.0067)		(0.0070)	(0.0026)		(0.0028)	(0.0100)		(0.0108)
Father's Education	-0.0072***	-0.0070***		-0.0053***	-0.0213***		-0.0182***	-0.0043*		-0.001	-0.0201**		-0.0141
	(0.0014)	(0.0018)		(0.0018)	(0.0062)		(0.0063)	(0.0025)		(0.0025)	(0.0090)		(0.0091)
Mother's Educ*Age of Child when Father Died					-0.0011**		-0.0013***				-0.0002		-0.0005
					(0.0005)		(0.0005)				(0.0007)		(0.0007)
Father's Educ*Age of Child when Father Died					0.0011**		0.0010**				0.0012*		0.001
					(0.0004)		(0.0004)				(0.0006)		(0.0006)
Mother's Log Income Fixed-Effect			0	0.0013		-0.0202	-0.032		0.0152**	0.0132		-0.0111	-0.0101
			(0.0050)	(0.0054)		(0.0187)	(0.0206)		(0.0077)	(0.0086)		(0.0319)	(0.0362)
Father's Log Income Fixed-Effect			-0.0308***	-0.0205***		-0.0533***	-0.0402**		-0.0384***	-0.0257***		-0.0940***	-0.0698***
			(0.0044)	(0.0045)		(0.0164)	(0.0166)		(0.0064)	(0.0065)		(0.0215)	(0.0219)
Mother's Log Income Fixed-Effect*Age of Child when Father Died						0.0014	0.0023*					0.0017	0.0016
						(0.0013)	(0.0014)					(0.0022)	(0.0024)
Father's Log Income Fixed-Effect*Age of Child when Father Died						0.0016	0.0014					0.0041***	0.0033**
						(0.0011)	(0.0012)					(0.0015)	(0.0016)
Number of School Fixed Effects	848	848	849	848	848	849	848	841	841	841	841	841	841
Observations	646795	641397	652178	641397	641397	652178	641397	437316	437316	437316	437316	437316	437316
<u>Father Died when Child < 18 interacted with</u>:													
Mother's Educ - Father's Educ	0.0056***	0.0056***		0.0043***	0.0199***		0.0183***	0.003		0.0003	0.0138		0.01
	(0.0013)	(0.0016)		(0.0016)	(0.0057)		(0.0058)	(0.0023)		(0.0023)	(0.0084)		(0.0085)
(Mother's Educ - Father's Educ)*Age of Child when Father Died					-0.0011***		-0.0011***				-0.0008		-0.0007
					(0.0004)		(0.0004)				(0.0006)		(0.0006)
Mother's Income Fixed-Effect - Father's Income Fixed-Effect			0.0224***	0.0153***		0.0327**	0.017		0.0297***	0.0209***		0.0662***	0.0473**
			(0.0039)	(0.0039)		(0.0146)	(0.0145)		(0.0054)	(0.0054)		(0.0191)	(0.0192)
(Mother's Income Fixed-Effect - Father's Income Fixed-Effect)*Age of Child when Father Died						-0.0008	-0.0001					-0.0027**	-0.002
						(0.0010)	(0.0010)					(0.0013)	(0.0014)
Number of School Fixed Effects	848	848	849	848	848	849	848	841	841	841	841	841	841
Observations	646795	641397	652178	641397	641397	652178	641397	437316	437316	437316	437316	437316	437316
Controls for Each Parent's Income being Zero Directly and Interacted with Losing Parent < Age 18		Yes	Yes	Yes	Yes	Yes	Yes						
Controls for Each Parent's Income being Zero interacted with Age of Child when Parent Died					Yes	Yes	Yes						

Notes: For every column, the upper and lower panels represent separate regressions. The sample is restricted to individuals who lost a father. Standard errors appear in parentheses. Significance levels are indicated by one, two, or three stars which represent 10 percent, 5 percent, and 1 percent levels, respectively. All regressions are performed using OLS and include controls for mother's age at the time of the child's birth, father's age at the time of the child's birth, gender, number of siblings, birth order, and dummy variables for each cohort year. All specifications also control for socioeconomic status of the locality, a dummy for whether the father died before age 18, age father died, education of the mother, and education of the father.

Appendix Table A6: Father Loss Analysis - Results for Different Subjects

	Pass Matriculation	Math > 80	Math > 70	English > 80	English > 70	Bible > 80	Bible > 70	Hebrew > 80	Hebrew > 70
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Father Died when Child < 18 interacted with:									
Mother's Education	0.0073**	0.0035	0.0029	0.0114***	0.0077**	0.0111***	0.0144***	0.0044	0.0071**
	(0.0037)	(0.0036)	(0.0037)	(0.0033)	(0.0036)	(0.0031)	(0.0036)	(0.0031)	(0.0035)
Father's Education	-0.0137***	-0.0052	-0.0114***	-0.0126***	-0.0166***	-0.0118***	-0.0117***	-0.0112***	-0.0138***
	(0.0034)	(0.0033)	(0.0033)	(0.0030)	(0.0033)	(0.0028)	(0.0033)	(0.0028)	(0.0032)
Mother's Educ*Age of Child when Father Died	-0.0002	0.0000	0.0000	-0.0005**	-0.0004	-0.0006***	-0.0007***	-0.0001	-0.0001
	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0002)	(0.0003)	(0.0002)	(0.0003)
Father's Educ*Age of Child when Father Died	0.0006**	0.0002	0.0005*	0.0007***	0.0009***	0.0009***	0.0008***	0.0007***	0.0008***
	(0.0003)	(0.0003)	(0.0003)	(0.0002)	(0.0003)	(0.0002)	(0.0003)	(0.0002)	(0.0002)
Number of School Fixed-Effects	749	749	749	749	749	749	749	749	749
Observations	49,590	49,590	49,590	49,590	49,590	49,590	49,590	49,590	49,590

Notes: The sample includes only those that lost a father at any age. Standard errors appear in parentheses. Significance levels are indicated by one, two, or three stars which represent 10 percent, 5 percent, and 1 percent levels, respectively. All regressions are performed using OLS and include controls for mother's age at the time of the child's birth, father's age at the time of the child's birth, gender, number of siblings, birth order, and dummy variables for each cohort year. All specifications also control for socioeconomic status of the locality, a dummy for whether the parent died before age 18, age parent died, the interaction between age the parent died and losing the parent before age 18, education of the mother, and education of the father.