Heterogeneity in Expectations, Risk Tolerance, and Household Stock Shares: The Attenuation Puzzle

APPENDICES

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Appendix A. Data Description

We use responses to three VRI surveys, conducted in the fall of 2013, winter of 2014 and summer of 2014.[1] The main focus of the first survey was to inventory income, wealth and portfolio of households as well as to gather basis demographics. The second survey implemented Strategic Survey Questions (SSQs), which ask respondents to make choices under hypothetical situations designed to elicit meaningful preference data. This paper uses the questions about risk preference. The third survey includes the questions about beliefs about returns used for this paper, and also covers a number of issues not related to this paper. 4,730 respondents completed all the three surveys.[2] The item non-response rate of the VRI is remarkably low. Our analysis includes the 4,414 respondents with non-missing observations for all the variables used in the analysis.

The VRI sample frame is based on administrative account data for Vanguard. Having such data to create a sample is an important element of the VRI design. Additionally, administrative data are composed of monthly history of Vanguard assets, with information on types, balances and stock shares of the accounts linked to the survey measures. This paper uses both survey and administrative measures of assets and their composition. The survey measure

[1] VRI is designed as a longitudinal dataset. The first three surveys, however, cover distinctive topics with little longitudinal content. They were broken into three surveys of 40 to 60 minutes for the practical reason of not overwhelming respondents.

[2] The response rate for the first survey was 7.4%, with 8,950 responses. Using the administrative data, we can examine whether response rates vary by age, wealth held at Vanguard, and the line of business. They do not vary noticeably by age, but those with more wealth held at Vanguard and the individual sample (compared to the employer-sponsored sample) are slightly more likely to respond. See Ameriks, Caplin, Lee, Shapiro and Tonetti (2014b) for more details on the response analysis. Only those who completed the previous surveys and were still Vanguard clients were invited to the subsequent ones. The response rate was 64.9% for the second survey (5,741 responses) and 83.8% for the third survey (4,730 responses).

covers all assets, not just those held at Vanguard. See Ameriks, Caplin, Lee, Shapiro and Tonetti (2014a, 2014b) for a detailed discussion of the design of the VRI including sampling and response rates, and of the VRI's approach to wealth measurement.

The results in the text are based on the share of stock from the VRI survey. We also compute a stock share based on administrative account data. It differs in measurement, coverage (Vanguard versus all assets), and timing. The relationship between the survey and administrative measures are discussed in this part of appendix. In Appendix B, we present estimates of the main equations using administrative measurements. In brief, the results are little dependent on which measure is used.

The monthly history of accounts in the VRI administrative data breaks down the balance of each account into stock, bond and money market holdings. This break-down is not readily available for all accounts, so we imputed stock share when needed using information on the type of fund the account is invested in (e.g., for an account invested in a balanced fund, we assume 60% of stock share). The administrative stock share measure is available both at around the time when the stock expectation questions are asked and also at the time when the survey measure of household portfolio is obtained (the wealth survey took place in the fall of 2013, while expectations were asked in the summer of 2014). At the same time, the administrative stock share measure corresponds to the subset of financial wealth held at Vanguard.

Figure A1 Panel A compares the two measures. The figure shows that many observations are near the 45-degree line, so as a practical matter either measure may provide similar inferences for many respondents. At the same time, the two are often different. The correlation coefficient between the two measures is 0.51.[3] The two measures can be different for three main reasons. First, they are measured at different times, in summer 2014 versus fall 2013. Second, they refer to different sets of assets: Vanguard assets versus all financial assets. Third, they are measured in different ways: using administrative records versus answers to survey questions. A detailed analysis is included in Figure A1, Panel B – D. Its results show that imperfect perception is the most important source of the gap; the correlation coefficient between the survey and administrative measures of stock share of the wealth held at Vanguard in fall 2013 is 0.64. We also find very strong inertia in portfolio shares so the timing difference is not an important source of the overall gap; the correlation coefficient of portfolio shares in the administrative data in summer 2014 and in fall 2013 is 0.95. Both these findings—inertia in portfolio choice and limited perception of own portfolio—present challenges to standard theories of portfolio choice and therefore affect the interpretation of results relating portfolio choices to preferences and beliefs. We return to these issues after presenting the results.

[3] All figures and correlation coefficients are weighted by Vanguard wealth. The unweighted correlation is 0.40, indicating that deviations are somewhat larger if wealth is low.

Table A1. Distribution of Wealth in the VRI Data (N=4414)

	Mean	Std	p10	p25	Median	p75	p90
Financial wealth	1,147,525	1,516,575	164,835	363,000	759,750	1,403,843	2,467,899
Home stock	360,782	578,045	31,500	125,000	235,000	420,000	1,060,000

Table A2. Summary Statistics of the Control Variables (N=4414)

	Mean	Standard deviation
Single male	0.14	
Female in couple	0.17	
Single female	0.18	
Age	67.8	7.4
Age squared	4649	1023
In the employer-sponsored sample	0.21	
College degree	0.33	
MBA	0.07	
PhD	0.06	
Other higher degree	0.28	
Log(wealth)	13.4	1.09
Log(home equity)	11.5	3.37
Zero home equity	0.07	
Retired	0.60	
Log(Wage)	4.3	5.5
Log(Annuity Income)	6.5	5.3
Expected Log(Annuity Income)	4.3	5.3
Subjective probability of needing long-term care	0.43	0.30
Subjective probability of survival to target age	0.75	0.23

Notes.
Log variables are set to zero if the levels of the variables are zero. Zero home equity equals 1 (0) if home equity is zero (positive). Annuity income is the sum of Social Security income, defined benefit pension income and immediate annuity income, for retired households. It is set to zero for non-retired households. Expected annuity income is the sum of expected values of Social Security income, defined benefit pension income and immediate annuity income, for non-retired households. It is set to zero for retired households. Subjective probability of needing long-term care is the subjective probability chance that the respondent would need long-term care service at least for one year during her remaining life. The target age in subjective probability of survival question is set to 75 if the respondent is younger than 70, to 85 if the respondent is younger than 80, and to 95 if the respondent is younger than 90.

Table A3. The Risk Tolerance Strategic Survey Questions in VRI Survey 2

Set up	Suppose you are 80 years old. Suppose, further, that for the next year: • You live alone, rent your home, and pay all your own bills. • You are in good health and will remain in good health. • You will have no medical bills or other unexpected expenses. • You do not work.
Hypothetical financial products	• Plan A guarantees that you will have $c for spending next year. • Plan B will possibly provide you with more money, but is less certain. There is a 50% chance that Plan B would double your money, leaving you with $2W, and a 50% chance that it would cut it by x%, leaving you with \$ $(1-0.01\times x)c$.
Rules	• You have no other assets or income, and so the only money you have available for all your spending next year is from either Plan A or Plan B. • Any money that is not spent at the end of next year cannot be saved for the future. • You cannot give any money away or leave it as a bequest. • If you need anything next year, you have to pay for it. No one else can buy anything for you. • At the end of next year you will be offered the same choice with another $W for following year.
Parameters asked	c =100,000 and 50,000.
Question	Would you choose Plan A or Plan B?

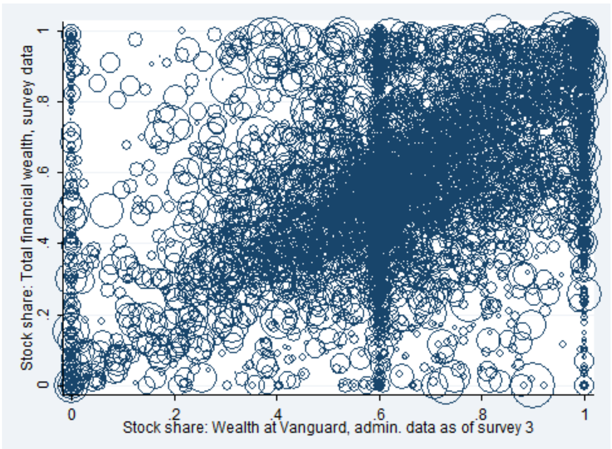
Table A4: The Stock Market Expectation Questions in VRI Survey 3.

Variable name	Survey question
Question Order *p-m*	
p0	What do you think is the percent chance that the stock market will be higher in twelve months than it is today? Think of a stock market index such as the Dow Jones Industrial Average and do not adjust for inflation.
p20	And what do you think is the percent chance that it will be at least 20% higher in twelve months than it is today? [If answer is greater than the p0 answer: "Please enter a response that is less than or equal to you previous response or change your previous response."]
m	Instead of probabilities, we are now interested in your expectation. By what percentage do you expect the stock market to increase or decrease in the next twelve months? Please enter a positive number for increase and negative number for decrease.
Question order *m-p*	
m	By what percentage do you expect the stock market to increase or decrease in the next twelve months? Think of a stock market index such as the Dow Jones Industrial Average and do not adjust for inflation. Please enter a positive number for increase and negative number for decrease.
p0	And what do you think is the percent chance that the stock market will be higher in twelve months than it is today?
p20	What do you think is the percent chance that it will be at least 20% higher in twelve months than it is today? [If answer is greater than the p0 answer: "Please enter a response that is less than or equal to you previous response or change your previous response."]

Note: The question orders are randomized in the survey instrument. The distributions of responses are slightly different depending on which sequence is used.

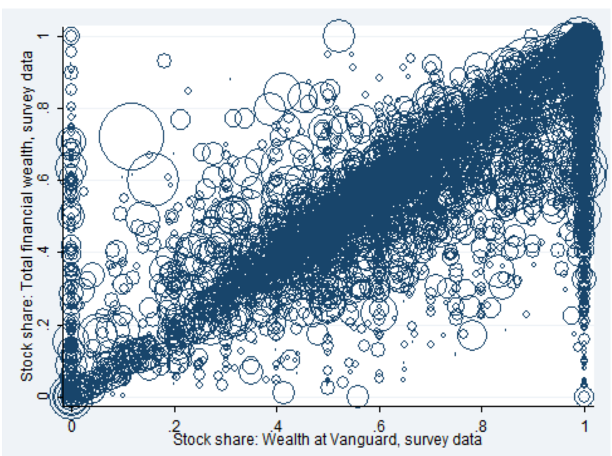
Figure A1. Stock Shares, Alternative Measurements Compared

A. Total Survey versus Vanguard Administrative
(Corr = 0.51)

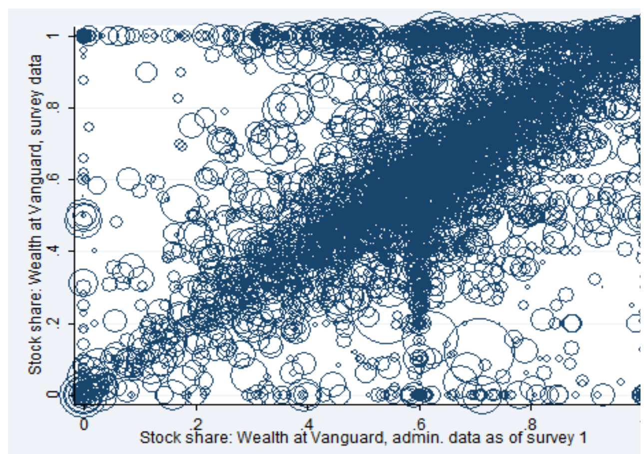


B. Different Time Periods, Vanguard Administrative
(Corr = 0.95)

C. Total versus Vanguard, Survey
(Corr = 0.81)



D.Vanguard Survey versus Vanguard Administrative
(Corr = 0.64)



Note: The variables on the horizontal and vertical axes are:
Panel A: the administrative measure of stock share at Vanguard at the time of Survey 3 (summer 2014) versus the survey measure of stock share overall at the time of Survey 1 (Summer 2014). These are the two main dependent variables in the analysis.
Panel B: the administrative measure of stock share at Vanguard at the time of Survey 1 (Fall 2013) versus the administrative measure of stock share at Vanguard at the time of Survey 3 (Summer 2014).
Panel C: the survey measure of stock share at Vanguard versus the survey measure of stock share overall, both measured at the time of Survey 1 (Fall 2013).
Panel D: the administrative measure of stock share at Vanguard versus the survey measure of stock share at Vanguard, both measured at the time of Survey 1 (Fall 2013).
The size of the marks on the figures is proportional to the Vanguard financial wealth of the respondents. The reported correlation coefficients are weighted by Vanguard financial wealth.
Due to the imputation used for the balanced funds, there is heaping at 60 percent in the administrative measure.

Appendix B. Additional Results

Table B1. Detailed Results of the Structural Estimation Model Without Covariates. (N=4,414)

	Preference		Beliefs		Bias in *p0*
	Θ	κ	μ	σ	ψ
constant	-1.148	-16.911	0.055	0.118	-0.539
	(0.027)	(1.033)	(0.002)	(0.002)	(0.017)
Heterogeneity					
ω_u	0.704	n.a.	0.063	0.032	n.a.
	(0.011)	n.a.	(0.001)	(0.001)	n.a.
			Correlation across latent variables		
β_θ			0.011	-0.004	
			(0.003)	(0.002)	
$\rho_{\mu\sigma}$			0.062		
			(0.021)		
Measurement error					
$\omega_{e\gamma 1}$			0.812		
			(0.015)		
$\omega_{e\gamma 2}$			0.544		
			(0.016)		
ω_{em}			0.079		
			(0.001)		
ω_{ep}			0.487		
			(0.008)		
Log-likelihood			-48006		

Notes. The third line reports how the latent risk tolerance parameter affects means of the belief parameter distributions. Statistics reported in Table 4 are calculated based on these parameters, where the means of belief parameter distributions are adjusted using the mean of the risk tolerance parameter multiplied with the numbers reported in the third row.
Standard errors in parentheses.

Table B2. Detailed Results of The Structural Estimation Model with Covariates. (N=4,414)

	Preference		Beliefs		Bias in *p0*
	θ	κ	μ	σ	ψ
Constant	-0.405	-91.817	0.067	0.171	-0.292
	(0.696)	(39.028)	(0.029)	(0.028)	(0.854)
Single male	0.110	-3.398	0.004	0.002	-0.017
	(0.060)	(1.771)	(0.004)	(0.003)	(0.042)
Female in couple	-0.153	-2.090	0.005	0.000	-0.183
	(0.061)	(1.975)	(0.004)	(0.003)	(0.038)
Single female	-0.215	1.181	0.011	0.001	-0.284
	(0.063)	(2.100)	(0.004)	(0.003)	(0.039)
Age	-0.032	1.632	0.000	-0.002	-0.027
	(0.018)	(1.077)	(0.000)	(0.000)	(0.023)
Age sq.	0.000	-0.013	0.000	0.000	0.000
	(0.000)	(0.008)	(0.000)	(0.000)	(0.000)
Employer-sponsored	-0.239	11.091	0.015	-0.005	-0.155
	(0.059)	(2.093)	(0.003)	(0.003)	(0.037)
College degree	-0.036	3.227	-0.008	0.006	0.276
	(0.058)	(2.076)	(0.003)	(0.002)	(0.035)
MBA	0.296	-6.509	-0.003	0.004	0.201
	(0.085)	(2.241)	(0.006)	(0.004)	(0.063)
PhD	-0.030	2.988	-0.017	0.022	0.442
	(0.096)	(3.122)	(0.007)	(0.006)	(0.068)
Other higher degree	0.041	1.530	-0.009	0.013	0.344
	(0.061)	(2.128)	(0.004)	(0.003)	(0.038)
log(wealth)	0.059	-1.147	-0.008	0.005	0.125
	(0.022)	(0.765)	(0.001)	(0.001)	(0.012)
log(home equity)	0.012	0.565	-0.002	0.000	-0.005
	(0.023)	(0.716)	(0.001)	(0.001)	(0.015)
No home equity	-0.039	15.715	-0.020	0.000	-0.097
	(0.286)	(9.209)	(0.014)	(0.011)	(0.180)
Retired	-0.466	37.180	-0.025	-0.027	-0.209
	(0.575)	(20.647)	(0.029)	(0.028)	(0.383)
Log(Wage)	-0.005	0.080	0.000	-0.001	-0.006
	(0.016)	(0.536)	(0.001)	(0.001)	(0.011)
Log(Annuity Income)	-0.047	0.622	0.008	-0.002	-0.040
	(0.043)	(1.546)	(0.001)	(0.002)	(0.024)
Expected Log(Annuity Income)	-0.069	3.517	0.005	-0.003	-0.057
	(0.042)	(1.321)	(0.003)	(0.002)	(0.029)
LTC probability	0.359	-15.308	-0.018	0.003	0.185
	(0.070)	(2.281)	(0.004)	(0.003)	(0.044)
Longevity probability	0.367	-8.842	0.026	-0.004	0.031
	(0.102)	(3.955)	(0.006)	(0.004)	(0.062)

Heterogeneity					
ω_u	0.679	n.a.	0.063	0.030	n.a.
	(0.011)	n.a.	(0.001)	(0.001)	n.a.
Correlation across latent variables					
β_θ			0.012	-0.004	
			(0.003)	(0.002)	
$\rho_{\mu\sigma}$				0.012	
				(0.023)	
Measurement error					
$\omega_{e\gamma 1}$				0.819	
				(0.015)	
$\omega_{e\gamma 2}$				0.567	
				(0.016)	
ω_{em}				0.078	
				(0.001)	
ω_{ep}				0.455	
				(0.008)	
Log-likelihood				-47650	

Note: Standard errors in parentheses.
Reference categories are male in couple, individual client sample, not having a college degree. See notes to Table A2 for detailed description of the right hand side variables.

Table B3. Detailed Results of The Structural Estimation Model Without Covariates Using CRRA Utility Function. (N=4,414)

	Preference	Beliefs		Bias in *p0*
	Θ	μ	σ	ψ
constant	-1.476	0.055	0.118	-0.539
	(0.013)	(0.002)	(0.002)	(0.017)
Heterogeneity				
ω_u	0.683	0.063	0.032	n.a.
	(0.011)	(0.001)	(0.001)	n.a.
Correlation across latent variables				
β_θ		0.016	-0.006	
		(0.005)	(0.003)	
$\rho_{\mu\sigma}$		0.058		
		(0.022)		
Measurement error				
$\omega_{e\gamma 1}$		0.868		
		(0.015)		
$\omega_{e\gamma 2}$		0.506		
		(0.015)		
ω_{em}		0.079		
		(0.001)		
ω_{ep}		0.488		
		(0.008)		
Log-likelihood		-48098		

Notes.
The third line reports how the latent risk tolerance parameter affects means of the belief parameter distributions. Standard errors in parentheses.

Table B4. Detailed Results of The Structural Estimation Model With Covariates Using CRRA Utility Function. (N=4,414)

	Preference	Beliefs		Bias in *p0*
	θ	μ	σ	ψ
Constant	-1.793	0.081	0.129	-0.738
	(0.119)	(0.032)	(0.019)	(0.834)
Single male	0.032	0.005	0.002	-0.017
	(0.041)	(0.004)	(0.003)	(0.041)
Female in couple	-0.202	0.006	-0.000	-0.189
	(0.038)	(0.004)	(0.002)	(0.038)
Single female	-0.189	0.011	0.000	-0.293
	(0.039)	(0.004)	(0.003)	(0.038)
Age	-0.014	-0.000	-0.001	-0.019
	(0.007)	(0.000)	(0.000)	(0.023)
Age sq.	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Employer-sponsored	-0.009	0.014	-0.004	-0.152
	(0.037)	(0.003)	(0.002)	(0.036)
College degree	0.031	-0.009	0.007	0.286
	(0.033)	(0.003)	(0.002)	(0.035)
MBA	0.102	-0.002	0.003	0.205
	(0.058)	(0.006)	(0.004)	(0.062)
PhD	0.031	-0.018	0.022	0.455
	(0.059)	(0.007)	(0.006)	(0.068)
Other higher degree	0.071	-0.009	0.013	0.351
	(0.037)	(0.004)	(0.002)	(0.037)
log(wealth)	0.037	-0.008	0.005	0.131
	(0.014)	(0.001)	(0.001)	(0.013)
log(home equity)	0.027	-0.002	0.000	-0.006
	(0.015)	(0.001)	(0.001)	(0.014)
No home equity	0.320	-0.022	-0.000	-0.121
	(0.179)	(0.014)	(0.010)	(0.177)
Retired	0.263	-0.042	-0.013	-0.055
	(0.373)	(0.030)	(0.019)	(0.368)
Log(Wage)	-0.003	0.001	-0.001	-0.007
	(0.010)	(0.001)	(0.001)	(0.011)
Log(Annuity Income)	-0.026	0.008	-0.002	-0.046
	(0.028)	(0.001)	(0.001)	(0.022)
Expected Log(Annuity Income)	0.006	0.004	-0.003	-0.047
	(0.027)	(0.003)	(0.002)	(0.028)
LTC probability	-0.001	-0.014	0.001	0.170
	(0.042)	(0.004)	(0.003)	(0.043)
Longevity probability	0.162	0.028	-0.004	0.033

	(0.057)	(0.006)	(0.004)	(0.062)
Heterogeneity				
ω_u	0.665	0.063	0.030	n.a.
	(0.011)	(0.001)	(0.001)	n.a.
Correlation across latent variables				
β_θ		0.017	-0.004	
		(0.005)	(0.003)	
$\rho_{\mu\sigma}$			-0.001	
			(0.024)	
Measurement error				
$\omega_{e\gamma 1}$			0.864	
			(0.015)	
$\omega_{e\gamma 2}$			0.524	
			(0.014)	
ω_{em}			0.078	
			(0.001)	
ω_{ep}			0.453	
			(0.008)	
Log-likelihood			-47748	

Note: Standard errors in parentheses.
Reference categories are male in couple, individual client sample, not having a college degree. See notes to Table A2 for detailed description of the right hand side variables.

Table B5. Stock Share Regressions with Raw Survey Answers on the Right Hand Side (with m_i as a Proxy for Beliefs of Mean Returns μ_i)

	Survey stock share		Administrative stock share	
m_i	0.126	0.153	0.180	0.192
	(0.037)	(0.037)	(0.038)	(0.038)
$p0_i$-$p20_i$	0.107	0.085	0.098	0.091
	(0.016)	(0.016)	(0.017)	(0.017)
SSQ1 cat=2	0.026	0.016	0.016	0.008
	(0.011)	(0.011)	(0.011)	(0.011)
SSQ1 cat=3	0.047	0.035	0.038	0.028
	(0.011)	(0.011)	(0.011)	(0.011)
SSQ1 cat=4	0.054	0.044	0.057	0.049
	(0.013)	(0.013)	(0.013)	(0.013)
SSQ1 cat=5	0.083	0.073	0.080	0.075
	(0.014)	(0.014)	(0.015)	(0.015)
SSQ1 cat=6	0.053	0.045	-0.023	-0.026
	(0.031)	(0.031)	(0.032)	(0.031)
Single male		0.045		0.013
		(0.031)		(0.012)
Female in couple		0.016		0.021
		(0.012)		(0.011)
Single female		-0.007		0.019
		(0.011)		(0.012)
Age		-0.007		-0.014
		(0.012)		(0.009)
Age sq.		0.000		0.000
		(0.001)		(0.000)
Employer-sponsored		-0.053		-0.042
		(0.011)		(0.011)
College degree		0.018		0.023*
		(0.010)		(0.010)
MBA		0.033		0.022
		(0.017)		(0.018)
PhD		0.009		0.068
		(0.017)		(0.018)
Other higher degree		0.015		0.029
		(0.011)		(0.011)
log(wealth)		0.017		-0.001
		(0.004)		(0.004)
log(home equity)		0.004		0.008
		(0.004)		(0.004)
No home equity		0.031		0.080
		(0.054)		(0.055)

Retired		-0.254		-0.318
		(0.116)		(0.119)
Log(Wage)		0.005		-0.001
		(0.003)		(0.003)
Log(Annuity Income)		0.002		0.023
		(0.008)		(0.008)
Expected Log(Annuity Income)		-0.023		-0.002
		(0.008)		(0.008)
LTC probability		-0.027		-0.035
		(0.013)		(0.013)
Longevity probability		0.042		0.034
		(0.018)		(0.019)
Constant		0.371		1.028
		(0.111)		(0.319)
R^2	0.023	0.040	0.023	0.043
Observations	4414	4414	4414	4414

Note: Standard errors in parentheses.
Reference categories are male in couple, individual client sample, not having a college degree. See notes to Table A2 for detailed description of the right hand side variables.

Table B6. Stock Share Regressions with Raw Survey Answers on the Right Hand Side (with $\left(p_{0,i}+p_{20,i}\right)/2$ as a Proxy for Beliefs of Mean Returns μ)

	Survey stock share		Administrative stock share	
$(p0_i+p20_i)/2$	0.115	0.118	0.097	0.089
	(0.024)	(0.024)	(0.025)	(0.025)
$p0_i - p20_i$	0.076	0.056	0.075	0.074
	(0.018)	(0.018)	(0.018)	(0.018)
SSQ1 cat=2	0.023	0.013	0.012	0.005
	(0.011)	(0.011)	(0.011)	(0.011)
SSQ1 cat=3	0.043	0.031	0.033	0.024
	(0.011)	(0.011)	(0.011)	(0.011)
SSQ1 cat=4	0.049	0.040	0.052	0.045
	(0.013)	(0.013)	(0.013)	(0.013)
SSQ1 cat=5	0.079	0.069	0.076	0.071
	(0.014)	(0.014)	(0.015)	(0.015)
SSQ1 cat=6	0.051	0.043	-0.024	-0.027
	(0.031)	(0.031)	(0.032)	(0.032)
Single male	0.051	0.043		0.014
	(0.031)	(0.031)		(0.012)
Female in couple		0.017		0.023
		(0.012)		(0.011)
Single female		-0.006		0.023
		(0.011)		(0.012)
Age		-0.004		-0.014
		(0.012)		(0.009)
Age sq.		0.001		0.000
		(0.001)		(0.000)
Employer-sponsored		-0.052		-0.041
		(0.011)		(0.011)
College degree		0.014		0.020
		(0.010)		(0.010)
MBA		0.029		0.019
		(0.017)		(0.018)
PhD		0.004		0.064
		(0.017)		(0.018)
Other higher degree		0.011		0.025
		(0.011)		(0.011)
log(wealth)		0.017		-0.001
		(0.004)		(0.004)
log(home equity)		0.004		0.008
		(0.004)		(0.004)
No home equity		0.033		0.080

		(0.054)		(0.055)
Retired		-0.256		-0.321
		(0.116)		(0.120)
Log(Wage)		0.005		-0.001
		(0.003)		(0.004)
Log(Annuity Income)		0.003		0.024
		(0.008)		(0.008)
Expected Log(Annuity Income)		-0.022		-0.001
		(0.008)		(0.008)
LTC probability		-0.028		-0.037
		(0.013)		(0.013)
Longevity probability		0.039		0.034
		(0.018)		(0.019)
Constant		0.340		1.010
		(0.111)		(0.319)
R^2	0.025	0.041	0.022	0.040
Observations	4414	4414	4414	4414

Note: Standard errors in parentheses.
Reference categories are male in couple, individual client sample, not having a college degree. See notes to Table A2 for detailed description of the right hand side variables.

Table B7. Stock Shares versus Cardinal Proxies for Preferences and Beliefs,
Dependent variable: Administrative Stock Share

	Optimal Proxies		Error-Ridden Proxies	
Expected return	0.052	0.047	0.020	0.021
	(0.008)	(0.008)	(0.004)	(0.004)
Perceived standard deviation	-0.068	-0.083	-0.025	-0.020
	(0.040)	(0.038)	(0.006)	(0.006)
Risk tolerance parameter	0.012	0.013	0.013	0.013
	(0.010)	(0.010)	(0.004)	(0.004)
constant	-0.001	0.803	-0.000	0.781
	(0.007)	(0.519)	(0.006)	(0.507)
covariates	N	Y	N	Y
R^2	0.013	0.038	0.012	0.038
N	4414	4414	4414	4414

Notes. Stock share Vanguard wealth (administrative measure) are regressed on proxies for the expected stock returns, perceived standard deviation of stock returns, and the risk tolerance parameter. For the first two columns, right-hand-side variables are the optimal cardinal proxies ($\hat{\mu}_i$, $\hat{\sigma}_i$ and $\hat{\theta}_i$) calculated from (10). For the next two columns, right-hand-side variables are the raw survey answers to the stock market expectation question (m_i), a crude transformation of the probability answers to approximate perceived risk ($\breve{\sigma}_i = 0.2 / \left(\Phi^{-1}(p_{0i}) - \Phi^{-1}(p_{20i}) \right)$), and the median value of the CRRA risk tolerance parameter that corresponds to the answers to the first set of the risk tolerance questions (κ set to zero). All variables are expressed as relative differences normalized to their mean values (as specified in equation (12)).
Control variables: married, male, age, whether respondent comes from the employer-sponsored subsample, education (below college; college; MBA; PhD, other higher degree); log financial wealth, log wage, dummy for owning a house, log annuity income (Social Security and DB pensions) for retired, log expected annuity income for non-retired; dummy for retired, log home stock; subjective probability of needing long-term care, and longevity expectations. Bootstrap standard errors in parentheses.

Table B8. Stock Share and Preference And Belief Proxies.
Detailed Results Corresponding to Table 5 and Table B7.

	Survey stock share		Administrative stock share	
Expected return	0.058	0.055	0.052	0.048
	(0.010)	(0.009)	(0.008)	(0.008)
Perceived standard deviation	-0.093	-0.083	-0.068	-0.083
	(0.046)	(0.051)	(0.040)	(0.038)
Risk tolerance parameter	0.034	0.033	0.012	0.013
	(0.009)	(0.010)	(0.010)	(0.010)
Single male		0.027		0.022
		(0.022)		(0.019)
Female in couple		-0.025		0.023
		(0.021)		(0.018)
Single female		-0.031		0.013
		(0.020)		(0.019)
Age		-0.042		-0.027
		(0.017)		(0.015)
Age sq.		0.000		0.000
		(0.000)		(0.000)
Employer-sponsored		-0.115		-0.081
		(0.020)		(0.018)
College degree		0.048		0.051
		(0.021)		(0.017)
MBA		0.072		0.048
		(0.027)		(0.032)
PhD		0.057		0.143
		(0.032)		(0.025)
Other higher degree		0.053		0.069
		(0.022)		(0.019)
log(wealth)		0.044		0.011
		(0.009)		(0.007)
log(home equity)		0.008		0.013
		(0.009)		(0.006)
No home equity		0.052		0.120
		(0.118)		(0.079)
Retired		-0.448		-0.496
		(0.244)		(0.196)
Log(Wage)		0.007		-0.002
		(0.005)		(0.005)
Log(Annuity Income)		-0.002		0.032
		(0.016)		(0.015)
Expected Log(Annuity Income)		-0.045		-0.006
		(0.019)		(0.012)

LTC probability		-0.032		-0.041
		(0.028)		(0.021)
Longevity probability		0.084		0.050
		(0.033)		(0.032)
Constant	-0.001	1.136	-0.001	0.803
	(0.007)	(0.649)	(0.007)	(0.519)
R^2	0.019	0.045	0.013	0.038
Observations	4414	4414	4414	4414

Standard errors in parentheses.
Reference categories are male in couple, individual client sample, not having a college degree. See notes to Table A2 for detailed description of the right hand side variables.

Table B9. Stock Shares versus Error-Ridden Cardinal Measures of Preferences and Beliefs. Estimation without Accounting for Measurement Error in the Cardinal Proxies.

	Survey stock share		Administrative stock share	
m_i	0.017	0.020	0.020	0.021
	(0.004)	(0.004)	(0.004)	(0.004)
$\hat{\sigma}_i$	-0.029	-0.019	-0.025	-0.020
	(0.006)	(0.007)	(0.006)	(0.006)
$\hat{\theta}_i$	0.021	0.020	0.013	0.013
	(0.005)	(0.005)	(0.004)	(0.004)
Single male		0.027		0.021
		(0.022)		(0.019)
Female in couple		-0.024		0.025
		(0.020)		(0.018)
Single female		-0.026		0.019
		(0.021)		(0.019)
Age		-0.040		-0.025
		(0.016)		(0.014)
Age sq.		0.000		0.000
		(0.000)		(0.000)
Employer- sponsored		-0.099		-0.067
		(0.019)		(0.017)
College degree		0.037		0.040
		(0.019)		(0.017)
MBA		0.066		0.041
		(0.031)		(0.028)
PhD		0.025		0.113
		(0.032)		(0.028)
Other higher degree		0.036		0.052
		(0.020)		(0.018)
log(wealth)		0.034		0.002
		(0.008)		(0.007)
log(home equity)		0.008		0.013
		(0.008)		(0.007)
No home equity		0.054		0.118
		(0.098)		(0.088)
Retired		-0.454		-0.497
		(0.212)		(0.190)
Log(Wage)		0.007		-0.002
		(0.005)		(-0.005)
Log(Annuity Income)		0.004		0.037
		(0.015)		(0.013)
Expected Log(Annuity Income)		-0.041		-0.002
		(0.015)		(0.013)
LTC probability		-0.043		-0.050

		(0.023)		(0.021)
Longevity probability		0.106		0.065
		(0.033)		(0.030)
constant	-0.001	1.120	-0.000	0.781
	(0.007)	(0.565)	(0.006)	(0.507)
R^2	0.013	0.039	0.012	0.038
N	4414	4414	4414	4414

Notes. In these regressions the cardinal proxies $\hat{\mu}_i, \hat{\sigma}_i, \hat{\theta}_i$ are replaced with $m_i, \widehat{\sigma}_i, \widehat{\theta}_i$, respectively, where m_i is the raw answer to the expected stock returns question, $\widehat{\sigma}_i = 0.2 / \left(\Phi^{-1}(p_{0i}) - \Phi^{-1}(p_{20i}) \right)$ (the denominator replaced with 0.2 if zero), and $\widehat{\theta}_i$ is the median valie of the CRRA risk tolerance parameter that corresponds to the answers to the first set of the risk tolerance questions (κ set to zero).
Standard errors in parentheses.
Reference categories are male in couple, individual client sample, not having a college degree. See notes to Table A2 for detailed description of the right hand side variables.

Table B10. Stock Shares versus Cardinal Proxies for Preferences and Beliefs Based on CRRA Utility Function

	Survey stock share		Administrative stock share	
Expected return	0.058	0.054	0.049	0.045
	(0.008)	(0.007)	(0.007)	(0.007)
Perceived standard deviation	-0.095	-0.081	-0.071	-0.084
	(0.042)	(0.042)	(0.038)	(0.038)
Risk tolerance parameter (CRRA)	0.036	0.035	0.014	0.015
	(0.008)	(0.009)	(0.008)	(0.008)
constant	-0.000	1.094	0.000	0.758
	(0.007)	(0.563)	(0.006)	(0.507)
covariates	N	Y	N	Y
R^2	0.018	0.040	0.013	0.038
N	4414	4414	4414	4414

Notes. Right-hand-side variables are the optimal cardinal proxies ($\hat{\mu}_i$, $\hat{\sigma}_i$ and $\hat{\theta}_i$) calculated from (10) except for that κ is set to be zero. All variables are expressed as relative differences normalized to their mean values (as specified in equation (12)).
Control variables: married, male, age, whether respondent comes from the employer-sponsored subsample, education (below college; college; MBA; PhD, other higher degree); log financial wealth, log wage, dummy for owning a house, log annuity income (Social Security and DB pensions) for retired, log expected annuity income for non-retired; dummy for retired, log home stock; subjective probability of needing long-term care, and longevity expectations. Bootstrap standard errors in parentheses.

Table B11. Stock Shares Versus Cardinal Proxies for Preferences and Beliefs. Employer-Sponsored Subsample

	Survey stock share		Administrative stock share	
Expected return	0.067	0.062	0.083	0.080
	(0.018)	(0.019)	(0.014)	(0.015)
Perceived standard deviation	-0.122	-0.037	-0.014	0.055
	(0.097)	(0.107)	(0.088)	(0.087)
Risk tolerance parameter	0.070	0.068	0.016	-0.007
	(0.029)	(0.031)	(0.032)	(0.040)
constant	-0.074	1.930	-0.030	3.388
	(0.017)	(1.896)	(0.015)	(1.836)
control variables	N	Y	N	Y
R^2	0.026	0.040	0.033	0.079
N	923	923	923	923

Notes.
Employer-sponsored sample are those who only have 401(k) type accounts at Vanguard.

Table B12. Stock Shares Versus Cardinal Proxies for Preferences and Beliefs. Individual-Client Subsample

	Survey stock share		Administrative stock share	
Expected return	0.059	0.055	0.041	0.036
	(0.010)	(0.012)	(0.011)	(0.009)
Perceived standard deviation	-0.075	-0.089	-0.091	-0.112
	(0.051)	(0.055)	(0.051)	(0.046)
Risk tolerance parameter	0.027	0.024	0.012	0.013
	(0.010)	(0.010)	(0.010)	(0.011)
constant	0.024	1.099	0.011	0.765
	(0.009)	(0.525)	(0.007)	(0.570)
control variables	N	Y	N	Y
R^2	0.016	0.032	0.008	0.028
N	3491	3491	3491	3491

Notes.
Individual-client sample is the complement of Employer-sponsored sample.

Table B13. Stock Shares Versus Cardinal Proxies for Preferences and Beliefs. Share of Wealth at Vanguard at least 50 percent

	Survey stock share		Administrative stock share	
Expected return	0.057	0.053	0.045	0.044
	(0.015)	(0.013)	(0.011)	(0.011)
Perceived standard deviation	-0.139	-0.131	-0.008	-0.018
	(0.067)	(0.076)	(0.055)	(0.053)
Risk tolerance parameter	0.035	0.038	0.029	0.029
	(0.012)	(0.015)	(0.012)	(0.014)
constant	0.005	0.776	-0.032	1.193
	(0.009)	(0.870)	(0.007)	(0.756)
control variables	N	Y	N	Y
R^2	0.020	0.034	0.018	0.042
N	1909	1909	1909	1909

Table B14. Stock Shares Versus Cardinal Proxies for Preferences and Beliefs. Share of Wealth at Vanguard at least 70 percent

	Survey stock share		Administrative stock share	
Expected return	0.058	0.054	0.060	0.058
	(0.016)	(0.017)	(0.013)	(0.013)
Perceived standard deviation	-0.127	-0.107	-0.018	-0.008
	(0.084)	(0.075)	(0.061)	(0.067)
Risk tolerance parameter	0.041	0.045	0.036	0.039
	(0.013)	(0.015)	(0.012)	(0.014)
constant	0.004	0.470	-0.046	0.698
	(0.015)	(1.225)	(0.012)	(1.032)
control variables	N	Y	N	Y
R^2	0.019	0.036	0.003	0.061
N	1241	1241	1241	1241

Table B15. Stock Shares Versus Cardinal Proxies for Preferences and Beliefs. Households with Directly Held Stocks

	Survey stock share		Administrative stock share	
Expected return	0.051	0.067	0.045	0.039
	(0.024)	(0.023)	(0.020)	(0.019)
Perceived standard deviation	-0.147	-0.136	-0.169	-0.095
	(0.156)	(0.126)	(0.149)	(0.107)
Risk tolerance parameter	0.023	0.022	-0.001	0.011
	(0.017)	(0.024)	(0.030)	(0.036)
constant	0.070	1.321	0.045	3.797
	(0.018)	(1.771)	(0.018)	(1.600)
control variables	N	Y	N	Y
R^2	0.013	0.026	0.011	0.042
N	639	639	639	639

Table B16. Stock Shares Versus Cardinal Proxies for Preferences and Beliefs. By Education Group

	Survey stock share			Administrative stock share		
	MBA	Other post-college	No post-college	MBA	Other post-college	No post-college
Expected return	0.136	0.052	0.055	0.050	0.062	0.043
	(0.029)	(0.014)	(0.012)	(0.032)	(0.014)	(0.010)
Perceived standard deviation	0.009	-0.098	-0.064	-0.129	-0.119	-0.055
	(0.151)	(0.069)	(0.053)	(0.165)	(0.063)	(0.047)
Risk tolerance parameter	0.040	0.023	0.031	0.028	0.033	-0.003
	(0.024)	(0.014)	(0.011)	(0.026)	(0.013)	(0.009)
constant	1.643	1.416	0.910	-1.348	1.165	0.488
	(2.163)	(0.921)	(0.764)	(2.370)	(0.845)	(0.667)
covariates	Y	Y	Y	Y	Y	Y
R^2	0.206	0.037	0.043	0.175	0.040	0.030
N	287	1,538	2,589	287	1,538	2,589

Table B17. Share of Risky Assets Versus Cardinal Proxies for Preferences and Beliefs. (Including Housing Wealth in the Share of Risky Asset Calculation)

	Housing wealth included as safe assets		Housing wealth included as risky assets	
Expected return	0.036	0.033	0.065	0.065
	(0.006)	(0.006)	(0.011)	(0.010)
Perceived standard deviation	-0.070	-0.062	-0.123	-0.106
	(0.031)	(0.031)	(0.057)	(0.051)
Risk tolerance parameter	0.018	0.018	0.030	0.022
	(0.006)	(0.006)	(0.010)	(0.011)
constant	-0.003	0.621	0.005	0.689
	(0.005)	(0.375)	(0.010)	(0.623)
control variables	N	Y	N	Y
R^2	0.014	0.042	0.014	0.052
N	4,414	4,414	4,414	4,414

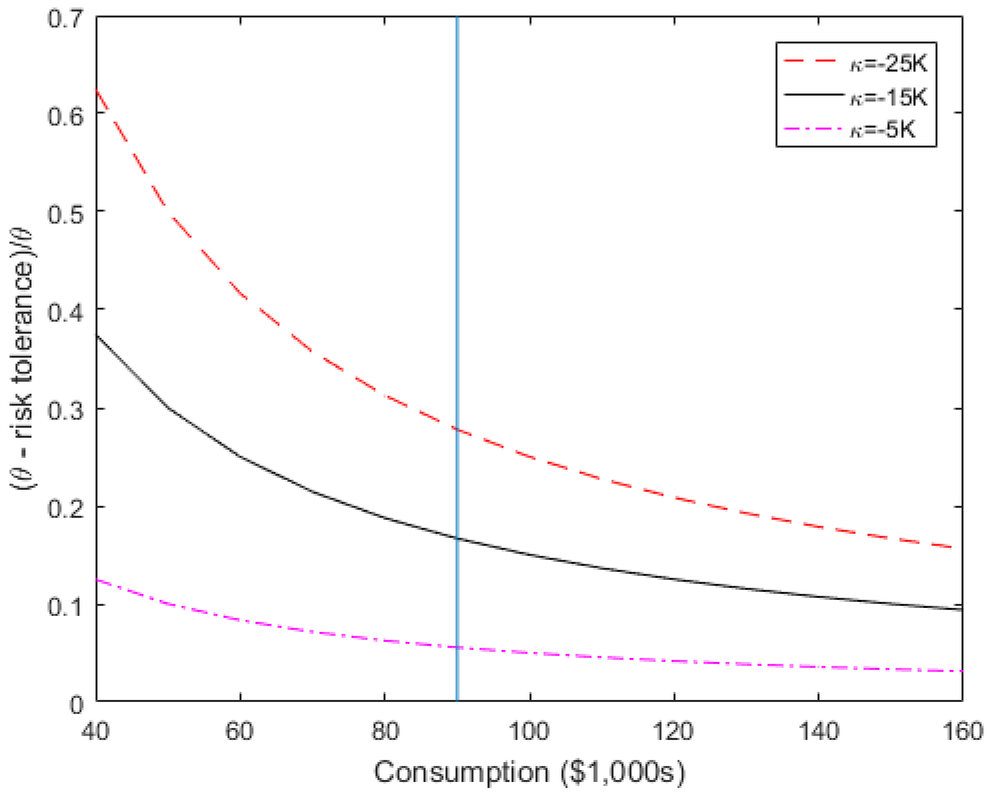
Notes.
For the first two columns, the LHS variable is calculated as the share of stock holdings (based on survey measure) out of the sum of total financial wealth and housing wealth.
For the last two columns, the LHS variable is calculated as the share of the sum of stock holding (based on survey measures) and housing wealth out of the sum of total financial wealth and housing wealth.

Table B18.
Dependent variable: Difference of Survey and Administrative Measures of Stock Share.

	Optimal Proxies		Error-Ridden Proxies	
Expected return	0.001	0.003	-0.005	-0.005
	(0.010)	(0.010)	(0.005)	(0.005)
Perceived standard deviation	-0.013	0.016	-0.001	0.005
	(0.056)	(0.053)	(0.008)	(0.007)
Risk tolerance parameter	0.018	0.016	0.006	0.005
	(0.011)	(0.011)	(0.006)	(0.006)
constant	-0.135	-0.551	-0.135	-0.564
	(0.009)	(0.184)	(0.009)	(0.184)
covariates	N	Y	N	Y
R^2	0.001	0.025	0.001	0.025
N	4414	4414	4414	4414

Note: Dependent variable: Difference in the stock share measure is normalized as the difference (survey – admin) divided by the sum of the average of survey measure and that of administrative measure (divided by 2).

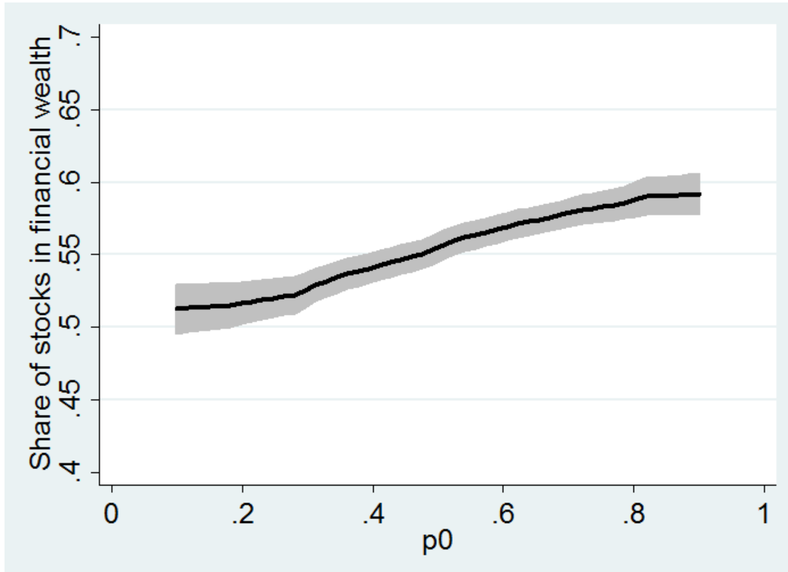
Figure B1. Difference between Relative Risk Tolerance and θ (as a fraction of θ) over Different Levels of Consumption and κ .



Notes.

The vertical line shows the mean level of household income in the VRI (before retirement), to approximate the average level of household consumption.

Figure B2. Bi-Variate Non-Parametric Regression of Stock Share in Total Financial Wealth on Each Probability Questions on Stock Market Expectation



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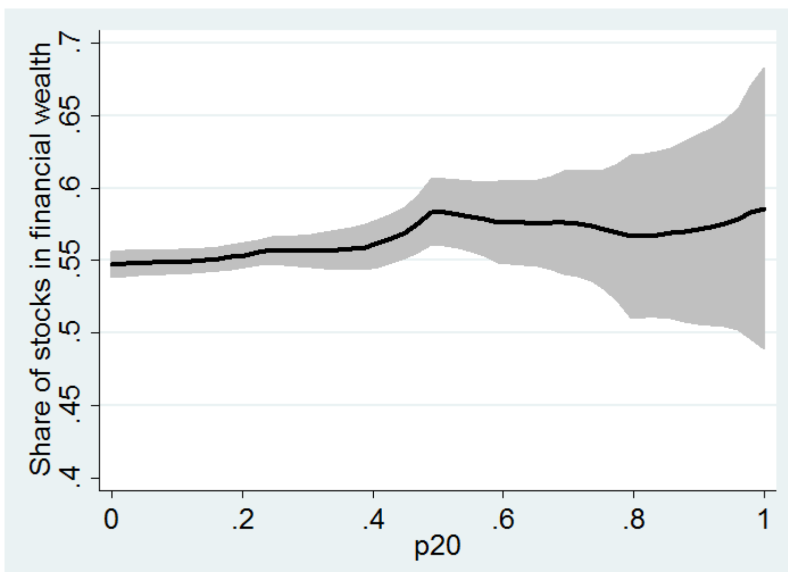

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Appendix C. Algorithm of Likelihood Function and Individual-Specific Cardinal Proxy Computation

Algorithm of likelihood function calculation

We use the Gaussian quadrature approximation of the normal distribution to numerically integrate the density functions over multiple dimensions. Let Ξ be the vector of parameters. Given a fixed Ξ_0, the likelihood function is calculated through the following algorithm:

1) Based on the parameter values that govern the true belief and preference parameter distributions in Ξ_0, and using Gaussian Quadrature approximation, generate K nodes $\{\mu_k, \sigma_k, \theta_k\}_{k=1}^{K}$ of belief and preference parameters, with corresponding probabilities $\{\pi_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} \pi_k = 1$.

2) For each $\{\mu_k, \sigma_k, \theta_k\}$ and each individual, calculate $[\varepsilon_{mi}^{low}, \varepsilon_{mi}^{high}]$, $[\varepsilon_{0i}^{low}, \varepsilon_{0i}^{high}]$, $[\varepsilon_{20i}^{low}, \varepsilon_{20i}^{high}]$, $[\varepsilon_{\theta 1i}^{low}, \varepsilon_{\theta 1i}^{high}]$, $[\varepsilon_{\theta 2i}^{low}, \varepsilon_{\theta 2i}^{high}]$ such that survey response error terms realized in these ranges generate the observed responses after rounding and corresponding constraints.

3) For each $\{\mu_k, \sigma_k, \theta_k\}$ and each individual, calculate the joint likelihood of the realization of the error terms in the range found in 2), using Gaussian CDF under the parameter values governing the error term distributions in Ξ_0. Let π_{ki}^{ε} denote this joint likelihood.

4) The likelihood for each individual is calculated as integration over *k* nodes as following:

$$L_i = \sum_{k=1}^{K} \pi_{ki}^{\varepsilon} \pi_k$$

Then the joint likelihood is calculated as products of L_i over individuals.

Algorithm of individual-specific cardinal proxy calculation

Individual-specific cardinal proxies ($\hat{\Omega}_i \equiv \{\hat{\mu}_i, \hat{\sigma}_i, \hat{\theta}_i\}$) are obtained as the conditional expectations based on the estimated distributions and observed individual responses. Under the same approximation used in the above algorithm, $\hat{\Omega}_i$ is calculated as:

$$\hat{\Omega}_i \equiv E[\Omega_i \mid \hat{\Xi}, Z_i] = \frac{1}{L_i}\sum_{k=1}^{K}\Omega_k \pi_{ki}^{\varepsilon}\pi_k \ ,$$

where Ω_k is the value of the latent variables that correspond to the k-th node ($\{\mu_k, \sigma_k, \theta_k\}$) in Gaussian quadrature approximation.

Appendix D. Details on Structural Life-Cycle Model of Portfolio Choice

Health Transition and Preferences The model starts from age 55, which is the lowest value in the VRI, and the household can live up to age 110 at most.[4] The probability of survival up to next period $(1-\pi_D)$ is a function of age. The household evaluate flow utility from the consumption using (1). It discounts next period utility by time discount factor β. When it dies, it leaves the bequest, and bequest utility is modeled as:

$$U_{Beq,i}(B)=\varsigma_{Beq}\frac{(B+\kappa_{Beq})^{1-1/\theta_i}}{1-1/\theta_i} \tag{D.1}$$

where ς_{Beq} determines the strength of the bequest motive and κ_{Beq} determines whether it is necessity or luxury, compared to its own consumption.

Labor Income Process The household retires at age 65. Until then, the labor income is exogenously determined as:

$$\log(Y_{it})=\log(\bar{y}_i)+\nu_{it},\ \ \nu_{it}\sim N(0,\sigma_\nu^2)\text{ for }t<65. \tag{D.2}$$

Given that households have only 10 years until retirement in this model, we abstract from permanent income shocks. After retirement, the household receives annuity income which captures Social Security income and defined benefit pension income and hence is not exposed to any uncertainty. This annuity income is modeled as a fraction (λ) of the mean income before retirement:

$$\log(Y_{it})=\log(\lambda)+\log(\bar{y}_i)\text{ for }t\geq 65. \tag{D.3}$$

[4] To avoid the complications arising from the joint survival process, we assume that the household dies when the head dies. Essentially, the model is looking at the single households' portfolio choice. Stock share regression using singles only give the essentially the same results as our baseline results using the full sample.

Financial Assets Households can invest in two different assets, a riskless asset and a risky asset where the latter represents stocks. The gross real return on a risk free asset is set as a constant $\bar{R}_f$. The subjective belief on distribution of the real gross return on a risky asset, R_t, is modeled as:

$$R_{t+1,i} = \mu_i + \eta_{t+1}, \ \ \eta_{t+1} \sim N(0, \sigma_i^2) \tag{D.4}$$

where η_{t+1} is an i.i.d. stock return shock. Note that this subjective belief process is heterogeneous across households. We assume that the aggregate stock return shock is uncorrelated with the idiosyncratic labor income shock, following Cocco, Gomes and Maenhout (2005).

Optimization problem of the households Let W_{it} be beginning-of-period cash in hand of a household and α_{it} be share of savings of this period invested to stocks. We assume that short sales and leveraged stock holdings are not allowed.[5] Then the household solves the following optimization problem (we drop the subscripts *i* and *t*):

$$\begin{aligned} V(W,t) = \max_{C,W',\alpha} \ & \{U(C) + \beta E[(1-\pi_D(t))V(W',t+1) + \pi_D(t)U_{Beq}(W')]\} \\ s.t. \ & W' = [(W-C)((1-\alpha)\bar{R}_f + \alpha R_s)] + y' \\ & C \le W \\ & \alpha \in [0,1] \end{aligned} \tag{D.5}$$

Computation We solve for the optimal policy function numerically using backward induction. The last period (at age 110) maximization is a static one so the value function is trivially

[5] Optimal stock share could go over 100% if we allowed leveraging, since labor earnings and retirement income are close substitutes to the risk-free asset, due to zero correlation with stock return for the former and the absence of risk for the latter. In addition, when we approximate the labor income process as a discrete process, even the worst possible realization of income guarantees positive resources net of the subsistence level of consumption (as in Cocco, Gomes and Maenhout (2005)) since mean level of labor income is much higher than the subsistence level of consumption.

obtained. This value function is used as a continuation value for the maximization program of the penultimate period. We repeat this until we solve for the maximization problem at the first period. For the choice over continuous spaces, i.e. over C and α, the optimization is done using grid search. With the curvature parameters the problem is no more homogenous to the scale, so it cannot be normalized as typically done in the literature (see Cocco, Gomes and Maenhout (2005) and Pang and Warshawsky (2010) for example). This does not increase computational burden too much since we abstract from permanent income shocks.

Calibration We solve this model for various sets of subjective belief and risk tolerance parameter values that are in the range supported by the evidence from the VRI, to understand the effects of heterogeneous belief and preference on the optimal stock share. The curvature parameter for the ordinary utility function (κ) is fixed at the value estimated from the VRI (-17K). Time discount factor (β) is set to be 0.96, a value that is typically used in the literature for annual models.

The probability of survival π_D is estimated from the HRS (1994 – 2010). For the parameters for the bequest utility function, we estimate these parameters using the methodology from Ameriks, Briggs, Caplin, Shapiro and Tonetti (2018) and the survey questions designed to estimate the strength of the bequest motive from the VRI ($\theta_{Beq} = 32,\ \kappa_{Beq} = 64K$). The parameters imply that a bequest is a luxury good compared to the ordinary consumption, but once the bequest motive kicks in for wealthy households the marginal utility from leaving bequest is large.[6] Risk free return ($\bar{R}_f$) is set to be 1.02. In the baseline model we use \$90,000 for the mean income before retirement ($\bar{y}$) and assume 0.5 for the replacement rate after

[6] Shutting of the bequest motive does not noticeably change the result from the model.

retirement (λ). These values are close to means from the VRI data. The variance of transitory income shocks (σ_v^2) is set to be 0.07, which is close to the value used in Cocco et al. (2005).[7]

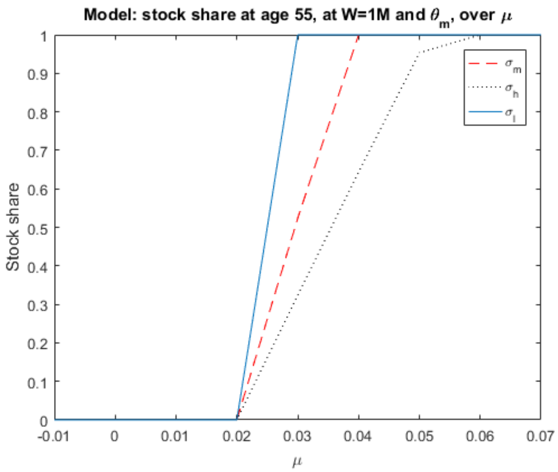
Table D1 summarizes the calibration of the parameters, and Figure D1 and D2 summarize the results.

Table D1. Calibration of Parameters for the Life-Cycle Model

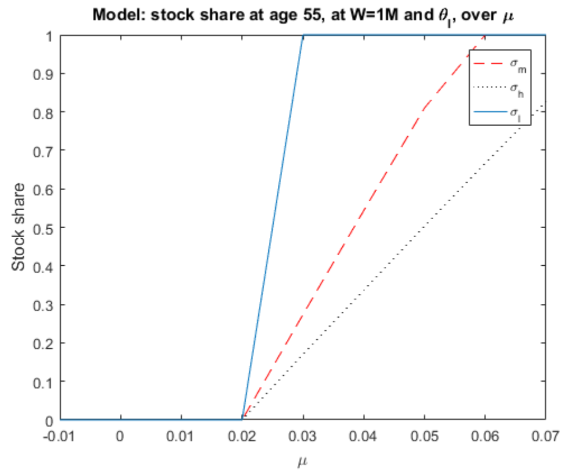
Parameters	Value	Target/Source
κ	-17K	VRI estimation
β	0.96	Standard
π_D		HRS estimation
$\bar{R}_f$	1.02	Cocco, Gomes and Maenhout (2005)
ς_{Beq}	32	VRI estimation
κ_{Beq}	64K	VRI estimation
$\bar{y}$	\$80,000	VRI data
λ	0.5	VRI data
σ_v^2	0.07	Cocco, Gomes and Maenhout (2005)

[7] They estimated it to be 0.058 for college graduates. We set it slightly larger here given that our model does not have permanent income shocks.

Figure D1. Stock share and the expected value of stock returns (μ) at different levels of the standard deviation of stock returns (σ) and risk tolerance (θ). Results from the life cycle portfolio choice model.

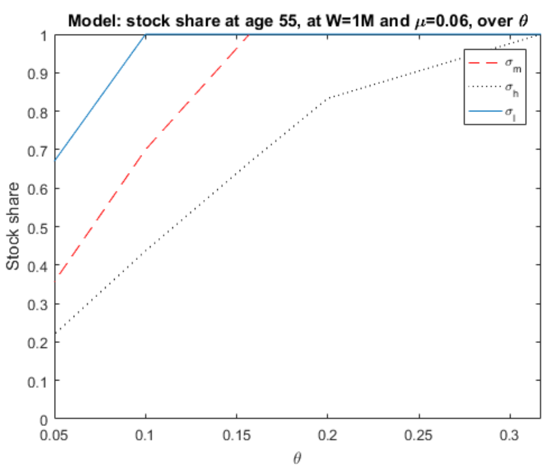


Portfolio choice model,
medium level of risk tolerance (θ = 0.32)

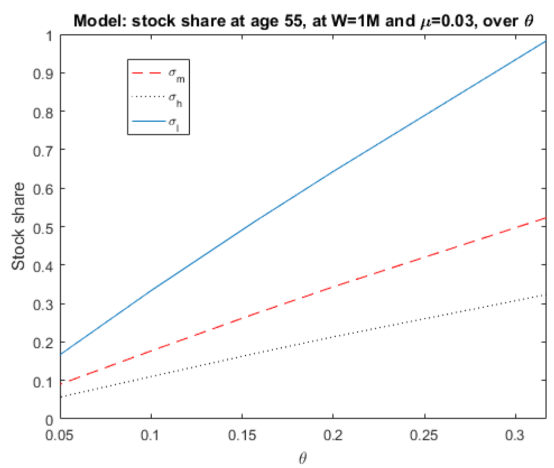


Portfolio choice model,
low level of risk tolerance (θ = 0.16)

Figure D2. Stock share and the risk tolerance (θ) at different levels of the standard deviation of stock returns (σ) and expected value of stock returns (μ). Results from the life cycle portfolio choice model.



Portfolio choice model,
medium level of expected return (μ = 0.06)



Portfolio choice model,
low level of expected return (μ = 0.03)

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