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Appendix to

Taking the Cochrane-Piazzesi Term Structure Model Out of Sample: More Data, Additional Currencies, and FX Implications

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1 A Free Constant Model

The Cochrane and Piazzesi (2005) empirical model in equation (6) in the main paper is inconsistent with the postulated ATSM of Cochrane and Piazzesi (2008) because it constrains the constant terms in each forecasting equation to have the same factor of proportionality as the time varying predictive variable. Examination of equation (14) in the main paper, which is the forecasting equation from the ATSM, shows that the Jensen's inequality terms for different horizons do not scale with B_{n-1} in the same way that the coefficients on the time-varying risk premiums scale.

To allow for different constant terms for each maturity in the empirical specification, consider the following system of equations that define error terms that are used in GMM:

$$\mathbf{rx}_{t+1} - \boldsymbol{\alpha} = \boldsymbol{\varepsilon}_{1,t+1} \quad (1)$$

$$\overline{\mathbf{fs}}_t - \boldsymbol{\delta} = \boldsymbol{\varepsilon}_{2,t+1} \quad (2)$$

$$(\mathbf{rx}_{t+1} - \boldsymbol{\alpha}) - \mathbf{b}\boldsymbol{\gamma}^{\intercal}\left(\overline{\mathbf{fs}}_t - \boldsymbol{\delta}\right) = \boldsymbol{\varepsilon}_{3,t+1} \quad (3)$$

The first set of orthogonality conditions specifies $\boldsymbol{\alpha}$ as the vector of unconditional means of the excess returns. The second set of orthogonality conditions specifies $\boldsymbol{\delta}$ as the vector of unconditional means of the average forward spreads where the notation $\overline{\mathbf{fs}}$ used here does not contain a constant term as it does in the main text. The third set of orthogonality conditions specifies that a linear combination of the demeaned forward spreads drives the demeaned expected excess returns.

For the orthogonality conditions, we specify that $\boldsymbol{\varepsilon}_{1,t+1}$ and $\boldsymbol{\varepsilon}_{2,t+1}$ are each orthogonal to a vector of ones and that $\varepsilon_{3,t+1}$ is orthogonal to $\overline{\mathbf{fs}}_t$. We estimate this system of orthogonality equations with traditional, two-step GMM, using an identity matrix as the weighting matrix in the first step and the estimated S^{-1} in the second step. We estimate the system of equations simultaneously subject to the constraint that the average of the b_n's equals one. The results are presented in Table 1 for the pre-2004 data and in Table 2 for the post-2003 data. An additional advantage of this specification is that it nests the specification of estimating the sample mean in the out-of-sample forecasting exercise. Examination of the coefficient estimates in these Tables versus the corresponding estimates in the Tables in the main paper indicates that the estimates for the $\mathbf{b}$'s or the $\boldsymbol{\gamma}$'s from this specification do not differ substantively from the estimates in the constrained specification of the main paper.

1.1 Out-of-Sample Forecasts of Cochrane-Piazzesi Model with Free Constants

Table 3 presents the out-of-sample forecast results for bond returns from the Cochrane-Piazzesi model with free constants as described above. The out-of-sample period begins in January 2004 and ends in December 2016. Freeing the constants does not improve the performance of the model as all out-of-sample R^2's, except those for three maturities of the CHF, are negative.

2 The Constrained OLS Estimator

Because the two-step estimator of Cochrane and Piazzesi (2005) uses OLS, it is straightforward to simultaneously estimate the excess return bond models for the nine currencies subject to the constraints that the parameters are the same across the different currencies. To see this, let the unconstrained generic specification for currency j be

$$y_{j,t+1} = \mathbf{x}_{j,t}^{\intercal}\boldsymbol{\beta}_j + \varepsilon_{j,t+1} \tag{4}$$

where $\mathbf{x}_{j,t}$ is $k \times 1$ and $j = 1, ..., N$. The OLS orthogonality conditions are $E\left(\mathbf{x}_{j,t}\varepsilon_{j,t+1}\right) = \mathbf{0}_k$. It is useful to define $\mathbf{g}_{j,t} \equiv \mathbf{x}_{j,t}\varepsilon_{j,t+1}$ in which case the sample mean of the orthogonality conditions for country j can be written as

$$\mathbf{g}_j = \frac{1}{T}\mathbf{X}_j^{\intercal}\left(\mathbf{y}_j - \mathbf{X}_j\boldsymbol{\beta}_j\right) \tag{5}$$

where $\mathbf{X}_j$ is the $T \times k$ matrix with $\mathbf{x}_{j,t}^{\intercal}$ stacked in the rows and $\mathbf{y}_j$ is the $T \times 1$ vector with $y_{j,t+1}$ stacked in the rows.

Then, for any arbitrary weighting matrix, $\mathbf{W}$, the value of $\boldsymbol{\beta}_j$ that would minimize the unconstrained

GMM objective function for currency j, $\mathbf{J}_j = \mathbf{g}_j^{\intercal}\mathbf{W}\mathbf{g}_j$, is the OLS estimator:

$$\hat{\boldsymbol{\beta}}_j = \left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)^{-1}\mathbf{X}_j^{\intercal}\mathbf{y}_j. \tag{6}$$

To estimate a common $\boldsymbol{\beta}$ for the N equations in (4) while imposing the same orthogonality conditions as in the N unconditional estimations, we stack the orthogonality conditions into an $Nk \times 1$ vector $\mathbf{g} = (\mathbf{g}_1^{\intercal}, ..., \mathbf{g}_N^{\intercal})^{\intercal}$ and use the identity matrix as the weighting matrix to derive the following constrained GMM objective function:

$$\mathbf{J} = \mathbf{g}^{\intercal}\mathbf{g} = \sum_{j=1}^{N}\mathbf{g}_j^{\intercal}\mathbf{g}_j = \frac{1}{T^2}\sum_{j=1}^{N}\left(\mathbf{y}_j^{\intercal}\mathbf{X}_j - \boldsymbol{\beta}^{\intercal}\left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)\right)\left(\mathbf{X}_j^{\intercal}\mathbf{y}_j - \left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)\boldsymbol{\beta}\right). \tag{7}$$

The minimized value of the constrained estimator is

$$\hat{\boldsymbol{\beta}} = \left[\sum_{j=1}^{N}\left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)^2\right]^{-1}\sum_{j=1}^{N}\left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)\mathbf{X}_j^{\intercal}\mathbf{y}_j. \tag{8}$$

By inserting $\left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)\left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)^{-1}$ after each of the $\left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)$ on the right-hand side of equation (8), we find that the constrained estimator is a weighted average of the unconstrained estimators:

$$\hat{\boldsymbol{\beta}} = \left[\sum_{j=1}^{N}\left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)^2\right]^{-1}\sum_{j=1}^{N}\left(\mathbf{X}_j^{\intercal}\mathbf{X}_j\right)^2\hat{\boldsymbol{\beta}}_j. \tag{9}$$

The asymptotic distribution of the constrained estimator is

$$\sqrt{T}\left(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right) \rightarrow N\left(0, \hat{\boldsymbol{\Omega}}\right), \tag{10}$$

where

$$\hat{\boldsymbol{\Omega}} = \left(\mathbf{D}^{\intercal}\mathbf{D}\right)^{-1}\mathbf{D}^{\intercal}\hat{\mathbf{S}}\mathbf{D}\left(\mathbf{D}^{\intercal}\mathbf{D}\right)^{-1}, \tag{11}$$

$\mathbf{D}$ is the gradient of the orthogonality conditions with respect to $\boldsymbol{\beta}$

$$\mathbf{D} = \frac{1}{T}\left(\mathbf{X}_1^{\intercal}\mathbf{X}_1, ..., \mathbf{X}_N^{\intercal}\mathbf{X}_N\right)^{\intercal} \tag{12}$$

and

$$\hat{\mathbf{S}} \equiv \mathbf{C}_0 + \sum_{k=1}^{K}\frac{K-k}{K}\left(\mathbf{C}_k + \mathbf{C}_k^{\intercal}\right), \tag{13}$$

We use $K = 18$ with

$$\mathbf{C}_k \equiv \frac{1}{T}\sum_{t=k+1}^{T}\mathbf{g}_t\mathbf{g}_{t-k}^{\intercal}. \tag{14}$$

2.1 Out-of-Sample Forecasts of Restricted Cochrane-Piazzesi Model

Table 4 presents the out-of-sample forecast results of the Cochrane and Piazzesi (2005) model where all the parameters are constrained to be equal across the nine currencies. The sample for the initial estimation begins in January 1988 for all currencies and ends in December 2003. The out-of-sample period begins in January 2004 and ends in December 2016. Once again, we find that 15 of the 32 R^2's are negative. Only the forecasts for the GBP, the CAD, and the AUD show any marginal improvements relative to forecasts based on the historical means.

3 Evolution of the Estimated Parameters of the Models

Figures 1 through 9 present the evolution of the estimated parameters from recursive estimation of the basic model for each of the nine currencies. In general, the $b(k)$ parameter estimates are quite stable for all currencies. The instability in the estimates of the $\gamma(k)$ parameters is worst for the USD, the AUD, and the NOK and best for the CAD, the JPY, and the SEK. Figures 10 through 18 present the evolution of the parameters from recursive estimation of the free-constant model for each of the nine currencies. Here, the $b(k)$ parameter estimates are slightly more unstable than before, while the estimates of the $\gamma(k)$ parameters appear a bit more stable.

References

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Cochrane, John H, and Monika Piazzesi. 2005. "Bond risk premia." *American Economic Review*, 95(1): 138–160.

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Table 1: The Single Factor Model with Free Constants (Pre-2004 Data)

The Table reports coefficient estimates for the single factor model with free constants. The errors of the model are defined by these equations:

$$
\begin{aligned}
\mathbf{rx}_{t_1} - \boldsymbol{\alpha} &= \boldsymbol{\varepsilon}_{1,t+1} \\
\overline{\mathbf{fs}}_t - \boldsymbol{\delta} &= \boldsymbol{\varepsilon}_{2,t+1} \\
(\mathbf{rx}_{t_1} - \boldsymbol{\alpha}) - \mathbf{b}\boldsymbol{\gamma}^{\intercal}\left(\overline{\mathbf{fs}}_t - \boldsymbol{\delta}\right) &= \boldsymbol{\varepsilon}_{3,t+1}
\end{aligned}
$$

The GMM orthogonality conditions specify that $\boldsymbol{\varepsilon}_{1,t+1}$ and $\boldsymbol{\varepsilon}_{2,t+1}$ are each orthogonal to a vector of ones and that $\boldsymbol{\varepsilon}_{3,t+1}$ is orthogonal to $\overline{\mathbf{fs}}_t$. We estimate this system of orthogonality equations with traditional, two-step GMM subject to the constraint that the average of the b_n's equals one. Standard errors are constructed with 18 Newey-West (1987) lags. The J-stat tests the overidentifying restrictions of the model. The R^2 corresponds to the projection of the average returns on the forward spreads.is from the first step regression. Standard errors are in parentheses, and p-values are in angled brackets. The sample periods for the dependent variables all end in 2003:12. The samples begin in 1974:12 for the USD, the GBP, and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK.

CUR	α_2	α_3	α_4	α_5	δ_2	δ_3	δ_4	δ_5	
USD	0.14	0.17	0.38	0.28	0.58	0.88	1.26	1.20	
	(0.21)	(0.40)	(0.56)	(0.67)	(0.09)	(0.15)	(0.19)	(0.20)	
GBP	0.45	0.88	1.36	1.90	0.30	0.44	0.53	0.64	
	(0.19)	(0.34)	(0.47)	(0.58)	(0.13)	(0.18)	(0.20)	(0.22)	
EUR	0.76	1.49	2.02	2.44	0.50	0.95	1.24	1.40	
	(0.22)	(0.40)	(0.53)	(0.65)	(0.11)	(0.16)	(0.19)	(0.22)	
CHF	0.12	0.51	0.90	1.17	-0.26	-0.13	0.10	0.30	
	(0.30)	(0.50)	(0.64)	(0.77)	(0.16)	(0.22)	(0.24)	(0.25)	
CAD	1.21	2.11	2.86	3.53	0.72	1.09	1.33	1.51	
	(0.19)	(0.36)	(0.53)	(0.68)	(0.11)	(0.17)	(0.20)	(0.21)	
JPY	0.44	1.00	1.55	2.07	0.27	0.64	0.99	1.27	
	(0.14)	(0.27)	(0.38)	(0.47)	(0.05)	(0.09)	(0.12)	(0.15)	
AUD	0.91	1.75	2.35	2.83	0.40	0.77	0.95	1.04	
	(0.23)	(0.42)	(0.59)	(0.74)	(0.12)	(0.17)	(0.20)	(0.22)	
SEK	0.79	1.32	1.73	2.09	0.28	0.41	0.50	0.59	
	(0.18)	(0.38)	(0.56)	(0.74)	(0.06)	(0.08)	(0.09)	(0.09)	
NOK	-0.06	0.02	0.10	0.18	-0.48	-0.61	-0.61	-0.58	
	(0.07)	(0.13)	(0.20)	(0.26)	(0.02)	(0.04)	(0.05)	(0.06)	
CUR	b_2	b_3	b_4	γ_2	γ_3	γ_4	γ_5	J-stat	R^2
USD	0.39	0.81	1.24	2.39	3.24	2.82	-5.23	34.79	0.51
	(0.01)	(0.01)	(0.01)	(0.81)	(0.65)	(0.47)	(0.41)	⟨0.00⟩	
GBP	0.47	0.83	1.17	-5.48	11.42	-6.32	0.54	50.96	0.18
	(0.03)	(0.02)	(0.02)	(2.03)	(6.88)	(8.22)	(3.34)	⟨0.00⟩	
EUR	0.38	0.80	1.22	-1.33	-4.43	16.37	-10.02	20.41	0.22
	(0.03)	(0.02)	(0.01)	(3.05)	(8.59)	(11.23)	(5.24)	⟨0.02⟩	
CHF	0.56	0.91	1.16	-1.49	-9.61	26.11	-14.48	86.93	0.33
	(0.02)	(0.01)	(0.01)	(5.52)	(14.43)	(18.30)	(8.35)	⟨0.00⟩	
CAD	0.41	0.80	1.19	-0.42	-7.56	19.93	-10.64	35.29	0.43
	(0.01)	(0.01)	(0.00)	(1.49)	(3.46)	(4.25)	(2.15)	⟨0.00⟩	
JPY	0.35	0.79	1.23	-14.61	16.77	-0.04	-4.98	15.52	0.25
	(0.02)	(0.02)	(0.00)	(1.60)	(2.03)	(0.60)	(0.87)	⟨0.08⟩	
AUD	0.33	0.72	1.21	-32.94	107.81	-138.80	61.28	29.72	0.13
	(0.03)	(0.02)	(0.01)	(7.18)	(21.53)	(27.27)	(12.04)	⟨0.00⟩	
SEK	0.27	0.75	1.24	24.26	-51.72	53.94	-21.14	57.98	0.11
	(0.04)	(0.02)	(0.01)	(9.60)	(21.30)	(22.52)	(8.89)	⟨0.00⟩	
NOK	0.24	0.74	1.26	34.31	-129.90	199.82	-100.07	53.56	0.42
	(0.04)	(0.02)	(0.01)	(5.33)	(18.09)	(24.94)	(11.63)	⟨0.00⟩	

Table 2: The Single Factor Model with Free Constants (Post-2003 Data)

The Table reports coefficient estimates for the single factor model with free constants. The errors of the model are defined by these equations:

$$\begin{aligned} \mathbf{rx}_{t_1} - \boldsymbol{\alpha} &= \boldsymbol{\varepsilon}_{1,t+1} \\ \overline{\mathbf{fs}}_t - \boldsymbol{\delta} &= \boldsymbol{\varepsilon}_{2,t+1} \\ (\mathbf{rx}_{t_1} - \boldsymbol{\alpha}) - \mathbf{b}\boldsymbol{\gamma}^{\intercal}\left(\overline{\mathbf{fs}}_t - \boldsymbol{\delta}\right) &= \boldsymbol{\varepsilon}_{3,t+1} \end{aligned}$$

The GMM orthogonality conditions specify that $\boldsymbol{\varepsilon}_{1,t+1}$ and $\boldsymbol{\varepsilon}_{2,t+1}$ are each orthogonal to a vector of ones and that $\boldsymbol{\varepsilon}_{3,t+1}$ is orthogonal to $\overline{\mathbf{fs}}_t$. We estimate this system of orthogonality equations with traditional, two-step GMM subject to the constraint that the average of the b_n's equals one. Standard errors are constructed with 18 Newey-West (1987) lags. The J-stat tests the overidentifying restrictions of the model. The R^2 corresponds to the projection of the average returns on the forward spreads.is from the first step regression. Standard errors are in parentheses, and p-values are in angled brackets. The sample periods for the dependent variables all end in 2016:12. The samples begin 2004:12 for all currencies.

CUR	α_2	α_3	α_4	α_5	δ_2	δ_3	δ_4	δ_5
USD	0.79	1.84	2.96	3.75	0.25	0.73	1.35	1.72
	(0.12)	(0.22)	(0.29)	(0.36)	(0.07)	(0.11)	(0.15)	(0.20)
GBP	0.50	1.32	2.13	2.88	0.26	0.74	1.16	1.48
	(0.07)	(0.17)	(0.29)	(0.43)	(0.07)	(0.12)	(0.16)	(0.19)
EUR	0.51	1.20	2.01	2.84	0.39	0.82	1.23	1.59
	(0.10)	(0.19)	(0.26)	(0.32)	(0.04)	(0.06)	(0.08)	(0.09)
CHF	0.09	0.46	0.97	1.47	0.20	0.51	0.87	1.19
	(0.04)	(0.09)	(0.15)	(0.20)	(0.05)	(0.07)	(0.09)	(0.10)
CAD	0.61	1.25	1.90	2.56	0.26	0.55	0.84	1.09
	(0.07)	(0.13)	(0.21)	(0.29)	(0.08)	(0.12)	(0.14)	(0.15)
JPY	0.08	0.20	0.42	0.57	0.07	0.18	0.40	0.49
	(0.01)	(0.03)	(0.04)	(0.06)	(0.02)	(0.04)	(0.05)	(0.06)
AUD	0.37	1.02	1.64	2.21	0.09	0.31	0.48	0.55
	(0.05)	(0.12)	(0.19)	(0.26)	(0.04)	(0.05)	(0.06)	(0.07)
SEK	0.64	1.36	1.98	2.49	0.44	0.86	1.12	1.29
	(0.10)	(0.22)	(0.35)	(0.49)	(0.07)	(0.12)	(0.15)	(0.17)
NOK	0.34	0.86	1.45	2.04	0.21	0.47	0.73	0.97
	(0.08)	(0.14)	(0.18)	(0.23)	(0.07)	(0.13)	(0.16)	(0.18)

CUR	b_2	b_3	b_4	γ_2	γ_3	γ_4	γ_5	J-stat	R^2
USD	0.16	0.49	1.21	-5.38	0.37	2.37	-0.16	23.13	0.31
	(0.05)	(0.07)	(0.01)	(1.01)	(0.63)	(0.38)	(0.20)	⟨0.01⟩	
GBP	0.21	0.67	1.25	-6.34	-6.61	25.98	-14.68	56.06	0.36
	(0.03)	(0.03)	(0.01)	(2.83)	(9.38)	(12.77)	(5.65)	⟨0.00⟩	
EUR	0.27	0.71	1.23	-19.00	33.70	-25.77	7.56	30.91	0.13
	(0.04)	(0.03)	(0.01)	(4.29)	(9.40)	(9.78)	(3.90)	⟨0.00⟩	
CHF	0.37	0.72	1.19	-10.02	15.36	-14.72	6.82	28.53	0.46
	(0.01)	(0.02)	(0.01)	(1.37)	(3.82)	(4.89)	(2.19)	⟨0.00⟩	
CAD	0.22	0.66	1.23	3.94	-14.04	21.19	-9.78	55.93	0.21
	(0.03)	(0.04)	(0.01)	(2.11)	(4.48)	(5.25)	(2.21)	⟨0.00⟩	
JPY	0.35	0.81	1.23	3.14	-2.08	4.28	-2.46	35.30	0.43
	(0.01)	(0.01)	(0.01)	(0.66)	(0.66)	(0.89)	(0.44)	⟨0.00⟩	
AUD	-0.06	0.56	1.32	1.49	2.31	-3.12	0.93	48.82	0.07
	(0.10)	(0.07)	(0.03)	(0.56)	(0.67)	(0.60)	(0.24)	⟨0.00⟩	
SEK	0.06	0.50	1.25	16.76	-41.77	46.26	-18.08	15.27	0.17
	(0.06)	(0.06)	(0.01)	(2.68)	(6.41)	(7.17)	(2.90)	⟨0.08⟩	
NOK	3.65	4.13	0.88	0.03	-0.41	0.76	-0.46	70.33	0.00
	(18.08)	(18.50)	(1.85)	(0.30)	(2.56)	(4.56)	(2.73)	⟨0.00⟩	

Table 3: Out-of-Sample Forecasts of Excess Bond Returns: Cochrane-Piazzesi Models with Free Constants vs. Historical Means

The Table reports two statistics that compare the out-of-sample forecasts from recursive estimations of an alternative version of the Cochrane and Piazzesi (2005) model, which allows for free constant terms, for the excess rates of returns on bonds denominated in different currencies to the forecasts based only on the historical mean excess rates of return. The first statistic is the R^2, which is calculated as one minus the ratio of the mean squared error of the CP forecasts to the mean squared error of the historical mean. The second statistic tests the equality of the forecasts and is the Clark and McCracken (2005) $MSE-F$ statistic. The sample periods for the dependent variables during the initial in-sample estimation all end in 2003:12, which is the end of the Cochrane and Piazzesi (2005) sample. The samples begin in 1974:12 for the USD, the GBP, and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. The out-of-sample periods are all 2004:01 to 2016:12.

	R^2				MSE-F			
CUR	$rx_{t+1}^{(2)}$	$rx_{t+1}^{(3)}$	$rx_{t+1}^{(4)}$	$rx_{t+1}^{(5)}$	$rx_{t+1}^{(2)}$	$rx_{t+1}^{(3)}$	$rx_{t+1}^{(4)}$	$rx_{t+1}^{(5)}$
USD	-1.36	-2.14	-2.23	-2.21	-83.51	-98.89	-100.06	-99.87
GBP	-0.07	-0.08	-0.10	-0.13	-9.01	-10.57	-13.42	-16.34
EUR	-0.08	-0.11	-0.20	-0.28	-10.21	-14.39	-23.80	-31.88
CHF	-0.36	0.04	0.10	0.07	-38.24	6.13	16.57	11.38
CAD	-0.21	-0.13	-0.16	-0.27	-25.50	-16.85	-20.13	-31.15
JPY	-0.17	-0.09	-0.14	-0.21	-21.34	-11.59	-18.35	-24.71
AUD	-1.89	-2.08	-2.34	-2.42	-94.90	-97.91	-101.57	-102.66
SEK	-0.15	-0.12	-0.08	-0.06	-18.49	-15.37	-11.31	-7.84
NOK	-0.60	-0.82	-0.98	-1.07	-54.17	-65.24	-71.82	-75.12

Table 4: Out-of-Sample Forecasts of Excess Bond Returns: Cochrane-Piazzesi Models with Free Constants vs. Historical Means

The Table reports two statistics that compare the out-of-sample forecasts from recursive estimations of the Cochrane and Piazzesi (2005) model in which the parameters of the model are constrained to be the same across the nine currencies to the forecasts based only on the historical mean excess rates of return. The first statistic is the R^2, which is calculated as one minus the ratio of the mean squared error of the CP forecasts to the mean squared error of the historical mean. The second statistic tests the equality of the forecasts and is the Clark and McCracken (2005) $MSE-F$ statistic. The sample periods for the dependent variables during the initial in-sample estimation all end in 2003:12, which is the end of the Cochrane and Piazzesi (2005) sample. The sample for the initial estimation begins in January 1988 for all currencies and ends in December 2003. The out-of-sample period begins in January 2004 and ends in December 2016.

	R^2				MSE-F			
CUR	$rx_{t+1}^{(2)}$	$rx_{t+1}^{(3)}$	$rx_{t+1}^{(4)}$	$rx_{t+1}^{(5)}$	$rx_{t+1}^{(2)}$	$rx_{t+1}^{(3)}$	$rx_{t+1}^{(4)}$	$rx_{t+1}^{(5)}$
USD	-0.05	-0.12	-0.08	-0.03	-6.66	-14.97	-10.73	-4.66
GBP	0.00	0.07	0.13	0.16	0.15	10.51	21.15	27.68
EUR	-0.01	0.03	0.07	0.10	-1.60	4.70	11.09	16.59
CHF	-0.48	-0.26	-0.16	-0.10	-47.35	-30.22	-20.44	-13.25
CAD	0.06	0.02	0.03	0.05	8.52	3.38	4.03	7.39
JPY	-0.49	-0.04	0.10	0.16	-47.45	-4.93	15.96	28.44
AUD	0.06	0.05	0.04	0.04	9.52	7.43	5.78	5.41
SEK	-0.09	-0.09	-0.05	-0.01	-11.90	-11.41	-6.84	-2.13

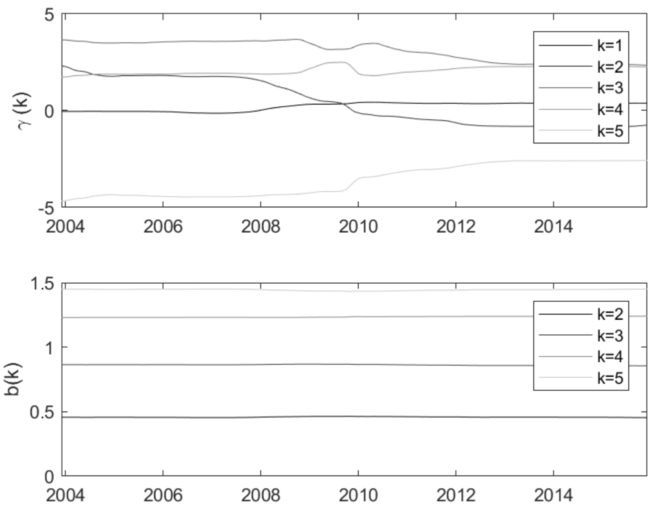

Figure 1: Evolution of parameters (USD)

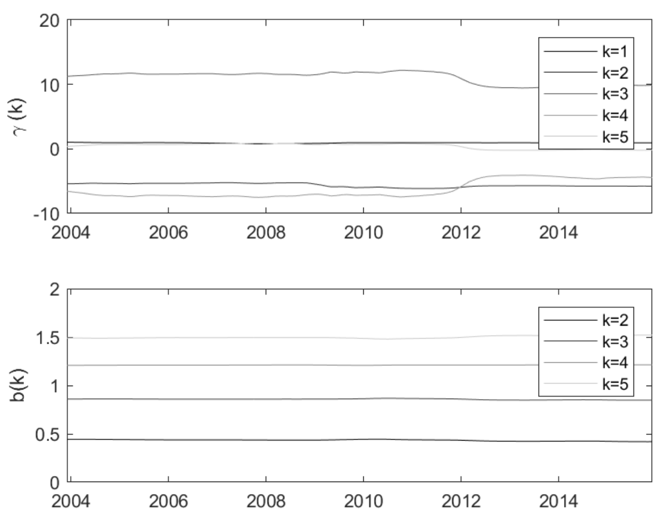

Figure 2: Evolution of parameters (GBP)

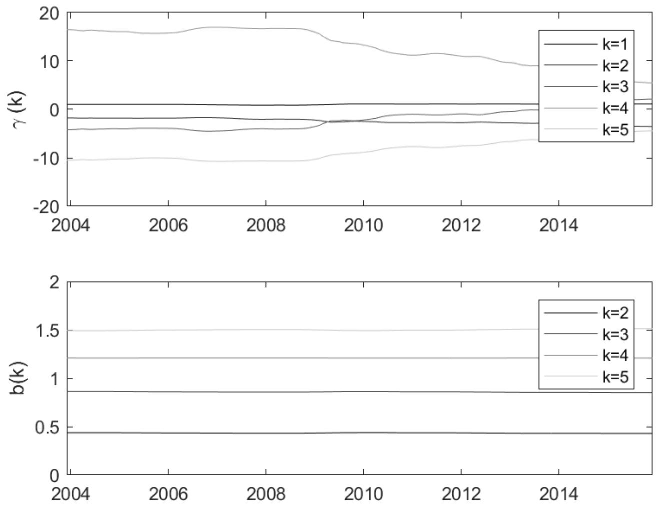

Figure 3: Evolution of parameters (EUR)

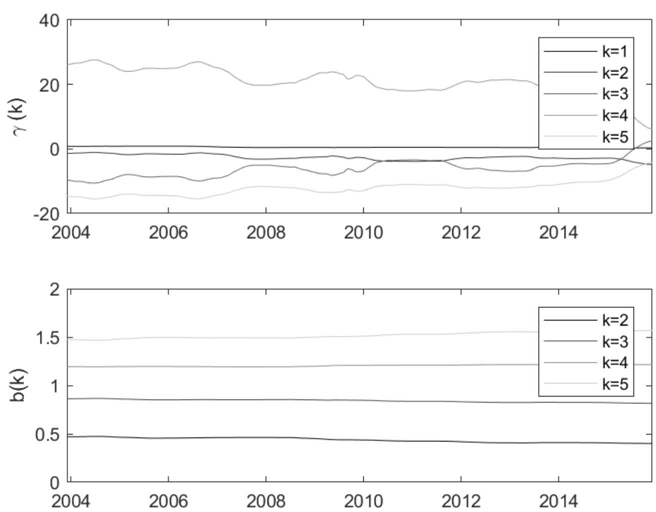

Figure 4: Evolution of parameters (CHF)

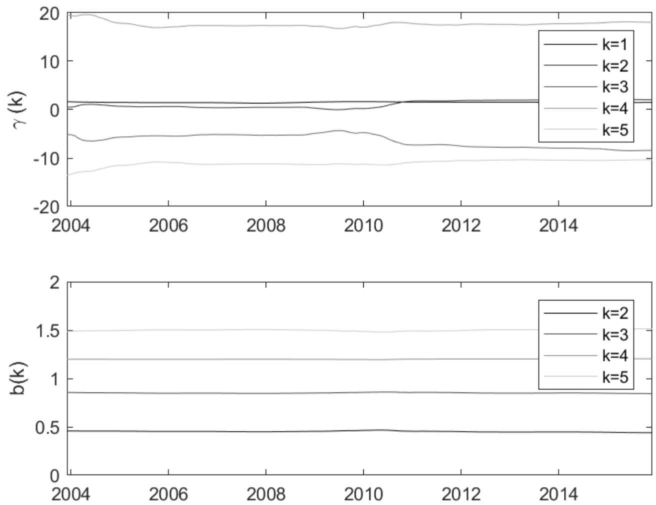

Figure 5: Evolution of parameters (CAD)

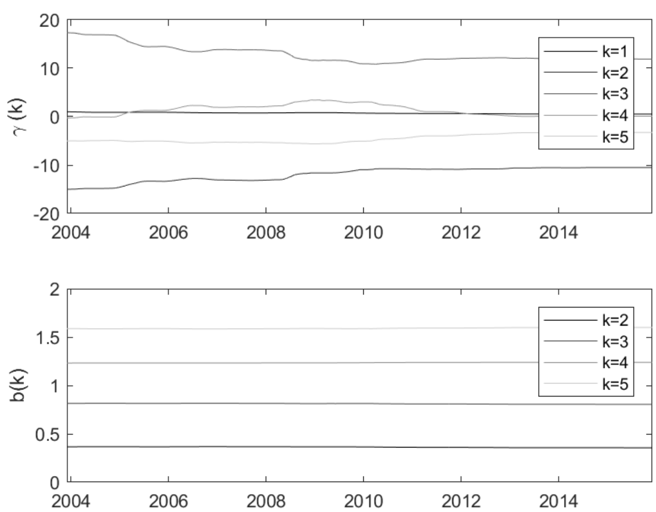

Figure 6: Evolution of parameters (JPY)

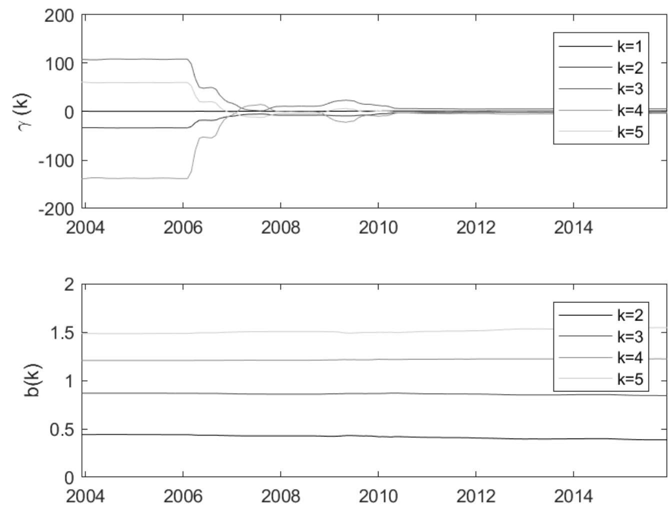

Figure 7: Evolution of parameters (AUD)

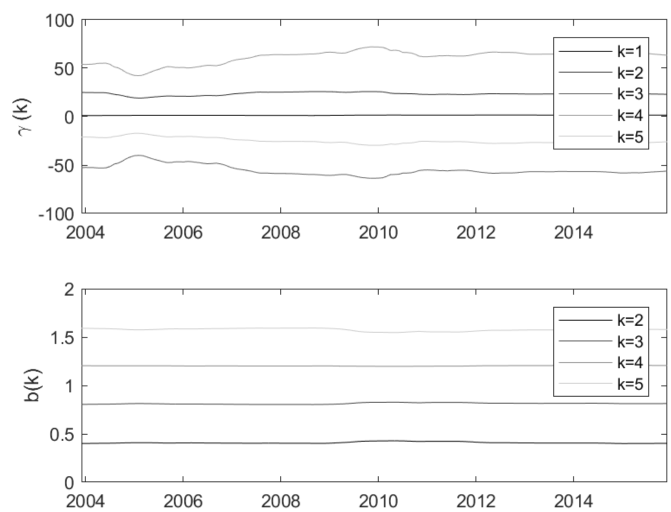

Figure 8: Evolution of parameters (SEK)

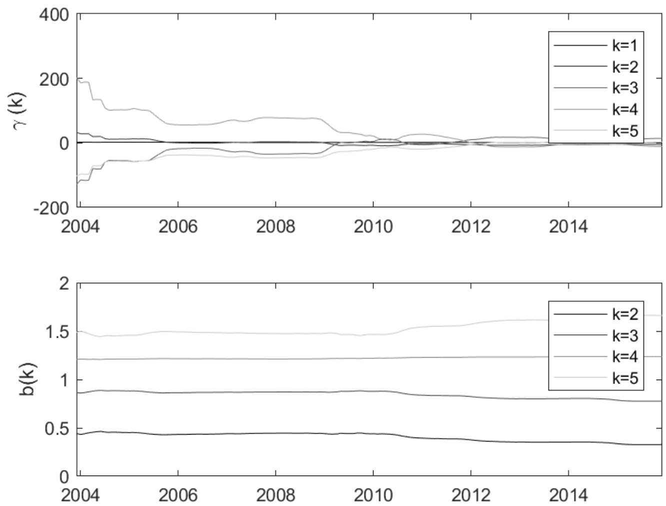

Figure 9: Evolution of parameters (NOK)

4 Evolution of free constant model parameters

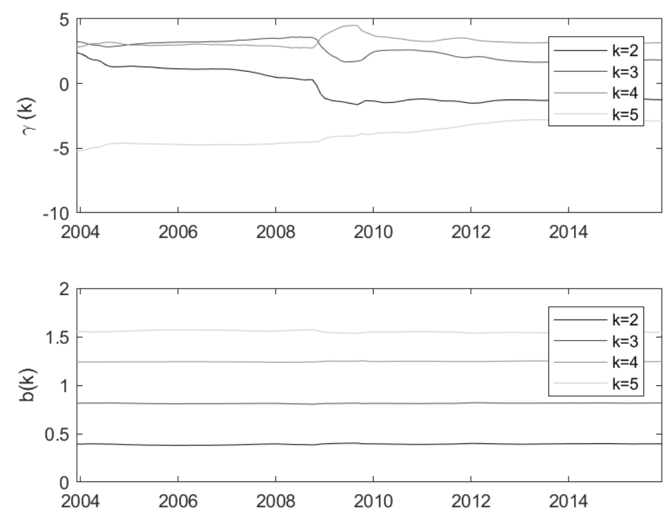

Figure 10: Evolution of parameters (USD)

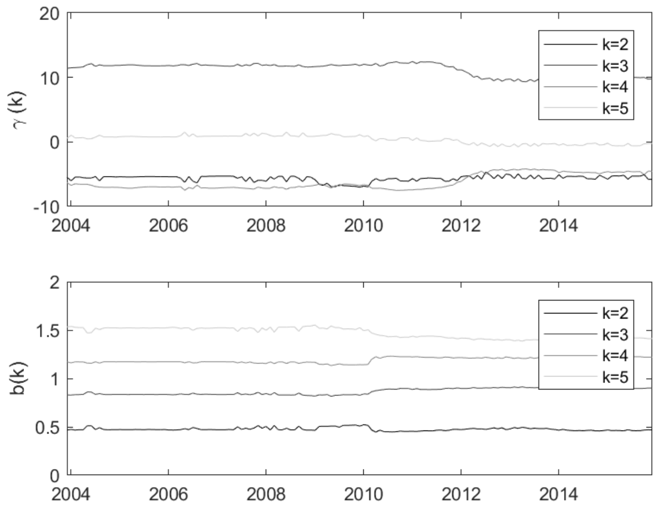

Figure 11: Evolution of parameters (GBP)

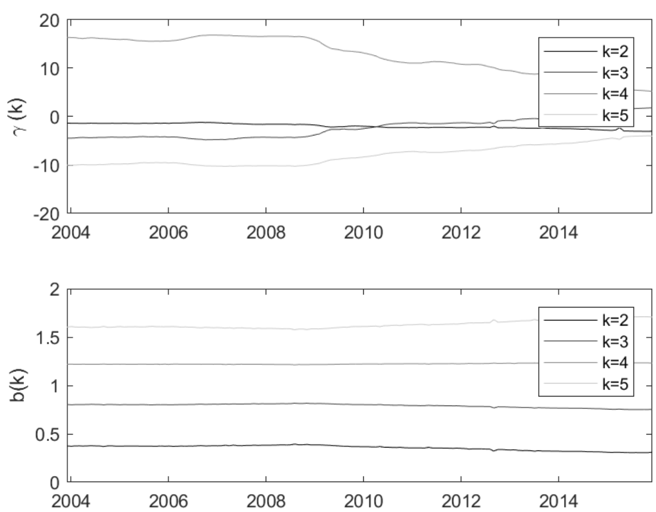

Figure 12: Evolution of parameters (EUR)

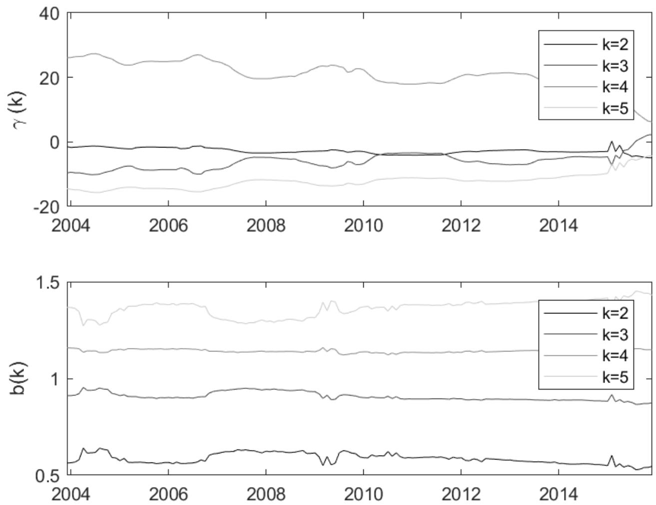

Figure 13: Evolution of parameters (CHF)

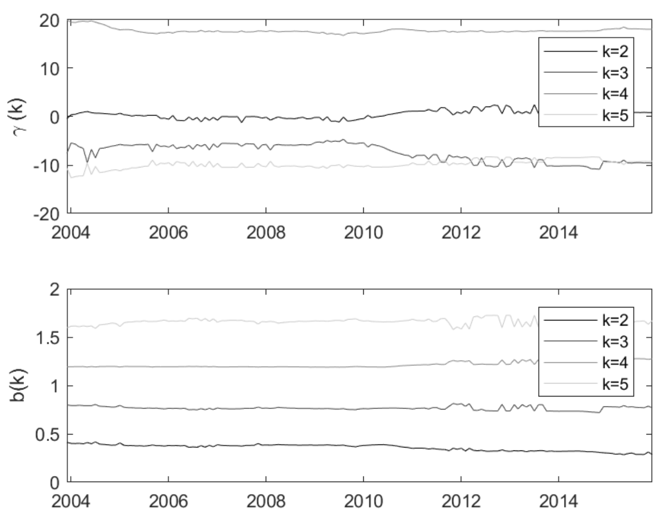

Figure 14: Evolution of parameters (CAD)

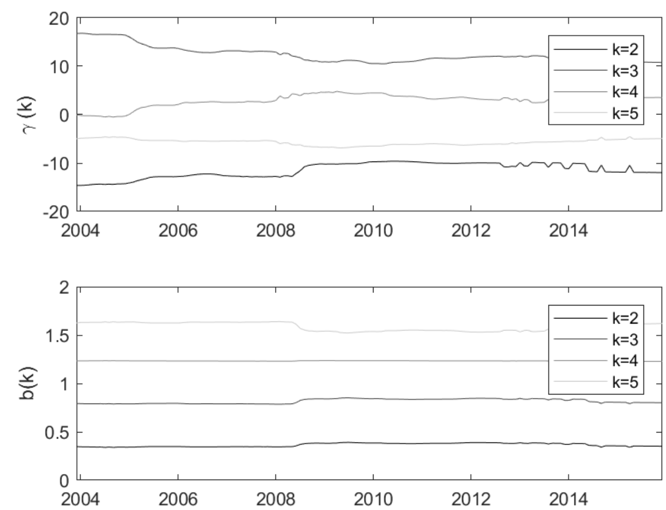

Figure 15: Evolution of parameters (JPY)

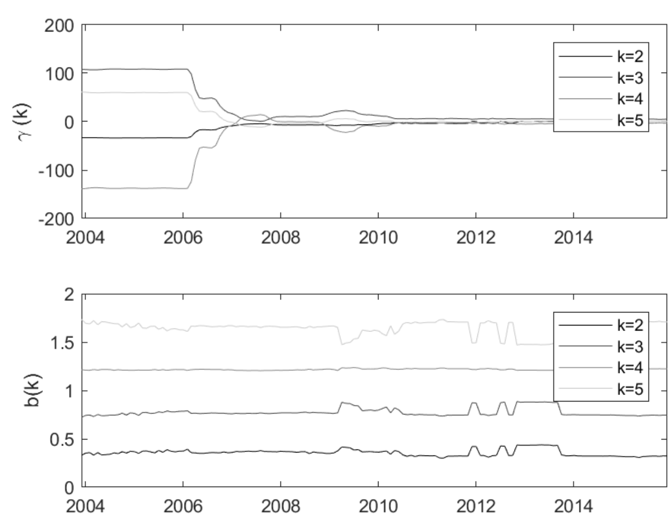

Figure 16: Evolution of parameters (AUD)

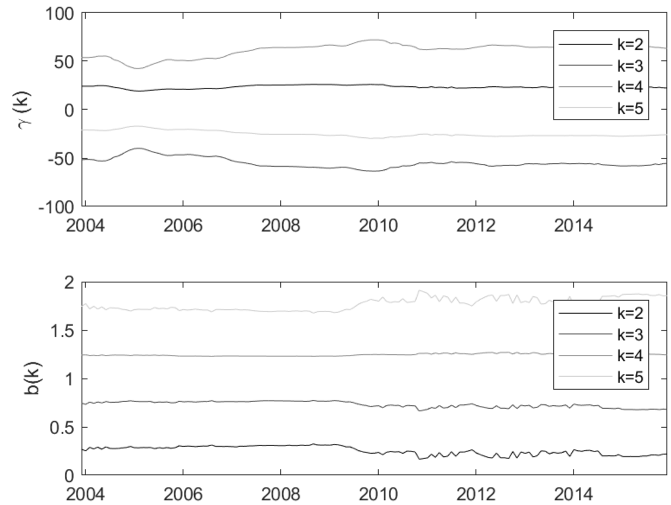

Figure 17: Evolution of parameters (SEK)

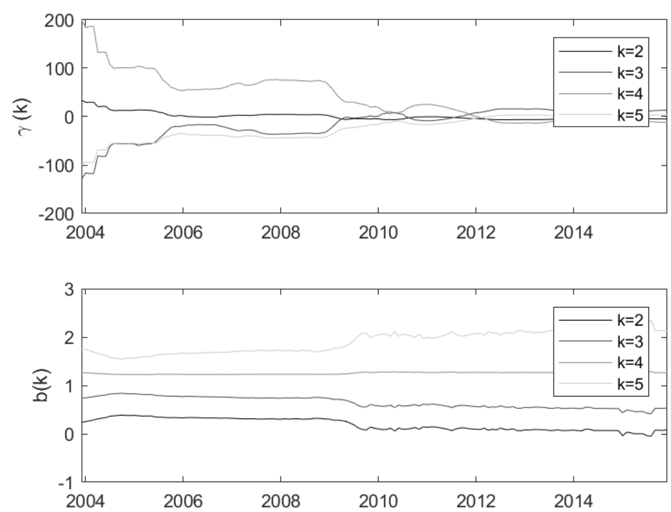

Figure 18: Evolution of parameters (NOK)