General Equilibrium Multipliers of Housing Wealth Effects and Fiscal Shocks

Online Supplement for "Housing Wealth Effects: The Long View"

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1 Overview

We show how the open-economy fiscal multiplier can be used to gauge the strength of the local general equilibrium multiplier on housing wealth effects on consumption. We use an estimate of the fiscal multiplier to remove these general equilibrium effects from an estimate of the housing wealth effect that comes from comparing consumption and home price changes across cities. The end result is an estimate of the partial equilibrium housing wealth effect and marginal propensity to consume out of housing wealth that corresponds to a change in home prices holding fixed wages other non-housing prices.

We present this argument in the context of a fully-specified general equilibrium model. For simplicity we work with a model where each region has a patient household and an impatient household rather than a model with rich microeconomic heterogeneity. This difference is not important for our argument. In Section 3 of this document, we show how to create an aggregate consumption function for each region. The same argument that we use there would apply to a model in the Bewley-Huggett-Aiyagari tradition.[1]

Finally, the argument we make here follows from the fact that different demand disturbances

[1] See Section 6 of Farhi and Werning (2017) for a similar aggregate consumption function in a Bewley-Huggett-Aiyagari model.

can have the same general equilibrium effects. This point has been made recently by Auclert et al. (2017) and our analysis bears some resemblance to theirs.

2 Model

Demographics There are two regions, "home" and "foreign." The population of the entire economy is normalized to one with a share n in the home region. Within each region there are two representative households, which we call "patient" and "impatient." Let $\omega(x)$ be the share of households that are $x \in \{\text{patient}, \text{impatient}\}$ in each region.

Preferences $\sum_{t=0} \beta\,(x)^t\, u(C_t(x), L_t(x), Q_{t+1}(x); \Omega_t)$, where the arguments are consumption, labor supply, units of housing Q_{t+1}, and Ω_t is an aggregate housing demand shock. x indexes a household's patience.

Commodities and technology There is a final good assembled out of intermediate inputs that is used locally for consumption, residential investment, and government purchases. The production of the final good satisfies:

$$\mathcal{Y}_{H,t} = \left[\phi_H^{\frac{1}{\eta}} Z_{H,t}^{\frac{\eta-1}{\eta}} + \phi_F^{\frac{1}{\eta}} Z_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$\mathcal{Y}_{F,t} = \left[\phi_H^{\frac{1}{\eta}} \left(Z_{F,t}^{*}\right)^{\frac{\eta-1}{\eta}} + \phi_F^{\frac{1}{\eta}} \left(Z_{H,t}^{*}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where $Z_{H,t}$ and $Z_{F,t}$ are the home and foreign inputs to the home production of final goods. These inputs are "composite" goods produced in the home and foreign regions and described shortly. Normalize so that $\phi_H + \phi_F = 1$. The bundle shows home bias if $\phi_H > n$. The cost-minimization problem implies:

$$Z_{H,t} = \phi_H \mathcal{Y}_{H,t} \left(\frac{P_{H,t}}{\mathcal{P}_{H,t}}\right)^{-\eta} \qquad \text{Home demand for home composite good}$$
$$Z_{F,t} = \phi_F \mathcal{Y}_{H,t} \left(\frac{P_{F,t}}{\mathcal{P}_{H,t}}\right)^{-\eta} \qquad \text{Home demand for foreign composite good}$$
$$Z_{H,t}^{*} = \phi_F \mathcal{Y}_{F,t} \left(\frac{P_{H,t}}{\mathcal{P}_{F,t}}\right)^{-\eta} \qquad \text{Foreign demand for home composite good}$$
$$Z_{F,t}^{*} = \phi_H \mathcal{Y}_{F,t} \left(\frac{P_{F,t}}{\mathcal{P}_{F,t}}\right)^{-\eta} \qquad \text{Foreign demand for foreign composite good}$$

where $\mathcal{P}_{r,t}$ is the price of the final good in region $r \in \{H, F\}$. The two composite goods are produced in amounts $Y_{H,t}$ and $Y_{F,t}$ and are themselves aggregates of intermediate inputs:

$$
\begin{aligned}
Y_{H,t} &= \left(\int_0^1 y_{H,t}(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}} \\
Y_{F,t} &= \left(\int_0^1 y_{F,t}(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}.
\end{aligned}
$$

The usual cost-minimization problem results in price indices

$$
\begin{aligned}
P_{H,t} &= \left(\int_0^1 p_{H,t}(z)^{1-\theta} dz\right)^{\frac{1}{1-\theta}} \\
P_{F,t} &= \left(\int_0^1 p_{F,t}(z)^{1-\theta} dz\right)^{\frac{1}{1-\theta}} \\
\mathcal{P}_{H,t} &= \left(\phi_H P_{H,t}^{1-\eta} + \phi_F P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
\mathcal{P}_{F,t} &= \left(\phi_H P_{F,t}^{1-\eta} + \phi_F P_{H,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}
\end{aligned}
$$

Each intermediate good is produced linearly out of labor $y_{H,t}(z) = L_{H,t}(z)$.

Housing supply The supply of housing satisfies:

$$
\begin{aligned}
Q_{H,t} &= (1-\delta)Q_{H,t-1} + I_{H,t}^{\alpha_H} M_{H,t}^{1-\alpha_H} \\
Q_{F,t} &= (1-\delta)Q_{F,t-1} + I_{H,t}^{\alpha_F} M_{F,t}^{1-\alpha_F}.
\end{aligned}
$$

where $I_{H,t}$ is resources (in goods) devoted to residential investment in the home region and $M_{H,t}$ is units of construction permits sold by the federal government. The region-specific parameter α allows the construction technology to differ across regions.

Markets The two regions share the same money, which serves as the numeraire. Composite intermediate goods markets are competitive and completely integrated across regions and each region faces the price $P_{H,t}$ for the home good and $P_{F,t}$ for the foreign good. Intermediate variety firms face Calvo price-setting frictions with probability of adjusting their price of $1-\chi$. The labor markets are local to each region and competitive with nominal wages $W_{H,t}$ and $W_{F,T}$. Households trade a nominal bond at nominal interest rate i_t. Units of housing trade at price $J_{r,t}$ in region $r \in \{H, F\}$.

Households face an LTV constraint:

$$-B_{t+1} \leq s \frac{J_{r,t+1} Q_{t+1}}{1+i_t}.$$

We assume this constraint is constantly binding on impatient households and never binding on patient households.

Intermediate goods firms produce profits, which are owned by the patient households in the region. We use $D_t(x)$ to denote the nominal profits received by household of type x.

Government There is a federal government that purchases goods in the two regions, sells construction permits in the two regions, and sets a common monetary policy. Let $G_{H,t}$ and $G_{F,t}$ be per capita spending in the two regions. These are exogenous and financed by national lump-sum taxes.

There is a monetary policy rule that sets the nominal interest rate:

$$1 + i_t = \frac{1}{\beta(\text{patient})} + \varphi_\pi \bar{\pi}_t + \varphi_y \bar{y}_t,$$

where $\bar{\pi}_t = n\pi_{H,t} + (1-n)\pi_{F,t}$ is the weighted average of the two inflation rates in the regions and $\bar{y}_t$ is the weighted average of the log deviations of output from their steady state values.

The government sells construction permits according to the rules:

$$M_{H,t} = \bar{M}_H \left(\frac{J_{H,t}}{\mathcal{P}_{H,t}} \right)^{\gamma_H}$$
$$M_{F,t} = \bar{M}_F \left(\frac{J_{F,t}}{\mathcal{P}_{F,t}} \right)^{\gamma_F}.$$

The region-specific parameter γ reflects the fact that regions differ in the elasticity of supply of vacant land.

Decision problems A household solves (we omit the region subscripts because all prices in the household's problem are local):

$$\sum_{t=0} \beta(x)^t \, u(C_t(x), L_t(x), Q_{t+1}(x); \Omega_t),$$

subject to:

$$\mathcal{P}_t C_t(x) + J_t Q_{t+1}(x) + B_{t+1}(x) = W_t L_t(x) - T_t + (1+i_{t-1})B_t(x) + J_t Q_t(x) + D_t(x)$$
$$-B_{t+1}(x) \leq s\frac{J_{t+1}Q_{t+1}(x)}{1+i_t}.$$

The Lagrangian is:

$$\begin{aligned}\mathcal{L} = &\sum_{t=0} \beta\,(x)^t \left\{u(C_t(x), L_t(x), Q_{t+1}(x); \Omega_t)\right. \\ &-\lambda_t(x)\left(\mathcal{P}_t C_t(x) + J_t Q_{t+1}(x) + B_{t+1}(x) - W_t L_t(x) + T_t - (1+i_{t-1})B_t(x) - J_t Q_t(x) - D_t(x)\right) \\ &\left.+\frac{\zeta_t(x)}{\mathcal{P}_t}\left[B_{t+1}(x) + \frac{J_{t+1}Q_{t+1}(x)}{1+i_t}\right]\right\}\end{aligned}$$

The first-order conditions can be rearranged to:

$$\begin{aligned} u_C &= \mathcal{P}_t \lambda_t \\ -u_L &= \frac{W_t}{\mathcal{P}_t} u_C \\ u_Q &= \frac{J_t}{\mathcal{P}_t} u_C - \beta\left[\frac{J_{t+1}}{\mathcal{P}_{t+1}} u_C'\right] - \zeta_t\left[\frac{\frac{J_{t+1}}{\mathcal{P}_{t+1}}\pi_{t+1}}{1+i_t}\right] \\ u_C &= \beta\left[\frac{1+i_t}{\pi_{t+1}} u_C'\right] + \zeta_t, \end{aligned}$$

where $\pi_{t+1} \equiv \mathcal{P}_{t+1}/\mathcal{P}_t$. For patient households we have $\zeta_t = 0$. For impatient households we have $\zeta_t > 0$ and the LTV constraint holds with equality.

The real estate developer solves:

$$\max_I J_t I^\alpha M_t^{1-\alpha} - \mathcal{P}_t I,$$

with first order condition:

$$\alpha J_t I^{\alpha-1} M_t^{1-\alpha} = \mathcal{P}_t.$$

The resources invested in housing are then:

$$I_t = \left(\alpha J_t M_t^{1-\alpha} \mathcal{P}_t^{-1}\right)^{\frac{1}{1-\alpha}}, \tag{1}$$

and the construction of new houses is:

$$I^\alpha M_t^{1-\alpha} = \left(\alpha \frac{J_t}{\mathcal{P}_t}\right)^{\frac{\alpha}{1-\alpha}} M_t.$$

Substitute in the rule for sales of construction permits:

$$I^{\alpha} M_t^{1-\alpha} = \left(\alpha \frac{J_t}{\mathcal{P}_t}\right)^{\frac{\alpha}{1-\alpha}} \bar{M} \left(\frac{J_t}{\mathcal{P}_t}\right)^{\gamma}.$$

It follows that the elasticity of new houses with respect to the price of housing is $\alpha/(1-\alpha)+\gamma$. This differs across regions because α and γ differ across regions. Finally, the resources invested in housing can be obtained by substituting the rule for permits into (1) to obtain:

$$I_t = \alpha^{\frac{1}{1-\alpha}} \bar{M} \left(\frac{J_t}{\mathcal{P}_t}\right)^{\gamma+\frac{1}{1-\alpha}}. \tag{2}$$

Notice that the housing supply elasticity is $\alpha/(1-\alpha)+\gamma$, but the elasticity of residential investment is $1/(1-\alpha)+\gamma$. Cities with more elastic housing supply (large α) will have residential investment respond more strongly to a given change in the price of housing. However, it is not clear which cities have more volatile residential investment because house prices will rise more in low elasticity cities.

The intermediate goods producer solves:

$$\max_{P_0^*} \sum_{t=0}^{\infty} \chi^t \lambda_t \left[P_0^* y_{H,t}(z) - W_{H,t} L_{H,t}(z)\right],$$

where:

$$\begin{aligned} y_{H,t}(z) &= Y_{H,t} \left(\frac{P_0^*}{P_{H,t}}\right)^{-\theta} \\ L_{H,t}(z) &= y_{H,t}(z). \end{aligned}$$

Substituting the constraints into the objective yields:

$$\max_{P_0^*} \sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^{\theta} \left(P_0^* - W_{H,t}\right) \left(P_0^*\right)^{-\theta}$$

with first-order conditions:

$$\begin{aligned} \sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^{\theta} \left(\theta - 1\right) P_0^{-\theta} &= \sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^{\theta} \theta W_{H,t} P_0^{-\theta-1} \\ P_0^* &= \frac{\theta}{\theta - 1} \frac{\sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^{\theta} W_{H,t}}{\sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^{\theta}}. \end{aligned}$$

Preference specification The preferences of the home households take the form:

$$u = \frac{1}{1-\sigma}\left[\left(C - \psi\frac{L^{1+\nu}}{1+\nu}\right)^{\kappa}(Q-\Omega_t)^{1-\kappa}\right]^{1-\sigma}.$$

We then have:

$$\begin{aligned}
u_C &= \left[\left(C - \psi\frac{L^{1+\nu}}{1+\nu}\right)^{\kappa}(Q-\Omega_t)^{1-\kappa}\right]^{-\sigma}\kappa\left(C - \psi\frac{L^{1+\nu}}{1+\nu}\right)^{\kappa-1}(Q-\Omega_t)^{1-\kappa}\\
u_L &= -\left[\left(C - \psi\frac{L^{1+\nu}}{1+\nu}\right)^{\kappa}(Q-\Omega_t)^{1-\kappa}\right]^{-\sigma}\kappa\left(C - \psi\frac{L^{1+\nu}}{1+\nu}\right)^{\kappa-1}(Q-\Omega_t)^{1-\kappa}\psi L^{\nu}.
\end{aligned}$$

Using these derivatives, the labor supply curve is:

$$\psi L_{H,t}(x)^{\nu} = \frac{W_{H,t}}{\mathcal{P}_{H,t}}. \tag{3}$$

Equilibrium definition Let lower-case denote real variables (i.e. normalized by $\mathcal{P}_t$). For $r \in \{H, F\}$ and $x \in \{\text{patient}, \text{impatient}\}$, the equilibrium variables are $j_{r,t}$, $w_{r,t}$, $\pi_{r,t}$, $\mathcal{Y}_{r,t}$, $Y_{r,t}$, $b_{r,t}(x)$, i_t, $Q_{r,t}(x)$, $Q_{r,t}$, $C_{r,t}(x)$, $L_{r,t}(x)$, $M_{r,t}$, $G_{r,t}$, t_t, $Z_{r,t}$, $Z^*_{r,t}$, $d_{r,t}(\text{patient})$, $I_{r,t}$, $E_t \equiv P_{H,t}/P_{F,t}$, where $Y^*_{r,t}$ is foreign demand for the composite good produced in r. The equilibrium conditions are:

$$\begin{aligned}
Q_{r,t} &= \sum_x \omega(x) Q_{r,t}(x) && \forall r && (4)\\
Y_{r,t}\int_0^1\left(\frac{p_{H,t}(z)}{P_t}\right)^{-\theta} dz &= \sum_x \omega(x) L_{r,t}(x) && \forall r && (5)\\
\psi L_{r,t}(x)^{\nu} &= w_{r,t} && \forall r, x && (6)\\
\mathcal{Y}_{r,t} &= \sum_x \omega(x) C_{r,t}(x) + G_{r,t} + I_{r,t} && \forall r && (7)\\
Y_{H,t} &= nZ_{H,t} + (1-n)Z^*_{H,t} && && (8)\\
Y_{F,t} &= nZ^*_{F,t} + (1-n)Z^*_{F,t} && && (9)\\
Z_{H,t} &= \phi_H \mathcal{Y}_{H,t}\left(\phi_H + \phi_F E_t^{-(1-\eta)}\right)^{\frac{\eta}{1-\eta}} && && (10)\\
Z_{F,t} &= \phi_F \mathcal{Y}_{H,t}\left(\phi_H E_t^{1-\eta} + \phi_F\right)^{\frac{\eta}{1-\eta}} && && (11)\\
Z^*_{H,t} &= \phi_F \mathcal{Y}_{F,t}\left(\phi_H E_t^{-(1-\eta)} + \phi_F\right)^{\frac{\eta}{1-\eta}} && && (12)\\
Z^*_{F,t} &= \phi_H \mathcal{Y}_{F,t}\left(\phi_H + \phi_F E_t^{1-\eta}\right)^{\frac{\eta}{1-\eta}}. && && (13)
\end{aligned}$$

Additionally there are two production functions for the final goods, two Phillips curves, four budget constraints (for the patient and impatient household in each region), the monetary policy rule, four housing first-order conditions, two Euler equations (for the patient agents), two LTV constraints (for the impatient agents), two exogenous G sequences, two housing permit supply rules, two dynamic equations for Q, one government budget, the definitions of dividend for each region, the first-order condition of the real estate developer in each region, and the evolution of the real exchange rate E_t given by:

$$
\begin{aligned}
\mathcal{P}_{H,t} &= \left(\phi_H + \phi_F E_t^{-(1-\eta)}\right)^{\frac{1}{1-\eta}} P_{H,t} \\
\mathcal{P}_{F,t} &= \left(\phi_H + \phi_F E_t^{1-\eta}\right)^{\frac{1}{1-\eta}} P_{F,t} \\
\pi_{H,t} &= \frac{\left(\phi_H + \phi_F E_t^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{\left(\phi_H + \phi_F E_{t-1}^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}} \frac{P_{H,t}}{P_{H,t-1}} \\
\pi_{F,t} &= \frac{\left(\phi_H + \phi_F E_t^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\phi_H + \phi_F E_{t-1}^{1-\eta}\right)^{\frac{1}{1-\eta}}} \frac{P_{F,t}}{P_{F,t-1}} \\
\frac{\pi_{H,t}}{\pi_{F,t}} &= \frac{\left(\phi_H + \phi_F E_t^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}}{\left(\phi_H + \phi_F E_{t-1}^{-(1-\eta)}\right)^{\frac{1}{1-\eta}}} \frac{\left(\phi_H + \phi_F E_{t-1}^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\phi_H + \phi_F E_t^{1-\eta}\right)^{\frac{1}{1-\eta}}} \frac{E_t}{E_{t-1}}.
\end{aligned}
$$

3 Preliminaries

Let's consider perfect foresight transitions lasting $\mathcal{T}$ dates in a linearized version of the model. For simplicity we will assume houses are created out of permits alone ($\alpha \to 0$) and complete home bias ($\phi_F \to 0$), which implies $\mathcal{Y}_{r,t} = Y_{r,t}$.

3.1 A dynamic consumption function

The first step is to establish that consumption and housing demand at date t along the transition can be written as a function of current and future prices and preference shocks and an endogenous state that summarizes the asset holdings of the patient and impatient households. Let $\mathcal{A}_t \equiv (1+i_{t-1})B_t + J_t Q_t$ be the wealth of an individual at date t. For an individual (patient or impatient) household, consumption at date t is the solution to:

$$
V_t(\mathcal{A}_t) = \max_{C_t, Q_{t+1}, L_t, B_{t+1}} u(C_t, L_t, Q_{t+1}; \Omega_t) + \beta V_{t+1}((1+i_t)B_{t+1} + J_{t+1}Q_{t+1}),
$$

subject to:

$$\mathcal{P}_t C_t + J_t Q_{t+1} + B_{t+1} = W_t L_t - T_t + \mathcal{A}_t + D_t$$
$$-B_{t+1} \leq s \frac{J_{t+1} Q_{t+1}}{1+i_t}.$$

$V_{\mathcal{T}+1}$ is the steady state value function associated with steady state prices and preference shifter Ω, which are fixed. $C_{\mathcal{T}}$ then depends on $\mathcal{A}_{\mathcal{T}}$ and the prices at $\mathcal{T}$. Recursing backwards, C_t depends on $\mathcal{A}_t$ and the prices at future dates. Similarly for the choices Q_{t+1} and B_{t+1}. For L_t we already know that labor supply only depends on the current real wage due to GHH preferences.

Given the initial, steady state value $\mathcal{A}_0$, we have already established that $\mathcal{A}_1$ depends on the whole path $\{\mathcal{P}_t, J_t, W_t, i_t, D_t, \Omega_t\}_{t=1}^{\mathcal{T}}$. We then recurse forwards and say $\mathcal{A}_{s+1}$ depends on current and future prices $\{\mathcal{P}_t, J_t, W_t, i_t, D_t, \Omega_t\}_{t=s}^{\mathcal{T}}$ and $\mathcal{A}_s$, which itself is a function of $\{\mathcal{P}_t, J_t, W_t, i_t, D_t, \Omega_t\}_{t=1}^{\mathcal{T}}$ so we can write $\mathcal{A}_{s+1}$ as a function of the whole path of prices.

Finally, substitute these "solutions" for asset holdings into the solution for C_t to write C_t as function of the full path of prices. The same argument applies to Q_{t+1} and B_{t+1}.

3.2 Price functions

Next we establish that there are functions that map past, current and future demand in region r, $\{\mathcal{Y}_{r,s}\}_{s=1}^{\mathcal{T}}$ to the current inflation rate, $\pi_{r,t}$, price level, $\mathcal{P}_{r,t}$, wage, $W_{r,t}$, and dividend, $D_{r,t}$. We use the linearized aggregate production function:

$$\mathcal{Y}_{r,t} = \sum_x \omega(x) L_{r,t}(x).$$

From the labor supply curve (3) we see that patient and impatient types work the same hours so the production function can be written as:

$$\mathcal{Y}_{r,t} = L_{r,t},$$

for the common labor supply $L_{r,t}$. Using the labor supply curve (3) this becomes:

$$\mathcal{Y}_{r,t} = \left(\frac{W_{r,t}}{\psi \mathcal{P}_{r,t}} \right)^{1/\nu}, \tag{14}$$

and we can use this to solve for the real and nominal wages as a function of current demand and the current price level. The definition of the dividend gives $D_{r,t}$ in terms of $\mathcal{Y}_{r,t}$, $W_{r,t}$, and

$L_{r,t}$. For the inflation rate and price level we work with the log-linearized Phillips curve,

$$\pi_t = \beta\pi_{t+1} + \frac{(1-\beta\chi)(1-\chi)}{\chi}\left[\log\left(\frac{W_{r,t}}{\mathcal{P}_{r,t}}\right) - \log\left(\frac{\theta}{1-\theta}\right).\right].$$

Solving this forward and using (14) gives

$$\pi_t = \sum_{s=t}^{\mathcal{T}} \beta^{s-t}\frac{(1-\beta\chi)(1-\chi)}{\chi}\left[\log\left(\psi\mathcal{Y}_{r,t}^{\nu}\right) - \log\left(\frac{\theta}{1-\theta}\right)\right].$$

We can then accumulate the inflation rate to find the path of the price level

$$\mathcal{P}_t = \left(\prod_{s=1}^{t}\pi_t\right)\mathcal{P}_0.$$

3.3 Summary

The key points from this section are (i) for any t and region r, we can write the equilibrium solutions for $C_{r,t}$ and $Q_{r,t+1}$ as functions of $\{\mathcal{P}_{r,t}, J_{r,t}, W_{r,t}, i_t, D_{r,t}, \Omega_t\}_{t=1}^{\mathcal{T}}$ and (ii) we can write the equilibrium solutions for $W_{r,t}$, $\mathcal{P}_{r,t}$, $D_{r,t}$ as functions of $\{\mathcal{Y}_{r,t}\}_{t=1}^{\mathcal{T}}$.

4 Our argument

We consider a perfect foresight transition lasting $\mathcal{T}$ dates. The previous section establishes that there are functions that map the paths for $\{\mathcal{P}_{r,t}, J_{r,t}, W_{r,t}, i_t, D_{r,t}, T_t, \Omega_t\}_{t=1}^{\mathcal{T}}$ to $C_{t,r}$ and $Q_{t,r}$ for each r and t. A first-order approximation of these functions gives:

$$\hat{C}_r = \mathbf{C}_J\hat{J}_r + \mathbf{C}_{\mathcal{P}}\hat{\mathcal{P}}_r + \mathbf{C}_i\hat{i} + \mathbf{C}_W\hat{W}_r + \mathbf{C}_D\hat{D}_r + \mathbf{C}_T\hat{T} + \mathbf{C}_\Omega\hat{\Omega}$$
$$\hat{Q}_r = \mathbf{Q}_J\hat{J}_r + \mathbf{Q}_{\mathcal{P}}\hat{\mathcal{P}}_r + \mathbf{Q}_i\hat{i} + \mathbf{Q}_W\hat{W}_r + \mathbf{Q}_D\hat{D}_r + \mathbf{Q}_T\hat{T} + \mathbf{Q}_\Omega\hat{\Omega},$$

where $\hat{C}$ is a $\mathcal{T}$ vector of consumption deviations from steady state for the $\mathcal{T}$ dates, $\mathbf{C}_J$ is a $\mathcal{T}\times\mathcal{T}$ matrix where the (i,j) element gives the effect of $\hat{J}_{t+j}$ on $\hat{C}_{t+i}$, and so on. We assume that the model is linearized around a symmetric steady state so the $\mathbf{C}$ matrices are the same for the two regions. While the two cities differ in their housing supply elasticities, the consequences of these differences affect consumption through the paths of prices. Similarly, government purchases affect consumption through the paths of prices and taxes.

Next, the previous section established that there are functions of $\{\mathcal{Y}_{r,t}\}_{t=1}^{\mathcal{T}}$ that give $\mathcal{P}_{r,t}$, $W_{r,t}$, and $D_{r,t}$ at each date t. The linearized version of these functions gives:

$$\hat{\mathcal{P}}_r = \mathbf{P}_{\mathcal{Y}} \hat{\mathcal{Y}}_r$$
$$\hat{W}_r = \mathbf{W}_{\mathcal{Y}} \hat{\mathcal{Y}}_r$$
$$\hat{D}_r = \mathbf{D}_{\mathcal{Y}} \hat{\mathcal{Y}}_r,$$

where $\mathbf{P}$, $\mathbf{W}$, and $\mathbf{D}$, are $\mathcal{T} \times \mathcal{T}$ matrices.

The aggregate resource constraint is:

$$\hat{\mathcal{Y}}_r = \hat{C}_r + \hat{G}_r.$$

Combining all these equations we have:

$$\hat{C}_r = \mathbf{C}_J \hat{J}_r + \underbrace{(\mathbf{C}_{\mathcal{P}} \mathbf{P}_{\mathcal{Y}} + \mathbf{C}_W \mathbf{W}_{\mathcal{Y}} + \mathbf{C}_D \mathbf{D}_{\mathcal{Y}})}_{\equiv \mathbf{m}} \left(\hat{C}_r + \hat{G}_r \right) + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \tag{15}$$

$$(I - \mathbf{m}) \hat{C}_r = \mathbf{C}_J \hat{J}_r + \mathbf{m} \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega}$$

$$\hat{C}_r = \underbrace{(I - \mathbf{m})^{-1}}_{\equiv \mathbf{M}} \left[\mathbf{C}_J \hat{J}_r + \mathbf{m} \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right]. \tag{16}$$

Finally, consider the effect on output:

$$\begin{aligned}
\hat{\mathcal{Y}}_r &= \hat{C}_r + \hat{G}_r \\
&= \mathbf{M} \left[\mathbf{C}_J \hat{J}_r + \mathbf{m} \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right] + \hat{G}_r \\
&= \mathbf{M} \left[\mathbf{C}_J \hat{J}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right] + [\mathbf{M}\mathbf{m} + I] \hat{G}_r \\
&= \mathbf{M} \left[\mathbf{C}_J \hat{J}_r + \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right],
\end{aligned} \tag{17}$$

where the last equality follows from the fact that $\mathbf{M}\mathbf{m} + I = I + \mathbf{m} + \mathbf{m}^2 + \cdots = (I - \mathbf{m})^{-1} = \mathbf{M}$. This makes clear that the consumption-income multiplier that applies to the partial equilibrium effects of the general equilibrium home price change is the same as the multiplier that applies to the government purchases shock.

5 Empirical specification

To relate (16) to our empirical specification it is convenient to simplify by assuming all the matrices in (16) are diagonal with a constant element on the diagonal. One interpretation of this assumption is that the economy is hit by a permanent shock to $\hat{\Omega}$ and undergoes an immediate transition to a new steady state so that at any date t along the transition the current value of, say, W_t is sufficient for the W_τ for all $\tau \in [1, \mathcal{T}]$. It then follows that a version of (16) holds period by period along the transition the only difference being that the matrices are now interpreted as scalars. Another way of putting it is that $\mathcal{T} = 1$ so the matrices are in fact scalars. Differencing (16) then yields:

$$\Delta \hat{C}_{r,t} = \underbrace{\mathbf{MC}_J}_{\text{housing wealth effect}} \times \Delta \hat{J}_{r,t} + \underbrace{\mathbf{Mm}\Delta \hat{G}_{r,t}}_{\text{city-specific shock}} + \underbrace{\mathbf{MC}_i \Delta \hat{i}_t + \mathbf{MC}_T \Delta \hat{T}_t + \mathbf{MC}_\Omega \Delta \hat{\Omega}_t}_{\text{time fixed effect}}.$$

6 Relationship between M and measured fiscal multiplier

M is likely smaller than what is measured as the open-economy fiscal multiplier. Part of the impact of the purchases shock on consumption comes through its effect on local house prices. This "asset-price" channel is omitted from **M**. We now explain this issue in detail although quantitatively it turns out to be a small difference because the effect of purchases on consumption that comes through home prices is small given our estimates of the housing wealth effect.

What we need to add to the preceding analysis is that $\hat{J}_r$ is an equilibrium outcome. To a first-order approximation, market clearing for housing requires:

$$\mathbf{Q}_J \hat{J}_r + \mathbf{Q}_\mathcal{P} \hat{\mathcal{P}}_r + \mathbf{Q}_i \hat{i} + \mathbf{Q}_W \hat{W}_r + \mathbf{Q}_D \hat{D}_r + \mathbf{Q}_T \hat{T} + \mathbf{Q}_\Omega \hat{\Omega} = \mathbf{Q}^S_{r,J} \hat{J}_r + \mathbf{Q}^S_{r,P} \hat{\mathcal{P}}_r,$$

where $\mathbf{Q}^S_{r,J}$ and $\mathbf{Q}^S_{r,P}$ determine the linearized supply curve. The argument that the supply curve can be written this way is as follows: at each date the government sells an amount of permits given by the real price of housing so the current supply of housing is discounted sum of the current and past real housing prices. Rearranging and substituting we have:

$$\left(\mathbf{Q}_J - \mathbf{Q}^S_{r,J}\right) \hat{J}_r + \left(\mathbf{Q}_\mathcal{P} - \mathbf{Q}^S_{r,P}\right) \mathbf{P}_\mathcal{Y} \hat{\mathcal{Y}}_r + \mathbf{Q}_W \mathbf{W}_\mathcal{Y} \hat{\mathcal{Y}}_r + \mathbf{Q}_D \mathbf{D}_\mathcal{Y} \hat{\mathcal{Y}}_r + \mathbf{Q}_i \hat{i} + \mathbf{Q}_T \hat{T} + \mathbf{Q}_\Omega \hat{\Omega} = 0$$

$$\left(\mathbf{Q}^S_{r,J} - \mathbf{Q}_J\right) \hat{J}_r = \left(\mathbf{Q}_\mathcal{P} \mathbf{P}_\mathcal{Y} - \mathbf{Q}^S_{r,P} \mathbf{P}_\mathcal{Y} + \mathbf{Q}_W \mathbf{W}_\mathcal{Y} + \mathbf{Q}_D \mathbf{D}_\mathcal{Y}\right) \hat{\mathcal{Y}}_r + \mathbf{Q}_i \hat{i} + \mathbf{Q}_T \hat{G}_r + \mathbf{Q}_\Omega \hat{\Omega}$$

$$\hat{J}_r = \mathbf{J}_{r,\mathcal{Y}} \hat{\mathcal{Y}}_r + \underbrace{\left(\mathbf{Q}^S_{r,J} - \mathbf{Q}_J\right)^{-1} \mathbf{Q}_i}_{\equiv \mathbf{J}_{r,i}} \hat{i} + \underbrace{\left(\mathbf{Q}^S_{r,J} - \mathbf{Q}_J\right)^{-1} \mathbf{Q}_T}_{\equiv \mathbf{J}_{r,T}} \hat{T} + \underbrace{\left(\mathbf{Q}^S_{r,J} - \mathbf{Q}_J\right)^{-1} \mathbf{Q}_\Omega}_{\equiv \mathbf{J}_{r,\Omega}} \hat{\Omega}, \qquad (18)$$

where:

$$\mathbf{J}_{r,\mathcal{Y}} \equiv \left(\mathbf{Q}^S_{r,J} - \mathbf{Q}_J\right)^{-1} \left(\mathbf{Q}_{\mathcal{P}}\mathbf{P}_{\mathcal{Y}} - \mathbf{Q}^S_{r,P}\mathbf{P}_{\mathcal{Y}} + \mathbf{Q}_W\mathbf{W}_{\mathcal{Y}} + \mathbf{Q}_D\mathbf{D}_{\mathcal{Y}}\right).$$

Now we substitute this expression for $\hat{J}_r$ into (17) to obtain:

$$\begin{aligned}
\hat{\mathcal{Y}}_r &= \mathbf{M}\left[\mathbf{C}_J\left(\mathbf{J}_{r,\mathcal{Y}}\hat{\mathcal{Y}}_r + \mathbf{J}_{r,i}\hat{i} + \mathbf{J}_{r,T}\hat{T} + \mathbf{J}_{r,\Omega}\hat{\Omega}\right) + \hat{G}_r + \mathbf{C}_i\hat{i} + \mathbf{C}_T\hat{T} + \mathbf{C}_\Omega\hat{\Omega}\right] \\
&= \left(I - \mathbf{M}\mathbf{C}_J\mathbf{J}_{r,\mathcal{Y}}\right)^{-1}\mathbf{M}\left[\hat{G}_r + \left(\mathbf{C}_i + \mathbf{C}_J\mathbf{J}_{r,i}\right)\hat{i} + \left(\mathbf{C}_T + \mathbf{C}_J\mathbf{J}_{r,T}\right)\hat{T} + \left(\mathbf{C}_\Omega + \mathbf{C}_J\mathbf{J}_{r,\Omega}\right)\hat{\Omega}\right]
\end{aligned}$$

There are several things to note here. (i) The fiscal multiplier is $\left(I - \mathbf{M}\mathbf{C}_J\mathbf{J}_{r,\mathcal{Y}}\right)^{-1}\mathbf{M}$, which exceeds $\mathbf{M}$ if consumption is increasing in home prices so $\mathbf{C}_J > 0$ and home prices increase in income so $\mathbf{J}_{r,\mathcal{Y}} > 0$. (ii) The fiscal multiplier differs across regions because the asset-price multiplier differs across regions. (ii) The measured fiscal multiplier does not necessarily fully control for monetary policy and national taxes because the regions are differentially exposed to these aggregate variables through the differential responses of home prices across regions.

The fiscal multiplier exceeds the multiplier of the housing wealth effect by a factor $\left(I - \mathbf{M}\mathbf{C}_J\mathbf{J}_{r,\mathcal{Y}}\right)^{-1}$. Given our estimate of the housing wealth effect, which implies a value of $\mathbf{C}_J$, this scaling factor is close to one. To show this we proceed from the following facts:

- Our estimated housing wealth effect is $\frac{J}{C}\mathbf{M}\mathbf{C}_J \approx 0.071$.
- Consumption is about 2/3 of GDP according to NIPA data.
- The open economy fiscal multiplier is about 1.5 according to Nakamura and Steinsson (2014). Local general equilibrium effects will be stronger if more of the income gains are spent locally. Nakamura and Steinsson's estimate is at the state level and our analysis is at the CBSA level so the relevant local government spending multiplier for our analysis is likely somewhat smaller than 1.5.

A final crucial ingredient to the calculation is the income elasticity of house prices is $\frac{Y}{J}\mathbf{J}_{\mathcal{Y}} \equiv x$. We leave this as a variable for the time being in order to consider a range of values. The measured fiscal multiplier is $\left(I - \mathbf{M}\mathbf{C}_J\mathbf{J}_{\mathcal{Y}}\right)^{-1}\mathbf{M} = 1.5$, where we have omitted the term $\mathbf{M}\mathbf{I}_{\mathcal{P}}\mathbf{P}_{\mathcal{Y}}$, which captures the response of the price level to income, because will treat J as the relative price of housing.

Step 1. Let Q be the quantity units of housing such that the value of housing is JQ. We then have $\frac{JQ}{C} = \frac{JQ}{Y}\frac{Y}{C} = \frac{3}{2}\frac{JQ}{Y}$.

Step 2. The housing wealth effect then implies:

$$\begin{aligned}\frac{J}{C}\mathbf{MC}_J &= 0.071\\ \frac{JQ}{C}\mathbf{MC}_J &= 0.071Q\\ \mathbf{MC}_J &= 0.071Q\frac{C}{Y}\frac{Y}{JQ}\\ \mathbf{MC}_J &= 0.071Q\frac{2}{3}\frac{Y}{JQ}.\end{aligned}$$

Step 3. Similarly, the income elasticity of house prices implies $\frac{Y}{JQ}\mathbf{J}_{\mathcal{Y}} = xQ^{-1}$ so $\mathbf{J}_{\mathcal{Y}} = xQ^{-1}\frac{JQ}{Y}$.

Step 5. Now put the pieces together:

$$\begin{aligned}(I - \mathbf{MC}_J\mathbf{J}_{\mathcal{Y}})^{-1}\mathbf{M} &= 1.5\\ \left(I - 0.071Q\frac{2}{3}\frac{Y}{JQ}xQ^{-1}\frac{JQ}{Y}\right)^{-1}\mathbf{M} &= 1.5\\ \left(I - 0.071\frac{2}{3}x\right)^{-1}\mathbf{M} &= 1.5\\ \mathbf{M} &= 1.5 - 0.071x.\end{aligned}$$

If house prices do not respond to income ($x = 0$) then we have $\mathbf{M} = 1.5$, which is the same as the fiscal multiplier. Lamont and Stein (1999) provide estimates of the short-run income elasticity of house prices, which imply $x < 0.8$ and more likely near 0.3. For $x = 0.3$ we have $\mathbf{M} = 1.48$ and for $x = 0.8$ we have $\mathbf{M} = 1.44$.

7 Incorporating residential investment

So far we have assumed that houses are constructed without using any real resources. In the more general case with $\alpha > 0$, residential investment consumes resources and this has two consequences for our analysis. First, part of the housing wealth effect that we measure comes not from the response of consumption to home prices directly, but indirectly from the income gains associated with housing construction. Second, part of the measured fiscal multiplier reflects the consumption response to the construction response to the purchases shock. The latter consideration is related to the asset-price channel discussed in the previous section in that it concerns how we relate the measured fiscal multiplier to the multiplier for the housing wealth effect.

Starting from (15) we now have:

$$\hat{C}_r = \mathbf{C}_J \hat{J}_r + \mathbf{m}\left(\hat{C}_r + \hat{G}_r + \hat{I}_r\right) + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega},$$

where we have used the resource constraint $\hat{\mathcal{Y}}_r = \hat{C}_r + \hat{G}_r + \hat{I}_r$. Solving for $\hat{C}_r$, this becomes:

$$\hat{C}_r = \mathbf{M}\left[\mathbf{C}_J \hat{J}_r + \mathbf{m}\left(\hat{G}_r + \hat{I}_r\right) + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega}\right].$$

From (2), we can see that we can write:

$$\hat{I}_r = \mathbf{I}_{r,J} \hat{J}_r + \mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} \hat{\mathcal{Y}}_r, \tag{19}$$

and we substitute this in for $\hat{I}_r$ to arrive at:

$$\hat{C}_r = \mathbf{M}\left[\left(\mathbf{C}_J + \mathbf{m}\mathbf{I}_{r,J}\right) \hat{J}_r + \mathbf{m}\hat{G}_r + \mathbf{m}\mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} \hat{\mathcal{Y}}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega}\right]. \tag{20}$$

(20) shows the first issue: part of the consumption response to home price changes comes from $\mathbf{m}\mathbf{I}_{r,J}$, which is the consumption response to the income change generated by the change in residential investment.

Now plug (20) in for $\hat{C}_r$ in the aggregate resource constraint:

$$\hat{\mathcal{Y}}_r = \mathbf{M}\left[\left(\mathbf{C}_J + \mathbf{m}\mathbf{I}_{r,J}\right) \hat{J}_r + \mathbf{m}\hat{G}_r + \mathbf{m}\mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} \hat{\mathcal{Y}}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega}\right], + \hat{G}_r + \mathbf{I}_{r,J} \hat{J}_r + \mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} \hat{\mathcal{Y}}_r$$

where we have also used (19). Now collect terms involving $\hat{J}_r$ and substitute with (18):

$$\begin{aligned} \hat{\mathcal{Y}}_r = {} & \mathbf{M}\left[\mathbf{m}\hat{G}_r + \mathbf{m}\mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} \hat{\mathcal{Y}}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega}\right] + \hat{G}_r + \mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} \hat{\mathcal{Y}}_r \\ & + \left[\mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\mathbf{I}_{r,J}\right) + \mathbf{I}_{r,J}\right]\left(\mathbf{J}_{r,\mathcal{Y}} \hat{\mathcal{Y}}_r + \mathbf{J}_{r,i} \hat{i} + \mathbf{J}_{r,T} \hat{T} + \mathbf{J}_{r,\Omega} \hat{\Omega}\right). \end{aligned}$$

$$\begin{aligned} \hat{\mathcal{Y}}_r = {} & \left[I - \mathbf{M}\mathbf{m}\left(\mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} + \mathbf{I}_{r,J} \mathbf{J}_{r,\mathcal{Y}}\right) - \mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} - \mathbf{I}_{r,J} \mathbf{J}_{r,\mathcal{Y}} - \mathbf{M}\mathbf{C}_J \mathbf{J}_{r,\mathcal{Y}}\right]^{-1} \\ & \times \left\{\mathbf{M}\hat{G}_r + \mathbf{M}\left(\mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega}\right) + \left[\mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\mathbf{I}_{r,J}\right) + \mathbf{I}_{r,J}\right]\left(\mathbf{J}_{r,i} \hat{i} + \mathbf{J}_{r,T} \hat{T} + \mathbf{J}_{r,\Omega} \hat{\Omega}\right)\right\} \\ \hat{\mathcal{Y}}_r = {} & \left[I - \mathbf{M}\mathbf{I}_{r,\mathcal{P}} \mathbf{P}_{\mathcal{Y}} - \mathbf{M}\mathbf{I}_{r,J} \mathbf{J}_{r,\mathcal{Y}} - \mathbf{M}\mathbf{C}_J \mathbf{J}_{r,\mathcal{Y}}\right]^{-1} \\ & \times \left\{\mathbf{M}\hat{G}_r + \mathbf{M}\left(\mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega}\right) + \left[\mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\mathbf{I}_{r,J}\right) + \mathbf{I}_{r,J}\right]\left(\mathbf{J}_{r,i} \hat{i} + \mathbf{J}_{r,T} \hat{T} + \mathbf{J}_{r,\Omega} \hat{\Omega}\right)\right\} \end{aligned}$$

This expression shows that second issue: the measured fiscal multiplier is larger than $\mathbf{M}$ not only because of the term $\mathbf{MC}_J\mathbf{J}_{r,\mathcal{Y}}$, which is the asset-price channel discussed in the previous section, but also because of the term $\mathbf{MI}_{r,\mathcal{P}}\mathbf{P}_{\mathcal{Y}} + \mathbf{MI}_{r,J}\mathbf{J}_{r,\mathcal{Y}}$, which captures the residential investment response to the change in real home prices induced by the fiscal shock.

In this case, our estimated housing wealth effect as an elasticity is $\frac{J}{C}\mathbf{M}\left(\mathbf{C}_J + \mathbf{mI}_J\right)$. We would like to remove the general equilibrium multiplier and the omitted variable component coming from construction income to recover $\frac{J}{C}\mathbf{C}_J$, which is the partial equilibrium housing wealth effect written as an elasticity.

We proceed from the following additional facts in addition to those used in the previous section:

- In an pooled IV regression for 1990-2015, we estimate the response of construction employment to home prices to be 0.343. This estimate uses the same specification as the pooled specification in Column 1 of Table 1 of Guren et al. (2018) with construction and real estate employment as the outcome variable.
- Residential investment is about 5 percent of GDP according to NIPA data.

The measured fiscal multiplier is now $\left(I - \mathbf{MI}_J\mathbf{J}_{\mathcal{Y}} - \mathbf{MC}_J\mathbf{J}_{\mathcal{Y}}\right)^{-1}\mathbf{M} \approx 1.5$, where we have again omitted the term $\mathbf{MI}_{\mathcal{P}}\mathbf{P}_{\mathcal{Y}}$, which captures the response of the price level to income, because will treat J as the relative price of housing.

Step 1. Let Q be the quantity units of housing such that the value of housing is JQ. We then have $\frac{JQ}{C} = \frac{JQ}{Y}\frac{Y}{C} = \frac{3}{2}\frac{JQ}{Y}$.

Step 2. We assume the elasticity of residential investment to home prices is the same as the elasticity of construction employment so we have $\frac{JQ}{I}\mathbf{I}_J = 0.343Q$, which implies:

$$
\begin{aligned}
\frac{JQ}{Y}\frac{Y}{I}\mathbf{I}_J &= 0.343Q \\
\mathbf{I}_J &= 0.343 \times 0.05 \times Q\frac{Y}{JQ}.
\end{aligned}
$$

Step 3. The housing wealth effect then implies:

$$\frac{J}{C}\mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\mathbf{I}_{r,J}\right) = 0.071$$
$$\frac{JQ}{C}\mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\mathbf{I}_{r,J}\right) = 0.071Q$$
$$\mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\mathbf{I}_{r,J}\right) = 0.071Q\frac{2}{3}\frac{Y}{JQ}$$
$$\mathbf{M}\mathbf{C}_J = 0.071Q\frac{2}{3}\frac{Y}{JQ} - \mathbf{M}\mathbf{m}\mathbf{I}_J$$
$$\mathbf{M}\mathbf{C}_J = 0.071Q\frac{2}{3}\frac{Y}{JQ} - \mathbf{M}\mathbf{m}0.343 \times 0.05 \times Q\frac{Y}{JQ}.$$

Step 4. Similarly, the income elasticity of house prices is defined as $\frac{Y}{JQ}\mathbf{J}_{\mathcal{Y}} = x$, which implies $\frac{Y}{JQ}\mathbf{J}_{\mathcal{Y}} = xQ^{-1}$ so $\mathbf{J}_{\mathcal{Y}} = xQ^{-1}\frac{JQ}{Y}$.

Step 5. Now put the pieces together to find $\mathbf{M}$:

$$\left(I - \mathbf{M}\mathbf{I}_J\mathbf{J}_{\mathcal{Y}} - \mathbf{M}\mathbf{C}_J\mathbf{J}_{\mathcal{Y}}\right)^{-1}\mathbf{M} = 1.5$$
$$\left[I - \left(\mathbf{M}\mathbf{I}_J + \mathbf{M}\mathbf{C}_J\right)\mathbf{J}_{\mathcal{Y}}\right]^{-1}\mathbf{M} = 1.5$$
$$\left[I - \left(\mathbf{M}0.343 \times 0.05 \times Q\frac{Y}{JQ} + 0.071Q\frac{2}{3}\frac{Y}{JQ} - \mathbf{M}\mathbf{m}0.343 \times 0.05Q\frac{Y}{JQ}\right)xQ^{-1}\frac{JQ}{Y}\right]^{-1}\mathbf{M} = 1.5$$
$$\left[I - \left(\mathbf{M}0.343 \times 0.05 + 0.071\frac{2}{3} - \mathbf{M}\mathbf{m}0.343 \times 0.05\right)x\right]^{-1}\mathbf{M} = 1.5.$$

$\mathbf{M}$ and $\mathbf{m}$ are related by $\mathbf{M} = (1 - \mathbf{m})^{-1}$ so $\mathbf{M} - I = \mathbf{M}\mathbf{m}$ and we have:

$$\left[I - \left(\mathbf{M}0.343 \times 0.05 + 0.071\frac{2}{3} - (\mathbf{M} - I)\,0.343 \times 0.05\right)x\right]^{-1}\mathbf{M} = 1.5$$
$$\left[I - \left(0.071\frac{2}{3} + 0.343 \times 0.05\right)x\right]^{-1}\mathbf{M} = 1.5$$

Solving this for $\mathbf{M}$ yields

$$\mathbf{M} = 1.5 - 0.064x.$$

For $x = 0.3$ we have $\mathbf{M} = 1.48$.

Step 6. Now remove the omitted variable effect. $\mathbf{M} = 1.48$ implies $\mathbf{m} = 0.325$. The estimated housing wealth effect then implies

$$\begin{aligned} \frac{J}{C}\mathbf{MC}_J + \frac{J}{C}\mathbf{MmI}_J &= 0.071 \\ \frac{J}{C}\mathbf{MC}_J + \frac{J}{C}\mathbf{Mm}0.343 \times 0.05 \times Q\frac{Y}{JQ} &= 0.071 \\ \frac{J}{C}\mathbf{MC}_J + \mathbf{Mm}0.343 \times 0.05 \times \frac{Y}{C} &= 0.071 \\ \frac{J}{C}\mathbf{C}_J &= 0.040. \end{aligned}$$

To summarize, the model presented in this appendix implies that the local general equilibrium multiplier for the housing wealth effect is close to the measured fiscal multiplier. It is slightly lower than the fiscal multiplier because our housing wealth effect takes home prices as given while part of the measured fiscal multiplier operates through changes in home prices. For reasonable values of the income elasticity of home prices, the two are very close in magnitude. The model presented in this appendix also implies that part of the measured housing wealth effect comes from income changes related to construction. Our pooled IV regression for 1990-2015 finds a response of construction employment to home prices to be 0.343, and using this we can remove the construction effects from the housing wealth effect. Making both of these adjustments yields a housing wealth effect elasticity of 0.040.

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