Online appendix for: International Evidence on Long-Run Money Demand

Luca Benati
University of Bern[*]

Robert E. Lucas, Jr.
University of Chicago[†]

Juan-Pablo Nicolini
Federal Reserve Bank
of Minneapolis[‡]

Warren Weber
University of South Carolina,
Federal Reserve Bank of Atlanta,
and Bank of Canada[§]

Throughout this entire online appendix, we will use (e.g.) 'Appendix A', 'Appendix B', etc. as a shorthand for 'online Appendix A', 'online Appendix B', etc. (to put it differently, nowhere we will make references to Appendices A and B *in the paper*).

A Integration Properties of the Data

Tables A.1*a*-A.1*c* report, for the series in our dataset, bootstrapped p-values for Elliot *et al.*'s (1996) tests (for technical details, see Section 5 in the text).

A.1 M_1 velocity and the short rate

Evidence of a unit root in M_1 velocity and the short rate is typically strong, with the bootstrapped p-values being almost uniformly greater than the 10 per cent significance level we take as our benchmark throughout the entire paper, and often significantly so. The following exceptions ought to be briefly discussed:

---

[*] Department of Economics, University of Bern, Schanzeneckstrasse 1, CH-3001, Bern, Switzerland. Email: luca.benati@vwi.unibe.ch

[†] Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, Illinois 60637, United States. Email: relucas@uchicago.edu

[‡] Federal Reserve Bank of Minneapolis, 90 Hennepin Avenue, Minneapolis, MN 55401, United States. Email: juanpa@minneapolisfed.org

[§] Darla Moore School of Business, University of South Carolina, 1014 Greene Street, Columbia, SC 29208. Email: weweber@gmail.com

First, and least importantly, in a few cases results based on the two alternative lag orders we consider produce contrasting evidence. This is the case, e.g., for the logarithms of velocity and the short rate for Israel; for log velocity for Chile for the period 1940-1995; and for the short rate for West Germany. In these cases we regard the null of a unit root as not having been convincingly rejected, and in what follows we therefore proceed under the assumption that these series are I(1). For Israel and Chile there are two main reasons for doing so:

(*i*) even if the tests performed perfectly, still, as a simple matter of probability, they would incorrectly reject the null at the x per cent level x per cent of the times. This means that a small fraction of fluke rejections of the null of a unit root *should* be expected even if *all* of the series we are dealing with are indeed I(1).

(*ii*) The visual evidence from Figure 2 in the online appendix provides indeed support to the notion that these results are simply part of the 'unavoidable flukes' associated with statistical testing. This is clearly the case for Israel, with the logarithms of both velocity and the short rate exhibiting an obvious I(1) behavior (keep in mind that, as discussed in Section 5, these series should *not* exhibit a trend, which on the contrary they do). Evidence for Chile's velocity is just slighty less strong, but still it suggests that the series can be regarded as I(1).

Finally, for Chile a unit root in log velocity is not rejected for the longer sample period (1941-2012), which suggests that ambiguity of the results for the shorter period may just be a small-sample issue.

Ambiguity of the results for West Germany's short rate, on the other hand, cannot be explained along the same lines. On the one hand, the visual evidence from Figure 6 in the online appendix by no means suggests that the series may be I(1), and on the contrary, if anything it suggests the opposite. Further, we do indeed have reasons for entertaining the possibility that post-WWII West German short rates might be I(0). By the Fisher effect, short-term rates should be equal to the Wicksellian (or 'natural') rate of interest plus expected inflation. Given the *Bundesbank*'s strong counter-inflationary stance, we might logically expect post-WWII German inflation to be I(0). In turn, absent a significant extent of permanent variation in Germany's natural rate of interest,[1] this would imply that German short-term rates should be stationary, too. Evidence on the integration properties of post-WWII German inflation is mixed. For the period considered herein, a unit root in CPI inflation cannot be rejected, with bootstrapped p-values for Elliot *et al.*'s tests without a time trend being equal to 0.374 and 0.131. For the longer sample 1948-1998, however, rejection is strong, with the p-values equal to 0.016 and 0.001. So the bottom line is that

---

[1] Labor productivity data from the Ohanian and Raffo (2011) dataset, however, point towards a significant slowdown in post-WWII German labor productivity growth. Evidence for real GDP growth is qualitatively the same. (All of these results are available upon request.) As discussed, e.g., in Laubach and Williams (2003), under very general conditions this should be expected to map (although not one-for-one) into a corresponding decrease in the natural rate of interest. (Specifically, this mapping holds. e.g., within the Ramsey and Solow growth models.)

although we regard a unit root in West Germany's short rate as not having been convingly rejected, and in what follows we therefore proceed under the assumption that the short rate is I(1), it has to be kept in mind that this may not be the case.

Second—and more importantly—under the Gold Standard a unit root in both the level and the logarithm of the short rate (either with, or without the 1% correction) is rejected for Canada, Finland, France, and Spain, and it is rejected for Switzerland based on the logarithm of the short rate with the 1% correction. In all of these cases, stationarity of the short rate precludes it from being entered in any cointegrated system, or cointegrating regression. On the other hand, it is not rejected for Italy, and Portugal, whereas for Japan results based on the two alternative lag orders we consider produce contrasting evidence, and therefore, as we did before, we regard the null of a unit root as not having been convincingly rejected, and we therefore treat Japan's short rate for the period 1885-1913 as being I(1). By the same token, a unit root in velocity under the Gold Standard is strongly rejected for Finland and Italy, whereas it is not rejected for Canada, Japan, and Spain. As for Italy's velocity for the period 1861-1913, visual evidence from Figure 6 in the online appendix clearly suggests that rejection of the null of a unit root should be regarded as one of those fluke results which, as previously mentioned, are all but unavoidable when performing a large number of statistical tests, such as in the present case. We therefore proceed under the assumption that Italy's velocity under the Gold Standard was I(1).

Third, by the same token, a unit root in the level of the short rate is rejected for Argentina, Brazil for the period 1934-2012, and Chile for the period 1941-2012, whereas in neither case it is rejected based on the logarithms. For these three cases we will therefore eschew the Selden-Latané specification. Under the Gold Standard a unit root in either the level or the logarithm of velocity is rejected for Italy: In this case we will therefore uniquely consider unrestricted specifications for GDP, M_1, and the short rate.

Third, for Taiwan a unit root in velocity is rejected based on the level, but not based on the logarithm. In this case we will eschew the Selden-Latané specification.

A.2 GDP and M_1

Evidence of a unit root in the logarithms of nominal GDP and nominal M_1 is, likewise, typically strong.[2] For GDP, a unit root is rejected only for Bolivia, and for France under the Gold Standard (the latter rejection is ultimately irrelevant, since, as previously mentioned, for France the interest rate is stationary, so that it is not possible to analyze cointegrated systems). As for M_1, it is rejected only for Israel, Canada (1967-2013), and Finland (1914-1985). For Bolivia, Israel, Canada (1967-2013), and Finland (1914-1985) we will therefore eschew unrestricted specifications for GDP, M_1,

---

[2] Again, in those few cases in which results based on the two alternative lag orders produce contrasting evidence, we regard the null of a unit root as not having been convincingly rejected, and we proceed under the assumption that the series is I(1).

and the short rate, and we will uniquely focus on bivariate systems with velocity and the short rate.

B Why Not Using Shin's (1994) Asymptotic Critical Values in Performing Tests of the Null of Cointegration?

As discussed in the text, all cointegration tests in this paper have been performed based on bootstrapped p-values. As for Johansen's tests of the null of no cointegration, the rationale for doing so was provided by Johansen (2002) himself, who showed how, in small samples, trace and maximum eigenvalue tests based on asymptotic critical values typically tend to perform poorly. Since this is a small-sample issue, as a matter of simple logic we should expect Shin's (1994) tests of the null of cointegration to suffer from an analogous poor performance, thus justifiying the use of a bootstrapping procedure to compute critical and p-values.

In this appendix we provide an additional rationale for bootstrapping Shin's tests: As we show, even in very large samples (in the following experiments we use samples of length $T = 100{,}000$) the Monte Carlo distributions of Shin's test statistics coincide with the asymptotic distribution of the critical values reported in Shin's (1994) Table 1 *only* if the cointegration residual has no persistence (specifically: it is white noise). This holds for either of the four kernels we use for computing Shin's test statistics. Further, the greater the persistence of the cointegration residual, the more the Monte Carlo distributions of Shin's test statistics in very large samples deviate from the asymptotic distribution reported in Shin (1994). Since, as we discuss in the text, the 'candidate cointegration residuals' produced by either Shin's or Johansen's procedure based on the actual data typically exhibit very high persistence, this logically implies that, if cointegration truly is there, performing Shin's tests based on his asymptotic critical values would automatically bias such tests towards rejection of the null of cointegration. This provides a further rationale for bootstrapping Shin's tests.

The model we use for the Monte Carlo experiments is given by

$$y_{i,t} = y_{i,t-1} + \epsilon_{i,t},\ i = 1,\ 2,\ \text{with } \epsilon_{i,t} \sim i.i.d.\ N(0,1) \tag{B.1}$$

$$x_t = \frac{y_{1,t} + y_{2,t}}{2} + u_t \tag{B.2}$$

$$u_t = \rho u_{t-1} + v_t,\ \text{with } 0 \leq \rho < 1,\ v_t \sim i.i.d.\ N(0,1) \tag{B.3}$$

As for ρ, we consider six possible values, corresponding to alternative extents of persistence of the cointegration residual between the three series, that is, $\rho = 0$, 0.25, 0.5, 0.75, 0.9, 0.95. There are two reasons for using this specific data generation process (henceforth, DGP). *First*, it captures the essence of the problem at hand.

Here we have three typically[3] I(1) series—GDP, M_1, and a short term nominal rate—whose long-run dynamics might obey a long-run equilibrium relationship (that is: they might be cointegrated). *Second*, by parameterizing the extent of persistence of the deviation from the long-run equilibrium relationship, we can effectively explore how the performance of Shin's (1994) test depends on such persistence, even in very large samples. This is key because, as already pointed out, real-world ('candidate') cointegration residuals from long-run money demand relationships are indeed very highly persistent. Intuitively, for the reasons discussed by Engle and Granger (1987), we would expect that, *ceteris paribus*, the higher the persistence of the cointegration residual, the more difficult it is for any statistical test to detect cointegration. As we will see this is indeed the case.

Details of the Monte Carlo simulations are as follows. For each value of ρ, we perform N = 100,000 Monte Carlo simulations of samples of length T = 100,000. For each individual simulation, we run a pre-sample of length 10,000 which we then discard, in order to eliminate dependence on initial conditions (which we set to $y_{1,0} = y_{2,0} = x_0 = 0$). Following Shin (1994, Section 5, pages 103-105), we set the number of leads and lags used in the dynamic OLS estimation of Shin's regression to $K = [T^{1/3}]$, where $[x]$ stands for 'the largest integer of x'.[4] In computing Shin's test statistic—which we do based on Shin's (1994) equation (2), page 93, i.e. for a model with an intercept, but no time trend—we consider the following four kernels, which are described, e.g., in Andrews (1991, expressions (2.7), page 821): Bartlett, Parzen, Tukey-Hanning, and 'Quadratic Spectral' (which, for the reasons we discuss below, is our kernel of choice in the entire paper). We select the spectral bandwidth parameter (in Shin's notation, ℓ) *via* the 'plug-in' method discussed in Andrews (1991) (see his Section 6, 'Automatic Bandwidth Estimators').

The results are reported in Table B.1. Several key facts are immediately apparent from the table. Specifically,

first, a comparison between the second column, and the columns from the third to the sixth, shows that the critical values reported in Shin's (1994) Table 1 are valid *only* in the case of a cointegrated DGP in which *the cointegration residual is white noise*. This holds true for either of the four kernels we consider. Since in most real-world applications cointegration residuals are typically very highly persistent, this implies that Shin's critical values are essentially irrelevant for practical purposes.[5]

---

[3] We say 'typically' because, as reported in Tables A.1*a*-A.1*c*, in a few cases (most of the times, under the Gold Standard), the null of a unit root can be rejected for M_1 velocity and/or the short term nominal rate.

[4] We have experimented with alternative values of K, either larger or smaller than $[T^{1/3}]$, and the results reported below are robust to using such alternative values, as long as they are not 'too small' (e.g., K = 1), or 'too large'.

[5] To be precise: For a few fractiles (specifically, the 1, 70, 90, 95, 97.5, and 99 per cent ones) our Monte Carlo critical values for the case in which ρ=0 are slightly different from Shin's (1994) asymptotic critical values. For the 90 per cent fractile, for example, our critical value is 0.161 based on either kernel, whereas the value reported by Shin is 0.163. Since Shin generated his critical values

Second—and crucially—as the persistence of the cointegration residual increases, the Monte Carlo distributions of Shin's test statistic get systematically 'shifted upwards' for each individual fractile. Once again, this holds, albeit with some differences (see the third point below), for either of the four kernels we consider. At the 10 per cent level we use in the paper, for example, the critical value is equal to 0.161 for ρ=0 for either of the four kernels, but it ranges between 0.311 and 0.429 for ρ=0.5; it ranges between 0.622 and 0.940 for ρ=0.75; and it ranges between 2.866 and 4.471 for ρ=0.95 (which is, in many cases, an empirically plausible value). This implies that, *even asymptotically*, relying on Shin's (1994, Table 1) critical values would induce a researcher to incorrectly reject the null hypothesis of cointegration too often in any situation in which the cointegration residual is characterized by at least some moderate extent of persistence. The practical implication of all this is that if in real-world situations—in which we do not even have the advantage of using samples of length T = 100,000—we were to perform Shin's tests based on his asymptotic critical values, we would most likely end up rejecting cointegration at the x per cent level much more than x per cent of the times.

Third, among the four kernels we consider, there is a clear 'ranking' in terms of how much the fractiles of the associated Monte Carlo distributions deviate from Shin's asymptotic critical values. Specifically, for each individual fractile, the lowest extent of upward deviation is associated with the quadratic spectral kernel, whereas the largest pertains to the Bartlett kernel. This is the reason why all of our empirical work based on Shin's tests is based on the quadratic spectral kernel.[6]

C Methodological Issues Pertaining Bootstrapping Cointegrated Processes

C.1 Why bootstrapping critical and p-values

As for Johansen's tests, the rationale for bootstrapping critical and p-values was provided by Johansen (2002) himself, who showed how, in small samples, trace and maximum eigenvalue tests based on asymptotic critical values typically tend to perform poorly.[7] Since this is a small-sample issue, as a matter of logic we should expect

---

'using a sample size of 2000' (see Shin (1994, page 99)), rather than 100,000 as we did, we believe our critical values should regarded as more reliable.

[6] To be precise, this does not represent a very strong rationale for preferring this kernel to either of the other three. Since we are going to bootstrap all of the tests, it is reasonable to expect that either of the other three kernels will produce *both* larger test statistics based on the actual data, *and* more upward-shifted bootstrapped distributions, but in the end, the resulting bootstrapped p-values should be the same. Still, however, since at the end of the day we ought to choose one kernel, our choice appears as logical to us.

[7] This provided indeed the motivation behind the boostrapping procedures proposed by Swensen (2006), and then by Cavaliere, Rahbek, and Taylor (2012), which improves upon Swensen's (and

Shin's (1994) tests to suffer from an analogous poor performance, thus justifiying the use of a bootstrapping procedure.

Appendix B provides an additional rationale for bootstrapping Shin's tests: As we show there, even in very large samples the distributions of Shin's test statistics coincide with the asymptotic distribution reported in Shin's (1994) Table 1 *only* if the cointegration residual has no persistence. Further, the greater the persistence of the cointegration residual, the more the Monte Carlo distributions of Shin's test statistics in very large samples is shifted upwards compared to the asymptotic distribution reported in Shin (1994). Since, as we document in the text (see Section 7.1), the 'candidate cointegration residuals' produced by either Shin's or Johansen's procedure based on the actual data typically exhibit very high persistence, this logically implies that, if cointegration truly is there, performing Shin's tests based on his asymptotic critical values would automatically bias such tests towards rejection of the null of cointegration.

C.2 Details of the bootstrapping procedures

As for Johansen's tests, we bootstrap trace and maximum eigenvalue statistics *via* the procedure proposed by Cavaliere *et al.* (2012; henceforth, CRT). In a nutshell, CRT's procedure is based on the notion of computing critical and p-values by bootstrapping the model which is *relevant under the null hypothesis*. This means that for tests of the null of no cointegration against the alternative of one or more cointegrating vectors the model which is being bootstrapped is a simple, non-cointegrated VAR in differences. For the maximum eigenvalue tests of h versus h+1 cointegrating vectors, on the other hand, the model which ought to be bootstrapped is the VECM estimated under the null of h cointegrating vectors. All of the technical details can be found in CRT, which the reader is referred to. We select the VAR lag order as the maximum[8] between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria[9] for the VAR in levels.

As for Shin's tests, to the very best of our knowledge nobody has yet provided anything comparable to what CRT did for Johansen's procedure (in fact, we were not even able to find a single paper discussing how to bootstrap Shin's test statistic). The bootstrap procedure we propose is based on exactly the same idea underlying CRT, that is: Computing critical and p-values by bootstrapping the process which is relevant under the null hypothesis. Within the present context, this implies that the

---

will be used in what follows).

[8] We consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model mis-specification) is more serious than the one resulting from choosing a lag order greater than the true one (over-fitting).

[9] On the other hand, we do not consider the Akaike Information Criterion since, as discussed (e.g.) by Luetkepohl (1991), for systems featuring I(1) series the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

process to be bootstrapped is the VECM estimated under the null of one cointegration vector.

C.3 Monte Carlo evidence on the performance of the two bootstrapping procedures

Table 1 in the text reports Monte Carlo evidence on the performance of the bootstrapping procedure for Johansen's trace tests[10] proposed by Cavaliere, Rahbek, and Taylor (2012),[11] whereas Table 2 reports evidence for the bootstrapping procedure for Shin's tests proposed herein. In either case, we perform the Monte Carlo simulations based on two types of DGP, featuring *no cointegration* and *cointegration*, respectively. The rationale for doing this is that, *first*, Johansen's and Shin's tests are based on different null hypotheses (no cointegration for the former, and cointegration for the latter); and *second*, in order to properly interpret the results from either test based on the actual data, it is necessary to see how the two bootstrapping procedures perform conditional on the two possible alternative states of the world.

As for the DGP featuring *no cointegration*, we simply consider three independent random walks. As for the one featuring *cointegration*, we consider the one discussed in Appendix B, which allows us to explore how the two procedures perform conditional on alternative extents of persistence of the cointegration residuals (this is conceptually in line with some of the the evidence reported by Engle and Granger (1987)). For either DGP, we consider five alternative sample lengths, T = 50, 100, 200, 500, and 1,000. The results reported in either table are based on 1,000 Monte Carlo simulations. For each simulation we generate a sample of length T+100, and we then discard the first 100 observations in order to eliminate dependence on initial conditions (which we set to 0 for either series). For each individual simulation we perform bootstrapping based on 5,000 replications.

C.3.1 Evidence for Johansen's test of the null of no cointegration

Table 1 reports evidence for Johansen's trace test of the null of no cointegration against the alternative of one or more cointegration vectors. Specifically, the table reports, for either DGP, sample length, and (for the DGP featuring cointegration) value of ρ, the fraction of replications for which no cointegration is rejected at the 10 per cent level. The following main findings clearly emerge from the table:

---

[10] Numerically near-identical evidence for Johansen's maximum eigenvalue tests is not reported for reasons of space, but it is available upon request.

[11] Extensive Monte Carlo evidence on the good performance of Cavaliere *et al.*'s (2012) procedure was already provided by Cavaliere *et al.* themselves in their original paper. Benati (2015) also provided some (much more limited) evidence conditional on the specific DGPs he was interestd in. The rationale for providing additional evidence here is the same as Benati (2015), that is: looking at how the procedure performs conditional on the DGPs we are here interested in.

first, in line with the evidence reported by both CRT and Benati (2015), the procedure performs remarkably well conditional on DGPs featuring *no cointegration.* A key point which ought to be stressed is that the specific sample length used in the simulations does not appear to make any material difference for the final results, with the fractions of rejections ranging between 0.098 and 0.119 (with the ideal one being 0.1). This is testimony to the power of bootstrapping, which is capable of automatically controlling for the specific characteristics of the DGP under investigation.

Second, when the DGP does feature *cointegration*, ideally we would like the test to reject as much as possible. As the lower part of the table shows, the procedure performs indeed very well if ρ is small. If $\rho = 0$, for example, cointegration is already detected 100 per cent of the times for $T = 100$, whereas if $\rho = 0.5$ it is detected 88.2 per cent of the times for $T = 100$, and a sample length of $T = 200$ is already sufficient to detect cointegration 100 per cent of the times. As ρ increases, however, the performance deteriorates. The intuition for this is straightforward: as the cointegration residual becomes more and more persistent, it gets closer and closer to a random walk (in which case there would be no cointegration), and the procedure needs therefore larger and larges samples to detect the truth (that the residual is highly persistent, but ultimately stationary). In particular, as ρ increases, the fraction of rejections tends to converge, for each sample size, to the fraction of rejections under the DGP featuring no cointegration. This is especially apparent for $T = 50$ or 100, with the fractions being equal to 0.114 and 0.120, respectively. In the limit, for $\rho \rightarrow 1$, the procedure will tend to reject 10 per cent of the times.

Comparison with the Monte Carlo evidence of Cavaliere *et al.* (2012) This evidence is qualitatively and also quantitatively in line with the Monte Carlo evidence reported in CRT's Tables I and II, pp. 1731-1732. Although the DGPs they used (either non-cointegrated VARs, or cointegrated VECMs featuring one cointegration vector), were different from the DGPs used herein, their results and ours turn out to be very close. Specifically,

- the results in panel (b) of their Table I illustrate the excellent performance of their bootstrapping procedure for tests of the null of no cointegration when the true DGP features no cointegration. In line with the evidence reported in the first row of our Table 1, their results illustrate how, at the 5 per cent level, the empirical rejection frequencies (henceforth, ERF) are quite close to 5 per cent irrespective of the sample size.

- Panel (a) in the same table reports qualitatively and quantitatively similar evidence for the maximum eigenvalue test of 1 versus 2 cointegrating vectors, conditional on DGPs featuring one cointegrating vector.

- Finally, Table II reports evidence on the ability of the sequential bootstrapped procedure to select the correct cointegration rank, which in their experiments is one (see the columns under the heading ‘Bootstrap (CRT)’). Those results are in line with the ones reported in our Table 1 conditional on DGPs featuring one cointegration vector. In either case, the larger the sample size, the more frequently CRT’s procedure detects the truth, with ERFs converging towards 1 for sufficiently large sample. In comparatively small samples (e.g., for $T = 50$), ERFs are typically much below one—as we show, the more so, the more persistent is the cointegration residual.[12]

So the bottom line is that that our Monte Carlo evidence, although based on a set of DGPs which have been specifically tailored to the problem at hand, is in fact exactly in line, both qualitatively and quantitatively, with the evidence reported in CRT.

Summing up There are two things to take away from all this, and to keep in mind in order to correctly interpret the results from Johansen’s bootstrapped tests performed on the actual data:

- if the true DGP features *no cointegration*, Cavaliere *et al.*’s (2012) procedure performs remarkably well irrespective of the sample size.
- If, however, the true DGP does feature *cointegration*, the procedure performs well only if the persistence of the cointegration residual is sufficiently low, and/or the sample size is sufficiently large.[13] If, on the other hand, the cointegration residual is persistent, and the sample size is small, the procedure will fail to detect cointegration a non-negligible fraction of the times. For example, with $T = 100$, cointegration will be detected 43.3 per cent of the times if $\rho = 0.75$, and just 12.0 per cent of the times if $\rho = 0.95$.

All of this means that if Johansen’s tests do detect cointegration, we should have a reasonable presumption that cointegration is indeed there. If, on the other hand, they do not detect it, a possible explanation is that the sample period is too short, and/or the cointegration residual is highly persistent.

---

[12] Different from the present work, CRT do not explore how the persistence of the cointegration residual affects the performance of their procedure. The results reported in their Table II, however, are quantitatively in line with ours. We found out this in the following way. We simulated their VECM conditional on one cointegration vector 10,000 times for samples of lenghth $T = 10{,}000$, and for each simulation we computed the implied cointegration residual, and based on it we estimated an AR(4) process (in fact, given the nature of their DGP, an AR(2) would have been enough). The sum of the AR coefficients is our measure of persistence. For their benchmark case of δ=0.1, both the mean and the median of the distribution werte equal to 0.61. From their Table II we can see that for δ=0.1 and $T = 50$, the ERF is 49.0 per cent. In Table 1 we report, for $T = 50$ and ρ=0.5, an ERF of 35 per cent.

[13] Since cointegration is a property of a system pertaining, in principle, to the infinite long-run, Faust and Leeper (1997)’s point about the intrinsic difficulty of identifying such features of the data based on finite samples directly applies.

C.3.2 Evidence for Shin's (1994) test of the null of cointegration

Before delving into the Monte Carlo evidence reported in Table 2, we spend some time discussing two technical issues.

Two technical issues A first strictly technical issue pertains how to estimate the VECM which is going to be bootstrapped conditional on one cointegration vector. Here there are (at least) two possibilites. A first one is to simply use Johansen's estimator of the VECM, as detailed e.g. in Hamilton (1994).[14] A second possibility is to follow the two-step procedure discussed in Luetkepohl (1991),[15] which calls for (*i*) getting the residual from the cointegrating regression (which Shin's procedure produces automatically), and then (*ii*) estimating the VECM *via* OLS conditional on such cointegration residual.

A key difference between the two procedures is that, as discussed by Luetkepohl (1991), whereas Johansen's estimator is valid for any number of cointegration vectors in the system, this second approach is only valid in the case in which there is just one cointegration vector. This is not an issue when we test for cointegration between (log) M_1 velocity and (the log of) a short rate: clearly, within such a system there can be at most one cointegration vector. It is potentially an issue, however, when we consider trivariate, 'unrestricted' systems with a short rate and nominal GDP and nominal M_1. In these cases, we preliminarly perform Johansen's tests for the null of no cointegration between the short rate and nominal GDP. The rationale for preliminarly performing Johansen's is that, for the short rate and nominal GDP, we regard no cointegration as the 'natural null hypothesis'.

Evidence Table 2 reports evidence for the proposed bootstrapping procedure. Details of the Monte Carlo simulations are exactly as before under all respects (types of DGP considered, sample lengths, etc.). Once again, the table reports, for either DGP, sample length, and (for the DGP featuring cointegration) value of ρ, the fraction of replications for which no cointegration is rejected at the 10 per cent level. For the DGP featuring *cointegration*—that is: under the null hypothesis for which the test was designed—the following main findings emerge from the table:

first, the proposed bootstrapping procedure improves upon Shin's asymptotic critical values as long as the cointegration residual exhibits some mild extent of persistence. If ρ is equal to either 0, or 0.25, the performance of tests based on our proposed procedure, and on Shin's asymptotic critical values, is essentially equivalent. For larger values of ρ, however, the two performances diverge, with the proposed bootstrapping procedure outperforming tests based on Shin's asymptotic critical values the more, the larger the value of ρ. Focusing, just to fix ideas, on the comparative performance of tests based on samples of length $T = 100$, the two procedures are still

---

[14] See Hamilton (1994, chapter 20).

[15] See Luetkepohl (1991, pp. 370-371).

essentially equivalent when $\rho = 0.5$, with the fraction of simulations for which the null hypothesis is incorrectly rejected being equal to 0.143 and 0.136. For $\rho = 0.75$, however, the two performances already start to significantly diverge, with tests based on asymptotic critical values incorrectly rejecting the null 33.6 per cent of the times, and our procedure only rejecting 17.2 per cent of the times. For very highly persistent cointegration residuals, the difference becomes large: for $\rho = 0.95$, in particular, asymptotic critical values would lead a researcher to reject the null of cointegration 72.1 per cent of the times, whereas the bootstrap-based procedure only rejects 25.1 per cent of the times.

Second—and counter-intuitively—increasing the sample size does *not* improve the comparative performance of tests based on asymptotic critical values. On the contrary: The larger the sample size, the *worse* the comparative performance of tests based on asymptotic critical values becomes. This is uniformly the case for all values of ρ. For $\rho = 0.95$, for example, tests based on Shin's asymptotic critical values incorrectly reject cointegration 72.1 per cent of the times for $T = 100$, 93.2 per cent of the times for $T = 200$, and *100.0 per cent of the times* for $T = 1{,}000$. The corresponding fractions of rejections based on the bootstrap procedure, on the other hand, are equal to 25.1, 23.3, and 15.1 per cent, respectively. The reason for this counter-intuitive phenomenon is straightforward, and it has to do with the results reported in Table B.1 of Appendix B: Even in very large samples (there we worked with samples of length $T = 100{,}000$) the Monte Carlo distribution of Shin's test statistic gets 'shifted upwards', compared to the asymptotic distribution whose fractiles are reported in Shin's (1994) Table 1, the more the larger the value of ρ. For the present purposes, this implies that, the larger ρ, the more tests based on asymptotic critical values will reflect such very-large-samples distortion. As a result, the larger T, the *worse* tests based on asymptotic critical values will become. On the other hand, the performance of the proposed bootstrapping procedure, although not as good as that of Cavaliere *et al.*'s procedure for Johansen's tests, is uniformly superior to that of tests based on asymptotic critical values.

Third, the performance of the bootstrap procedure follows a 'hump-shaped' pattern as a function of the sample size. For each value of ρ, the fraction of rejections reaches a maximum (among the sample sizes considered herein) for $T = 100$, and it then declines monotonically, reaching the minimum for $T = 1{,}000$. We do not have a clear intuition for why this may be the case, but the pattern is a robust one, so it ought to be the case that some deep underlying cause is at work here. In particular, it is reassuring that, for $T = 1{,}000$—that is: under the circumstances in which Shin's asymptotical critical values perform worse—the bootstrap procedure works best, with the fractions of false rejections ranging between 0.111 and 0.115 for ρ smaller than or equal to 0.75, and increasing to 0.13 and 0.151 for ρ equal to 0.9 and 0.95, respectively.

Turning to the DGP featuring *no cointegration*, here, ideally, we would want the tests to reject as much as possible. At first blush, it would appear that, conditional on this DGP, Shin's asymptotic critical values perform uniformly much better than our bootstrapping procedure. Upon a moment's reflection, however, it clearly appears that such apparently superior performance is nothing but a fluke, and it simply originates from the fact that—as we just discussed—the higher the persistence of the cointegration residual, the more the Monte Carlo distribution of Shin's test statistic gets shifted upwards compared to the asymptotic distribution reported by Shin, with the result that, based on Shin's asymptotic critical values you reject more and more frequently. Another way of putting this is that the results reported in the one-to-last row of Table 2 are nothing but the limit of what is reported in the previous part of the table, based on Shin's asymptotic critical values, as ρ progressively increased. So the bottom line is that such apparently superior performance is just a fluke. As for the proposed bootstrapped procedure, even in very large samples the ERFs it produces by no means approach the ideal one of 100 per cent. For $T = 1000$, for example, cointegration is rejected only about 38 per cent of the times, and based on smaller sample lengths, much less than that.

Overall, the Monte Carlo evidence reported in Table 2 clearly shows how the bootstrapping procedure we are here proposing significantly improves upon tests based on Shin's asymptotic critical values. At the same time, however, the performance is far from perfect: conditional on the DGP featuring cointegration, for example, if $\rho = 0.95$ and $T = 200$ we incorrectly reject the null of about 23 per cent of the times.

Summing up There are two things to take away from all this, and to keep in mind in order to correctly interpret the results from Shin's bootstrapped tests performed on the actual data:

- if the true DGP features *no cointegration*, our procedure rejects the null of cointegration much less than it should.
- If, on the other hand, the true DGP does feature *cointegration*, the procedure—although it represents an improvement upon using Shin's asymptotic critical values—still does not perform optimally, the more so the more persistent the cointegration residual is.

Key implications from all this are that,

first, Shin's asymptotic critical values should not be used.

Second, *lack of rejection* of the null of cointegration based on Shin's tests and our bootstrapping procedure does not represent strong evidence that cointegration truly is there. As the last row of Table 2 shows, if the true DGP does not feature cointegration, our procedure will capture the truth between 17.5 and 37.8 per cent of the times.

Third, rejection of the null of cointegration does not appear to be especially informative about the true nature of the DGP, as the ERFs are not significantly different conditional on the two possible states of the world.

Another way to put all this is that results from Shin's tests appear, overall, as less informative than the corresponding results produced by Johansen's tests bootstrapped as in CRT.

D Are GDP and Short-Term Nominal Interest Rates Cointegrated?

As discussed by Shin (1994), when the number of I(1) regressors in his cointegrating regressions is greater than one, a necessary condition for his approach to be valid is that they are not cointegrated. This means that for either of the unrestricted cointegrating regressions we run in Appendix E, that is

$$\ln(M_{1,t}) = \beta_0 + \beta_1 \ln(NGDP_t) + \beta_2 R_t + \varepsilon_t, \tag{D.1}$$

or

$$\ln(M_{1,t}) = \beta_0 + \beta_1 \ln(NGDP_t) + \beta_2 \ln(R_t) + \varepsilon_t, \tag{D.2}$$

where the notation is obvious, with $NGDP_t$ and R_t being nominal GDP and the short rate, the two right-hand side variables ought not to share a common stochastic trend.

Tables D.1.a-D.1.c report results from Johansen[16] cointegration tests for (the log of) the short rate and the logarithm of nominal GDP. Out of 34 tests, we fail to reject the null in 25 cases. Taken at face value, these results would imply that either (D.1) or (D.2) can indeed be run in those cases, whereas in the remaining nine cases Shin's approach cannot be applied. In fact, we regard those nine rejections as (quite obvious) flukes, which is why in Tables E.1a-E.1c in Appendix E we report results from either (D.1) or (D.2) for all 34 samples. The reason for this is that, based on economic logic, the notion that the short-term rate might share a common stochastic trend with nominal GDP is manifestly absurd. Further, as we discuss in the next paragraph, it is not uncommon for Johansen's tests to 'detect' cointegration among variables which, based on either economic logic—of just simple, plain logic—cannot possibly by cointegrated.

Most (or all) economists would likely think that the long-run evolution of the price level has nothing to do with global warming. Whereas the latter is driven by CO_2 emissions, the former depends on the nature of the monetary regime. This is not, however, what Johansen's cointegration tests—taken at face value—would seem to suggest. As Table D.1 shows, for two countries (Bolivia and Brazil) out of the four we consider, the very same Johansen tests we use to explore the presence of a long-run money demand 'detect' cointegration between the logarithm of the price level and

---

[16] We only consider Johansen tests because, as we will discuss shortly, no cointegration between GDP and the short rate is the 'natural null hypothesis'.

either the ocean or the land 'global temperature anomaly', the two most commonly-used indicators of the strength of global warming.[17] These results are much more common than it might be thought: the very first draft of Benati (2015), which was presented at a conference at the *Swiss National Bank*, and it is still available from the SNB's website, documented how the very same Johansen tests used herein 'detect' cointegration between Canada's unemployment rate and the concentration of CO_2 in the atmosphere, and between the ocean 'global temperature anomaly' and the unemployment rate in the Euro area, the U.K., and Canada. All of this is to bring home the point that (economic) logic should take the precedence over results from statistical tests, and when the two collide, the former should prevail.

E Searching for a Long-Run Money Demand

Tables SELA.2a, SL.2a, LL.2a, and LLCO.2a in the online appendix report, for the six high-inflation countries, results from either Johansen or Shin tests for cointegration between (log) velocity and (the log of) the short rate based on the four specifications considered herein: Selden-Latané, semi-log, log-log, and log-log with the 1% correction to the short rate along the lines of Alvarez and Lippi (2009). Tables SELA.2b, SL.2b, LL.2b, and LLCO.2b report the corresponding sets of results for all other countries.

Figures SELA.1-SELA.6, SL.1-SL.6, LL.1-LL.6, and LLCO.1-LLCO.6 in the online appendix report, in the top rows, the candidate cointegration residuals produced by either Johansen's or Stock and Watson's (1993) estimators, and, in the bottom rows, the bias-corrected bootstrapped distributions[18] of the corresponding estimates of the coefficient on (the log of) the short rate (so, to be clear, e.g., what Figures LL.1-LL.6 and LLCO.1-LLCO.6 show are the bootstrapped distributions of the estimated elasticities). For each bootstrapped distribution we also report the mean, the median, and the 5th and 95th percentiles. For the reasons discussed in Sections 6 and 7, and especially in Section 7.2, we report both candidate cointegration residuals, and

---

[17] The 'global temperature anomaly'—either for the Earth's landmass, or for its oceans—is defined as the temperature's deviation from a reference level, which is essentially an average since January 1880. Monthly, seasonally unadjusted series for the Earth's global land and ocean temperature anomalies are from the U.S. *National Oceanic and Atmospheric Administration*'s website. The series are available since January 1880, and they have been converted to the annual frequency by taking simple annual averages.

[18] Bootstrapping has been implemented as in Cavaliere *et al.* (2012) based on the estimated VECM conditonal on one cointegration vector. The bias-correction has been implemented as in Kilian (1998). The only difference between what Kilian did, and what we are doing here, is that whereas he applied his proposed methodology to bias-correcting impulse-response functions, we are here using it to bias-correct the elements of the cointegration vector (this is conceptually in line with Cavaliere, Taylor, and Trenkler (2015)). In general, however, the extent of the bias was small (the non-bias-corrected estimates are available upon request), so that bias-correcting does not make any material difference to the results.

estimates of the coefficients on the short rate, for all countries, rather than only for those for which statistical tests detect evidence of cointegration.

Tables LL.3 and LLCO.3 report bootstrapped p-values for testing the null hypothesis that the elasticity of money demand is equal to the Baumol-Tobin benchmark value, -0.5, based on bivariate systems featuring the logarithms of velocity and the short rate. Table SL.3 report p-values for testing the null hypothesis that the semi-elasticity is equal to -0.1 based on bivariate systems featuring the short rate and the logarithm of velocity. As for the Selden-Latané specification, since nobody has estimated it since 1960, we do not have any benchmark value that we can use to perform statistical tests. In Table SELA.3 we therefore report bootstrapped p-values for testing the null hypothesis that the coefficient on the short rate be equal to -0.4, which is roughly equal to the median or modal estimates we obtain for the United States based on the Lucas-Nicolini aggregate (see Figure SELA.6).

Tables SL.4, LL.4, and LLCO.4 report results from Johansen's cointegration tests based on unrestricted specifications featuring (the log of) the short rate, and the logarithms of nominal GDP and M_1, whereas Tables SL.5, LL.5, and LLCO.5 report the corresponding results from Shin's tests.

Finally, Tables SL.6, LL.6, and LLCO.6 report bootstrapped p-values for testing the null hypothesis that the income elasticity of money demand is equal to 1, based on the same unrestricted specifications of Tables SL.4, LL.4, and LLCO.4, and Tables SL.5, LL.5, and LLCO.5.[19]

E.1 Unrestricted tests of the null of cointegration

Although this paper mostly focuses on the results produced by bivariate systems, let's start by briefly discussing those produced by Shin's tests applied to unrestricted specifications featuring (the logarithm of) the short rate, and the logarithms of nominal GDP and M_1. The reason for doing so is that they represent one 'extreme end' of the spectrum within the full set of our results: As Tables SL.5, LL.5, and LLCO.5 show, based on unrestricted three-variables systems it is extremely difficult to reject the null of cointegration.[20] At the 10 per cent level we obtain *just four* rejections

---

[19] We do not consider specifications featuring the levels of either GDP or M_1. For the reasons discussed in Section 3 in the text, it is not possible to meaningfully test for a unit root based on the level of either series, and, as a consequence, it is not possible to run cointegration tests based on them.

[20] To be precise: As discussed by Shin (1994), when the number of I(1) regressors in his cointegrating regressions is greater than one, a necessary condition for his approach to be valid is that they are not cointegrated. This means that, within the present context, the (log of the) short rate and either nominal or real GDP ought not to be cointegrated. We address this issue in appendix D. As we discuss there, even if in a few cases Johansen's tests reject the null of cointegration between the short rate and GDP, based on economic logic those results should be regarded as flukes. Further, as we show, it is not uncommon for Johansen's tests to 'detect' cointegration among trending series which cannot possibly be cointegrated, such as the price level and global warming indicators. All of this is to bring home the point that (economic) logic should take the precedence over results from

of the null out of 33 tests (that is, 12.1 per cent of the total) based on the semi-log specification, whereas based on the log-log specification with the 1% correction to the short rate we obtain *only one* rejection out of 31 tests (3.2 per cent of the total). These figures are in line with the number of 'fluke rejections' we should expect from the tests even in the best of circumstances, and in fact, for the log-log specification, they are much smaller.[21] This means that, in principle, these rejections could well be 'explained away' as flukes.

This is however *not* the position we want to take here: Rather, we want to downplay these results. The reasons for this have already been discussed in Sections 6.2.2 and 6.2.3 in the text, and Section C.3.2 in Appendix C: As we stressed there, lack of rejection of the null of cointegration based on Shin's tests and our bootstrapping procedure does not represent strong evidence that cointegration truly is there. This means that the results reported in Tables SL.5, LL.5, and LLCO.5 in the online appendix do not truly represent *strong* evidence in favor of cointegration, and we should not read too much into them.

E.2 Testing for cointegration between velocity and the short rate

Turning to the set of results based on bivariate systems, the evidence reported in Tables SELA.2, SL.2, LL.2, and LLCO.2 can be usefully classified as follows.

E.2.1 Cases in which evidence of cointegration is strong

The United States We start from the United States, which has been the focus of the most intensive investigation, and for which researchers such as Friedman and Kuttner (1992; henceforth, FK) have documented the disappearance, starting from the 1980s, of any previously identified stable relationship between monetary aggregates, GDP, and interest rates. As the results based on the standard M_1 aggregate show, based on (log) velocity and (the log of) the short rate FK's results for M_1 [22] are still valid, with Johansen's test not rejecting the null of no cointegration, and Shin's test strongly rejecting the null of cointegration, and *based on either specification.*

Things are very different, however, based on Lucas and Nicolini's (2015) 'New M_1' aggregate. *(For the sake of simplicity, from now on, when we refer to the United States, 'M_1' will mean 'Lucas and Nicolini's New M_1 aggregate', whereas the traditional aggregate will be explicitly referred to as the 'standard M_1 aggregate'.)* In line with the visual evidence in the second panel of Figure 1 in the online appendix, both

---

statistical tests, and when the two collide, the former should prevail.

[21] To fix ideas for the less econometrically inclined readers: Under ideal circumstances, any statistical test should incorrectly reject the null hypothesis at the x per cent level x per cent of the times.

[22] FK considered several monetary and even credit aggregates.

Johansen and Shin tests in Table SELA.2*b* point towards the presence of cointegration between the two series, with *p*-values for the trace and maximum eigenvalue tests equal to 0.038 and 0.048, and the *p*-value for Shin's test equal to 0.121. These results provide statistical backing to Lucas and Nicolini's (2015) point that, once MMDAs are properly classified, on the basis of the economic function they perform, as part of M_1, the puzzle highlighted (e.g.) by FK of the lack of a stable demand for M_1 simply disappears.

Further, a comparison between the results based on the Selden-Latané specification and those based on the semi-log and log-log ones confirms the visual impression from Figure 1 in the online appendix, with the null of no cointegration not being rejected based on the log-log specification, and with the semi-log specification producing weak and inconclusive evidence (with the corresponding *p*-values being equal to 0.101 and 0.081). This suggests that, at least for the United States, the data would seem to prefer the Selden-Latané specification, as opposed to the semi-log and log-log which have dominated the literature on money demand. As we will see, this appears to be the case for several other low-inflation countries, notably the United Kingdom and Canada.

Other countries Moving to other countries, due to the sheer size of the overall set of results reported in the online appendix, in what follows we will mostly focus on the Selden-Latané specification, and on the log-log one with the 1% correction to the short rate, which the data seem to favor compared to the semi-log one.

Among the very high-inflation countries, evidence of cointegration is strong for Argentina, Brazil, Chile, Israel, and Mexico. Further, for Chile it is important to keep in mind that, as shown in Figure 2 in the online appendix, Chilean log velocity had exhibited, in the early 1970s, a dramatic temporary *fall* at a time when the short rate was spiking *upwards*, which distorts any attempt—based on cointegration, or otherwise—to detect a positive relationship between the two series. Although we do not have any solid, comprehensive explanation for this phenomenon, it is worth recalling that those years (the fall in velocity was especially marked between 1971 and 1974) had been characterized by the economic and political turmoil which culminated with Augusto Pinochet's military *coup d'etat* of September 1973. Although we have no hard proof of this, it is therefore highly likely that the fall in velocity of the early 1970s had been mostly unrelated to interest rates fluctuations, and it had been instead caused by the turmoil which was ravaging the country.

Among other countries, evidence of cointegration is strong for Australia, Canada, Korea, New Zealand, Norway, and Portugal (1914-1965) based on the Selden-Latané specification; and for Canada (1967-2012), New Zealand, Norway, Portugal, South Africa, and Switzerland based on the log-log specification with the 1% correction to the short rate (based on the semi-log specification, on the other hand, evidence of cointegration is strong only for Canada and Portugal (1914-1965)).

Let's now turn to the symmetric case in which both Johansen's and Shin's tests produce weak or non-existent evidence of cointegration.

E.2.2 Cases in which evidence of cointegration is weak, or non-existent

This is the case for Finland (1914-1985) and Japan under the Gold Standard based on either specification; for Portugal (1966-1998) based on either the Selden-Latané specification or the semi-log; and for West Germany based on the log-log.

The main issue which is worth exploring here is to which extent these results might reasonably be regarded as flukes due to a short sample period and/or a highly persistent cointegration residual. For Portugal the sample period is indeed very short, but the CCRs shown in Figures SELA.5 and SL.5 appear as hardly stationary. For Finland the CCR in Figures SELA.3 and LLCO.3 also looks hardly stationary, whereas the length of the sample period (72 years) cannot be invoked as an explanation for not having detected a long-run relationship between the series. Finally, for Japan evidence is mixed. On the one hand, the sample period is just 28 years long, and the CCRs produced by Johansen's procedure mostly appear as stationary, and are moderately persistent (see Tables SELA.1, ..., LLCO.1). On the other hand, the CCRs produced by Stock and Watson's estimator appear as all but stationary, and in fact the point estimate of $\hat{\rho}_{MUB}$ in Tables SELA.1, ..., LLCO.1 are borderline explosive.

Let's now turn to the case in which Johansen's and Shin's tests give conflicting signals, thus producing ambiguous results.

E.2.3 Cases in which evidence is ambiguous

An important point to keep in mind here is that, as discussed in Section 6 and Appendix C, Johansen's tests exhibit an overall better performance, and appear as more informative than Shin's.

Johansen: Cointegration, Shin: No cointegration Nowhere this is better illustrated than in the case of the United Kingdom: Whereas the visual evidence in Figure 3 in the online appendix points towards a strong relationship between velocity and the short rate, and Johansen's tests detect cointegration based on the Selden-Latané specification at a very high confidence level (with both p-values smaller than 0.02), the p-value for Shin's test is equal to 0.046, thus strongly rejecting the null of cointegration. (Qualitatively similar evidence is produced by the semi-log specification, whereas the results based on the log-log specification are uniformly weak.) This result is most likely a fluke: Although the sample period is quite long (91 years), the estimates of the persistence of the CCRs reported in Table SELA.1 are quite high (at 0.64 and 0.69). The results from Monte Carlo simulations reported in Table 2 show that, with $T = 100$ and $\rho = 0.75$, Shin's test incorrectly rejects cointegration 17.2 per cent of the times. This position is reinforced by the strong look of stationarity exhibited by the CCRs in Figure 15. Taking into account of the fact that Shin's tests are less informative than Johansen's, it can be reasonably concluded that, overall, evidence does indeed point towards cointegration.

The same holds—to an even greater extent—for Switzerland for the period 1948-2005: Based on *either specification*, Johansen's tests detect very strong evidence of cointegration, whereas Shin's tests consistently reject the null. In this case, too, cointegration residuals uniformly look stationary, and they are moderately persistent. Taking into account that the sample period, at 58 years, is not especially long, it is fair to conclude that evidence points quite strongly towards cointegration.

Turning to Norway, on the other hand, although the statistical evidence is qualitatively in line with that for the U.K., here we want to *downplay* it, and to argue that some skepticism is instead in order. The main reason for this is that the CCR appears as very highly persistent and possibly non-stationary, which is conceptually related to the visual evidence in the last panel of Figure 4 in the online appendix.

Johansen: No cointegration, Shin: Cointegration Turning to the opposite case, in Section 7.2 in the main text we already discussed the case of Turkey. As we argued there, a possible explanation for the failure, on the part of Johansen's procedure, to detect evidence of cointegration based on either the Selden-Latané or the semi-log specifications is the high persistence of the CCR, coupled with the comparatively short sample length. The same argument holds for Colombia, Germany (1876-1913), Japan (1955-2013), Italy, the Netherlands, Spain, and Switzerland (1851-1906) based on the Selden-Latané specification; and for Australia, Canada (1934-2006), Colombia, Germany (1876-1913), Japan (1955-2013), Korea, the Netherlands, Spain, and Taiwan based on the log-log specification with the 1% correction to the short rate. In most of these cases sample periods are quite short, and estimates of the persistence of the CCRs in Tables SELA.1 and LLCO.1 are moderate-to-high, thus suggesting that failure to detect cointegration based on Johansen's tests may simply originate from the problem discussed by Engle and Granger (1987), which we discussed in Sections 6 and 7 in the main text, and in Appendix C.

Let's now turn to the evidence produced by Johansen's procedure applied to unrestricted specifications for M_1, GDP, and the short rate.

E.3 Unrestricted tests of the null of no cointegration

Tables SL.4, LL.4, and LLCO.4 report results from Johansen's tests of the null hypothesis of no cointegration based on unrestricted specifications for the logarithms of GDP and M_1, and (the logarithm of) the short rate.

Based on the semi-log specification, at the ten per cent level cointegration is detected based on both the trace and the maximum eigenvalue tests for Argentina, Bolivia, Brazil (1934-2012), Chile, Israel, Mexico, the Netherlands, Norway, Switzerland (1948-2005), Taiwan, and the United Kingdom, whereas the two tests produce opposite results for Australia, Brazil (1974-2012), Canada, Germany (1876-2013), Japan

(1955-2013), Korea, Portugal (1914-1965), South Africa, and Spain. Only for the remaining few cases do Johansen tests clearly not reject the null of no cointegration, although, as previously discussed, in a number of instances a plausible explanation is the short sample period and/or the persistence of the cointegration residual.

Based on the log-log specification with the 1% correction to the short rate, cointegration is detected based on both the trace and the maximum eigenvalue tests for Argentina, Brazil (1974-2012), Canada (1967-2013), Japan (1955-2013), Korea, Israel, Mexico, the Netherlands, Norway, Portugal (1914-1965), and Switzerland (1948-2005), whereas the two tests produce opposite results for Bolivia, Germany (1876-2013), New Zealand, and South Africa.

E.3.1 Is the income elasticity of money demand equal to 1?

Tables SL.6, LL.6, and LLCO.6. report bootstrapped p-values for testing the null hypothesis that the income elasticity of money demand be equal to one, based on unrestricted specifications for the logarithms of GDP and M_1, and (the logarithm of) the short rate. Overall, results are mixed. Based on the semi-log specification, a unitary income elasticity is rejected in 13 cases out of 31 (i.e., 43.3 per cent of the times) based on Johansen's estimator of the cointegration vector, and in 21 cases (70.0 per cent of the times) based on Stock and Watson's estimator. The corresponding figures based on the log-log specification with the 1% correction on the short rate are 11 cases (36.7 per cent of the times) based on Johansen's estimator, and 19 cases (63.3 per cent of the times) based on Stock and Watson's.

E.4 Two cases in which the short rate is stationary

Finally, for two cases—Canada and Spain under the Gold Standard—it is not possible to find any evidence pointing towards cointegration. Since in either case the short rate is I(0)—see Table C.1b—cointegration tests can only be applied to the bivariate system comprising the logarithms of nominal GDP and nominal M_1. For Canada Johansen's trace statistic is equal to 10.932 with a bootstrapped p-value equal to 0.398, whereas the maximum eigenvalue statistic is equal to 8.647, with a bootstrapped p-value of 0.484. For Spain, the corresponding figures are the following (bootstrapped p-values in parentheses): for the trace test, 5.192 (0.895), and for the maximum eigenvalue test, 4.214 (0.914). As for Shin's tests, they are equal to 0.218 (0.074) for Canada, and to 0.486 (0.049) for Spain.

Table A.1*a* Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a] for very high inflation countries

	Logarithm of:													
	nominal GDP		nominal M_1		M_1 velocity		short rate		short rate+1		M_1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Argentina, 1914-2009	0.227	0.903	0.201	0.847	0.686	0.746	0.316	0.332	0.302	0.314	0.562	0.724	0.012	0.009
Bolivia, 1980-2013	0.090	0.067	0.114	0.062	0.915	0.849	0.866	0.837	0.834	0.803	0.627	0.674	0.139	0.188
Brazil														
1974-2012	0.935	0.839	0.955	0.913	0.239	0.225	0.627	0.711	0.613	0.696	0.300	0.409	0.084	0.324
1934-2012	0.294	0.589	0.256	0.651	0.735	0.779	0.377[b]	0.357[b]	0.378	0.333	0.567	0.718	0.009[b]	0.053[b]
Chile														
1940-1995	0.399	0.544	0.374	0.261	0.134	0.050	0.341	0.263	0.341	0.248	0.212	0.124	0.133	0.090
1941-2012	0.907	0.865	0.901	0.596	0.198	0.100	0.413[b]	0.459[b]	0.345	0.388	0.290	0.231	0.050[b]	0.021[b]
Israel, 1983-2013	0.000	0.163	0.002	0.000	0.542	0.007	0.181	0.010	0.159	0.005	0.231	0.000	0.114	0.000
Mexico, 1985-2014	0.013	0.021	0.066	0.016	0.767	0.100	0.629	0.289	0.603	0.238	0.679	0.027	0.346	0.023

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M_1, and with an intercept and no time trend for the other series.

[b] For this period we consider inflation, rather than the short rate.

Table A.1*b* Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Logarithm of:													
	nominal GDP		nominal M_1		M_1 velocity		short rate		short rate+1		M_1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Australia, 1969-2014	0.115	0.591	0.987	0.989	0.817	0.883	0.746	0.890	0.733	0.885	0.805	0.872	0.537	0.775
Canada														
1872-1913	0.618	0.549	0.571	0.758	0.564	0.491	0.019	0.010	0.020	0.038	0.532	0.472	0.000	0.000
1934-2006	0.891	0.927	0.407	0.355	0.712	0.749	0.772	0.775	0.734	0.745	0.730	0.760	0.455	0.527
1967-2013	0.137	0.446	0.021	0.043	0.906	0.929	0.733	0.767	0.777	0.804	0.781	0.761	0.594	0.659
Colombia, 1959-2011	0.990	0.838	0.995	0.993	0.719	0.794	0.440	0.473	0.449	0.468	0.671	0.741	0.416	0.268
Finland														
1867-1913	0.790	0.840	0.071	0.364	0.008	0.039	0.094	0.043	0.095	0.042	0.004	0.038	0.074	0.042
1914-1985	0.277	0.099	0.056	0.078	0.765	0.574	0.538	0.513	0.373	0.519	0.904	0.886	0.504	0.520
France, 1852-1913	0.001	0.001	0.896	0.891	0.642	0.803	0.051	0.037	0.047	0.041	0.522	0.743	0.027	0.040
Germany, 1876-1913	0.111	0.896	0.008	0.433	0.021	0.185	0.126	0.236	0.125	0.228	0.043	0.230	0.144	0.257
Guatemala, 1980-2012	0.959	0.967	0.976	0.987	0.726	0.660	0.630	0.584	0.631	0.576	0.675	0.582	0.597	0.580
Hong Kong, 1985-2012	0.052	0.263	0.517	0.731	0.938	0.958	0.754	0.716	0.662	0.546	0.812	0.888	0.505	0.464
Japan														
1885-1913	0.637	0.817	0.452	0.596	0.761	0.864	0.034	0.230	0.034	0.235	0.792	0.874	0.035	0.251
1955-2013	0.217	0.716	0.131	0.438	0.946	0.928	0.726	0.752	0.757	0.766	0.791	0.770	0.598	0.571
Korea, 1970-2014	0.107	0.322	0.182	0.548	0.567	0.539	0.546	0.654	0.424	0.565	0.387	0.317	0.084	0.301
Italy														
1861-1913	0.955	0.995	0.116	0.723	0.016	0.007	0.767	0.802	0.766	0.798	0.006	0.005	0.756	0.780
1949-1996	0.794	0.889	0.993	0.945	0.333	0.648	0.857	0.899	0.861	0.897	0.234	0.643	0.805	0.848

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M_1, and with an intercept and no time trend for the other series.

Table A.1*b* (continued) Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Logarithm of:													
	nominal GDP		nominal M_1		M_1 velocity		short rate		short rate+1		M_1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Morocco, 1985-2008	0.298	0.255	0.797	0.869	0.904	0.761	0.859	0.747	0.867	0.752	0.434	0.267	0.896	0.755
Netherlands, 1950-1992	0.985	0.996	0.703	0.783	0.100	0.194	0.194	0.450	0.211	0.438	0.232	0.297	0.243	0.347
New Zealand, 1934-2004	0.933	0.930	0.613	0.630	0.860	0.888	0.627	0.627	0.609	0.602	0.867	0.832	0.373	0.362
Norway, 1946-2013	0.969	0.990	0.099	0.118	0.883	0.868	0.511	0.545	0.538	0.575	0.826	0.820	0.601	0.605
Portugal														
1891-1913	0.621	0.764	0.255	0.941	0.015	0.503	0.392	0.336	0.384	0.321	0.010	0.492	0.503	0.330
1914-1998	0.634	0.614	0.209	0.145	0.594	0.407	0.716	0.714	0.704	0.693	0.607	0.430	0.596	0.469
South Africa, 1967-2014	0.985	0.987	0.875	0.863	0.884	0.919	0.367	0.457	0.369	0.464	0.853	0.887	0.316	0.332
Spain														
1874-1913	0.953	0.951	0.462	0.318	0.601	0.569	0.056	0.019	0.061	0.020	0.649	0.598	0.059	0.020
1941-1989	0.632	0.504	0.154	0.505	0.187	0.440	0.828	0.878	0.822	0.871	0.363	0.512	0.589	0.720
Switzerland														
1851-1906	0.152	0.497	0.851	0.863	0.838	0.560	0.069	0.103	0.023	0.067	0.796	0.433	0.062	0.104
1948-2005	0.949	0.930	0.498	0.712	0.425	0.359	0.156	0.177	0.242	0.165	0.453	0.417	0.186	0.120
Taiwan, 1962-2013	0.502	0.844	0.216	0.645	0.264	0.229	0.609	0.671	0.606	0.667	0.053	0.033	0.427	0.524
Turkey, 1968-2014	0.869	0.826	0.412	0.639	0.839	0.766	0.643	0.666	0.662	0.683	0.776	0.753	0.735	0.764
United Kingdom, 1922-2014	0.076	0.679	0.082	0.571	0.925	0.842	0.814	0.851	0.707	0.816	0.837	0.768	0.333	0.572
United States, 1915-2014	0.642	0.315	0.609^b	0.380^b	0.657^b	0.572^b	0.639	0.598	0.569	0.498	0.737^b	0.551^b	0.296	0.317
Venezuela, 1962-1999	0.521	0.752	0.738	0.817	0.574	0.729	0.744	0.730	0.749	0.721	0.543	0.786	0.691	0.706
West Germany, 1960-1989	0.844	0.963	0.662	0.840	0.752	0.739	0.067	0.137	0.068	0.142	0.721	0.719	0.069	0.138

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M_1, and with an intercept and no time trend for the other series. [b] Based on Lucas and Nicolini's 'New M_1' aggregate. p-values based on the 'standard' M_1 series are 0.499 and 0.314 for log M_1, 0.811 and 0.889 for log M_1 velocity, and 0.597 and 0.707 for M_1 velocity.

Table A.2*a* Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a] for very high inflation countries

	Log-difference of:										*First-difference of:*			
	nominal GDP		nominal M_1		M_1 velocity		short rate		short rate+1		M_1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Argentina, 1914-2004	0.038	0.050	0.023	0.035	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Bolivia, 1980-2013	0.125	0.157	0.135	0.150	0.044	0.085	0.007	0.032	0.012	0.037	0.019	0.051	0.017	0.054
Brazil														
1974-2012	0.410	0.530	0.266	0.495	0.001	0.012	0.004	0.005	0.003	0.005	0.001	0.013	0.000	0.001
1934-2012	0.132	0.207	0.054	0.176	0.000	0.001	0.000[b]	0.000[b]	0.000	0.000	0.000	0.000	0.000[b]	0.000[b]
Chile														
1940-1995	0.153	0.079	0.361	0.317	0.000	0.002	0.001	0.001	0.002	0.001	0.001	0.004	0.005	0.002
1941-2012	0.138	0.056	0.272	0.277	0.000	0.000	0.000[b]	0.002[b]	0.000	0.000	0.000	0.000	0.000[b]	0.000[b]
Israel, 1983-2013	0.010	0.000	0.061	0.000	0.003	0.020	0.004	0.023	0.005	0.023	0.009	0.018	0.014	0.054
Mexico, 1985-2014	0.239	0.002	0.100	0.129	0.009	0.029	0.009	0.036	0.007	0.033	0.013	0.019	0.006	0.009

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M_1, and with an intercept and no time trend for the other series.

[b] For this period we consider inflation, rather than the short rate.

Table A.2*b* Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Log-difference of:										*First-difference of:*			
	nominal GDP		nominal M_1		M_1 velocity		short rate		short rate+1		M_1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Australia, 1969-2014	0.268	0.482	0.002	0.027	0.003	0.050	0.000	0.002	0.002	0.002	0.005	0.036	0.000	0.002
Canada														
1872-1913	0.031	0.043	0.011	0.042	0.002	0.003	0.000	0.000	0.000	0.001	0.001	0.004	0.000	0.000
1934-2006	0.000	0.005	0.010	0.028	0.003	0.023	0.000	0.003	0.000	0.002	0.003	0.032	0.000	0.000
1967-2013	0.173	0.400	0.011	0.008	0.026	0.039	0.000	0.000	0.000	0.000	0.031	0.028	0.000	0.001
Colombia, 1959-2011	0.479	0.711	0.021	0.276	0.001	0.008	0.001	0.005	0.001	0.004	0.001	0.005	0.005	0.012
Finland														
1867-1913	0.003	0.016	0.001	0.032	0.000	0.013	0.006	0.028	0.006	0.027	0.001	0.025	0.006	0.027
1914-1985	0.012	0.049	0.010	0.006	0.001	0.001	0.000	0.003	0.005	0.006	0.000	0.003	0.000	0.002
Germany, 1876-1913	0.132	0.148	0.003	0.071	0.001	0.043	0.000	0.013	0.002	0.001	0.002	0.051	0.001	0.015
Guatemala, 1980-2012	0.070	0.129	0.013	0.041	0.007	0.032	0.003	0.060	0.003	0.058	0.011	0.032	0.002	0.099
Hong Kong, 1985-2012	0.340	0.375	0.044	0.158	0.082	0.237	0.012	0.024	0.016	0.014	0.084	0.197	0.009	0.013
Japan														
1885-1913	0.012	0.039	0.015	0.025	0.002	0.014	0.003	0.025	0.002	0.001	0.003	0.020	0.002	0.026
1955-2013	0.513	0.748	0.164	0.389	0.012	0.064	0.000	0.003	0.000	0.000	0.005	0.015	0.000	0.000
Korea, 1970-2014	0.696	0.755	0.101	0.296	0.001	0.009	0.001	0.001	0.001	0.001	0.001	0.002	0.004	0.001
Italy														
1861-1913	0.000	0.001	0.001	0.007	0.000	0.010	0.001	0.001	0.000	0.000	0.000	0.020	0.001	0.001
1949-1996	0.205	0.565	0.152	0.394	0.002	0.031	0.002	0.120	0.001	0.104	0.001	0.057	0.000	0.031

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M_1, and with an intercept and no time trend for the other series.

Table A.2*b* (continued) Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Log-difference of:										*First-difference of:*			
	nominal GDP		nominal M_1		M_1 velocity		short rate		short rate+1		M_1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Morocco, 1985-2008	0.016	0.389	0.141	0.336	0.056	0.455	0.164	0.431	0.150	0.418	0.015	0.192	0.096	0.316
Netherlands, 1950-1992	0.068	0.437	0.007	0.099	0.001	0.057	0.000	0.000	0.000	0.000	0.001	0.042	0.000	0.000
New Zealand, 1934-2004	0.001	0.026	0.002	0.041	0.000	0.014	0.000	0.000	0.000	0.000	0.000	0.017	0.000	0.000
Norway, 1946-2013	0.001	0.019	0.004	0.040	0.002	0.026	0.000	0.000	0.000	0.000	0.001	0.027	0.000	0.000
Portugal														
1891-1913	0.030	0.100	0.000	0.368	0.000	0.172	0.092	0.235	0.000	0.000	0.000	0.196	0.100	0.238
1914-1998	0.026	0.039	0.006	0.010	0.000	0.000	0.003	0.089	0.003	0.075	0.000	0.000	0.001	0.003
South Africa, 1967-2014	0.027	0.079	0.002	0.021	0.001	0.013	0.000	0.002	0.000	0.003	0.002	0.015	0.000	0.000
Spain														
1874-1913	0.008	0.097	0.007	0.037	0.012	0.053	0.004	0.004	0.003	0.003	0.013	0.058	0.003	0.005
1941-1989	0.001	0.006	0.011	0.011	0.011	0.051	0.000	0.000	0.000	0.000	0.027	0.095	0.000	0.002
Switzerland														
1851-1906	0.001	0.018	0.002	0.049	0.001	0.005	0.000	0.000	0.001	0.002	0.000	0.001	0.000	0.000
1948-2005	0.028	0.087	0.000	0.005	0.000	0.009	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.001
Taiwan, 1962-2013	0.222	0.556	0.020	0.032	0.003	0.003	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000
Turkey, 1968-2014	0.494	0.598	0.297	0.518	0.008	0.081	0.006	0.114	0.006	0.116	0.003	0.029	0.002	0.060
United Kingdom, 1922-2014	0.006	0.052	0.017	0.053	0.007	0.024	0.000	0.000	0.000	0.000	0.002	0.032	0.000	0.000
United States, 1915-2014	0.000	0.000	0.000[b]	0.000[b]	0.000[b]	0.000[b]	0.000	0.000	0.000	0.000	0.000[b]	0.001[b]	0.000	0.000
Venezuela, 1962-1999	0.171	0.305	0.035	0.344	0.001	0.051	0.031	0.037	0.037	0.04	0.000	0.064	0.061	0.039
West Germany, 1960-1989	0.106	0.243	0.011	0.175	0.007	0.090	0.007	0.077	0.006	0.072	0.005	0.114	0.005	0.054

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M_1, and with an intercept and no time trend for the other series. [b] Based on Lucas and Nicolini's 'New M_1' aggregate. p-values based on the 'standard' M_1 series are 0.002 and 0.001 for log M_1, 0.000 and 0.002 for log M_1 velocity, and 0.000 and 0.002 for M_1 velocity.

Table B.1 Simulated fractiles of the distribution in very large samples of Shin's (1994) tests of the null hypothesis of cointegration,[a] based on alternative kernels, and for alternative values of the persistence of the cointegration residual

Fractile	Asymptotic critical values from Shin's (1994) Table 1	Persistence of the cointegration residual:											
		$\rho = 0$[b]				$\rho = 0.25$[b]				$\rho = 0.5$[b]			
		BAR	PAR	TH	QS	BAR	PAR	TH	QS	BAR	PAR	TH	QS
0.01	0.017	0.018	0.018	0.018	0.018	0.027	0.026	0.026	0.022	0.047	0.045	0.044	0.034
0.025	0.021	0.021	0.021	0.021	0.021	0.032	0.031	0.031	0.026	0.055	0.053	0.052	0.040
0.05	0.024	0.024	0.024	0.024	0.024	0.037	0.037	0.036	0.031	0.065	0.062	0.061	0.047
0.1	0.029	0.029	0.029	0.029	0.029	0.046	0.044	0.044	0.037	0.078	0.075	0.074	0.057
0.2	0.037	0.037	0.037	0.037	0.037	0.058	0.057	0.056	0.048	0.100	0.096	0.094	0.072
0.3	0.045	0.045	0.045	0.045	0.045	0.070	0.068	0.067	0.057	0.110	0.115	0.113	0.087
0.4	0.053	0.053	0.053	0.053	0.053	0.082	0.080	0.079	0.067	0.142	0.136	0.134	0.103
0.5	0.063	0.063	0.063	0.063	0.063	0.100	0.095	0.094	0.080	0.168	0.162	0.158	0.122
0.6	0.074	0.074	0.075	0.075	0.075	0.116	0.113	0.112	0.095	0.199	0.191	0.188	0.144
0.7	0.090	0.091	0.091	0.091	0.091	0.141	0.137	0.136	0.115	0.242	0.233	0.228	0.175
0.8	0.115	0.115	0.115	0.115	0.115	0.178	0.174	0.172	0.146	0.306	0.294	0.288	0.222
0.9	0.163	0.161	0.161	0.161	0.161	0.249	0.243	0.240	0.205	0.429	0.412	0.404	0.311
0.95	0.221	0.215	0.215	0.215	0.214	0.332	0.325	0.321	0.272	0.573	0.550	0.538	0.415
0.975	0.285	0.280	0.280	0.280	0.282	0.434	0.423	0.418	0.355	0.747	0.718	0.703	0.538
0.99	0.38	0.395	0.395	0.396	0.390	0.611	0.597	0.592	0.501	1.048	1.006	0.987	0.765

[a] Tests are based on Shin's (1994) equation (2), page 93, i.e. for a model with an intercept, but no time trend. For details on the data generation process used in the Monte Carlo simulations, see Appendix B. [b] $K = [T^{1/3}]$; ℓ is selected *via* Andrews' (1991) 'plug-in' method. BAR = Bartlett kernel; PAR = Parzen kernel; TH = Tukey-Hanning kernel; QS = quadratic spectral kernel

Table B.1 (continued) Simulated fractiles of the distribution in very large samples of Shin's (1994) tests of the null hypothesis of cointegration,[a] based on alternative kernels, and for alternative values of the persistence of the cointegration residual

Fractile	Asymptotic critical values from Shin's (1994) Table 1	Persistence of the cointegration residual:											
		$\rho = 0.75^b$				$\rho = 0.9^b$				$\rho = 0.95^b$			
		BAR	PAR	TH	QS	BAR	PAR	TH	QS	BAR	PAR	TH	QS
0.01	0.017	0.103	0.098	0.095	0.067	0.259	0.251	0.242	0.164	0.504	0.496	0.478	0.318
0.025	0.021	0.121	0.116	0.112	0.080	0.307	0.296	0.286	0.193	0.594	0.585	0.560	0.371
0.05	0.024	0.142	0.136	0.132	0.094	0.357	0.346	0.334	0.227	0.695	0.682	0.654	0.436
0.1	0.029	0.171	0.163	0.159	0.113	0.429	0.415	0.401	0.272	0.833	0.821	0.789	0.526
0.2	0.037	0.218	0.208	0.202	0.144	0.548	0.530	0.512	0.348	1.063	1.045	1.004	0.672
0.3	0.045	0.263	0.251	0.244	0.174	0.663	0.641	0.620	0.422	1.282	1.262	1.214	0.815
0.4	0.053	0.310	0.296	0.289	0.205	0.779	0.754	0.728	0.497	1.504	1.482	1.426	0.960
0.5	0.063	0.369	0.352	0.343	0.244	0.927	0.898	0.867	0.591	1.783	1.758	1.691	1.142
0.6	0.074	0.436	0.417	0.405	0.289	1.093	1.058	1.022	0.700	2.104	2.073	1.994	1.349
0.7	0.090	0.531	0.508	0.493	0.352	1.331	1.289	1.246	0.852	2.565	2.528	2.432	1.648
0.8	0.115	0.668	0.639	0.621	0.444	1.661	1.613	1.558	1.066	3.211	3.166	3.048	2.070
0.9	0.163	0.940	0.898	0.873	0.622	2.335	2.263	2.191	1.505	4.471	4.408	4.244	2.886
0.95	0.221	1.252	1.198	1.166	0.831	3.139	3.032	2.929	2.021	5.983	5.893	5.676	3.862
0.975	0.285	1.620	1.559	1.520	1.087	4.111	3.987	3.850	2.625	7.873	7.745	7.499	5.069
0.99	0.38	2.288	2.190	2.127	1.517	5.761	5.568	5.360	3.673	11.064	10.931	10.498	7.096

[a] Tests are based on Shin's (1994) equation (2), page 93, i.e. for a model with an intercept, but no time trend. For details on the data generation process used in the Monte Carlo simulations, see see Appendix B. [b] $K = [T^{1/3}]$; ℓ is selected *via* Andrews' (1991) 'plug-in' method. BAR = Bartlett kernel; PAR = Parzen kernel; TH = Tukey-Hanning kernel; QS = quadratic spectral kernel

Table D.1*a* Results from Johansen's cointegration tests between the logarithms of nominal GDP and of a short-term rate[a]

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegrating vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegrating vectors:*
Argentina, 1914-2004	38.246 (0.001)	37.348 (0.000)
Bolivia, 1980-2013	25.248 (0.032)	25.145 (0.019)
Brazil		
1974-2012	19.461 (0.142)	14.1757 (0.231)
1934-2012	20.974 (0.037)	18.993 (0.030)
Chile		
1940-1995	19.854 (0.062)	19.833 (0.033)
1941-2012	11.096 (0.380)	9.328 (0.416)
Israel, 1983-2013	11.678 (0.605)	9.845 (0.620)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table D.1*b* Results from Johansen's cointegration tests between the logarithm of nominal GDP and a short-term rate[a]

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*
Canada, 1934-2006	8.724 (0.504)	6.045 (0.693)
Colombia, 1959-2011	16.117 (0.213)	13.102 (0.253)
Finland, 1914-1985	6.299 (0.773)	4.663 (0.863)
Hong Kong, 1985-2012	20.509 (0.114)	18.521 (0.096)
Japan		
1885-1913	12.058 (0.318)	11.475 (0.268)
1955-2013	22.656 (0.048)	17.674 (0.069)
Korea, 1970-2014	43.262 (0.001)	27.120 (0.010)
Italy		
1861-1913	6.650 (0.737)	5.377 (0.781)
1949-1966	13.751 (0.253)	13.496 (0.182)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table D.1*b* (continued) Results from Johansen's cointegration tests between the logarithm of nominal GDP and a short-term rate[a]

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*
Morocco, 1985-2008	9.589 (0.673)	9.012 (0.614)
Netherlands, 1950-1992	21.231 (0.049)	17.593 (0.066)
Norway, 1946-2013	4.789 (0.881)	2.962 (0.970)
Portugal		
1914-1965	16.733 (0.132)	11.138 (0.324)
1966-1998	12.740 (0.482)	9.122 (0.638)
South Africa, 1967-2014	13.547 (0.236)	8.671 (0.493)
Spain, 1941-1989	14.726 (0.108)	14.566 (0.076)
Switzerland, 1851-1906	10.084 (0.358)	9.838 (0.286)
Taiwan, 1962-2013	33.729 (0.003)	26.519 (0.006)
Turkey, 1968-2014	16.251 (0.205)	10.935 (0.400)
United Kingdom, 1922-2014	8.134 (0.591)	7.593 (0.544)
United States, 1915-2014	3.885 (0.947)	3.749 (0.923)
West Germany, 1960-1989	11.381 (0.524)	7.730 (0.709)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table D.2 Results from Johansen's cointegration tests between the logarithm of the price level and either the ocean or the land 'global temperature anomaly'[a]

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegrating vectors:		*Maximum eigenvalue tests of 0 versus 1 cointegrating vectors:*	
	Ocean	Land	Ocean	Land
Argentina, 1914-1969	11.413 (0.333)	16.876 (0.348)	7.318 (0.592)	13.205 (0.474)
Bolivia, 1980-2011	24.997 (0.045)	23.472 (0.048)	21.688 (0.041)	20.729 (0.044)
Brazil, 1914-1991	23.214 (0.021)	32.901 (0.001)	15.037 (0.097)	21.095 (0.016)
Chile, 1940-2011	14.256 (0.188)	13.945 (0.192)	12.297 (0.195)	11.524 (0.234)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

References

Alvarez, F., and F. Lippi (2009): “Financial Innovation and the Transaction Demand for Cash,” *Econometrica*, 77(2), 363–402.

Andrews, D. K. (1991): “Heteroskedasticity and Autocorrelation-Consistent Covariance Matrix Estimation,” *Econometrica*, 59(3), 817–858.

Benati, L. (2015): “The Long-Run Phillips Curve: A Structural VAR Investigation,” *Journal of Monetary Economics*, 76(November), 15–28.

Cavaliere, G., A. Rahbek, and R. Taylor (2012): “Bootstrap Determination of the Co-Integration Rank in Vector Autoregressive Models,” *Econometrica*, 80(4), 1721–1740.

Cavaliere, G., R. Taylor, and C. Trenkler (2015): “Bootstrap Co-Integration Rank Testing: The Effect of Bias-Correcting Parameter Estimates,” *Oxford Bulletin of Economics and Statistics*, forthcoming, –.

Engle, R., and C. Granger (1987): “Cointegration and Error Correction: Representation, Estimation, and Testing,” *Econometrica*, 55, 251–276.

Faust, J., and E. Leeper (1997): “When Do Long-Run Identifying Restrictions Give Reliable Results?,” *Journal of Business and Economic Statistics*, 15(3), 345–353.

Hamilton, J. (1994): *Time Series Analysis.* Princeton, NJ, Princeton University Press.

Johansen, S. (2002): “A Small Sample Correction of the Test for Cointegrating Rank in the Vector Autoregressive Model,” *Econometrica*, 70, 1929–1961.

Kilian, L. (1998): “Small-Sample Confidence Intervals for Impulse-Response Functions,” *Review of Economics and Statistics*, pp. 218–230.

Laubach, T., and J. Williams (2003): “Measuring the Natural Rate of Interest,” *The Review of Economics and Statistics*, 85(4), 1063–1070.

LucasJr., R. E. (1988): “Money Demand in the United States: A Quantitative Review,” *Carnegie-Rochester Conference Series on Public Policy*, 29, 137–168.

LucasJr., R. E., and J.-P. Nicolini (2015): “On the Stability of Money Demand,” *Journal of Monetary Economics*, forthcoming.

Luetkepohl, H. (1991): *Introduction to Multiple Time Series Analysis, II edition.* Springer-Verlag.

Meltzer, A. H. (1963): "The Demand for Money: The Evidence from the Time Series," *Journal of Political Economy*, 71(3), 219–246.

Ohanian, L., and A. Raffo (2011): "Aggregate Hours Worked in OECD Countries: New Measurement and Implications for Business Cycles," *Journal of Monetary Economics*, 59(1), 40–56.

Shin, Y. (1994): "A Residual-Based Test of the Null of Cointegration against the Alternative of No Cointegration," *Econometric Theory*, 10(1 (Mar., 1994)), 91–115.

Swensen, A. (2006): "Bootstrap Algorithms for Testing and Determining the Cointegration Rank in VAR Models," *Econometrica*, 74, 1699–1714.

Tables for the online appendix

I: Full set of results based on the Selden-Latané specification

Table SELA.1 Assessing the persistence of candidate cointegration residuals: Hansen (1999) 'grid bootstrap' estimates of the sum of the autoregressive coefficients based on AR(2) models (median, and 90 per cent bootstrapped confidence interval)[a]

I: Based on Johansen's estimator of the cointegration vector

Australia, 1969-2014	0.30 [0.01; 0.59]	Mexico, 1985-2014	0.46 [0.26; 0.68]
Bolivia, 1980-2013	0.44 [0.17; 0.76]	Netherlands, 1950-1992	0.61 [0.37; 0.89]
Brazil, 1974-2012	0.59 [0.36; 0.84]	New Zealand, 1934-2004	0.75 [0.62; 0.89]
Canada		Norway, 1946-2013	0.97 [0.89; 1.02]
1934-2006	0.77 [0.63; 0.92]	Portugal	
1967-2013	0.33 [0.13; 0.54]	*1914-1965*	0.67 [0.48; 0.90]
Chile, 1940-1995	0.87 [0.72; 1.02]	*1966-1998*	1.00 [0.90; 1.02]
Colombia, 1959-2011	0.85 [0.68; 1.02]	South Africa, 1967-2014	0.86 [0.73; 1.01]
Finland, 1914-1985	0.93 [0.83; 1.01]	Spain, 1941-1989	0.59 [0.39; 0.80]
Germany, 1876-1913	0.59 [0.32; 0.95]	Switzerland, 1948-2005	0.74 [0.56; 0.95]
Guatemala, 1980-2012	0.63 [0.34; 1.02]	Turkey, 1968-2014	0.92 [0.75; 1.03]
Japan		United Kingdom, 1922-2014	0.64 [0.49; 0.81]
1885-1913	0.45 [0.08; 0.89]	United States, 1915-2014	
1955-2013	0.81 [0.68; 0.97]	*standard* M_1	0.92 [0.84; 1.01]
Korea, 1970-2014	0.56 [0.32; 0.81]	*Lucas-Nicolini 'New* M_1*'*	0.61 [0.47; 0.75]
Israel, 1983-2013	0.36 [0.32; 0.40]	Venezuela, 1962-1999	0.91 [0.74; 1.03]
Italy, 1949-1996	0.98 [0.80; 1.03]		

[a] Based on 2,000 bootstrap replications for each value of ρ in the grid. Candidate cointegration residuals have been computed based on the bivariate model for velocity and the short rate, and Johansen's estimator.

Table SELA.1 (continued) Assessing the persistence of candidate cointegration residuals: Hansen (1999) 'grid bootstrap' estimates of the sum of the autoregressive coefficients based on AR(2) models (median, and 90 per cent bootstrapped confidence interval)[a]

II: Based on Stock and Watson's estimator of the cointegration vector			
Australia, 1969-2014	0.31 [0.03; 0.61]	Mexico, 1985-2014	0.52 [0.31; 0.72]
Bolivia, 1980-2013	0.57 [0.28; 0.95]	Netherlands, 1950-1992	0.71 [0.48; 1.01]
Brazil, 1974-2012	0.79 [0.62; 1.01]	New Zealand, 1934-2004	0.81 [0.68; 0.95]
Canada		Norway, 1946-2013	1.00 [0.96; 1.02]
1934-2006	0.81 [0.69; 0.96]	Portugal	
1967-2013	0.34 [0.13; 0.56]	*1914-1965*	0.67 [0.47; 0.91]
Chile, 1940-1995	0.85 [0.71; 1.02]	*1966-1998*	1.02 [0.99; 1.10]
Colombia, 1959-2011	0.87 [0.70; 1.02]	South Africa, 1967-2014	1.01 [0.96; 1.03]
Finland, 1914-1985	0.97 [0.91; 1.01]	Spain, 1941-1989	0.61 [0.41; 0.82]
Germany, 1876-1913	0.99 [0.87; 1.03]	Turkey, 1968-2014	0.94 [0.76; 1.03]
Guatemala, 1980-2012	0.65 [0.34; 1.02]	Switzerland, 1948-2005	0.84 [0.67; 1.02]
Japan		United Kingdom, 1922-2014	0.69 [0.54; 0.84]
1885-1913	1.01 [0.87; 1.06]	United States, 1915-2014	
1955-2013	0.87 [0.74; 1.01]	*standard* M_1	1.00 [0.96; 1.02]
Korea, 1970-2014	0.57 [0.34; 0.82]	*Lucas-Nicolini 'New* M_1*'*	0.64 [0.51; 0.79]
Israel, 1983-2013	0.35 [0.32; 0.39]	Venezuela, 1962-1999	0.89 [0.69; 1.03]
Italy, 1949-1996	0.98 [0.85; 1.03]		

[a] Based on 2,000 bootstrap replications for each value of ρ in the grid. Candidate cointegration residuals have been computed based on the bivariate model for velocity and the short rate, and Johansen's estimator.

Table SELA.2a Results from cointegration tests between M_1 velocity and a short-term rate[a] for very high inflation countries

	I: Johansen's tests of the null of no cointegration		II: Shin's tests of the null of cointegration
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegrating vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegrating vectors:*	
Bolivia, 1980-2013	19.339 (0.089)	18.519 (0.053)	0.090 (0.976)
Brazil, 1974-2012	30.987 (0.005)	25.024 (0.008)	0.640 (0.018)
Chile			
1940-1995	24.191 (0.024)	14.026 (0.133)	0.696 (0.024)
1941-2012	23.304 (0.020)	18.084 (0.035)	0.411 (0.307)
Israel, 1983-2013	154.166 (0.000)	154.098 (0.000)	0.137 (0.282)
Mexico, 1985-2014	47.085 (3.0e-4)	29.609 (0.007)	0.110 (0.312)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SELA.2b Results from cointegration tests between M_1 velocity and a short-term rate[a]

	I: Johansen's tests of the null of no cointegration		II: Shin's tests of the null of cointegration
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*	
Australia, 1969-2014	16.903 (0.116)	15.890 (0.063)	0.278 (0.227)
Canada			
1934-2006	23.157 (0.017)	21.380 (0.009)	0.545 (0.039)
1967-2013	26.139 (0.016)	25.195 (0.007)	0.090 (0.558)
Colombia, 1959-2011	8.435 (0.673)	6.439 (0.717)	0.251 (0.433)
Finland, 1914-1985	6.825 (0.742)	6.765 (0.622)	1.391 (0.071)
Germany, 1876-1913	9.882 (0.571)	8.996 (0.503)	0.490 (0.197)
Guatemala, 1980-2012	20.282 (0.058)	18.014 (0.049)	0.053 (0.872)
Japan			
1885-1913	11.870 (0.408)	10.834 (0.333)	0.455 (0.094)
1955-2013	9.846 (0.511)	9.240 (0.427)	0.141 (0.888)
Korea, 1970-2014	18.407 (0.074)	16.909 (0.060)	0.175 (0.351)
Italy, 1949-1996	15.767 (0.145)	12.474 (0.171)	0.457 (0.230)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SELA.2b (continued) Results from cointegration tests between M_1 velocity and a short-term rate[a]

	I: Johansen's tests of the null of no cointegration		II: Shin's tests of the null of cointegration
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*	
Netherlands, 1950-1992	14.491 (0.211)	10.052 (0.349)	0.253 (0.381)
New Zealand, 1934-2004	16.696 (0.106)	15.823 (0.060)	0.647 (0.291)
Norway, 1946-2013	22.770 (0.021)	17.992 (0.031)	0.932 (0.084)
Portugal			
1914-1965	26.827 (0.012)	25.749 (0.004)	0.086 (0.495)
1966-1998	11.733 (0.422)	8.818 (0.511)	0.278 (0.004)
South Africa, 1967-2014	17.877 (0.117)	16.635 (0.068)	0.489 (0.109)
Spain, 1941-1989	14.260 (0.183)	13.569 (0.120)	0.272 (0.272)
Switzerland			
1851-1906	15.883 (0.109)	12.625 (0.158)	0.635 (0.225)
1948-2005	38.892 (0.000)	35.289 (0.000)	0.985 (0.033)
Turkey, 1968-2014	6.817 (0.814)	4.614 (0.896)	0.164 (0.523)
United Kingdom, 1922-2014	23.261 (0.019)	21.680 (0.011)	0.900 (0.046)
United States, 1915-2014			
based on the standard M_1 aggregate	7.152 (0.767)	4.822 (0.870)	3.507 (0.007)
based on Lucas and Nicolini's 'New M_1' aggregate	20.769 (0.038)	16.557 (0.048)	0.554 (0.121)
Venezuela, 1962-1999	7.635 (0.724)	5.836 (0.776)	0.412 (0.112)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SELA.3 Bootstrapped p-values[a] for testing the null hypothesis that the coefficient on the short rate is equal to -0.4, based on bivariate systems featuring velocity and the short rate

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.002	Israel, 1983-2013	0.007
Australia, 1969-2014	0.120	Italy, 1949-1996	0.021
Bolivia, 1980-2013	0.426	Mexico, 1985-2014	0.001
Brazil		Netherlands, 1950-1992	0.344
1974-2012	3.0e-4	New Zealand, 1934-2004	0.141
1934-2012	0.002	Norway, 1946-2013	0.084
Canada		Portugal	
1934-2006	0.001	*1914-1965*	0.006
1967-2013	0.005	*1966-1998*	0.028
Chile		South Africa, 1967-2014	0.323
1940-1995	1.0e-3	Spain, 1941-1989	0.010
1941-2012	0.014	Switzerland, 1948-2005	0.001
Colombia, 1959-2011	0.435	Taiwan, 1962-2013	0.214
Finland, 1914-1985	0.268	Turkey, 1968-2014	0.433
Germany, 1876-1913	0.136	United Kingdom, 1922-2014	0.101
Guatemala, 1980-2012	0.030	United States, 1915-2014	
Japan		*standard* M_1	0.331
1885-1913	0.445	Venezuela, 1962-1999	0.039
1955-2013	0.185	West Germany, 1960-1989	0.446
Korea, 1970-2014	0.011		

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SELA.3 (continued) Bootstrapped p-values[a] for testing the null hypothesis that the coefficient on the short rate is equal to -0.4, based on bivariate systems featuring velocity and the short rate

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.000	Israel, 1983-2013	0.002
Australia, 1969-2014	0.450	Italy, 1949-1996	0.000
Bolivia, 1980-2013	0.000	Mexico, 1985-2014	0.000
Brazil		Netherlands, 1950-1992	0.001
1974-2012	0.000	New Zealand, 1934-2004	0.048
1934-2012	0.000	Norway, 1946-2013	0.007
Canada		Portugal	
1934-2006	0.033	*1914-1965*	1.0e-4
1967-2013	0.019	*1966-1998*	0.000
Chile		South Africa, 1967-2014	3.0e-4
1940-1995	0.000	Spain, 1941-1989	0.000
1941-2012	0.000	Switzerland, 1948-2005	0.001
Colombia, 1959-2011	0.004	Taiwan, 1962-2013	0.071
Finland, 1914-1985	0.011	Turkey, 1968-2014	0.240
Germany, 1876-1913	0.008	United Kingdom, 1922-2014	0.351
Guatemala, 1980-2012	0.107	United States, 1915-2014	
Japan		*standard* M_1	0.347
1885-1913	0.017	Venezuela, 1962-1999	0.000
1955-2013	0.112	West Germany, 1960-1989	0.006
Korea, 1970-2014	0.047		

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

II: Full set of results based on the semi-log specification

Table SL.1 Assessing the persistence of candidate cointegration residuals: Hansen (1999) 'grid bootstrap' estimates of the sum of the autoregressive coefficients based on AR(2) models (median, and 90 per cent bootstrapped confidence interval)[a]

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.42 [0.29; 0.54]	Israel, 1983-2013	0.37 [0.33; 0.41]
Australia, 1969-2014	0.41 [0.13; 0.69]	Italy, 1949-1966	0.95 [0.78; 1.03]
Bolivia, 1980-2013	0.50 [0.23; 0.82]	Mexico, 1985-2014	0.58 [0.40; 0.79]
Brazil		Netherlands, 1950-1992	0.61 [0.39; 0.89]
1974-2012	0.64 [0.42; 0.91]	New Zealand, 1934-2004	0.78 [0.65; 0.90]
1934-2012	0.54 [0.36; 0.73]	Norway, 1946-2013	0.97 [0.89; 1.02]
Canada		Portugal	
1934-2006	0.75 [0.61; 0.90]	*1914-1965*	0.76 [0.61; 1.00]
1967-2013	0.75 [0.61; 0.90]	*1966-1998*	1.00 [0.91; 1.02]
Chile		South Africa, 1967-2014	0.86 [0.73; 1.01]
1940-1995	0.77 [0.63; 0.91]	Spain, 1941-1989	0.67 [0.47; 0.89]
1941-2012	0.74 [0.64; 0.85]	Switzerland, 1948-2005	0.74 [0.55; 0.94]
Colombia, 1959-2011	0.84 [0.69; 1.02]	Taiwan, 1962-2013	0.82 [0.70; 0.95]
Finland, 1914-1985	0.88 [0.77; 1.01]	Turkey, 1968-2014	0.86 [0.68; 1.17]
Germany, 1876-1913	0.61 [0.34; 0.91]	United Kingdom, 1922-2014	0.69 [0.54; 0.86]
Guatemala, 1980-2012	0.64 [0.32; 1.02]	United States, 1915-2014	
Japan		*standard* M_1	0.91 [0.83; 1.00]
1885-1913	0.45 [0.09; 0.89]	*Lucas-Nicolini 'New* M_1*'*	0.64 [0.51; 0.77]
1955-2013	0.86 [0.75; 1.01]	Venezuela, 1962-1999	0.93 [0.75; 1.03]
Korea, 1970-2014	0.60 [0.40; 0.82]	West Germany, 1960-1989	0.39 [0.07; 0.72]

[a] Based on 2,000 bootstrap replications for each value of ρ in the grid. Candidate cointegration residuals have been computed based on the bivariate model for log velocity and (the log of) the short rate, and Johansen's estimator.

Table SL.1 (continued) Assessing the persistence of candidate cointegration residuals: Hansen (1999) 'grid bootstrap' estimates of the sum of the autoregressive coefficients based on AR(2) models (median, and 90 per cent bootstrapped confidence interval)[a]

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.62 [0.50; 0.75]	Israel, 1983-2013	0.36 [0.32; 0.40]
Australia, 1969-2014	0.42 [0.15; 0.69]	Italy, 1949-1996	0.98 [0.85; 1.13]
Bolivia, 1980-2013	0.66 [0.37; 1.02]	Mexico, 1985-2014	0.58 [0.36; 0.82]
Brazil		Netherlands, 1950-1992	0.71 [0.50; 1.13]
1974-2012	0.84 [0.69; 1.02]	New Zealand, 1934-2004	0.83 [0.72; 0.97]
1934-2012	0.85 [0.73; 1.01]	Norway, 1946-2013	1.01 [0.97; 1.02]
Canada		Portugal	
1934-2006	0.79 [0.65; 0.94]	*1914-1965*	0.76 [0.59; 0.98]
1967-2013	0.44 [0.22; 0.67]	*1966-1998*	1.03 [0.99; 1.13]
Chile		South Africa, 1967-2014	1.01 [0.97; 1.04]
1940-1995	0.83 [0.68; 1.01]	Spain, 1941-1989	0.69 [0.50; 0.89]
1941-2012	0.84 [0.71; 1.01]	Switzerland, 1948-2005	0.82 [0.67; 1.01]
Colombia, 1959-2011	0.89 [0.72; 1.16]	Taiwan, 1962-2013	0.83 [0.73; 0.95]
Finland, 1914-1985	0.93 [0.83; 1.01]	Turkey, 1968-2014	0.88 [0.70; 1.03]
Germany, 1876-1913	0.99 [0.87; 1.03]	United Kingdom, 1922-2014	0.75 [0.61; 0.90]
Guatemala, 1980-2012	0.67 [0.35; 1.02]	United States, 1915-2014	
Japan		*standard* M_1	1.00 [0.94; 1.02]
1885-1913	1.01 [0.85; 1.05]	*Lucas-Nicolini 'New* M_1*'*	0.68 [0.55; 0.82]
1955-2013	0.91 [0.79; 1.01]	Venezuela, 1962-1999	0.92 [0.72; 1.03]
Korea, 1970-2014	0.63 [0.42; 0.86]	West Germany, 1960-1989	1.01 [0.82; 1.03]

[a] Based on 2,000 bootstrap replications for each value of ρ in the grid. Candidate cointegration residuals have been computed based on the bivariate model for log velocity and (the log of) the short rate, and Johansen's estimator.

Table SL.2a Results from cointegration tests between the logarithm of M_1 velocity and a short-term rate[a] for very high inflation countries

	I: Johansen's tests of the null of no cointegration		
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegrating vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegrating vectors:*	II: Shin's tests of the null of cointegration
Argentina, 1914-2009	26.061 (0.014)	24.401 (0.010)	1.145 (0.138)
Bolivia, 1980-2013	11.373 (0.526)	10.630 (0.423)	0.098 (0.984)
Brazil			
1974-2012	24.126 (0.026)	18.618 (0.042)	0.710 (0.015)
1934-2012	30.346 (0.005)	29.179 (0.004)	2.041 (0.012)
Chile			
1940-1995	24.654 (0.021)	16.839 (0.065)	0.712 (0.019)
1941-2012	17.068 (0.110)	13.001 (0.151)	0.386 (0.401)
Israel, 1983-2013	162.338 (0.000)	161.736 (0.000)	0.155 (0.231)
Mexico, 1985-2014	44.438 (4.0e-4)	35.513 (0.002)	0.110 (0.333)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SL.2b Results from cointegration tests between the logarithm of M_1 velocity and a short-term rate[a]

	I: Johansen's tests of the null of no cointegration		
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*	II: Shin's tests of the null of cointegration
Australia, 1969-2014	15.445 (0.168)	14.513 (0.099)	0.341 (0.188)
Canada, 1934-2006	19.801 (0.064)	17.675 (0.042)	0.550 (0.184)
Colombia, 1959-2011	8.275 (0.683)	6.601 (0.692)	0.275 (0.429)
Finland, 1914-1985	8.331 (0.642)	6.771 (0.659)	1.317 (0.029)
Germany, 1876-1913	9.839 (0.572)	8.794 (0.534)	0.498 (0.195)
Guatemala, 1980-2012	20.076 (0.055)	18.069 (0.043)	0.070 (0.779)
Japan			
1885-1913	11.681 (0.425)	10.425 (0.365)	0.462 (0.082)
1955-2013	12.868 (0.237)	12.710 (0.154)	0.172 (0.882)
Korea, 1970-2014	17.188 (0.106)	16.609 (0.070)	0.152 (0.491)
Italy, 1949-1996	15.265 (0.163)	12.130 (0.182)	0.441 (0.233)

[a] Bootstrapped *p*-values (in parentheses) are based on 10,000 bootstrap replications.

Table SL.2b (continued) Results from cointegration tests between the logarithm of M_1 velocity and a short-term rate[a]

	I: Johansen's tests of the null of no cointegration		
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*	II: Shin's tests of the null of cointegration
Netherlands, 1950-1992	15.988 (0.143)	10.743 (0.286)	0.257 (0.360)
New Zealand, 1934-2004	15.313 (0.160)	14.477 (0.091)	0.591 (0.345)
Norway, 1946-2013	23.540 (0.021)	19.644 (0.021)	1.102 (0.058)
Portugal			
1914-1965	20.171 (0.073)	19.072 (0.038)	0.117 (0.383)
1966-1998	9.385 (0.580)	6.676 (0.722)	0.267 (0.004)
South Africa, 1967-2014	18.114 (0.108)	17.183 (0.060)	0.535 (0.071)
Spain, 1941-1989	12.394 (0.279)	11.458 (0.215)	0.300 (0.274)
Switzerland			
1851-1906	13.641 (0.162)	13.071 (0.115)	0.630 (0.239)
1948-2005	35.641 (0.001)	32.258 (0.000)	1.048 (0.023)
Taiwan, 1962-2013	7.561 (0.690)	6.064 (0.742)	0.325 (0.222)
Turkey, 1968-2014	11.058 (0.450)	9.009 (0.444)	0.167 (0.508)
United Kingdom, 1922-2014	20.169 (0.051)	19.443 (0.021)	1.208 (0.022)
United States, 1915-2014			
based on the standard M_1 aggregate	7.214 (0.747)	5.637 (0.777)	3.658 (0.005)
based on Lucas and Nicolini's 'New M_1' aggregate	16.867 (0.101)	14.791 (0.081)	0.612 (0.116)
Venezuela, 1962-1999	6.917 (0.785)	5.128 (0.844)	0.399 (0.150)
West Germany, 1960-1989	7.118 (0.879)	6.316 (0.857)	0.409 (0.141)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SL.3 Bootstrapped p-values[a] for testing the null hypothesis that the semi-elasticity is equal to -0.1, based on bivariate systems featuring the short rate and the logarithm of velocity

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.005	Israel, 1983-2013	0.490
Australia, 1969-2014	0.058	Italy, 1949-1996	0.019
Bolivia, 1980-2013	0.114	Mexico, 1985-2014	0.002
Brazil		Netherlands, 1950-1992	0.270
1974-2012	1.0e-4	New Zealand, 1934-2004	0.336
1934-2012	0.003	Norway, 1946-2013	0.095
Canada		Portugal	
1934-2006	0.184	*1914-1965*	0.015
1967-2013	0.227	*1966-1998*	0.035
Chile		South Africa, 1967-2014	0.438
1940-1995	1.0e-4	Spain, 1941-1989	0.028
1941-2012	0.006	Switzerland, 1948-2005	8.0e-4
Colombia, 1959-2011	0.084	Taiwan, 1962-2013	0.167
Finland, 1914-1985	0.062	Turkey, 1968-2014	0.011
Germany, 1876-1913	0.180	United Kingdom, 1922-2014	0.090
Guatemala, 1980-2012	0.072	United States, 1915-2014	
Japan		*standard* M_1	0.320
1885-1913	0.408	*Lucas-Nicolini 'New* M_1*'*	0.146
1955-2013	0.019	Venezuela, 1962-1999	0.021
Korea, 1970-2014	0.409	West Germany, 1960-1989	0.447

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SL.3 (continued) Bootstrapped p-values[a] for testing the null hypothesis that the semi-elasticity is equal to -0.1, based on bivariate systems featuring the short rate and the logarithm of velocity

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.000	Israel, 1983-2013	0.000
Australia, 1969-2014	0.000	Italy, 1949-1996	0.000
Bolivia, 1980-2013	0.000	Mexico, 1985-2014	0.000
Brazil		Netherlands, 1950-1992	0.029
1974-2012	0.000	New Zealand, 1934-2004	0.001
1934-2012	0.000	Norway, 1946-2013	0.010
Canada		Portugal	
1934-2006	0.047	*1914-1965*	0.003
1967-2013	0.003	*1966-1998*	0.000
Chile		South Africa, 1967-2014	0.000
1940-1995	0.000	Spain, 1941-1989	0.000
1941-2012	0.000	Switzerland, 1948-2005	0.385
Colombia, 1959-2011	0.000	Taiwan, 1962-2013	0.004
Finland, 1914-1985	0.149	Turkey, 1968-2014	0.000
Germany, 1876-1913	0.000	United Kingdom, 1922-2014	0.267
Guatemala, 1980-2012	0.001	United States, 1915-2014	
Japan		*standard* M_1	0.115
1885-1913	0.040	*Lucas-Nicolini 'New* M_1*'*	0.109
1955-2013	0.340	Venezuela, 1962-1999	0.000
Korea, 1970-2014	0.123	West Germany, 1960-1989	0.001

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SL.4 Results from Johansen's tests of the null hypothesis of no cointegration based on unrestricted specifications: test statistics and bootstrapped p-values[a] (in parentheses)

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*
Argentina, 1914-2009	36.376 (0.056)	22.701 (0.098)
Australia, 1969-2014	35.978 (0.074)	16.929 (0.411)
Bolivia, 1980-2013	62.673 (0.004)	47.797 (0.002)
Brazil		
1934-2012	46.869 (0.012)	33.021 (0.015)
1974-2012	48.543 (0.031)	25.648 (0.140)
Canada		
1934-2006	32.906 (0.112)	27.505 (0.032)
1967-2013	43.774 (0.018)	21.241 (0.186)
Chile		
1940-1995	56.195 (0.002)	41.492 (0.002)
1941-2012	41.229 (0.026)	33.379 (0.009)
Colombia, 1959-2011	32.154 (0.281)	20.063 (0.326)
Finland, 1914-1985	22.329 (0.543)	15.485 (0.468)
Germany, 1876-1913	43.838 (0.060)	25.442 (0.153)
Japan		
1885-1913	28.025 (0.352)	15.520 (0.567)
1955-2013	39.105 (0.106)	27.673 (0.074)
Korea, 1970-2014	61.329 (0.004)	25.450 (0.153)
Israel, 1983-2013	178.839 (0.000)	161.626 (0.000)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SL.4 (continued) Results from Johansen's tests of the null hypothesis of no cointegration based on unrestricted specifications: test statistics and bootstrapped p-values[a] (in parentheses)

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*
Italy		
1861-1913	26.450 (0.278)	18.466 (0.246)
1949-1996	31.042 (0.181)	20.028 (0.215)
Netherlands, 1950-1992	50.330 (0.011)	32.681 (0.025)
Norway, 1946-2013	41.599 (0.012)	23.932 (0.056)
New Zealand, 1934-2004	23.785 (0.481)	14.548 (0.564)
Portugal		
1914-1965	43.099 (0.042)	24.022 (0.150)
1966-1998	41.925 (0.177)	25.013 (0.268)
South Africa, 1967-2014	42.015 (0.038)	19.876 (0.303)
Spain, 1941-1989	25.630 (0.300)	16.105 (0.368)
Switzerland		
1851-1906	22.402 (0.439)	12.118 (0.674)
1948-2005	53.629 (0.002)	46.195 (0.000)
Taiwan, 1962-2013	50.721 (0.013)	38.018 (0.005)
Turkey, 1968-2014	34.006 (0.232)	24.365 (0.163)
United Kingdom, 1922-2014	37.354 (0.047)	28.997 (0.021)
United States, 1915-2014		
standard M_1	16.148 (0.849)	11.595 (0.739)
Lucas-Nicolini 'New M_1*'*	18.903 (0.682)	16.203 (0.346)
West Germany, 1960-1989	35.618 (0.172)	25.394 (0.124)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SL.5 Results from Shin's (1994) tests of the null hypothesis of cointegration: test statistics and bootstrapped *p*-values[a] (in parentheses) based on unrestricted specifications[a]

Argentina, 1914-2009	0.233 (0.216)	Italy	
Australia, 1969-2014	0.057 (0.406)	*1861-1913*	0.096 (0.494)
Bolivia, 1980-2013	0.056 (0.804)	*1949-1996*	0.092 (0.188)
Brazil		Norway, 1946-2013	0.090 (0.639)
1934-2012	0.037 (0.853)	New Zealand, 1934-2004	0.115 (0.497)
1974-2012	0.126 (0.056)	Portugal	
Canada		*1914-1965*	0.069 (0.284)
1934-2006	0.080 (0.774)	*1966-1998*	0.046 (0.957)
1967-2013	0.061 (0.518)	South Africa, 1967-2014	0.039 (0.792)
Chile		Spain, 1941-1989	0.104 (0.287)
1940-1995	0.119 (0.126)	Switzerland	
1941-2012	0.125 (0.368)	*1851-1906*	0.065 (0.827)
Colombia, 1959-2011	0.115 (0.076)	*1948-2005*	0.090 (0.243)
Finland, 1914-1985	0.085 (0.609)	Taiwan, 1962-2013	0.158 (0.081)
Germany, 1876-1913	0.062 (0.538)	Turkey, 1968-2014	0.030 (0.894)
Japan		United Kingdom, 1922-2014	0.134 (0.476)
1885-1913	0.051 (0.555)	United States, 1915-2014	
1955-2013	0.116 (0.476)	*standard* M_1	0.310 (0.149)
Korea, 1970-2014	0.071 (0.225)	*Lucas-Nicolini 'New* M_1*'*	0.125 (0.511)
Israel, 1983-2013	0.038 (0.755)	West Germany, 1960-1989	0.115 (0.084)
Netherlands, 1950-1992	0.062 (0.524)		

[a] Bootstrapped *p*-values are based on 10,000 bootstrap replications of the VECM estimated under the null hypothesis of one cointegration vector. The estimated regression is

$$\ln(M_{1,t}) = \beta_0 + \beta_1 \ln(NGDP_t) + \beta_2 R_t + u_t.$$

$M_{1,t}$ = nominal M_1; $NGDP_t$ = nominal GDP; R_t = short rate.

Table SL.6 Bootstrapped p-values[a] for testing the null hypothesis that the income elasticity is equal to 1

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.315	Korea, 1970-2014	0.453
Australia, 1969-2014	0.094	Israel, 1983-2013	0.389
Bolivia, 1980-2013	0.210	Netherlands, 1950-1992	0.253
Brazil		New Zealand, 1934-2004	0.440
1974-2012	0.097	Norway, 1946-2013	0.164
1934-2012	0.005	Portugal	
Canada		*1914-1965*	0.196
1934-2006	0.456	*1966-1998*	0.021
1967-2013	0.186	South Africa, 1967-2014	0.393
Chile		Spain, 1941-1989	0.215
1940-1995	0.032	Switzerland, 1948-2005	0.052
1941-2012	0.097	Taiwan, 1962-2013	0.265
Colombia, 1959-2011	0.409	United Kingdom, 1922-2014	0.064
Finland, 1914-1985	0.037	United States, 1915-2014	
Germany, 1876-1913	0.046	*standard* M_1	0.056
Japan		*Lucas-Nicolini 'New* M_1*'*	0.325
1885-1913	0.055	West Germany, 1960-1989	0.175
1955-2013	0.489		

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table SL.6 (continued) Bootstrapped *p*-values[a] for testing the null hypothesis that the income elasticity is equal to 1

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.043	Korea, 1970-2014	0.026
Australia, 1969-2014	0.053	Israel, 1983-2013	0.019
Bolivia, 1980-2013	0.145	Netherlands, 1950-1992	0.099
Brazil		New Zealand, 1934-2004	0.023
1974-2012	0.023	Norway, 1946-2013	0.085
1934-2012	6.0e-4	Portugal	
Canada		*1914-1965*	0.038
1934-2006	0.033	*1966-1998*	0.013
1967-2013	0.426	South Africa, 1967-2014	0.032
Chile		Spain, 1941-1989	0.009
1940-1995	0.048	Switzerland, 1948-2005	0.053
1941-2012	0.213	Taiwan, 1962-2013	0.161
Colombia, 1959-2011	0.229	United Kingdom, 1922-2014	0.046
Finland, 1914-1985	0.001	United States, 1915-2014	
Germany, 1876-1913	0.006	*standard* M_1	0.006
Japan		*Lucas-Nicolini 'New* M_1*'*	0.364
1885-1913	0.003	West Germany, 1960-1989	0.239
1955-2013	0.454		

[a] Bootstrapped *p*-values (in parentheses) are based on 10,000 bootstrap replications.

III: Full set of results based on the log-log specification

Table LL.1 Assessing the persistence of candidate cointegration residuals: Hansen (1999) 'grid bootstrap' estimates of the sum of the autoregressive coefficients based on AR(2) models (median, and 90 per cent bootstrapped confidence interval)[a]

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.82 [0.72; 0.93]	Korea, 1970-2014	0.82 [0.61; 1.02]
Australia, 1969-2014	0.57 [0.30; 0.93]	Israel, 1983-2013	0.66 [0.49; 0.83]
Bolivia, 1980-2013	0.73 [0.53; 0.96]	Mexico, 1985-2014	1.00 [0.61; 1.04]
Brazil		Netherlands, 1950-1992	0.64 [0.38; 0.98]
1974-2012	0.93 [0.79; 1.02]	New Zealand, 1934-2004	0.85 [0.74; 1.01]
1934-2012	0.93 [0.83; 1.02]	Norway, 1946-2013	0.94 [0.86; 1.01]
Canada		Portugal	
1934-2006	0.90 [0.79; 1.02]	*1914-1965*	0.82 [0.67; 1.01]
1967-2013	0.42 [0.23; 0.61]	*1966-1998*	1.01 [0.95; 1.06]
Chile		South Africa, 1967-2014	0.90 [0.78; 1.01]
1940-1995	0.78 [0.63; 1.00]	Spain, 1941-1989	0.84 [0.68; 1.01]
1941-2012	0.83 [0.71; 0.97]	Switzerland, 1948-2005	0.76 [0.58; 0.96]
Colombia, 1959-2011	0.88 [0.70; 1.02]	Taiwan, 1962-2013	0.85 [0.74; 0.97]
Finland, 1914-1985	0.89 [0.79; 1.01]	United Kingdom, 1922-2014	0.84 [0.72; 0.98]
Germany, 1876-1913	0.63 [0.35; 0.99]	United States, 1915-2014	
Guatemala, 1980-2012	0.57 [0.27; 0.99]	*standard* M_1	0.96 [0.88; 1.01]
Japan		*Lucas-Nicolini 'New* M_1*'*	0.84 [0.74; 0.95]
1885-1913	0.44 [0.10; 0.85]	Venezuela, 1962-1999	0.95 [0.80; 1.03]
1955-2013	0.91 [0.82; 1.01]	West Germany, 1960-1989	0.39 [0.08; 0.71]

[a] Based on 2,000 bootstrap replications for each value of ρ in the grid. Candidate cointegration residuals have been computed based on the bivariate model for log velocity and (the log of) the short rate, and Johansen's estimator.

Table LL.1 (continued) Assessing the persistence of candidate cointegration residuals: Hansen (1999) 'grid bootstrap' estimates of the sum of the autoregressive coefficients based on AR(2) models (median, and 90 per cent bootstrapped confidence interval)[a]

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.86 [0.77; 0.99]	Korea, 1970-2014	0.83 [0.60; 1.02]
Australia, 1969-2014	0.58 [0.27; 0.91]	Israel, 1983-2013	0.60 [0.34; 0.89]
Bolivia, 1980-2013	0.71 [0.52; 0.92]	Mexico, 1985-2014	0.74 [0.54; 1.01]
Brazil		Netherlands, 1950-1992	0.69 [0.45; 0.99]
1974-2012	0.91 [0.77; 1.02]	New Zealand, 1934-2004	0.88 [0.77; 1.01]
1934-2012	1.00 [0.93; 1.03]	Norway, 1946-2013	1.00 [0.96; 1.02]
Canada		Portugal	
1934-2006	0.91 [0.80; 1.02]	*1914-1965*	0.82 [0.66; 1.01]
1967-2013	0.43 [0.24; 0.62]	*1966-1998*	1.02 [0.96; 1.08]
Chile		South Africa, 1967-2014	1.01 [0.97; 1.04]
1940-1995	0.80 [0.64; 1.00]	Spain, 1941-1989	0.83 [0.68; 1.01]
1941-2012	0.82 [0.71; 0.96]	Switzerland, 1948-2005	0.86 [0.72; 1.01]
Colombia, 1959-2011	0.92 [0.73; 1.03]	Taiwan, 1962-2013	0.86 [0.75; 0.99]
Finland, 1914-1985	0.93 [0.84; 1.01]	United Kingdom, 1922-2014	0.86 [0.74; 1.00]
Germany, 1876-1913	0.99 [0.86; 1.03]	United States, 1915-2014	
Guatemala, 1980-2012	0.59 [0.30; 0.94	*standard* M_1	1.01 [0.99; 1.03]
Japan		*Lucas-Nicolini 'New* M_1*'*	0.86 [0.76; 0.96]
1885-1913	1.01 [0.85; 1.05]	Venezuela, 1962-1999	0.93 [0.75; 1.04]
1955-2013	0.92 [0.82; 1.01]	West Germany, 1960-1989	1.01 [0.86; 1.09]

[a] Based on 2,000 bootstrap replications for each value of ρ in the grid. Candidate cointegration residuals have been computed based on the bivariate model for log velocity and (the log of) the short rate, and Johansen's estimator.

Table LL.2a Results from cointegration tests between the logarithms of M_1 velocity and of a short-term rate[a] for very high inflation countries

	I: Johansen's tests of the null of no cointegration		II: Shin's tests of the null of cointegration
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegrating vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegrating vectors:*	
Argentina, 1914-2009	21.471 (0.027)	18.988 (0.020)	0.549 (0.301)
Bolivia, 1980-2013	12.745 (0.410)	12.509 (0.279)	0.179 (0.192)
Brazil			
1974-2012	20.760 (0.052)	15.175 (0.095)	0.319 (0.108)
1934-2012	20.165 (0.032)	16.607 (0.037)	2.014 (0.013)
Chile			
1940-1995	26.274 (0.011)	19.076 (0.030)	0.165 (0.274)
1941-2012	16.822 (0.096)	12.958 (0.136)	0.122 (0.758)
Israel, 1983-2013	27.703 (0.012)	26.547 (0.008)	0.157 (0.224)
Mexico, 1985-2014	15.082 (0.261)	14.091 (0.201)	0.134 (0.275)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LL.2b Results from cointegration tests between the logarithm of M_1 velocity and of a short-term rate[a]

	I: Johansen's tests of the null of no cointegration		
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*	II: Shin's tests of the null of cointegration
Australia, 1969-2014	9.268 (0.594)	8.387 (0.503)	0.240 (0.436)
Canada, 1934-2006	11.260 (0.438)	8.582 (0.496)	0.402 (0.267)
Colombia, 1959-2011	6.558 (0.824)	4.852 (0.871)	0.226 (0.507)
Finland, 1914-1985	9.003 (0.586)	7.299 (0.611)	1.382 (0.027)
Germany, 1876-1913	9.959 (0.560)	8.653 (0.544)	0.528 (0.171)
Guatemala, 1980-2012	18.851 (0.079)	17.208 (0.054)	0.073 (0.724)
Japan			
1885-1913	11.980 (0.403)	10.787 (0.336)	0.433 (0.098)
1955-2013	16.010 (0.148)	14.415 (0.121)	0.145 (0.874)
Korea, 1970-2014	4.769 (0.908)	4.011 (0.914)	0.345 (0.252)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LL.2b (continued) Results from cointegration tests between the logarithm of M_1 velocity and of a short-term rate[a]

	I: Johansen's tests of the null of no cointegration		
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*	II: Shin's tests of the null of cointegration
Netherlands, 1950-1992	15.143 (0.169)	9.214 (0.400)	0.206 (0.440)
New Zealand, 1934-2004	17.900 (0.075)	16.683 (0.043)	0.535 (0.299)
Norway, 1946-2013	24.171 (0.015)	21.084 (0.013)	0.634 (0.219)
Portugal			
1914-1965	21.471 (0.052)	20.634 (0.026)	0.124 (0.339)
1966-1998	20.147 (0.075)	15.350 (0.117)	0.065 (0.645)
South Africa, 1967-2014	16.667 (0.145)	15.560 (0.091)	0.319 (0.176)
Spain, 1941-1989	7.606 (0.670)	7.429 (0.563)	0.253 (0.256)
Switzerland			
1851-1906	13.664 (0.160)	13.090 (0.113)	0.664 (0.219)
1948-2005	24.175 (0.017)	20.705 (0.015)	0.928 (0.107)
Taiwan, 1962-2013	6.107 (0.816)	5.587 (0.786)	0.401 (0.128)
United Kingdom, 1922-2014	13.702 (0.255)	13.592 (0.135)	0.795 (0.115)
United States, 1915-2014			
based on the standard M_1 aggregate	12.119 (0.271)	10.096 (0.280)	2.738 (0.031)
based on Lucas and Nicolini's 'New M_1' aggregate	9.626 (0.514)	8.926 (0.409)	0.571 (0.237)
Venezuela, 1962-1999	6.650 (0.759)	4.366 (0.888)	0.362 (0.094)
West Germany, 1960-1989	12.206 (0.420)	12.182 (0.256)	0.440 (0.071)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LL.3 Bootstrapped p-values[a] for testing the null hypothesis that the elasticity is equal to -0.5, based on bivariate systems featuring the logarithms of velocity and the short rate

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.442	Korea, 1970-2014	0.287
Australia, 1969-2014	0.333	Israel, 1983-2013	0.085
Bolivia, 1980-2013	0.099	Mexico, 1985-2014	0.012
Brazil		Netherlands, 1950-1992	0.411
1974-2012	0.017	New Zealand, 1934-2004	0.143
1934-2012	0.318	Norway, 1946-2013	0.079
Canada		Portugal	
1934-2006	0.056	*1914-1965*	0.042
1967-2013		*1966-1998*	0.009
Chile		South Africa, 1967-2014	0.188
1940-1995	0.001	Spain, 1941-1989	0.109
1941-2012	0.030	Switzerland, 1948-2005	0.330
Colombia, 1959-2011	0.304	Taiwan, 1962-2013	0.176
Finland, 1914-1985	0.055	United Kingdom, 1922-2014	0.383
Germany, 1876-1913	0.223	United States, 1915-2014	
Guatemala, 1980-2012	0.142	*standard* M_1	0.476
Japan		*Lucas-Nicolini 'New* M_1*'*	0.010
1885-1913	0.393	Venezuela, 1962-1999	0.220
1955-2013	0.482	West Germany, 1960-1989	0.333

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LL.3 (continued) Bootstrapped *p*-values[a] for testing the null hypothesis that the elasticity is equal to -0.5, based on bivariate systems featuring the logarithms of velocity and the short rate

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.001	Korea, 1970-2014	0.484
Australia, 1969-2014	0.333	Israel, 1983-2013	0.003
Bolivia, 1980-2013	0.208	Mexico, 1985-2014	0.023
Brazil		Netherlands, 1950-1992	0.049
1974-2012	0.000	New Zealand, 1934-2004	0.083
1934-2012	0.000	Norway, 1946-2013	0.003
Canada		Portugal	
1934-2006	0.002	*1914-1965*	0.007
1967-2013		*1966-1998*	0.001
Chile		South Africa, 1967-2014	0.002
1940-1995	0.000	Spain, 1941-1989	0.014
1941-2012	0.000	Switzerland, 1948-2005	0.000
Colombia, 1959-2011	0.001	Taiwan, 1962-2013	0.011
Finland, 1914-1985	0.116	United Kingdom, 1922-2014	0.110
Germany, 1876-1913	0.000	United States, 1915-2014	
Guatemala, 1980-2012	0.427	*standard* M_1	0.000
Japan		*Lucas-Nicolini 'New* M_1*'*	0.000
1885-1913	0.092	Venezuela, 1962-1999	0.050
1955-2013	0.003	West Germany, 1960-1989	0.000

[a] Bootstrapped *p*-values (in parentheses) are based on 10,000 bootstrap replications.

Table LL.4 Results from Johansen's tests of the null hypothesis of no cointegration based on unrestricted specifications: test statistics and bootstrapped p-values[a] (in parentheses)

	Trace tests of the null of no cointegration against the alternative of 1 *or more cointegration vectors:*	*Maximum eigenvalue tests of* 0 *versus* 1 *cointegration vectors:*
Argentina, 1914-2009	45.317 (0.008)	32.284 (0.010)
Australia, 1969-2014	30.105 (0.214)	17.292 (0.380)
Bolivia, 1980-2013	38.879 (0.173)	29.734 (0.099)
Brazil		
1934-2012	39.152 (0.041)	23.270 (0.102)
1974-2012	60.575 (0.001)	37.313 (0.007)
Canada		
1934-2006	19.159 (0.753)	10.965 (0.844)
1967-2013	42.329 (0.071)	22.221 (0.248)
Chile		
1940-1995	35.087 (0.144)	21.051 (0.251)
1941-2012	27.165 (0.375)	16.053 (0.494)
Colombia, 1959-2011	22.914 (0.721)	12.444 (0.845)
Finland, 1914-1985	24.400 (0.413)	17.098 (0.347)
Germany, 1876-1913	43.543 (0.063)	24.709 (0.177)
Japan		
1885-1913	28.324 (0.333)	16.231 (0.511)
1955-2013	45.477 (0.035)	32.445 (0.023)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LL.4 (continued) Results from Johansen's tests of the null hypothesis of no cointegration based on unrestricted specifications: test statistics and bootstrapped p-values[a] (in parentheses)

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*
Korea, 1970-2014	39.642 (0.063)	25.422 (0.085)
Israel, 1983-2013	94.381 (0.000)	49.649 (0.005)
Italy, 1861-1913	23.648 (0.419)	16.554 (0.351)
Netherlands, 1950-1992	55.067 (0.004)	38.552 (0.005)
Norway, 1946-2013	43.964 (0.006)	24.616 (0.048)
New Zealand, 1934-2004	33.158 (0.070)	18.793 (0.212)
Portugal		
1914-1965	48.208 (0.016)	32.206 (0.026)
1966-1998	36.440 (0.180)	19.853 (0.383)
South Africa, 1967-2014	37.958 (0.089)	18.602 (0.390)
Spain, 1941-1989	21.283 (0.549)	13.341 (0.594)
Switzerland		
1851-1906	22.613 (0.429)	12.262 (0.657)
1948-2005	50.510 (0.003)	43.397 (0.001)
Taiwan, 1962-2013	30.972 (0.319)	18.733 (0.409)
United Kingdom, 1922-2014	24.252 (0.483)	16.632 (0.422)
United States, 1915-2014		
standard M_1	28.275 (0.167)	19.028 (0.183)
Lucas-Nicolini 'New M_1*'*	20.135 (0.582)	17.468 (0.262)
West Germany, 1960-1989	36.353 (0.141)	25.650 (0.114)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LL.5 Results from Shin's (1994) tests of the null hypothesis of cointegration: test statistics and bootstrapped p-values[a] (in parentheses) based on unrestricted specifications[a]

Argentina, 1914-2009	0.167 (0.300)	Israel, 1983-2013	0.047 (0.543)
Australia, 1969-2014	0.044 (0.702)	Italy, 1861-1913	0.089 (0.582)
Bolivia, 1980-2013	0.060 (0.709)	Netherlands, 1950-1992	0.061 (0.537)
Brazil		Norway, 1946-2013	0.070 (0.807)
1934-2012	0.070 (0.549)	New Zealand, 1934-2004	0.066 (0.752)
1974-2012	0.046 (0.522)	Portugal	
Canada		*1914-1965*	0.064 (0.179)
1934-2006	0.110 (0.624)	*1966-1998*	0.021 (0.984)
1967-2013	0.050 (0.660)	South Africa, 1967-2014	0.044 (0.675)
Chile		Spain, 1941-1989	0.110 (0.170)
1940-1995	0.205 (0.014)	Switzerland	
1941-2012	0.090 (0.436)	*1851-1906*	0.064 (0.828)
Colombia, 1959-2011	0.099 (0.178)	*1948-2005*	0.127 (0.062)
Finland, 1914-1985	0.081 (0.626)	Taiwan, 1962-2013	0.101 (0.211)
Germany, 1876-1913	0.072 (0.417)	United Kingdom, 1922-2014	0.134 (0.775)
Japan		United States, 1915-2014	
1885-1913	0.073 (0.333)	*standard M_1*	0.132 (0.676)
1955-2013	0.068 (0.758)	*Lucas-Nicolini 'New M_1'*	0.066 (0.948)
Korea, 1970-2014	0.082 (0.114)	West Germany, 1960-1989	0.105 (0.114)

[a] Bootstrapped p-values are based on 10,000 bootstrap replications of the VECM estimated under the null hypothesis of one cointegration vector. The estimated regression is

$$\ln(M_{1,t}) = \beta_0 + \beta_1 \ln(NGDP_t) + \beta_2 \ln(R_t) + u_t.$$

$M_{1,t}$ = nominal M_1; $NGDP_t$ = nominal GDP; R_t = short rate.

Table LL.6 Bootstrapped p-values[a] for testing the null hypothesis that the income elasticity is equal to 1

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.435	Korea, 1970-2014	0.404
Australia, 1969-2014	0.447	Israel, 1983-2013	0.078
Bolivia, 1980-2013	0.198	Netherlands, 1950-1992	0.099
Brazil		New Zealand, 1934-2004	0.147
1974-2012	0.003	Norway, 1946-2013	0.248
1934-2012	0.017	Portugal	
Canada		*1914-1965*	0.373
1934-2006	0.176	*1966-1998*	0.474
1967-2013	0.116	South Africa, 1967-2014	0.348
Chile		Spain, 1941-1989	0.068
1940-1995	0.143	Switzerland, 1948-2005	0.025
1941-2012	0.076	Taiwan, 1962-2013	0.396
Colombia, 1959-2011	0.402	United Kingdom, 1922-2014	0.230
Finland,1914-1985	0.021	United States, 1915-2014	
Germany, 1876-1913	0.044	*standard M_1*	0.065
Japan		*Lucas-Nicolini 'New M_1'*	0.445
1885-1913	0.059	West Germany, 1960-1989	0.180
1955-2013	0.344		

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LL.6 (continued) Bootstrapped p-values[a] for testing the null hypothesis that the income elasticity is equal to 1

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.017	Korea, 1970-2014	0.052
Australia, 1969-2014	0.171	Israel, 1983-2013	0.392
Bolivia, 1980-2013	0.182	Netherlands, 1950-1992	0.178
Brazil		New Zealand, 1934-2004	0.013
1974-2012	1.0e-3	Norway, 1946-2013	0.078
1934-2012	5.0e-4	Portugal	
Canada		*1914-1965*	0.119
1934-2006	0.150	*1966-1998*	0.091
1967-2013	0.347	South Africa, 1967-2014	0.031
Chile		Spain, 1941-1989	0.028
1940-1995	0.283	Switzerland, 1948-2005	0.008
1941-2012	0.054	Taiwan, 1962-2013	0.109
Colombia, 1959-2011	0.177	United Kingdom, 1922-2014	0.091
Finland, 1914-1985	0.002	United States, 1915-2014	
Germany, 1876-1913	0.007	*standard* M_1	0.009
Japan		*Lucas-Nicolini 'New* M_1*'*	0.322
1885-1913	0.003	West Germany, 1960-1989	0.194
1955-2013	0.428		

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

IV: Full set of results based on the log-log specification with the correction ln(1+R)

Table LLCO.1 Assessing the persistence of candidate cointegration residuals: Hansen (1999) 'grid bootstrap' estimates of the sum of the autoregressive coefficients based on AR(2) models (median, and 90 per cent bootstrapped confidence interval)[a]

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.82 [0.72; 0.92]	Korea, 1970-2014	0.74 [0.51; 1.01]
Australia, 1969-2014	0.55 [0.26; 0.87]	Israel, 1983-2013	0.69 [0.57; 0.81]
Bolivia, 1980-2013	0.73 [0.55; 0.92]	Mexico, 1985-2014	0.75 [0.56; 0.99]
Brazil		Netherlands, 1950-1992	0.64 [0.39; 0.95]
1974-2012	0.94 [0.79; 1.02]	New Zealand, 1934-2004	0.84 [0.73; 0.99]
1934-2012	0.92 [0.82; 1.02]	Norway, 1946-2013	0.95 [0.87; 1.01]
Canada		Portugal	
1934-2006	0.83 [0.70; 1.01]	*1914-1965*	0.81 [0.64; 1.01]
1967-2013	0.39 [0.20; 0.57]	*1966-1998*	1.01 [0.95; 1.06]
Chile		South Africa, 1967-2014	0.90 [0.78; 1.01]
1940-1995	0.78 [0.63; 0.94]	Spain, 1941-1989	0.83 [0.66; 1.01]
1941-2012	0.83 [0.72; 0.97]	Switzerland, 1948-2005	0.79 [0.63; 1.01]
Colombia, 1959-2011	0.88 [0.71; 1.02]	Taiwan, 1962-2013	0.85 [0.75; 0.97]
Finland, 1914-1985	0.94 [0.86; 1.01]	United Kingdom, 1922-2014	0.81 [0.68; 0.95]
Germany, 1876-1913	0.61 [0.35; 0.98]	United States, 1915-2014	
Guatemala, 1980-2012	0.59 [0.29; 0.98]	*standard* M_1	0.96 [0.89; 1.01]
Japan		*Lucas-Nicolini 'New* M_1*'*	0.77 [0.66; 0.88]
1885-1913	0.44 [0.12; 0.88]	Venezuela, 1962-1999	0.95 [0.80; 1.03
1955-2013	0.93 [0.85; 1.01]	West Germany, 1960-1989	0.39 [0.07; 0.70]

[a] Based on 2,000 bootstrap replications for each value of ρ in the grid. Candidate cointegration residuals have been computed based on the bivariate model for log velocity and (the log of) the short rate, and Johansen's estimator.

Table LLCO.1 (continued) Assessing the persistence of candidate cointegration residuals: Hansen (1999) ‘grid bootstrap’ estimates of the sum of the autoregressive coefficients based on AR(2) models (median, and 90 per cent bootstrapped confidence interval)[a]

II: Based on Stock and Watson’s estimator of the cointegration vector

Argentina, 1914-2009	0.86 [0.77; 0.99]	Korea, 1970-2014	0.75 [0.54; 1.01]
Australia, 1969-2014	0.55 [0.26; 0.85]	Israel, 1983-2013	0.65 [0.46; 0.84]
Bolivia, 1980-2013	0.72 [0.55; 0.91]	Mexico, 1985-2014	0.73 [0.51; 0.98]
Brazil		Netherlands, 1950-1992	0.70 [0.46; 0.99]
1974-2012	0.91 [0.77; 1.02]	New Zealand, 1934-2004	0.88 [0.77; 1.01]
1934-2012	1.00 [0.93; 1.03]	Norway, 1946-2013	1.00 [0.97; 1.02]
Canada		Portugal	
1934-2006	0.85 [0.71; 1.01]	*1914-1965*	0.80 [0.64; 1.01]
1967-2013	0.39 [0.20; 0.57]	*1966-1998*	1.02 [0.98; 1.10]
Chile		South Africa, 1967-2014	1.01 [0.98; 1.04]
1940-1995	0.80 [0.66; 1.00]	Spain, 1941-1989	0.83 [0.67; 1.01]
1941-2012	0.83 [0.71; 0.98]	Switzerland, 1948-2005	0.88 [0.75; 1.02]
Colombia, 1959-2011	0.92 [0.73; 1.03]	Taiwan, 1962-2013	0.86 [0.76; 0.98]
Finland, 1914-1985	0.97 [0.90; 1.01]	United Kingdom, 1922-2014	0.83 [0.71; 1.00]
Guatemala, 1980-2012	0.60 [0.31; 0.97]	United States, 1915-2014	
Germany, 1876-1913	0.99 [0.86; 1.03]	*standard* M_1	1.01 [0.97; 1.02]
Japan		*Lucas-Nicolini ‘New* M_1*’*	0.79 [0.68; 0.90]
1885-1913	1.01 [0.86; 1.05]	Venezuela, 1962-1999	0.94 [0.76; 1.03]
1955-2013	0.95 [0.85; 1.01]	West Germany, 1960-1989	1.01 [0.86; 1.10]

[a] Based on 2,000 bootstrap replications for each value of ρ in the grid. Candidate cointegration residuals have been computed based on the bivariate model for log velocity and (the log of) the short rate, and Johansen’s estimator.

Table LLCO.2a Results from cointegration tests between the logarithms of M_1 velocity and of a short-term rate[a] for very high inflation countries

	I: Johansen's tests of the null of no cointegration		
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegrating vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegrating vectors:*	II: Shin's tests of the null of cointegration
Argentina, 1914-2009	21.303 (0.032)	18.866 (0.023)	0.567 (0.288)
Bolivia, 1980-2013	15.480 (0.255)	15.134 (0.154)	0.156 (0.249)
Brazil			
1974-2012	20.904 (0.049)	15.221 (0.093)	0.325 (0.104)
1934-2012	20.270 (0.034)	16.842 (0.037)	2.043 (0.011)
Chile			
1940-1995	26.453 (0.013)	18.953 (0.033)	0.178 (0.244)
1941-2012	18.541 (0.059)	13.224 (0.119)	0.127 (0.725)
Israel, 1983-2013	41.66 (0.001)	40.773 (0.000)	0.135 (0.350)
Mexico, 1985-2014	15.569 (0.230)	14.027 (0.205)	0.132 (0.285)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LLCO.2b Results from cointegration tests between the logarithm of M_1 velocity and of a short-term rate[a]

	I: Johansen's tests of the null of no cointegration		
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*	II: Shin's tests of the null of cointegration
Australia, 1969-2014	10.268 (0.506)	9.373 (0.405)	0.245 (0.395)
Canada			
1934-2006	17.093 (0.125)	14.506 (0.110)	0.259 (0.400)
1967-2012	27.310 (0.010)	27.262 (0.003)	0.079 (0.705)
Colombia, 1959-2011	6.603 (0.830)	4.896 (0.872)	0.225 (0.502)
Finland, 1914-1985	7.225 (0.736)	5.019 (0.839)	1.447 (0.023)
Germany, 1876-1913	9.947 (0.559)	8.689 (0.532)	0.522 (0.177)
Guatemala, 1980-2012	18.939 (0.077)	17.261 (0.052)	0.072 (0.737)
Japan			
1885-1913	11.938 (0.408)	10.737 (0.331)	0.435 (0.099)
1955-2013	13.502 (0.199)	13.502 (0.120)	0.098 (0.975)
Korea, 1970-2014	6.698 (0.746)	6.075 (0.715)	0.282 (0.269)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LLCO.2b (continued) Results from cointegration tests between the logarithm of M_1 velocity and of a short-term rate[a]

	I: Johansen's tests of the null of no cointegration		II: Shin's tests of the null of cointegration
	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*	
Netherlands, 1950-1992	15.054 (0.166)	9.309 (0.401)	0.216 (0.413)
New Zealand, 1934-2004	17.535 (0.087)	16.340 (0.049)	0.500 (0.327)
Norway, 1946-2013	24.004 (0.016)	20.698 (0.015)	0.736 (0.157)
Portugal			
1914-1965	20.699 (0.061)	19.887 (0.032)	0.120 (0.360)
1966-1998	19.392 (0.086)	14.975 (0.125)	0.074 (0.546)
South Africa, 1967-2014	16.776 (0.131)	15.686 (0.080)	0.336 (0.160)
Spain, 1941-1989	7.850 (0.642)	7.632 (0.537)	0.261 (0.256)
Switzerland			
1851-1906	15.520 (0.094)	15.377 (0.057)	0.780 (0.192)
1948-2005	31.284 (0.001)	27.586 (0.001)	0.975 (0.064)
Taiwan, 1962-2013	6.108 (0.816)	5.508 (0.794)	0.387 (0.131)
United Kingdom, 1922-2014	15.684 (0.159)	15.361 (0.077)	0.951 (0.058)
United States, 1915-2014			
based on the standard M_1 aggregate	11.224 (0.342)	9.563 (0.320)	3.021 (0.015)
based on Lucas and Nicolini's 'New M_1' aggregate	14.623 (0.187)	13.107 (0.137)	0.369 (0.290)
Venezuela, 1962-1999	6.616 (0.771)	4.389 (0.888)	0.364 (0.094)
West Germany, 1960-1989	12.243 (0.419)	12.194 (0.261)	0.442 (0.076)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LLCO.3 Bootstrapped p-values[a] for testing the null hypothesis that the elasticity is equal to -0.5, based on bivariate systems featuring the logarithms of velocity and the short rate

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.332	Korea, 1970-2014	0.050
Australia, 1969-2014	0.143	Israel, 1983-2013	0.031
Bolivia, 1980-2013	0.039	Mexico, 1985-2014	0.023
Brazil		Netherlands, 1950-1992	0.169
1974-2012	0.017	New Zealand, 1934-2004	0.054
1934-2012	0.295	Norway, 1946-2013	0.073
Canada		Portugal	
1934-2006	0.225	*1914-1965*	0.019
1967-2013	0.058	*1966-1998*	0.008
Chile		South Africa, 1967-2014	0.162
1940-1995	0.000	Spain, 1941-1989	0.186
1941-2012	0.034	Switzerland, 1948-2005	0.011
Colombia, 1959-2011	0.395	Taiwan, 1962-2013	0.374
Finland, 1914-1985	0.053	United Kingdom, 1922-2014	0.037
Germany, 1876-1913	0.190	United States, 1915-2014	
Guatemala, 1980-2012	0.069	*standard* M_1	0.295
Japan		*Lucas-Nicolini 'New* M_1*'*	0.109
1885-1913	0.398	Venezuela, 1962-1999	0.245
1955-2013	0.023	West Germany, 1960-1989	0.377

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LLCO.3 (continued) Bootstrapped p-values[a] for testing the null hypothesis that the elasticity is equal to -0.5, based on bivariate systems featuring the logarithms of velocity and the short rate

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.002	Korea, 1970-2014	0.127
Australia, 1969-2014	0.470	Israel, 1983-2013	0.170
Bolivia, 1980-2013	0.081	Mexico, 1985-2014	0.061
Brazil		Netherlands, 1950-1992	0.272
1974-2012	0.000	New Zealand, 1934-2004	0.356
1934-2012	0.000	Norway, 1946-2013	0.012
Canada		Portugal	
1934-2006	0.367	*1914-1965*	0.002
1967-2013	0.189	*1966-1998*	0.001
Chile		South Africa, 1967-2014	0.002
1940-1995	0.000	Spain, 1941-1989	0.050
1941-2012	0.000	Switzerland, 1948-2005	0.005
Colombia, 1959-2011	0.003	Taiwan, 1962-2013	0.068
Finland, 1914-1985	0.091	United Kingdom, 1922-2014	0.259
Germany, 1876-1913	0.000	United States, 1915-2014	
Guatemala, 1980-2012	0.330	*standard M_1*	0.019
Japan		*Lucas-Nicolini 'New M_1'*	0.002
1885-1913	0.122	Venezuela, 1962-1999	0.067
1955-2013	0.365	West Germany, 1960-1989	0.000

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LLCO.4 Results from Johansen's tests of the null hypothesis of no cointegration based on unrestricted specifications: test statistics and bootstrapped p-values[a] (in parentheses)

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*
Argentina, 1914-2009	46.397 (0.006)	33.464 (0.007)
Australia, 1969-2014	30.979 (0.183)	17.289 (0.378)
Bolivia, 1980-2013	41.113 (0.135)	31.226 (0.074)
Brazil		
1934-2012	38.058 (0.049)	22.928 (0.116)
1974-2012	61.502 (0.000)	38.319 (0.004)
Canada		
1934-2006	24.417 (0.450)	17.906 (0.318)
1967-2013	54.466 (0.007)	31.823 (0.033)
Chile		
1940-1995	34.951 (0.153)	21.066 (0.249)
1941-2012	27.450 (0.364)	15.876 (0.510)
Colombia, 1959-2011	23.579 (0.680)	12.860 (0.817)
Finland, 1914-1985	28.168 (0.245)	20.887 (0.160)
Germany, 1876-1913	43.658 (0.064)	24.871 (0.178)
Japan		
1885-1913	28.281 (0.340)	16.137 (0.519)
1955-2013	43.780 (0.049)	31.012 (0.036)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LLCO.4 (continued) Results from Johansen's tests of the null hypothesis of no cointegration based on unrestricted specifications: test statistics and bootstrapped p-values[a] (in parentheses)

	Trace tests of the null of no cointegration against the alternative of 1 or more cointegration vectors:	*Maximum eigenvalue tests of 0 versus 1 cointegration vectors:*
Korea, 1970-2014	43.784 (0.032)	27.554 (0.056)
Israel, 1983-2013	95.473 (0.000)	50.929 (0.000)
Italy, 1861-1913	24.261 (0.385)	16.973 (0.323)
Netherlands, 1950-1992	54.319 (0.005)	37.561 (0.008)
Norway, 1946-2013	43.279 (0.007)	24.491 (0.050)
New Zealand, 1934-2004	31.801 (0.094)	17.733 (0.261)
Portugal		
1914-1965	47.608 (0.018)	31.104 (0.034)
1966-1998	35.621 (0.206)	19.491 (0.416)
South Africa, 1967-2014	38.292 (0.075)	18.660 (0.371)
Spain, 1941-1989	21.426 (0.541)	13.347 (0.596)
Switzerland		
1851-1906	26.095 (0.239)	15.927 (0.348)
1948-2005	52.240 (0.002)	45.854 (0.000)
Taiwan, 1962-2013	33.274 (0.249)	21.262 (0.268)
United Kingdom, 1922-2014	27.215 (0.313)	19.569 (0.225)
United States, 1915-2014		
standard M_1	21.101 (0.539)	13.896 (0.532)
Lucas-Nicolini 'New M_1*'*	21.1006 (0.539)	13.896 (0.532)
West Germany, 1960-1989	36.082 (0.154)	25.552 (0.120)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LLCO.5 Results from Shin's (1994) tests of the null hypothesis of cointegration: test statistics and bootstrapped p-values[a] (in parentheses) based on unrestricted specifications[a]

Argentina, 1914-2009	0.168 (0.299)	Israel, 1983-2013	0.049 (0.555)
Australia, 1969-2014	0.046 (0.653)	Italy, 1861-1913	0.090 (0.561)
Bolivia, 1980-2013	0.063 (0.675)	Netherlands, 1950-1992	0.062 (0.527)
Brazil		Norway, 1946-2013	0.071 (0.791)
1934-2012	0.071 (0.529)	New Zealand, 1934-2004	0.075 (0.691)
1974-2012	0.047 (0.520)	Portugal	
Canada		*1914-1965*	0.066 (0.181)
1934-2006	0.075 (0.801)	*1966-1998*	0.013 (0.983)
1967-2013	0.048 (0.560)	South Africa, 1967-2014	0.043 (0.701)
Chile		Spain, 1941-1989	0.108 (0.187)
1940-1995	0.202 (0.017)	Switzerland	
1941-2012	0.093 (0.430)	*1851-1906*	0.064 (0.824)
Colombia, 1959-2011	0.101 (0.161)	*1948-2005*	0.115 (0.123)
Finland, 1914-1985	0.082 (0.617)	Taiwan, 1962-2013	0.119 (0.142)
Germany, 1876-1913	0.070 (0.440)	United Kingdom, 1922-2014	0.132 (0.676)
Japan		United States, 1915-2014	
1885-1913	0.068 (0.369)	*standard* M_1	0.169 (0.517)
1955-2013	0.061 (0.846)	*Lucas-Nicolini 'New* M_1*'*	0.169 (0.517)
Korea, 1970-2014	0.080 (0.136)	West Germany, 1960-1989	0.109 (0.109)

[a] Bootstrapped p-values are based on 10,000 bootstrap replications of the VECM estimated under the null hypothesis of one cointegration vector. The estimated regression is

$$\ln(M_{1,t}) = \beta_0 + \beta_1\ln(NGDP_t) + \beta_2\ln(1+R_t) + u_t.$$

$M_{1,t}$ = nominal M_1; $NGDP_t$ = nominal GDP; R_t = short rate.

Table LLCO.6 Bootstrapped p-values[a] for testing the null hypothesis that the income elasticity is equal to 1

I: Based on Johansen's estimator of the cointegration vector

Argentina, 1914-2009	0.463	Korea, 1970-2014	0.456
Australia, 1969-2014	0.477	Israel, 1983-2013	0.076
Bolivia, 1980-2013	0.166	Netherlands, 1950-1992	0.117
Brazil		New Zealand, 1934-2004	0.162
1974-2012	0.003	Norway, 1946-2013	0.222
1934-2012	0.015	Portugal	
Canada		*1914-1965*	0.381
1934-2006	0.202	*1966-1998*	0.410
1967-2013	0.053	South Africa, 1967-2014	0.369
Chile		Spain, 1941-1989	0.072
1940-1995	0.187	Switzerland, 1948-2005	0.209
1941-2012	0.072	Taiwan, 1962-2013	0.442
Colombia, 1959-2011	0.350	United Kingdom, 1922-2014	0.141
Finland, 1914-1985	0.026	United States, 1915-2014	
Germany, 1876-1913	0.046	*standard* M_1	0.056
Japan		*Lucas-Nicolini 'New* M_1*'*	0.405
1885-1913	0.053	West Germany, 1960-1989	0.189
1955-2013	0.377		

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Table LLCO.6 (continued) Bootstrapped p-values[a] for testing the null hypothesis that the income elasticity is equal to 1

II: Based on Stock and Watson's estimator of the cointegration vector

Argentina, 1914-2009	0.017	Korea, 1970-2014	0.031
Australia, 1969-2014	0.143	Israel, 1983-2013	0.438
Bolivia, 1980-2013	0.178	Netherlands, 1950-1992	0.153
Brazil		New Zealand, 1934-2004	0.014
1974-2012	1.0e-3	Norway, 1946-2013	0.083
1934-2012	1.0e-4	Portugal	
Canada		*1914-1965*	0.092
1934-2006	0.060	*1966-1998*	0.084
1967-2013	0.215	South Africa, 1967-2014	0.031
Chile		Spain, 1941-1989	0.023
1940-1995	0.267	Switzerland, 1948-2005	0.029
1941-2012	0.045	Taiwan, 1962-2013	0.123
Colombia, 1959-2011	0.174	United Kingdom, 1922-2014	0.050
Finland, 1914-1985	0.002	United States, 1915-2014	
Germany, 1876-1913	0.006	*standard* M_1	0.006
Japan		*Lucas-Nicolini 'New* M_1*'*	0.283
1885-1913	0.003	West Germany, 1960-1989	0.207
1955-2013	0.396		

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.

Figures for the online appendix

I: The raw series

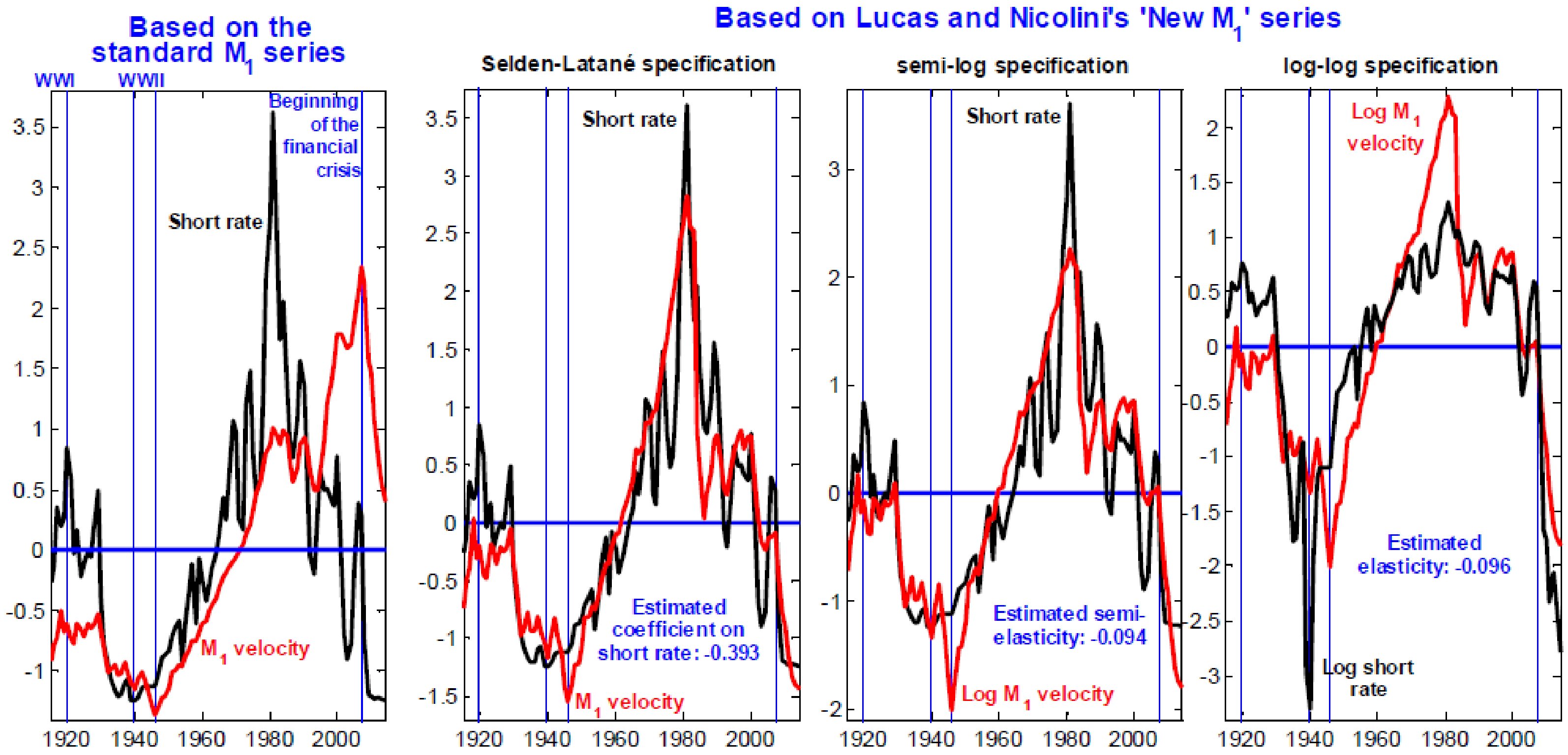


Figure 1 United States, 1915-2014: M_1 velocity and the short-term nominal interest rate (de-meaned and standardized)

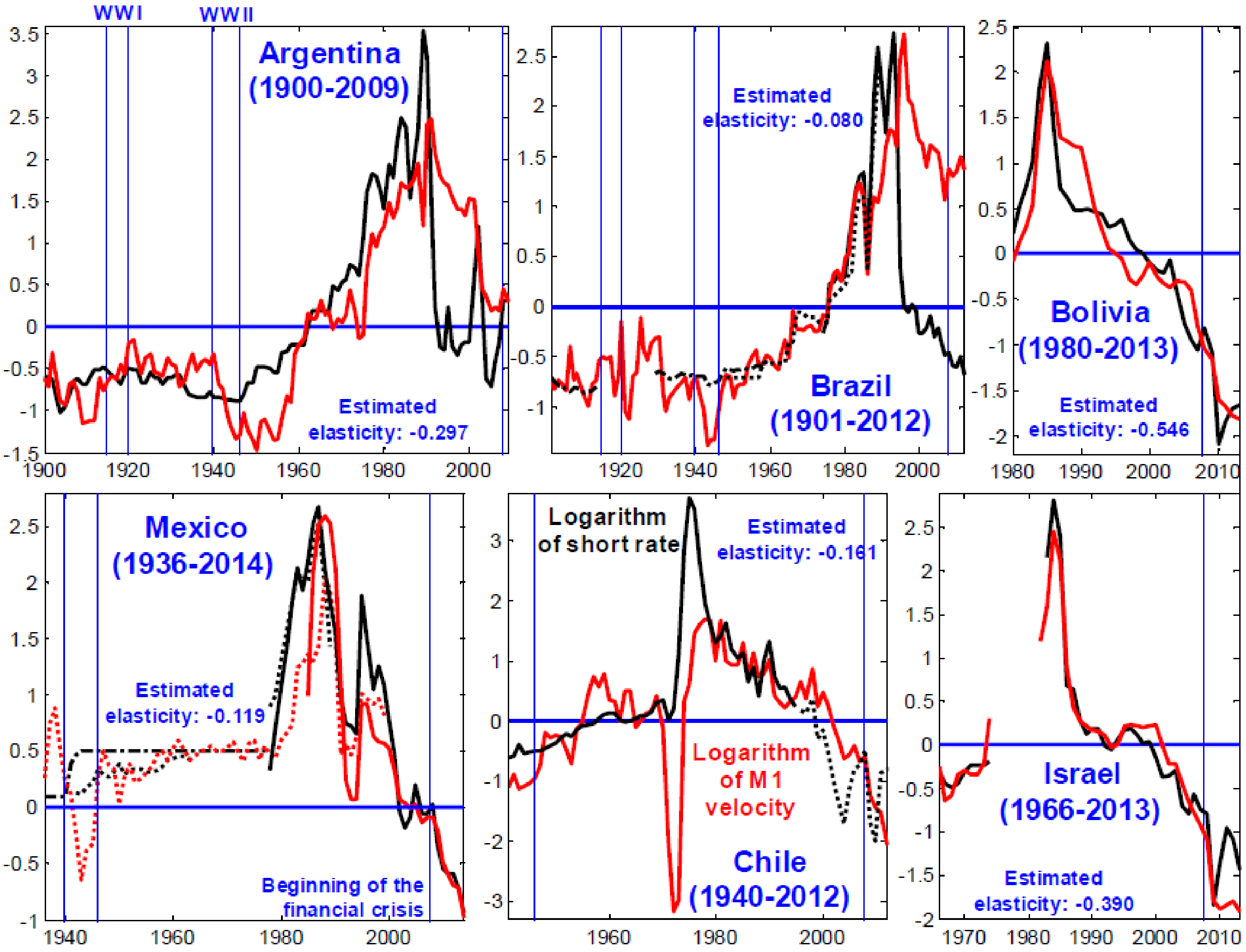


Figure 2 The logarithms of M_1 velocity and the short-term nominal interest rate, de-meaned and standardized (M_1 velocity computed as the ratio between nominal GDP and nominal M_1)

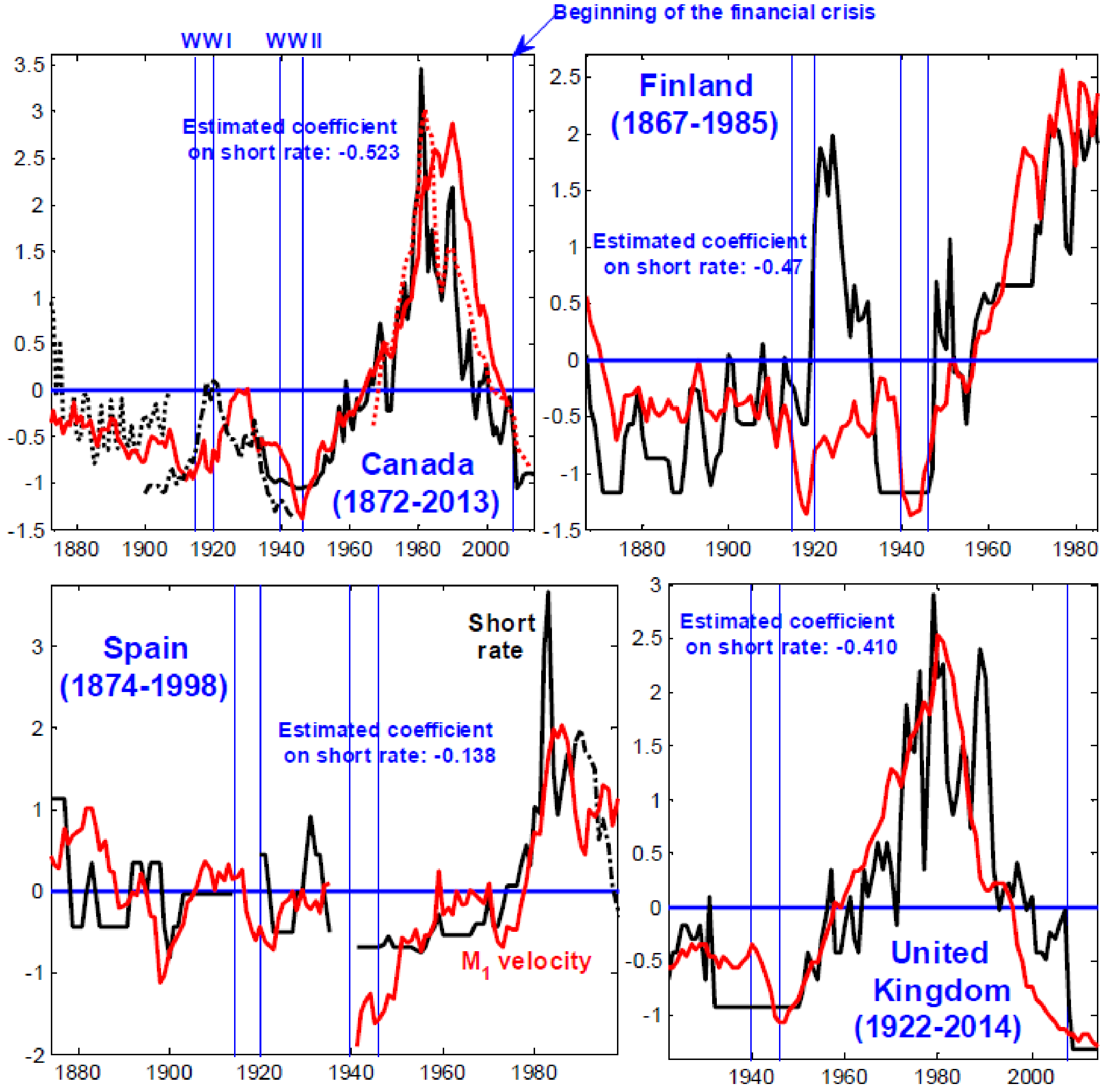


Figure 3 M_1 velocity and the short-term nominal interest rate, de-meaned and standardized (M_1 velocity computed as the ratio between nominal GDP and nominal M_1)

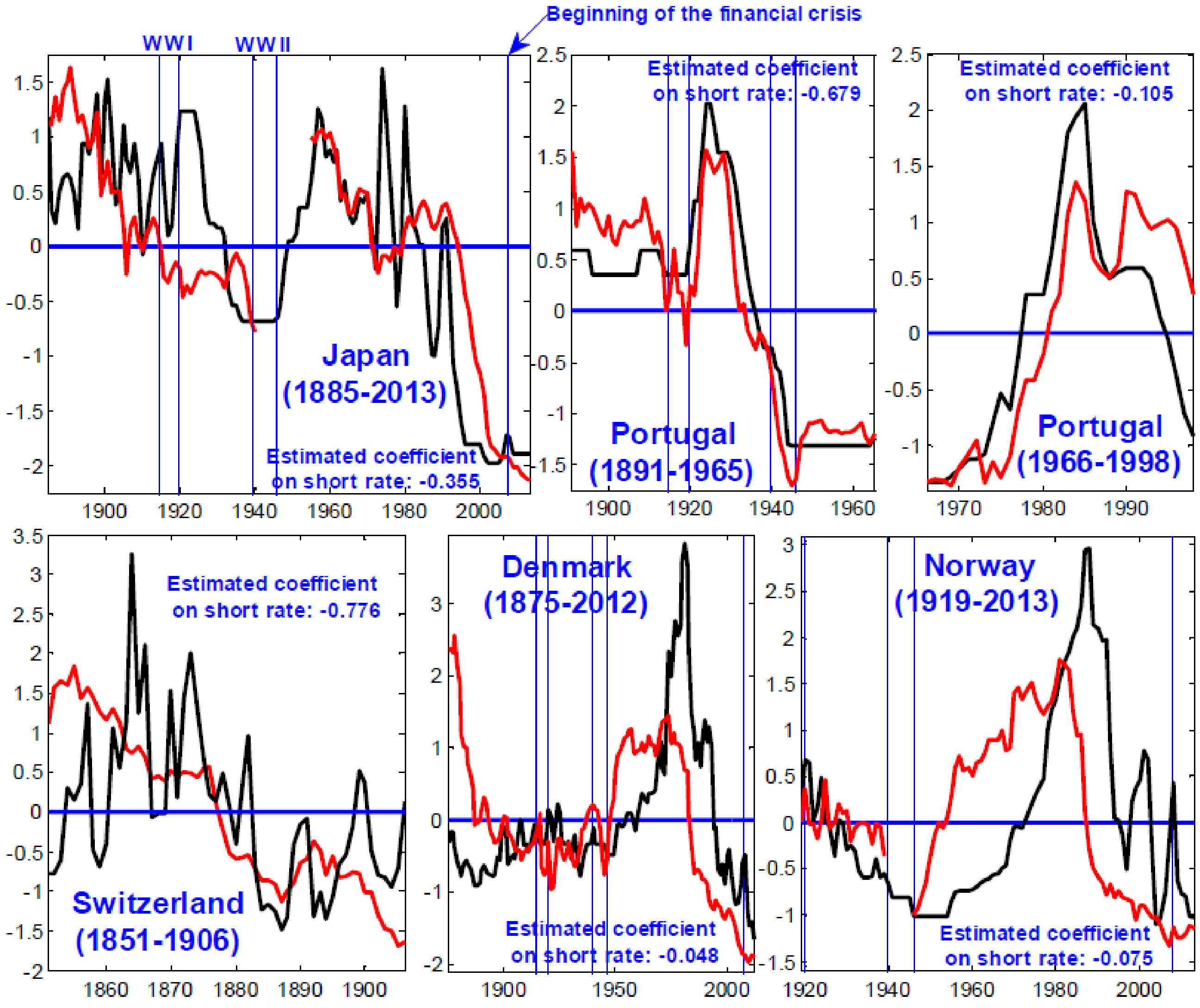


Figure 4 M$_1$ velocity and the short-term nominal interest rate, de-meaned and standardized (M$_1$ velocity computed as the ratio between nominal GDP and nominal M$_1$)

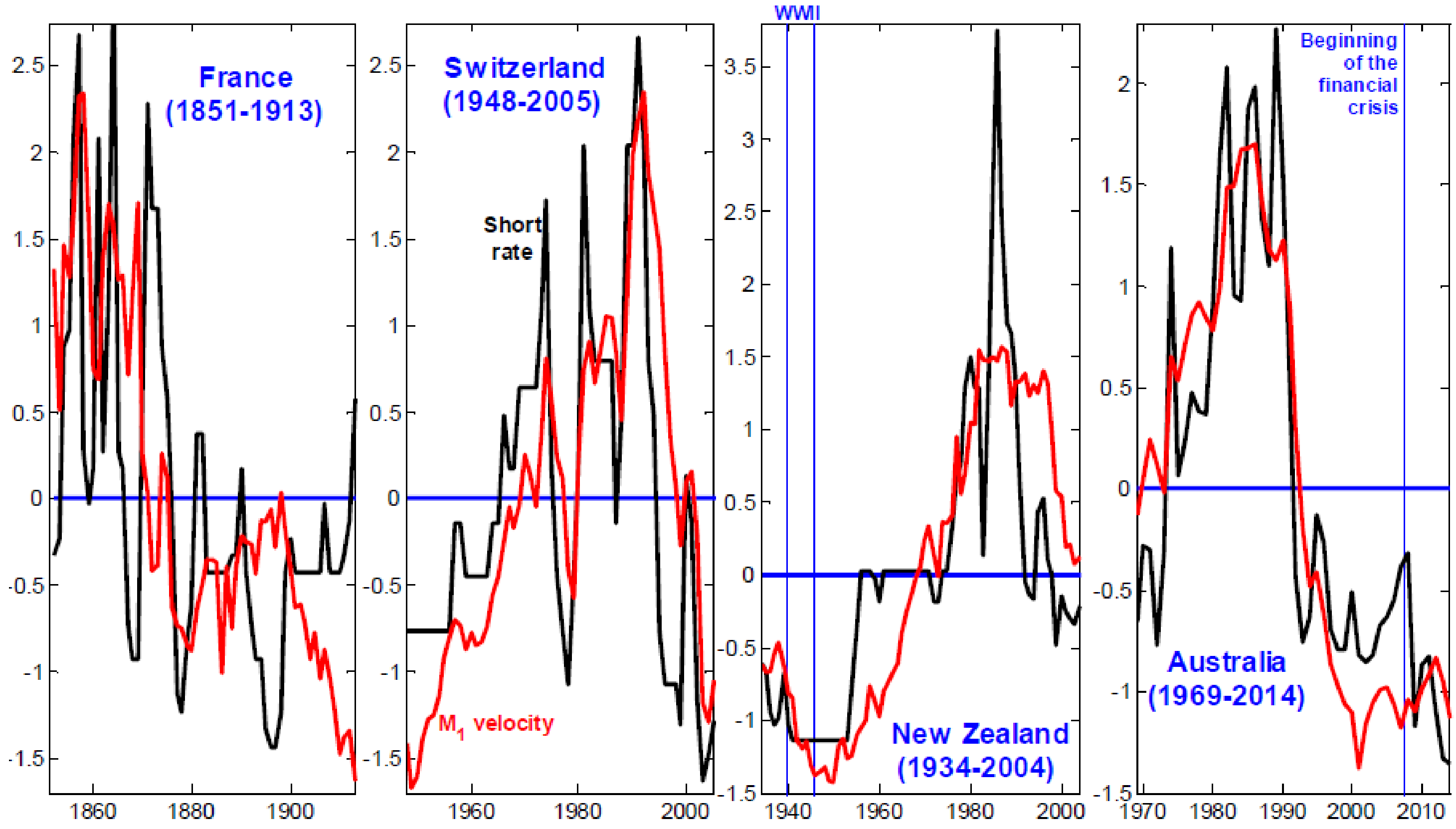


Figure 5 M_1 velocity and the short-term nominal interest rate, de-meaned and standardized (M_1 velocity computed as the ratio between nominal GDP and nominal M_1)

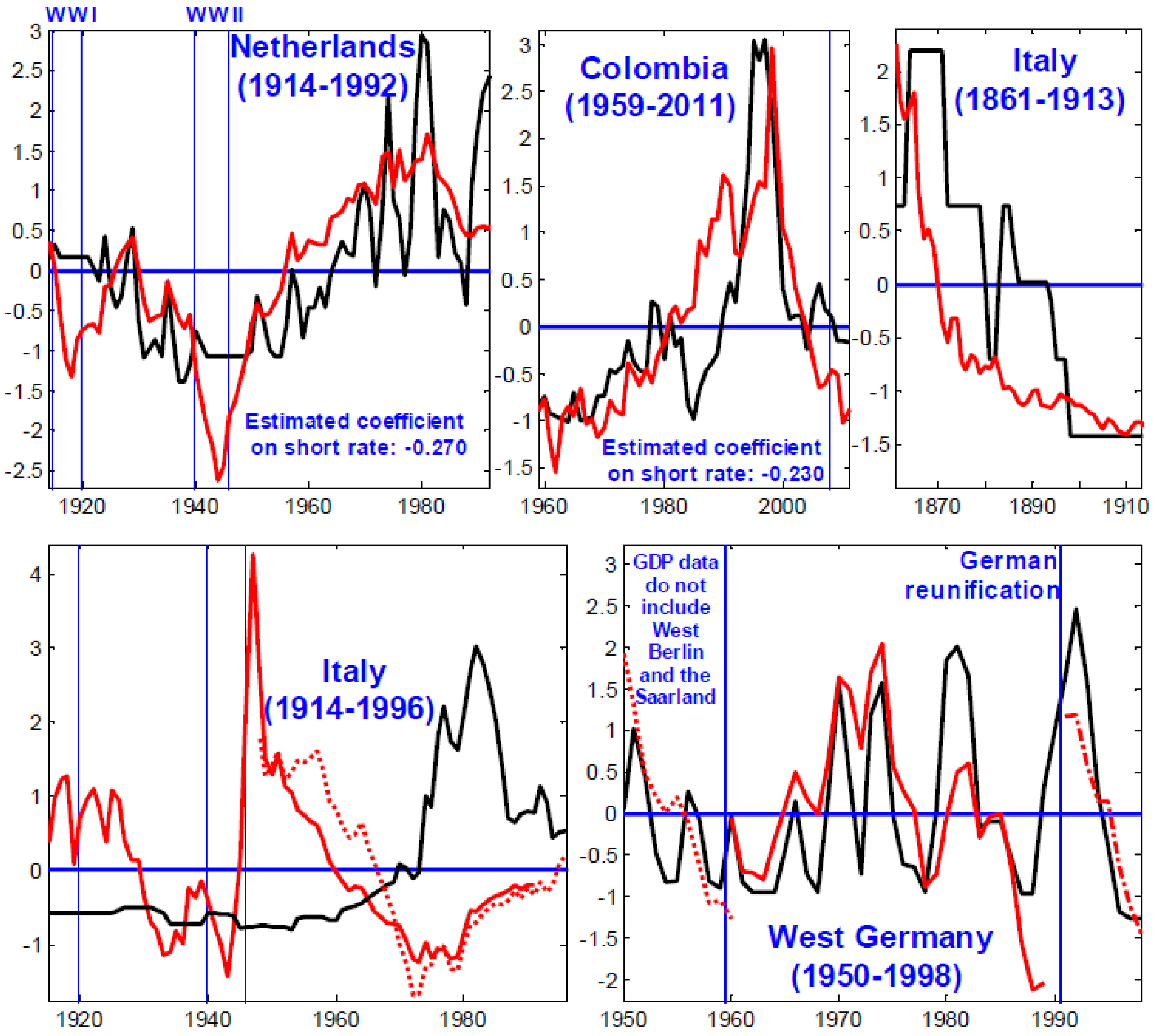


Figure 6 M_1 velocity and the short-term nominal interest rate, de-meaned and standardized (M_1 velocity computed as the ratio between nominal GDP and nominal M_1)

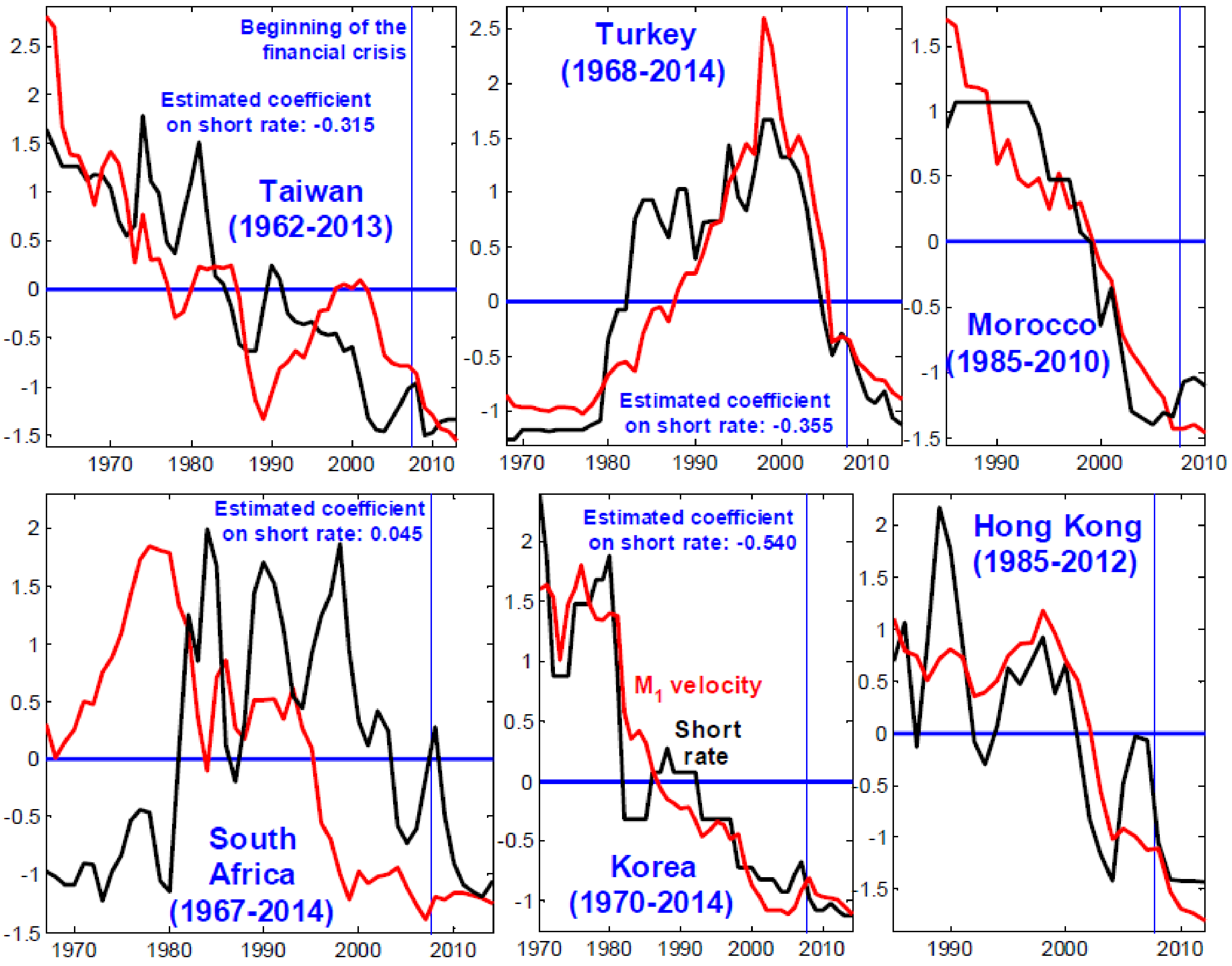


Figure 7 M_1 velocity and the short-term nominal interest rate, de-meaned and standardized (M_1 velocity computed as the ratio between nominal GDP and nominal M_1)

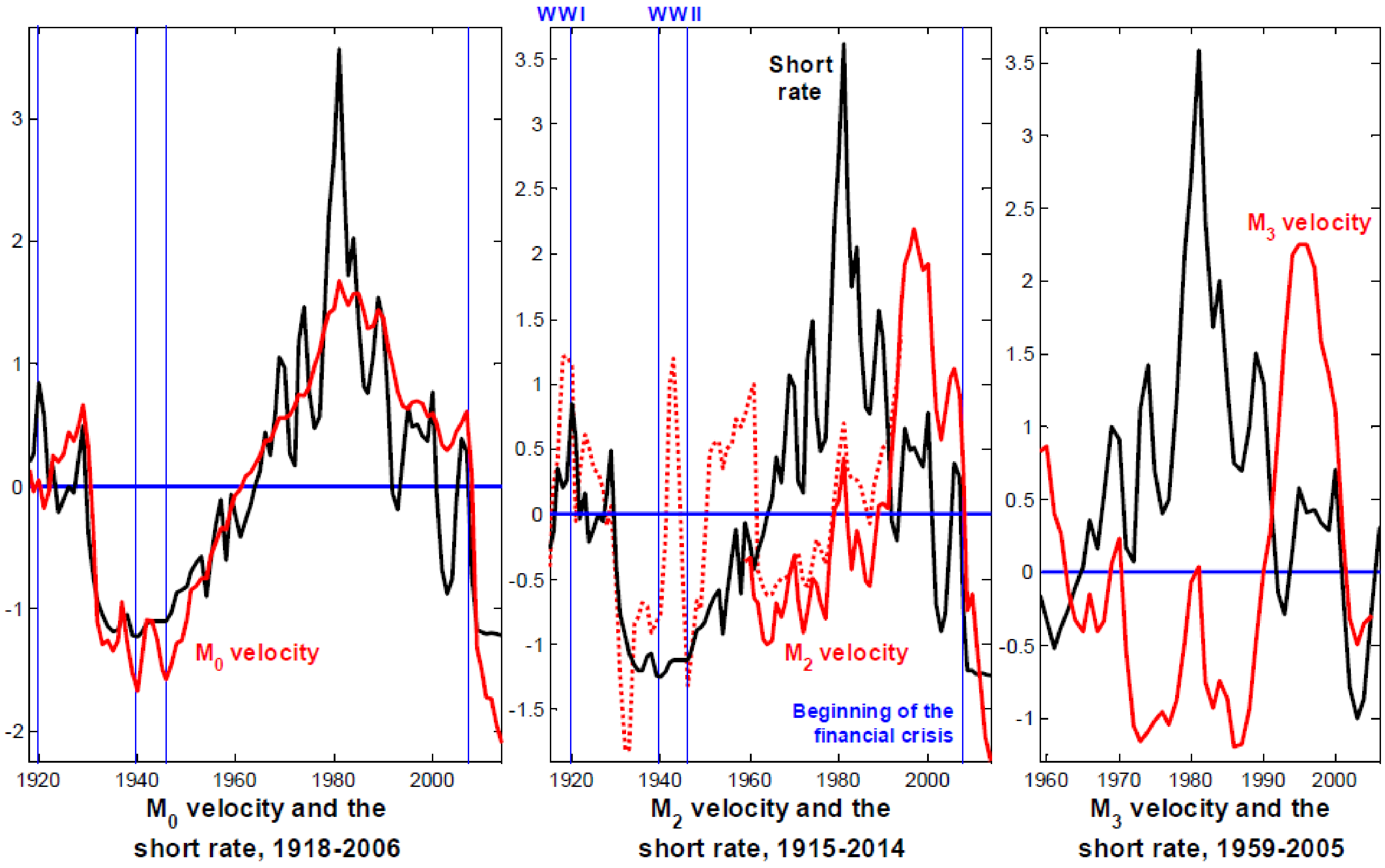


Figure 8 United States: M_0, M_2, and M_3, velocity and the short-term nominal interest rate (de-meaned and standardized)

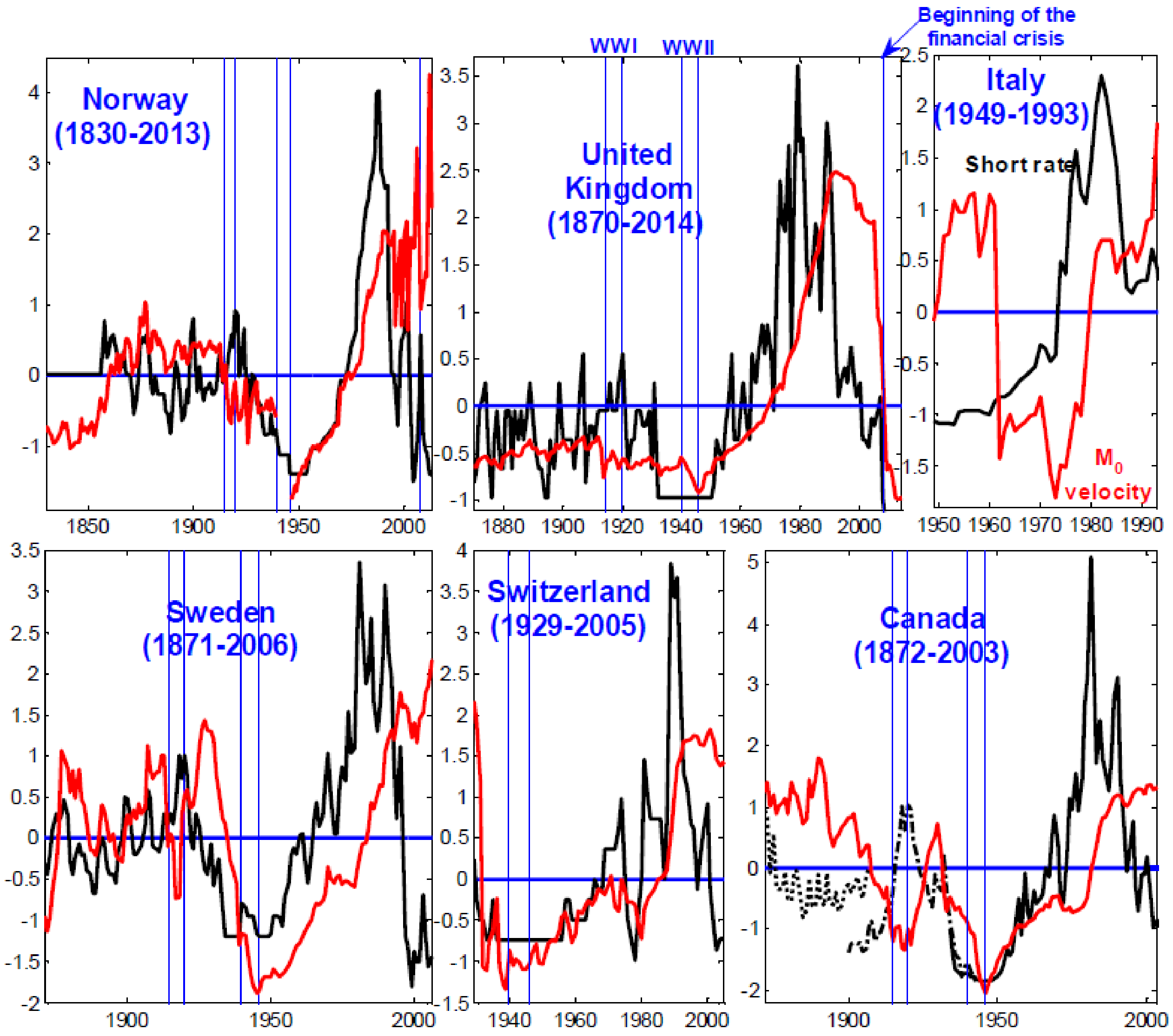


Figure 9 M_0 velocity and the short-term nominal interest rate for selected countries other than the United States (de-meaned and standardized)

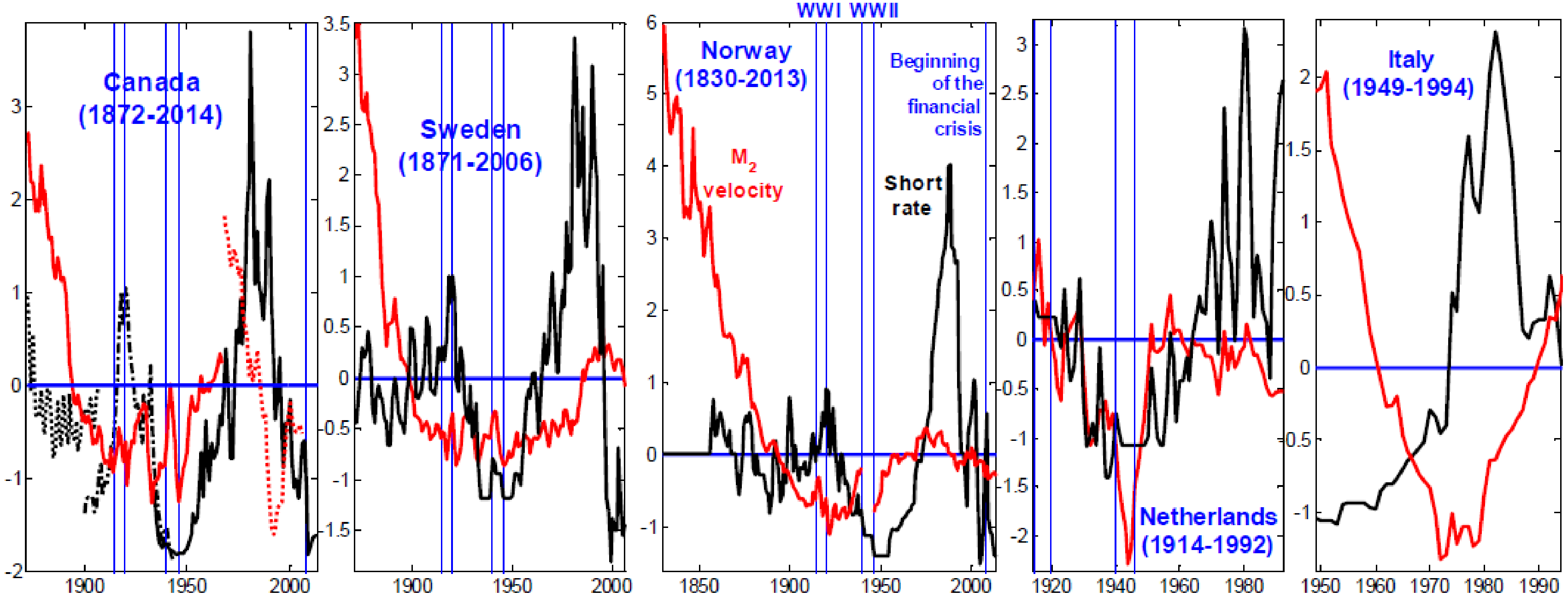


Figure 10 M$_2$ velocity and the short-term nominal interest rate for selected countries other than the United States (de-meaned and standardized)

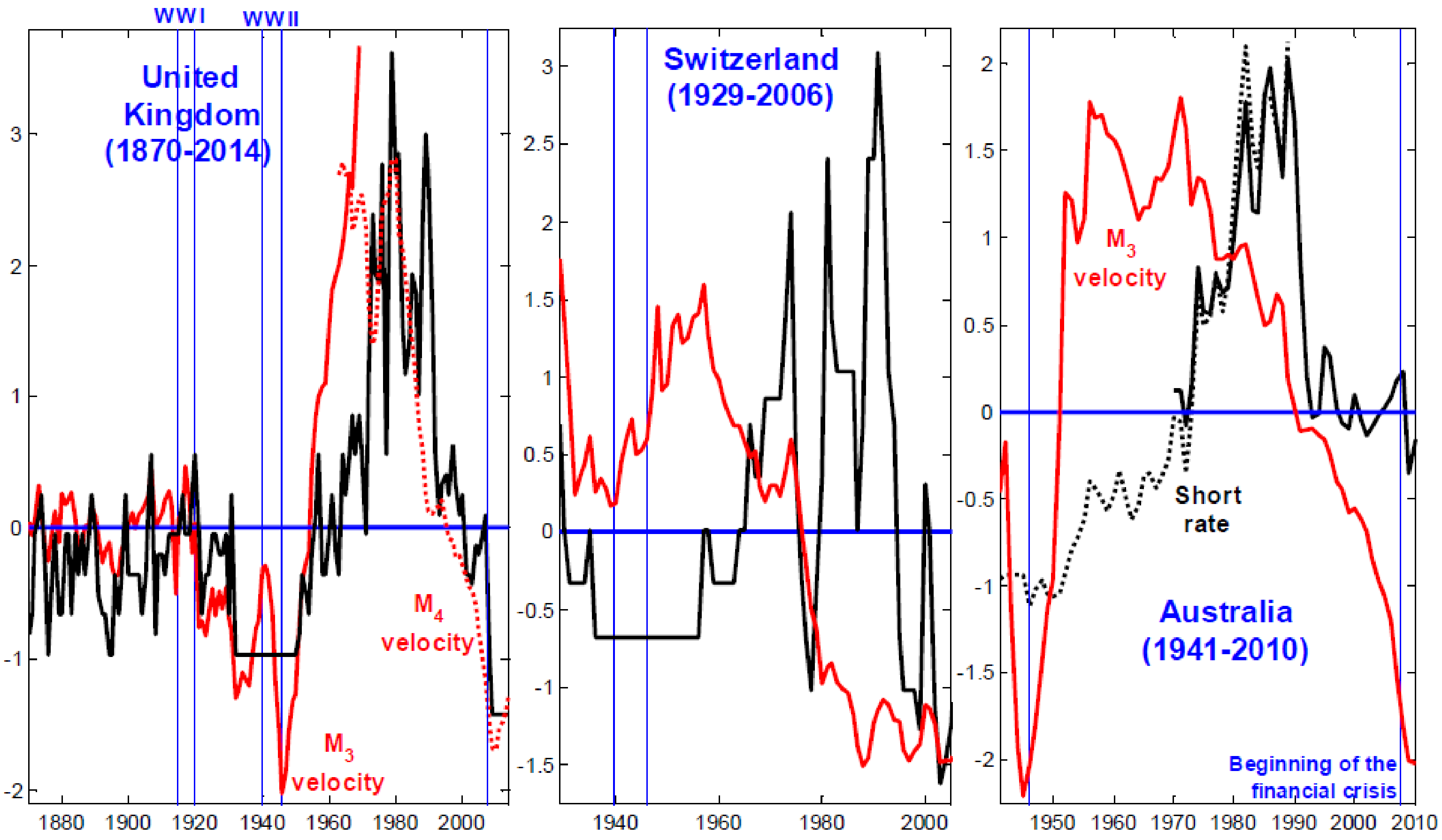
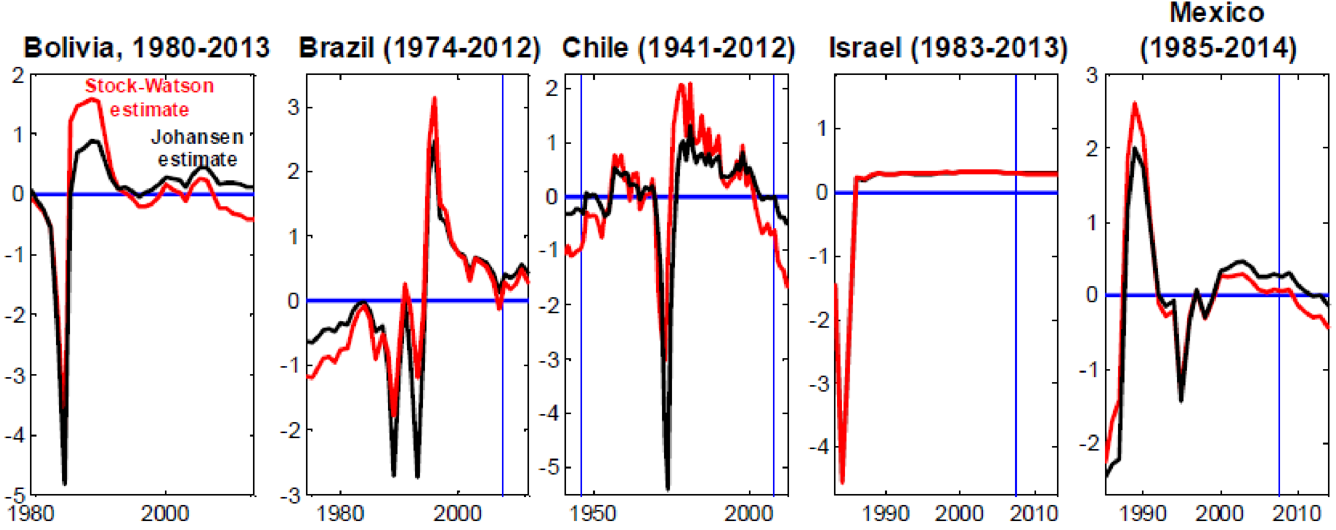


Figure 11 M3 velocity and the short-term nominal interest rate for selected countries other than the United States (de-meaned and standardized)

II: Full set of results based on the Selden-Latané specification

Candidate cointegration residuals, de-meaned and standardized



Bootstrapped distributions of the coefficient on the short rate

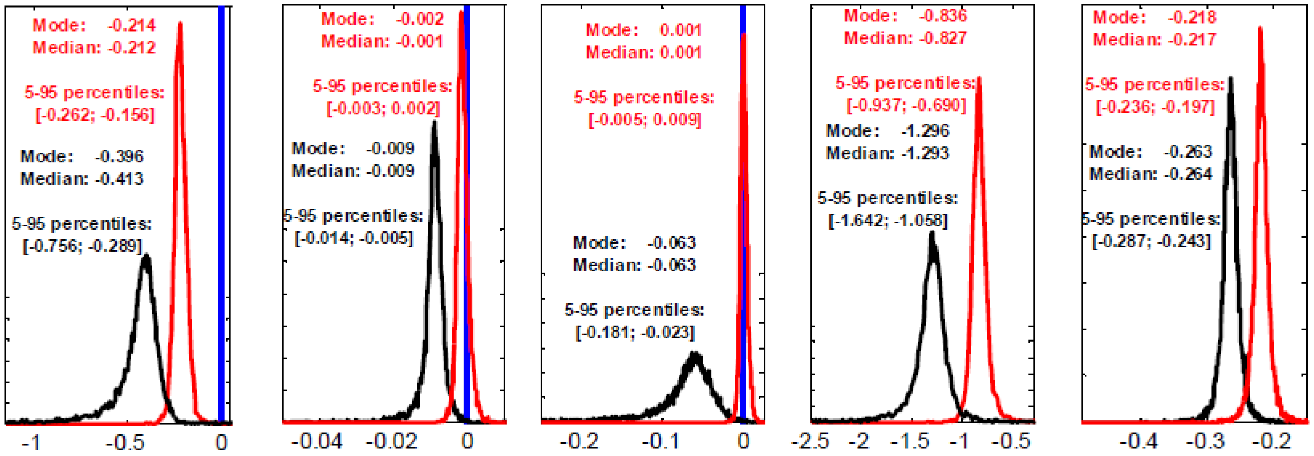


Figure SELA.1 Selden-Latané specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the short rate

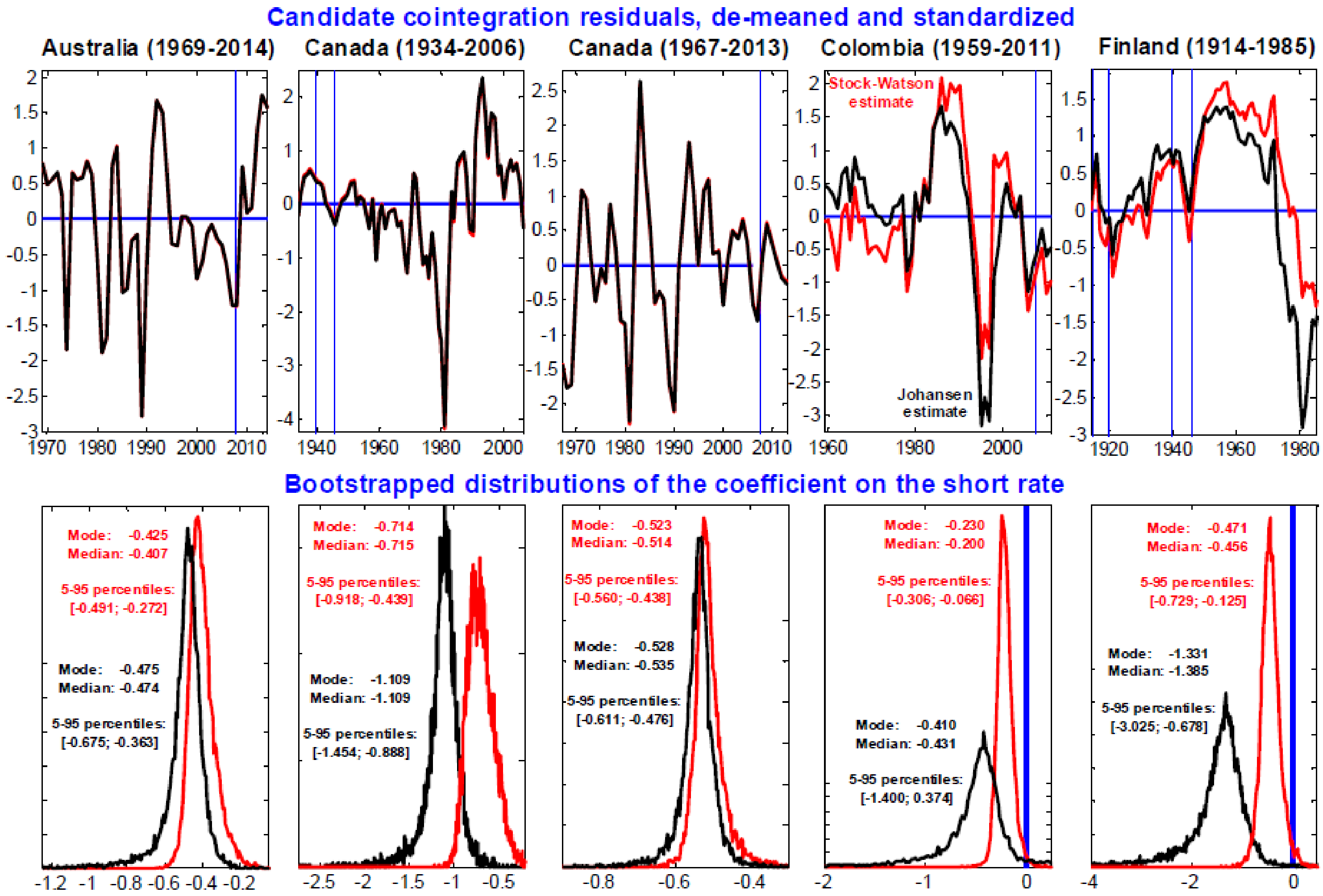


Figure SELA.2 Selden-Latané specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the short rate

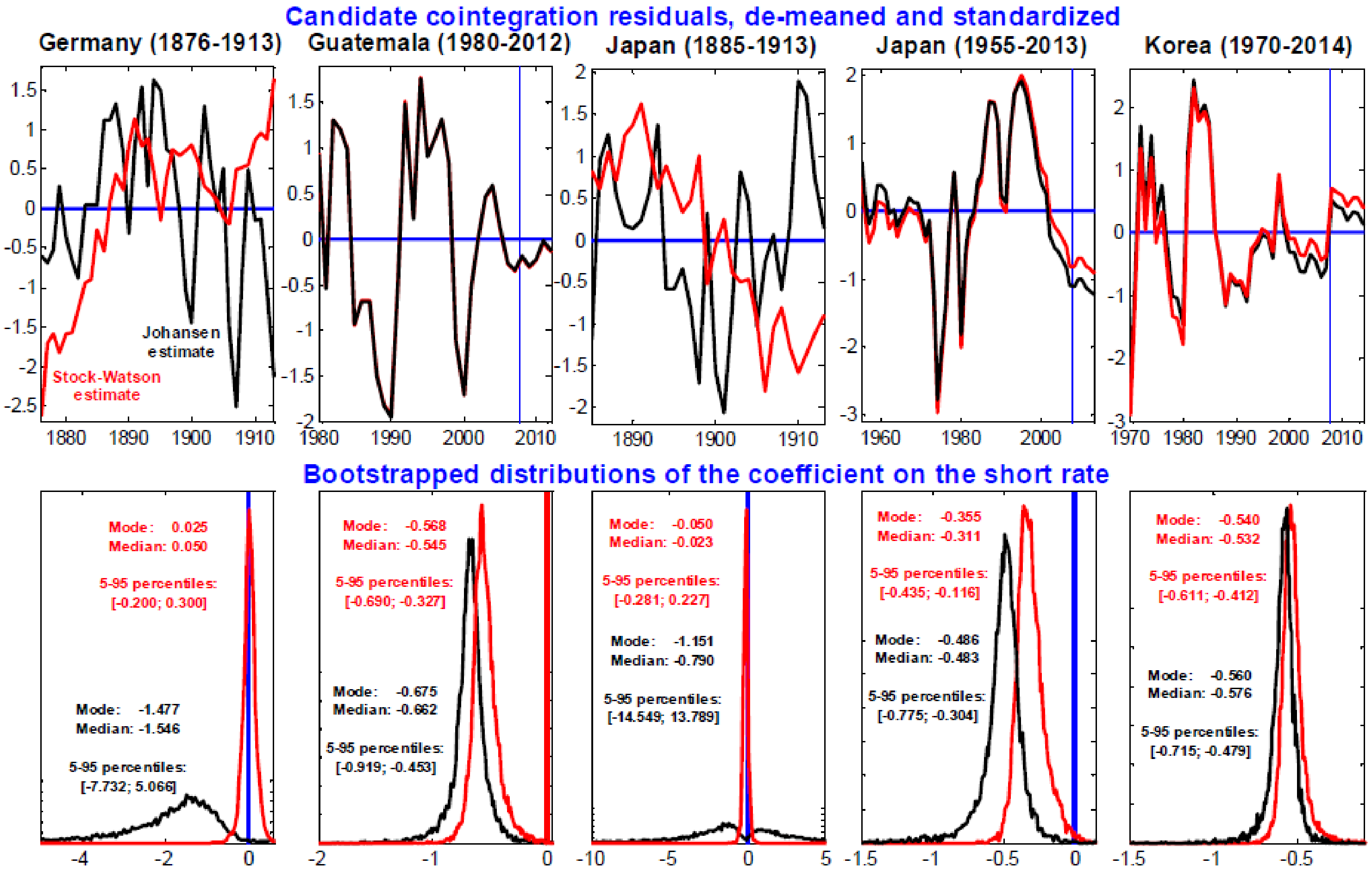


Figure SELA.3 Selden-Latané specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the short rate

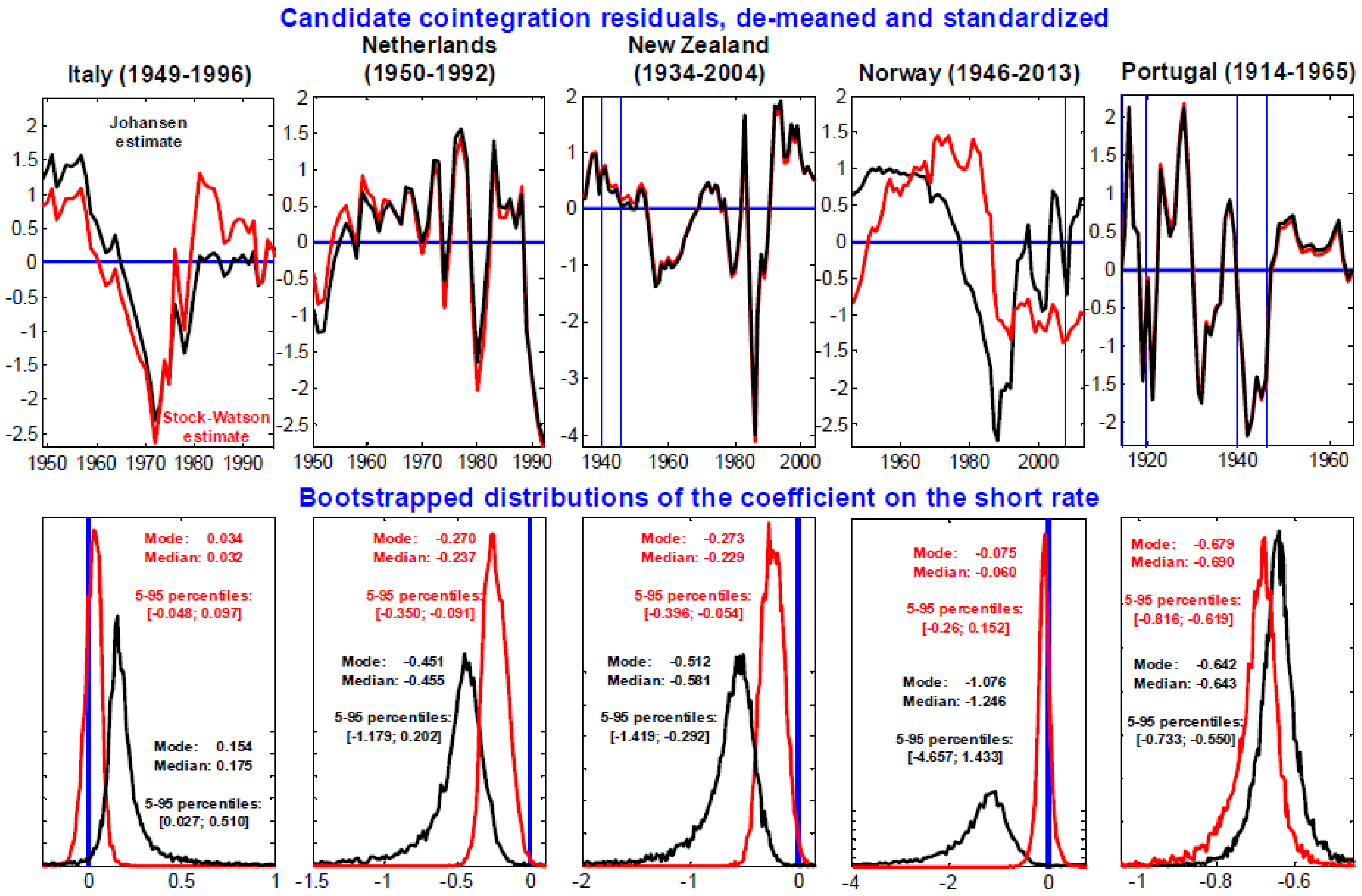


Figure SELA.4 Selden-Latané specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the short rate

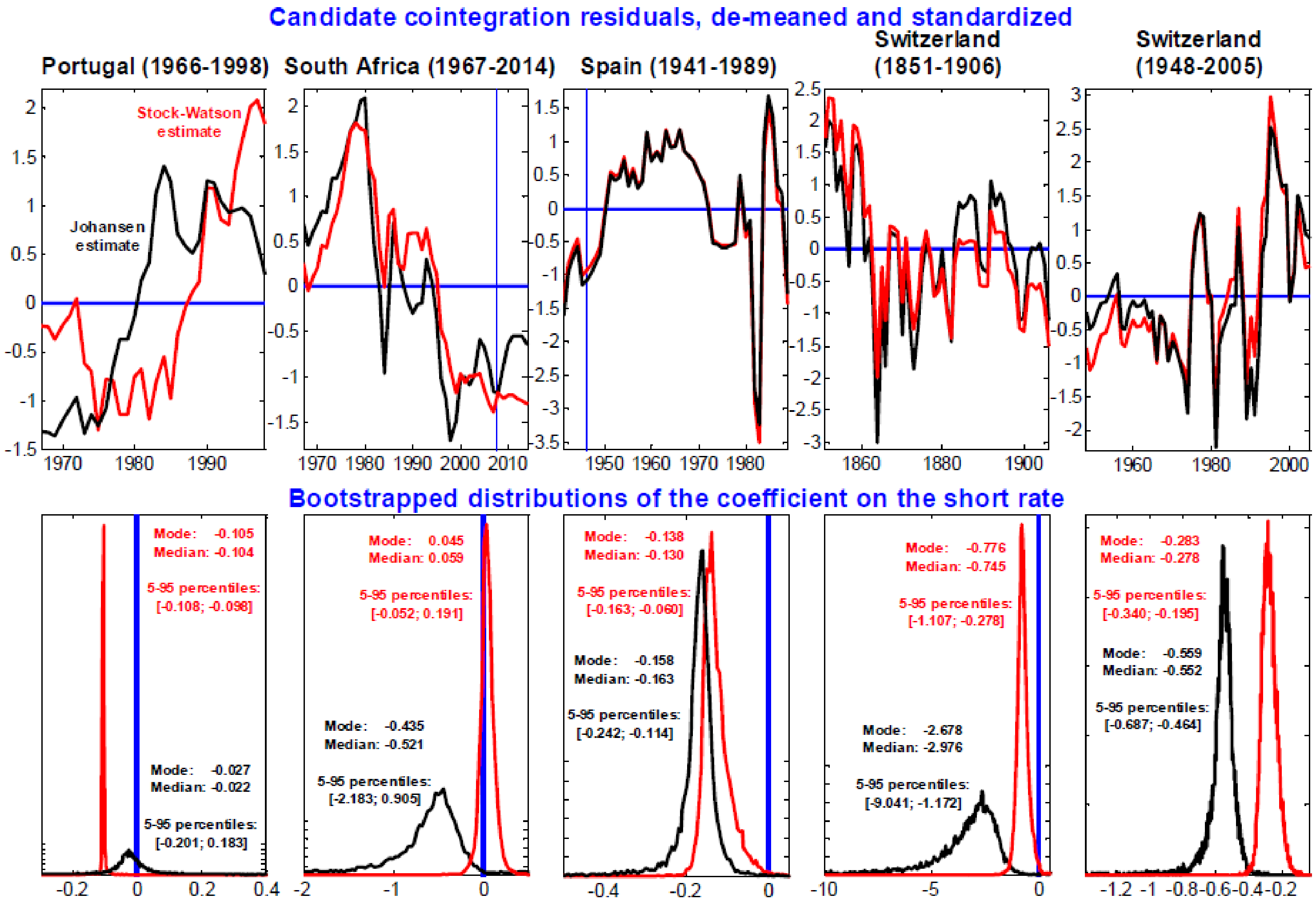


Figure SELA.5 Selden-Latané specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the short rate

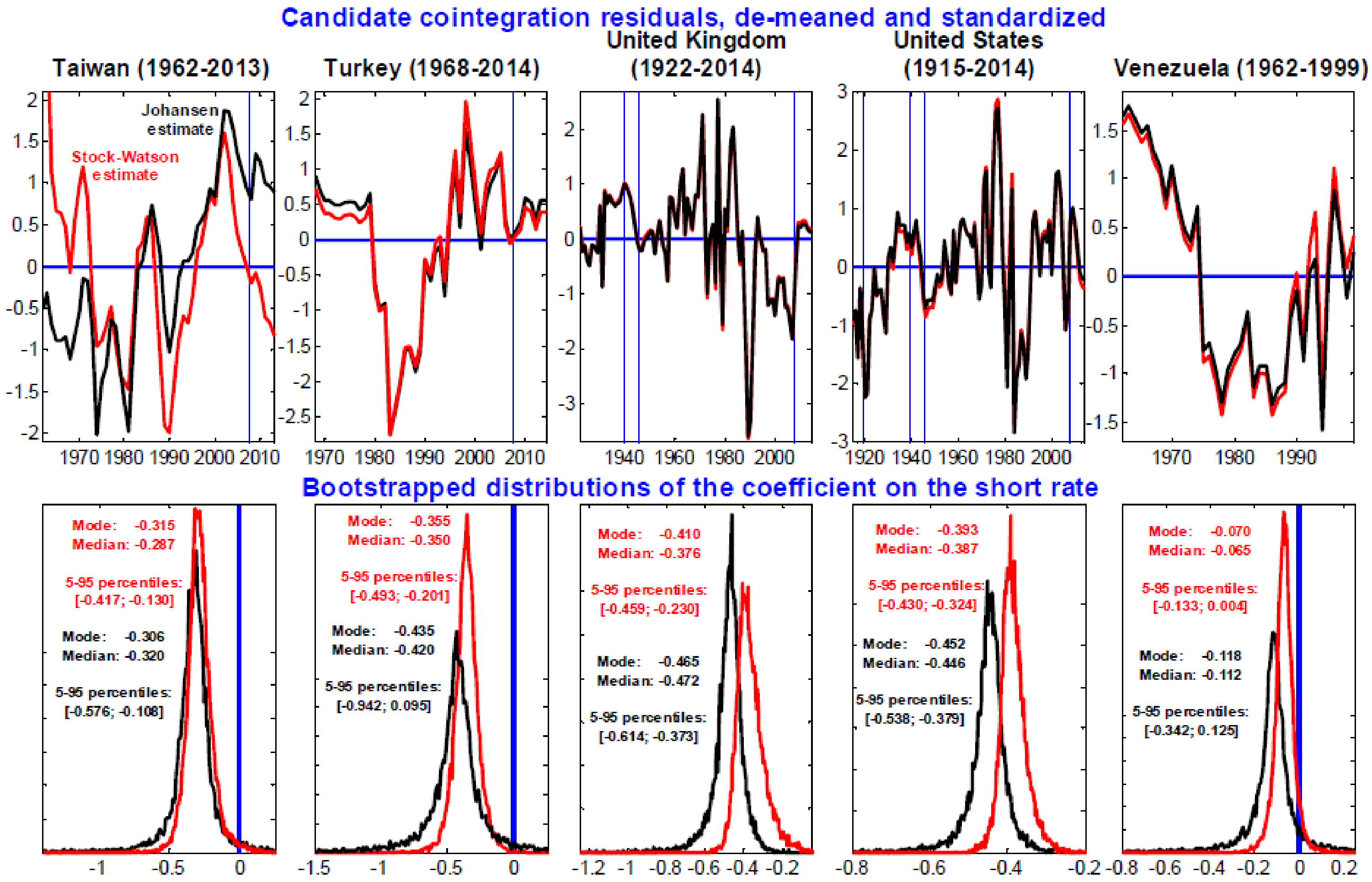


Figure SELA.6 Selden-Latané specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the short rate

III: Full set of results based on the semi-log specification

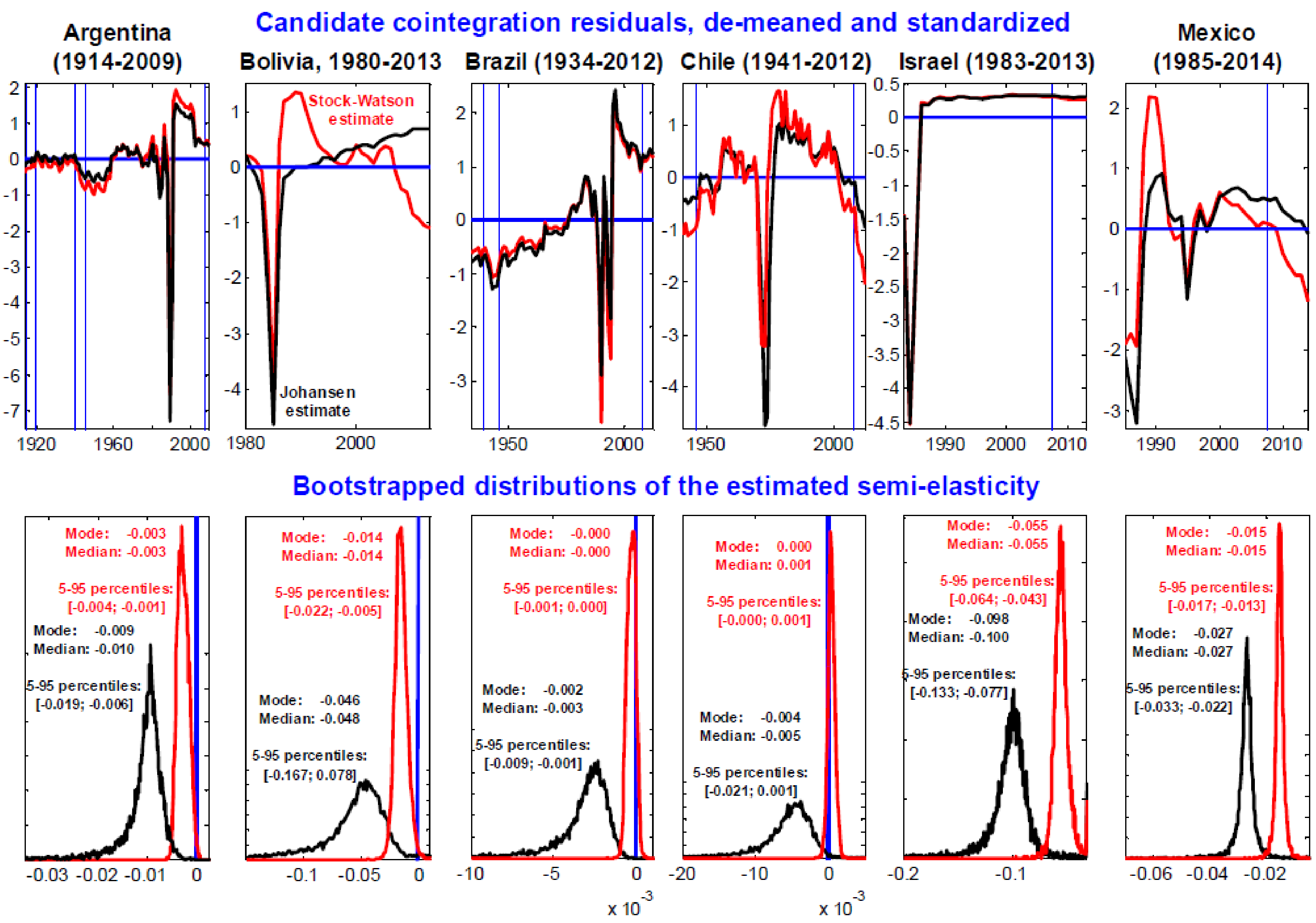


Figure SL.1 Semi-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

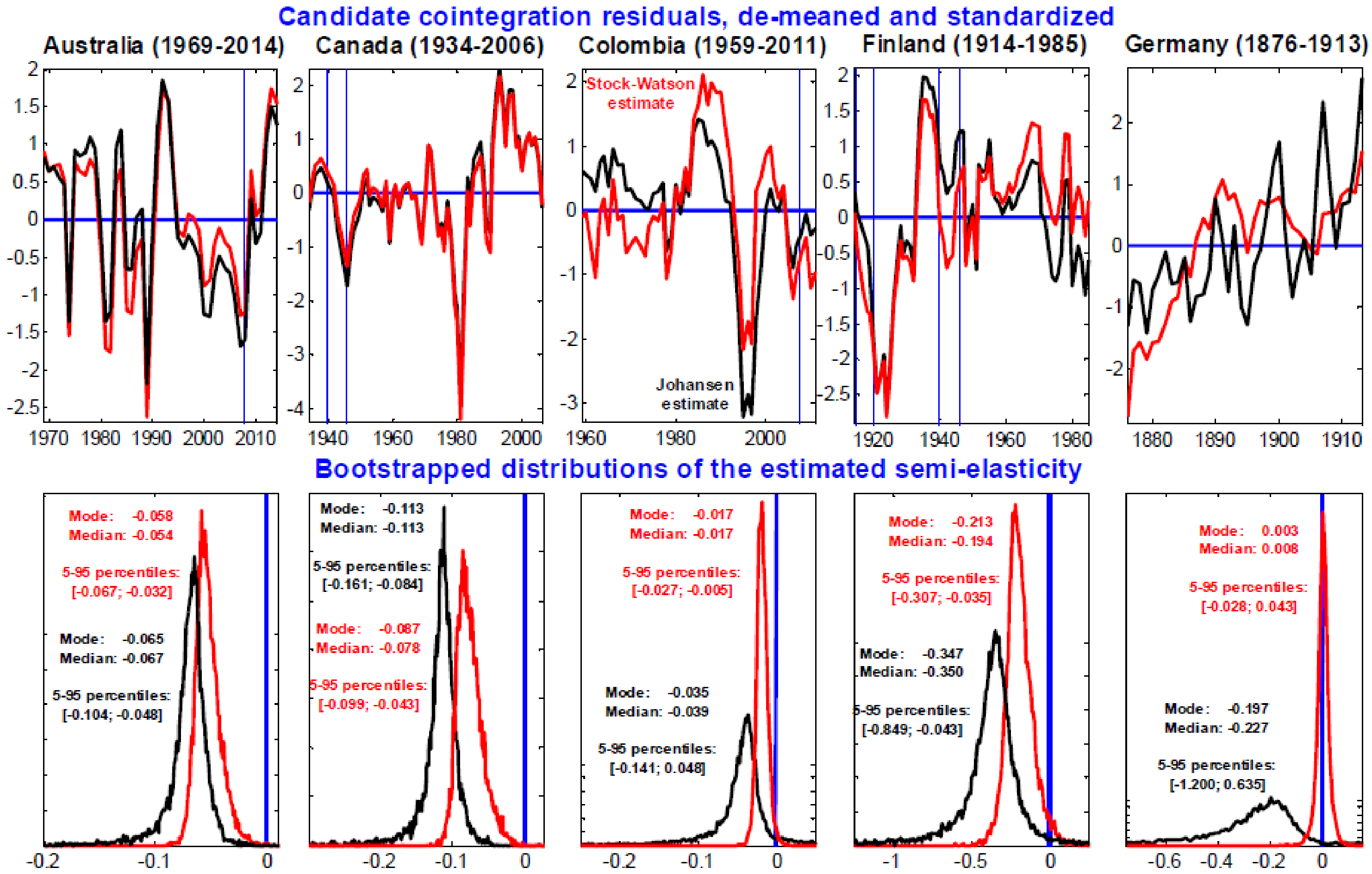


Figure SL.2 Semi-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

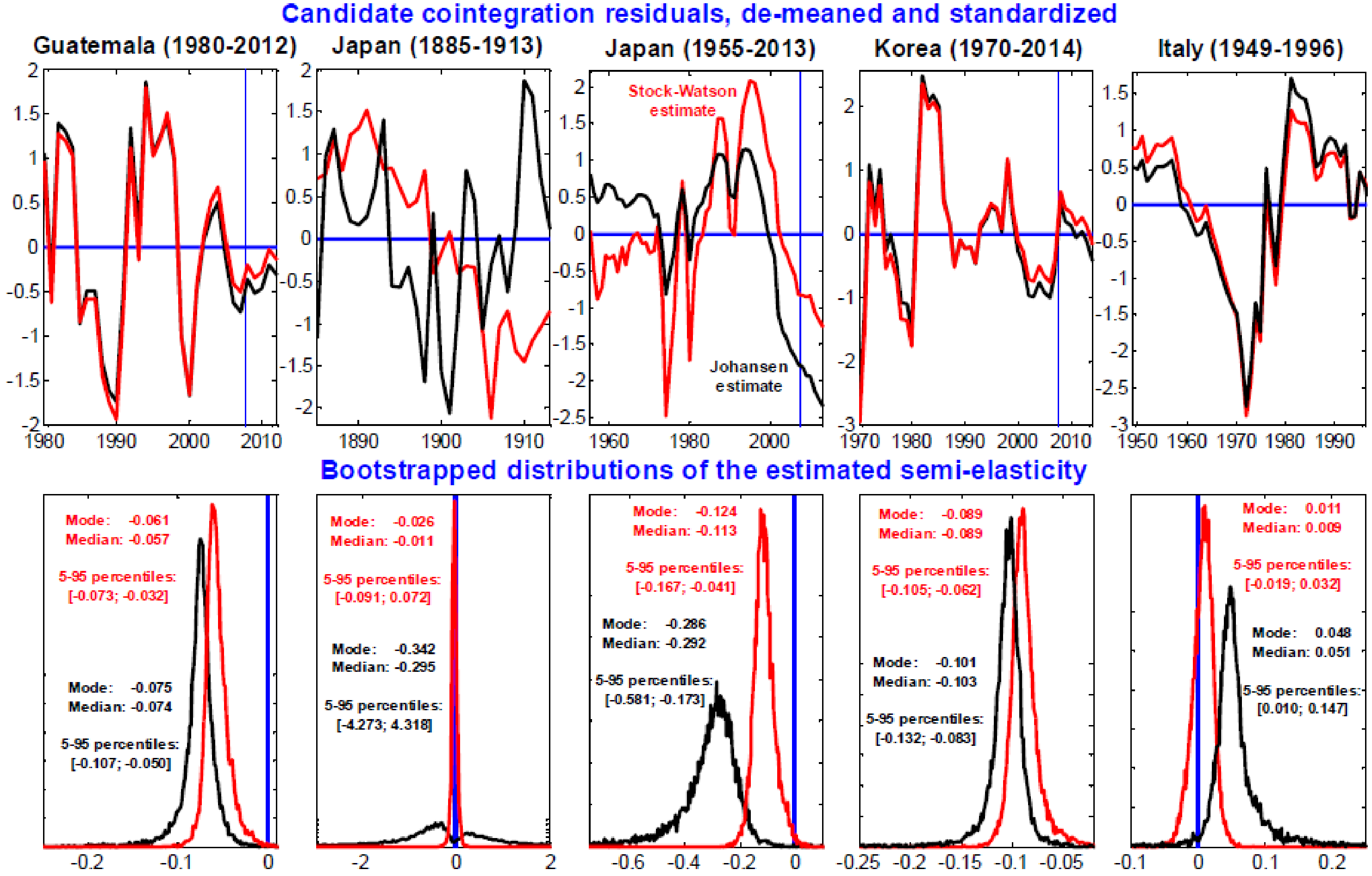


Figure SL.3 Semi-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

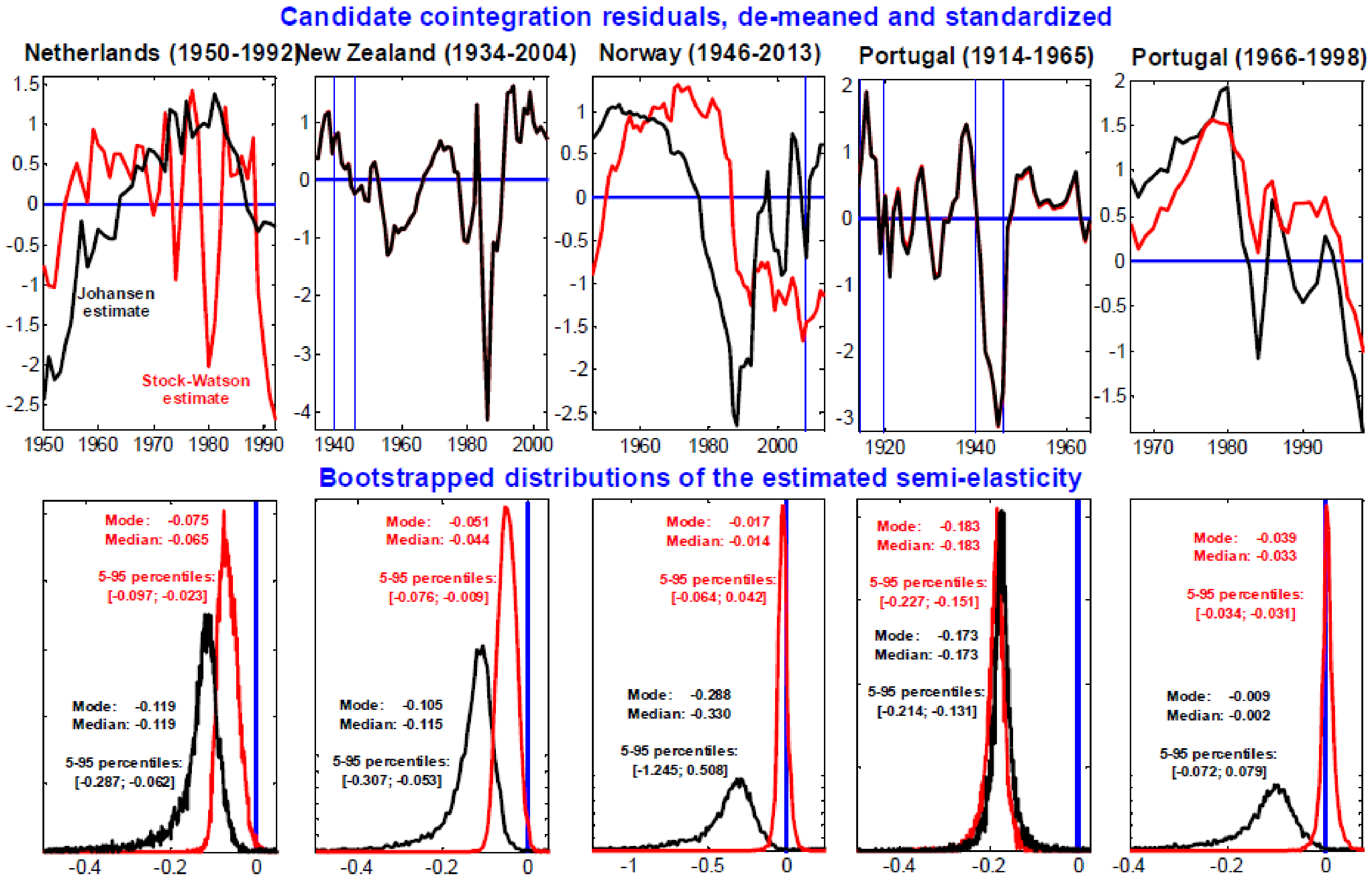


Figure SL.4 Semi-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

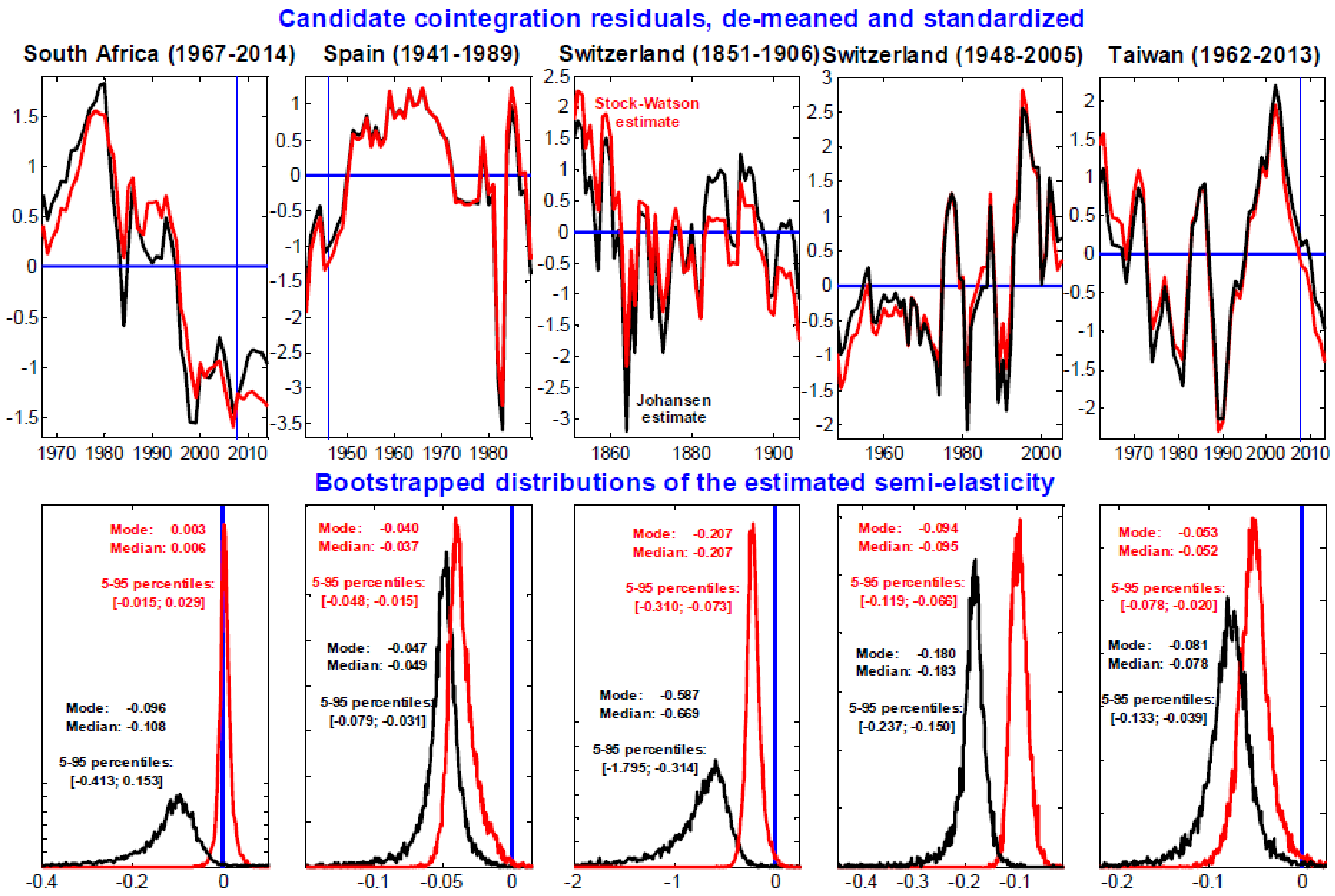


Figure SL.5 Semi-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

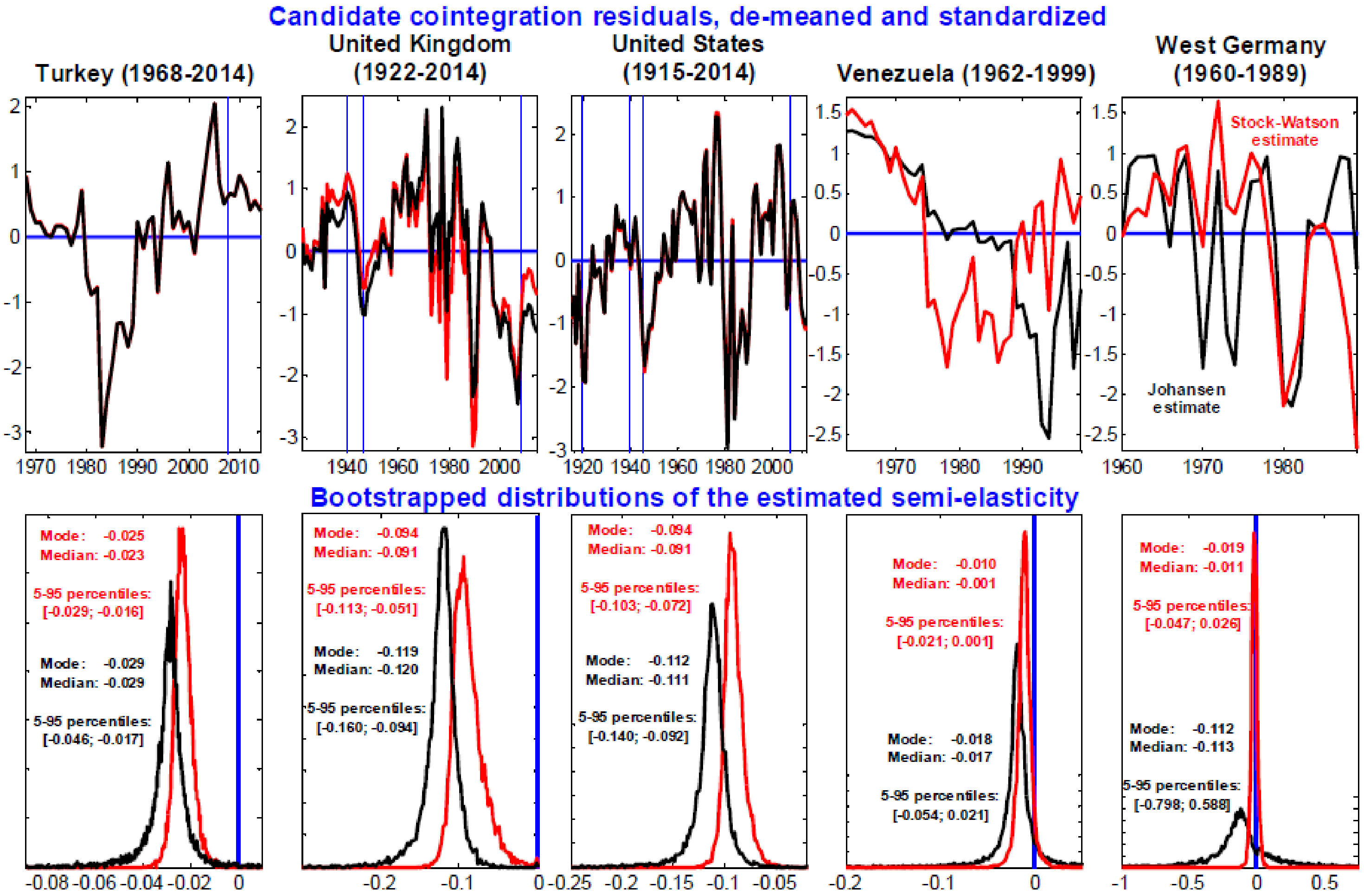


Figure SL.6 Semi-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

IV: Full set of results based on the log-log specification

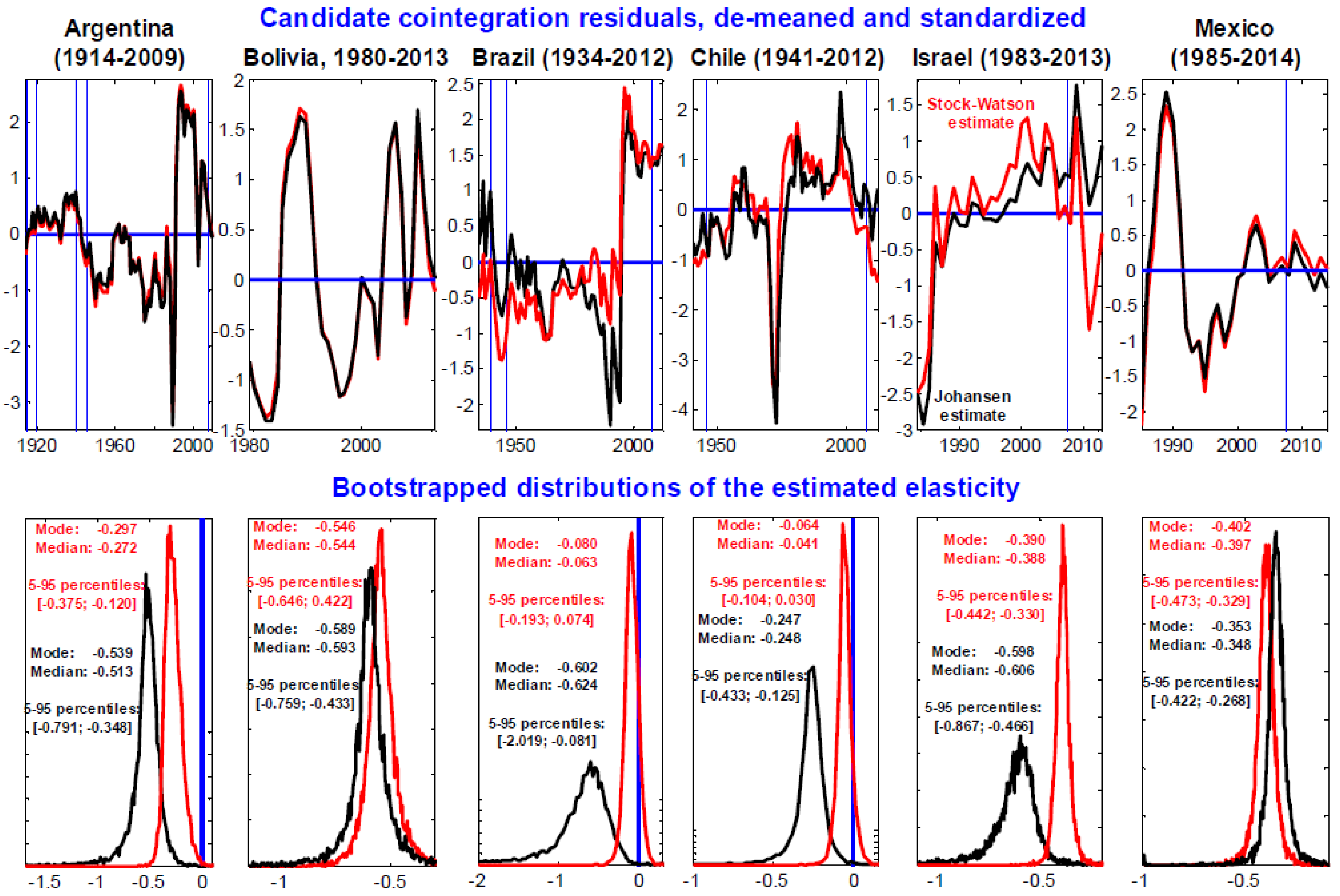


Figure LL.1 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

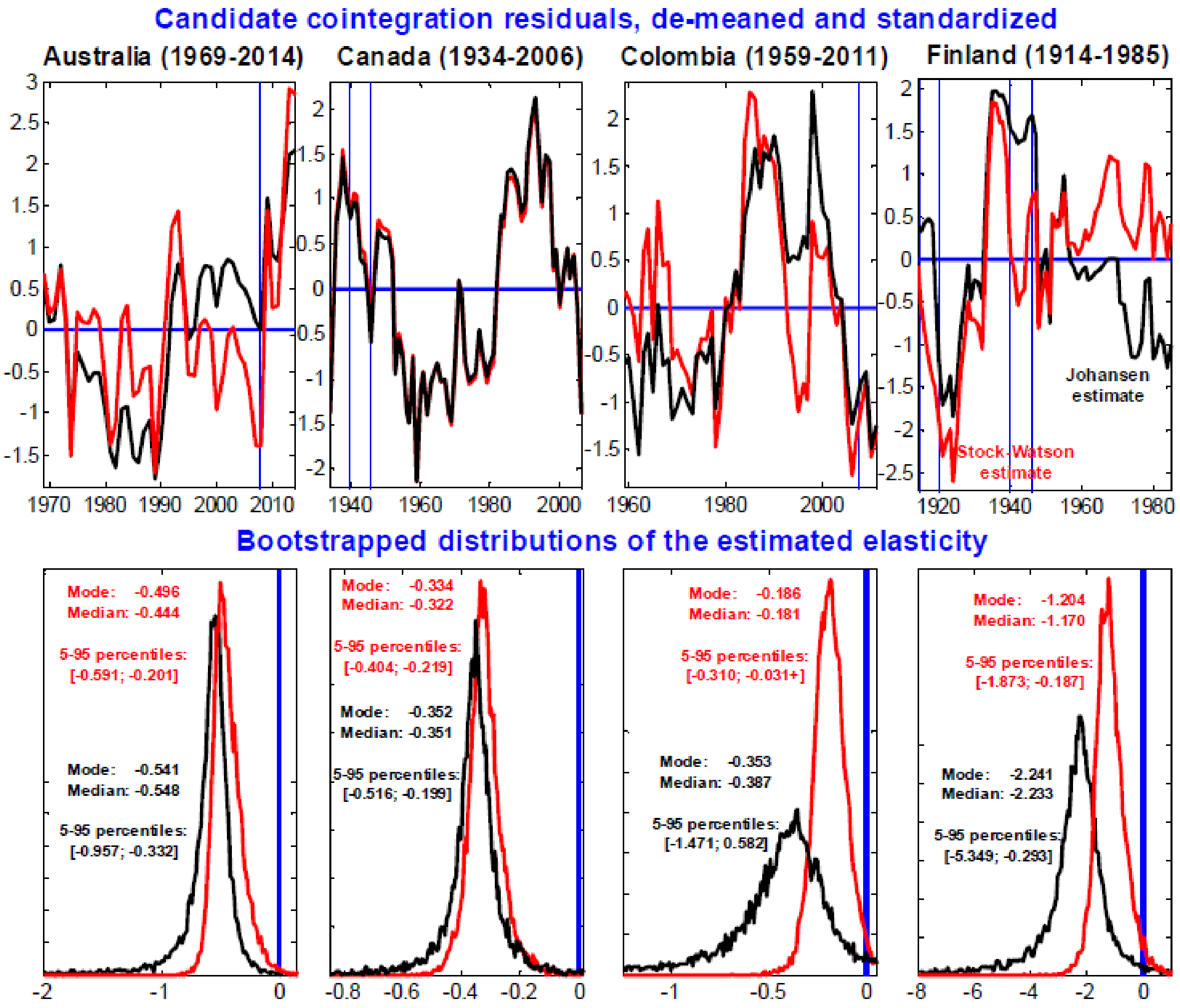


Figure LL.2 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

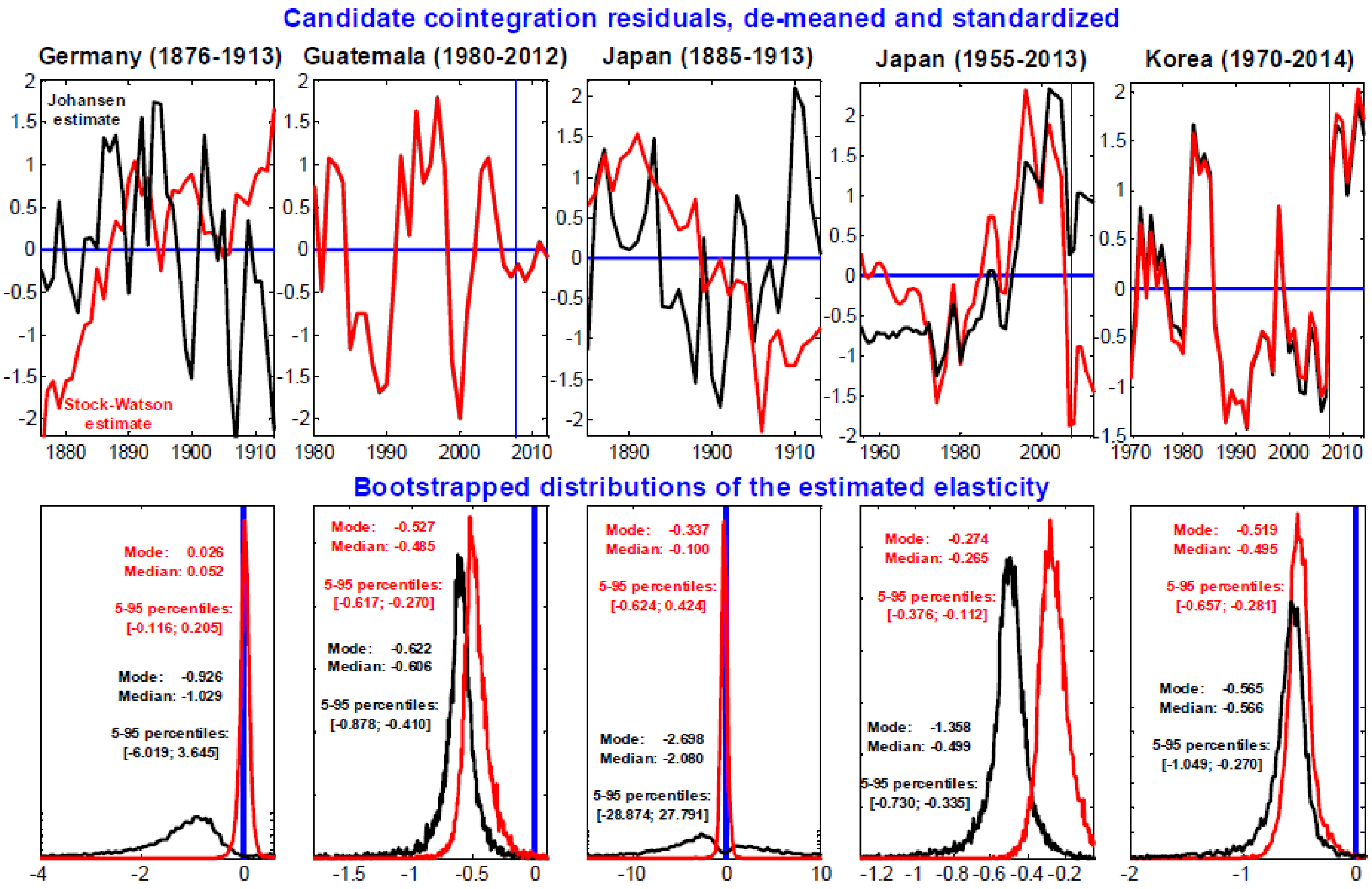


Figure LL.3 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

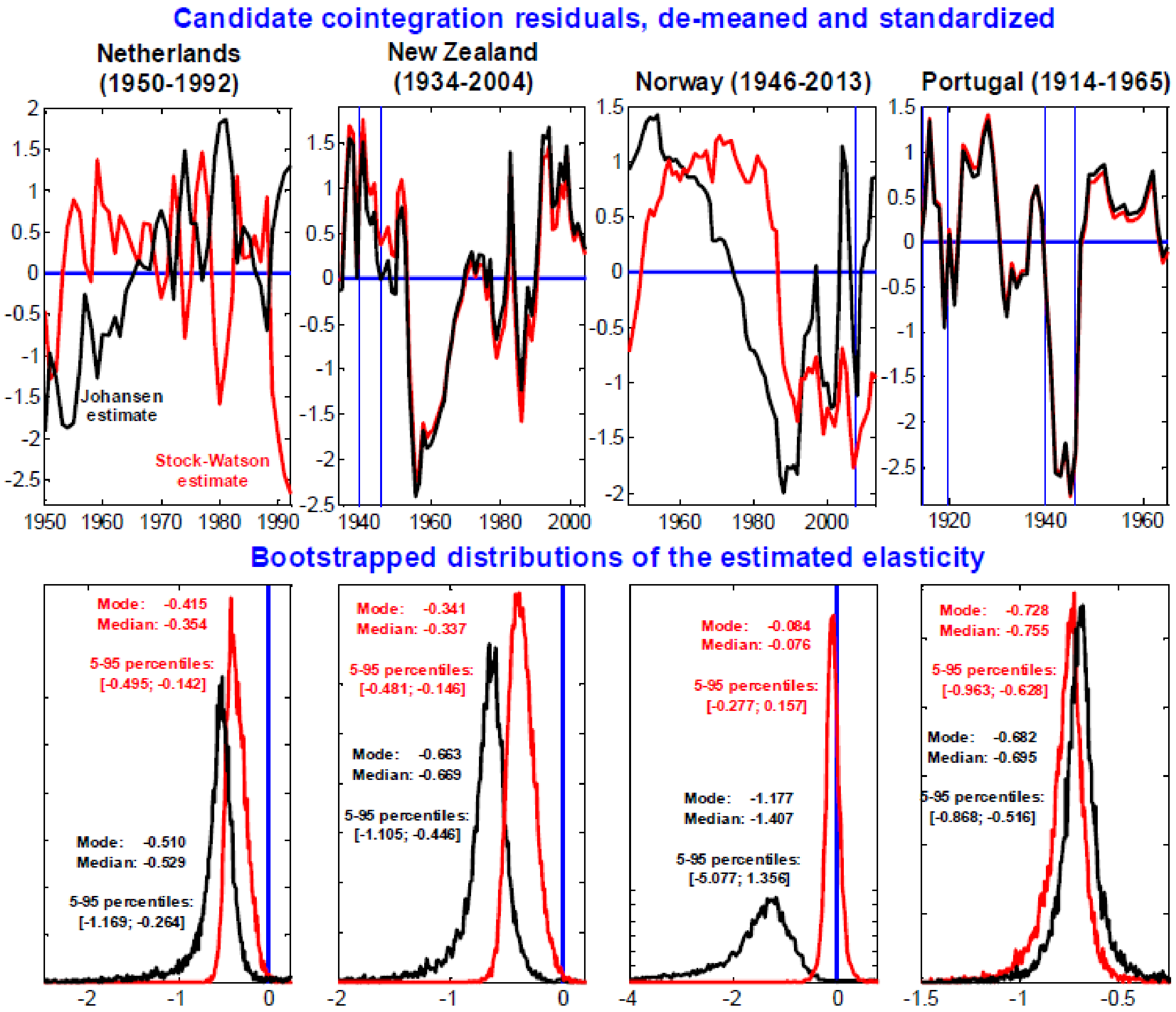


Figure LL.4 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

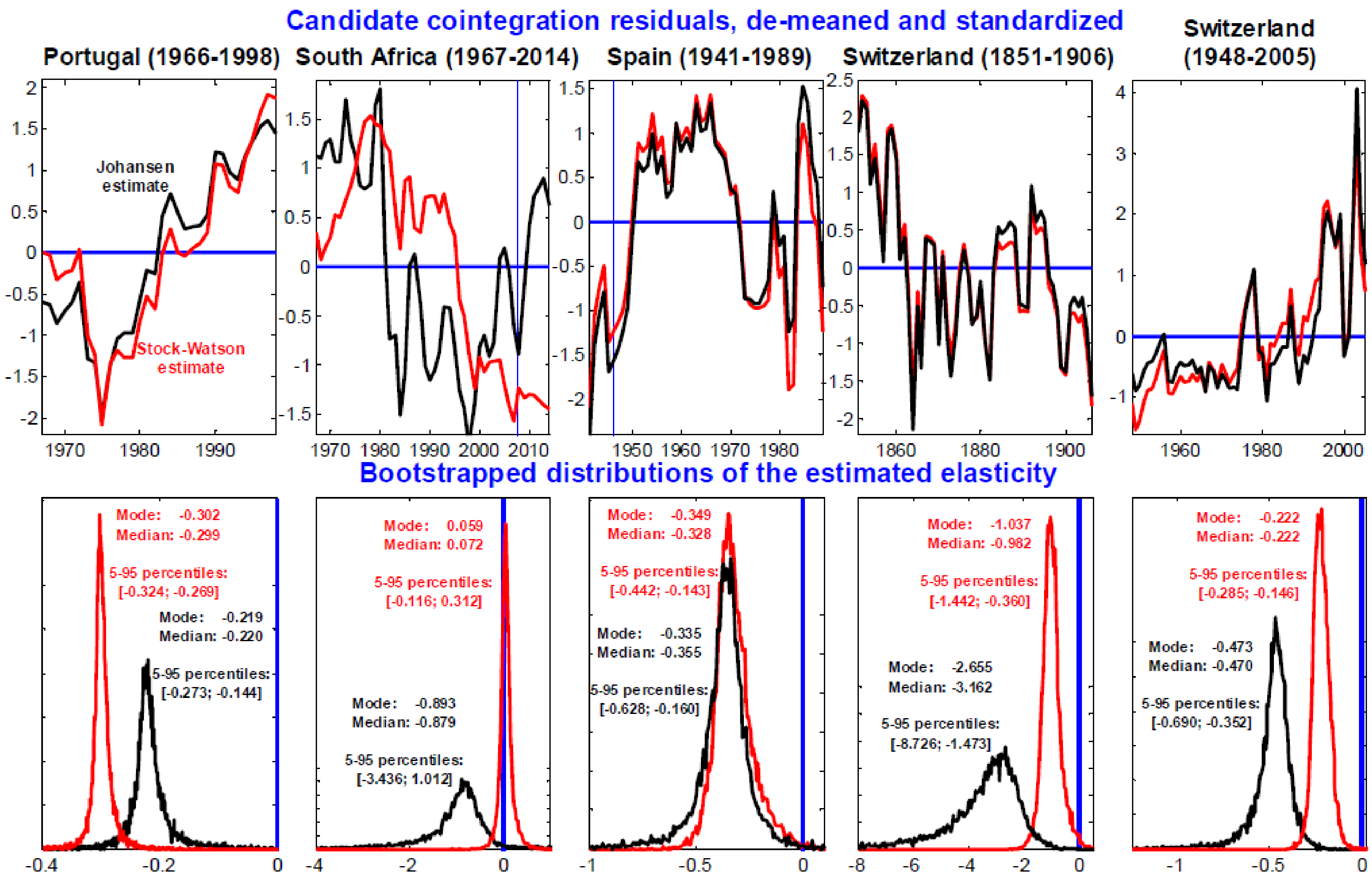


Figure LL.5 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

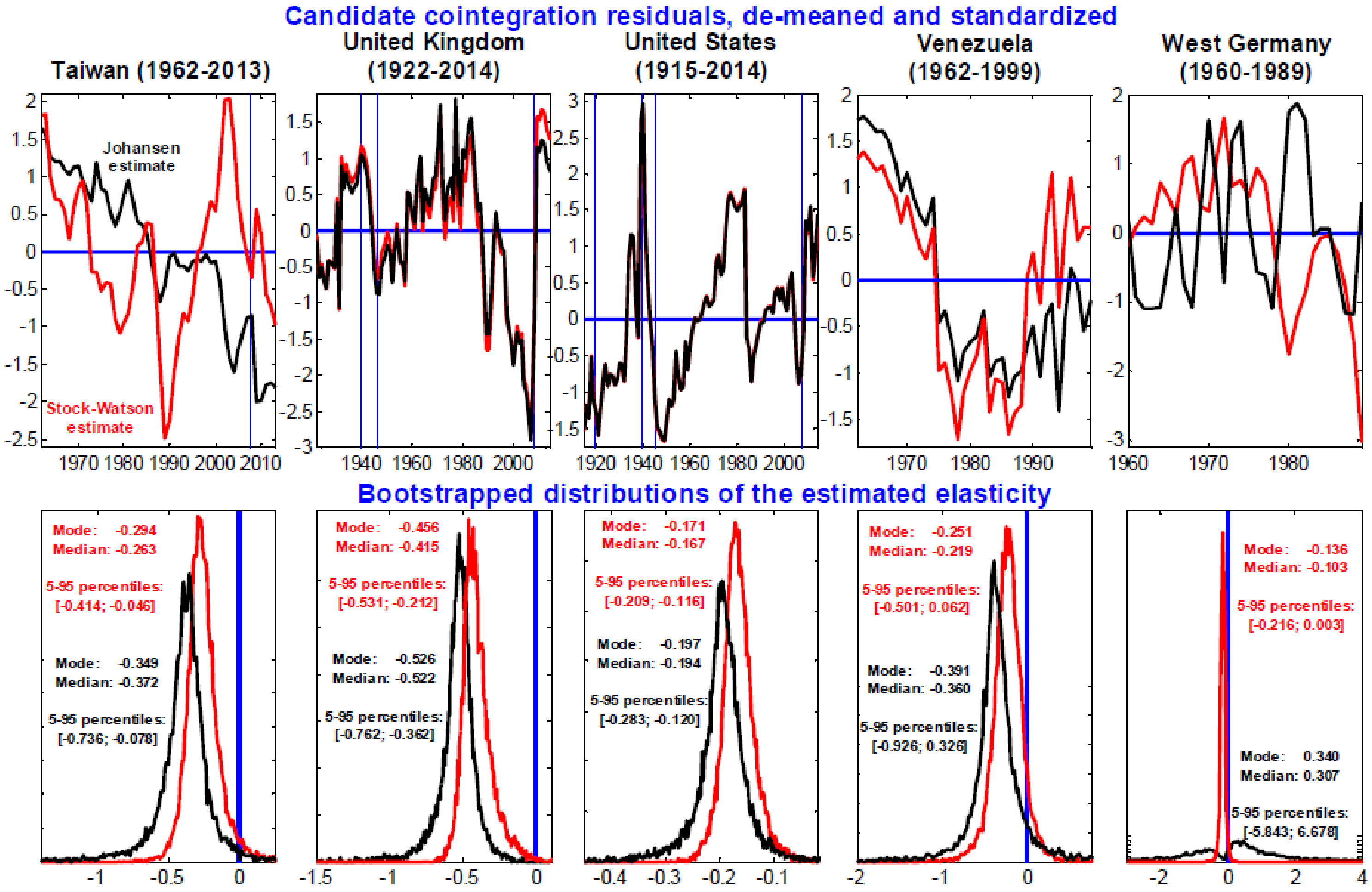


Figure LL.6 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

V: Full set of results based on the log-log specification with the correction ln(1+*R*)

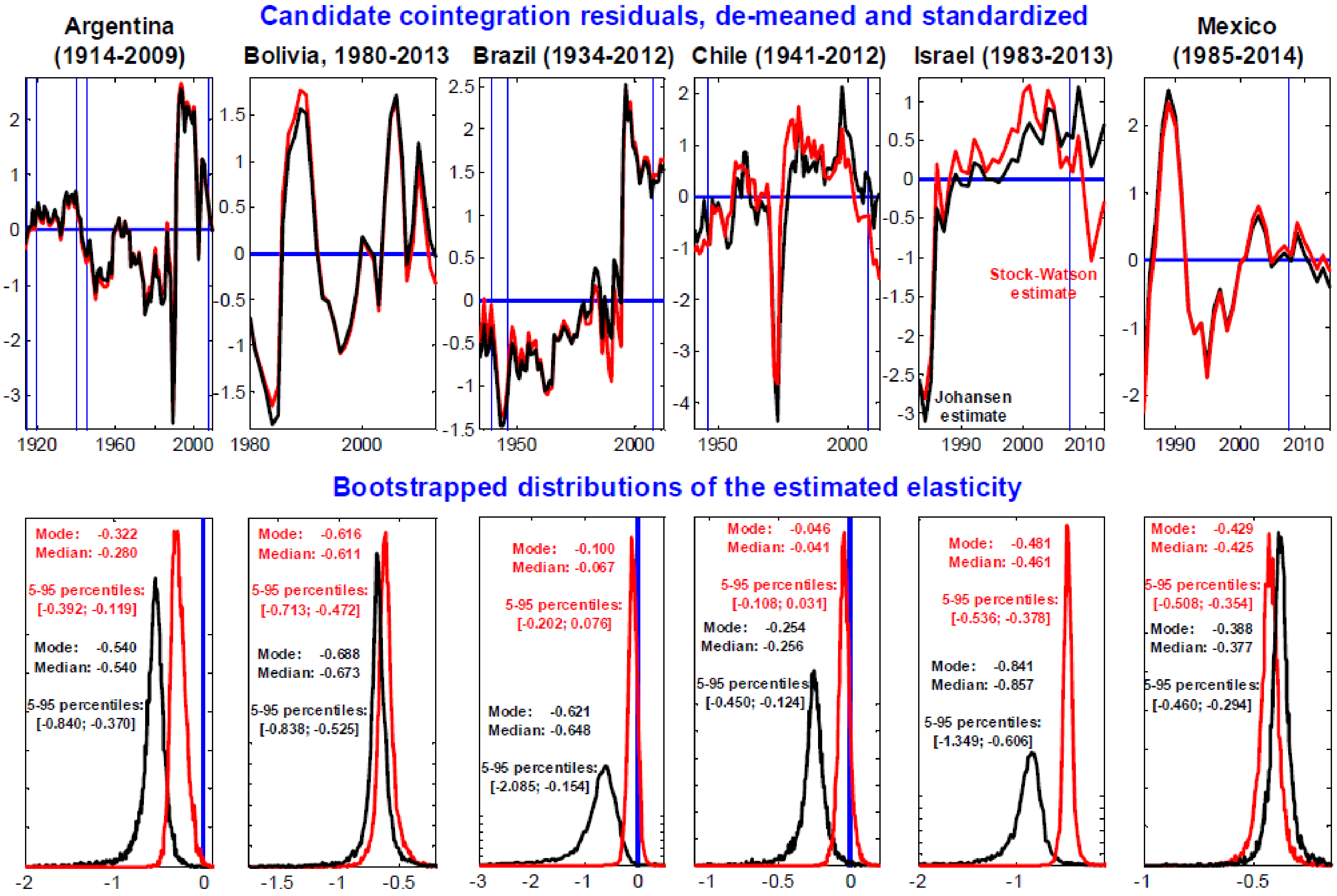


Figure LLCO.1 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

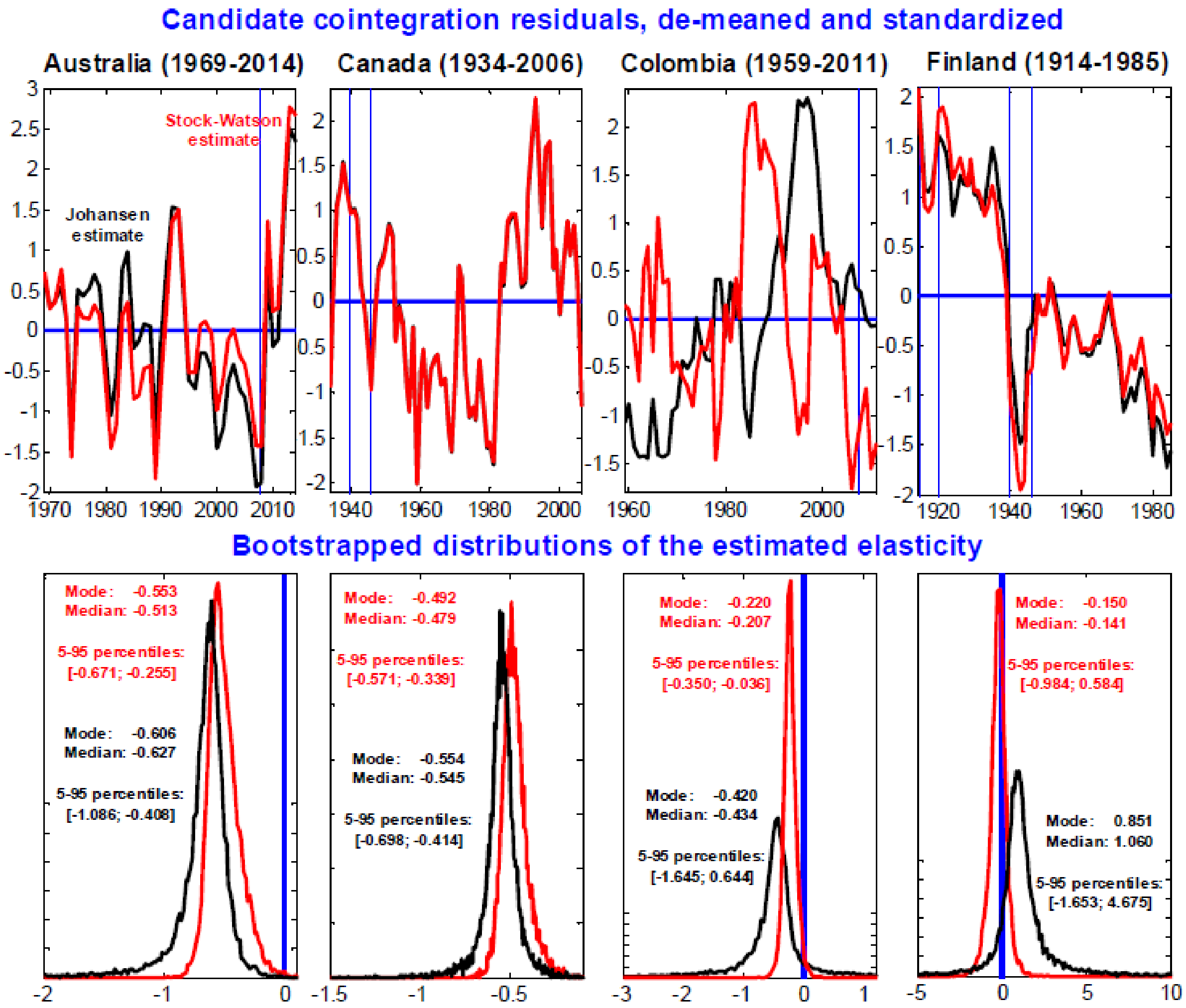


Figure LLCO.2 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

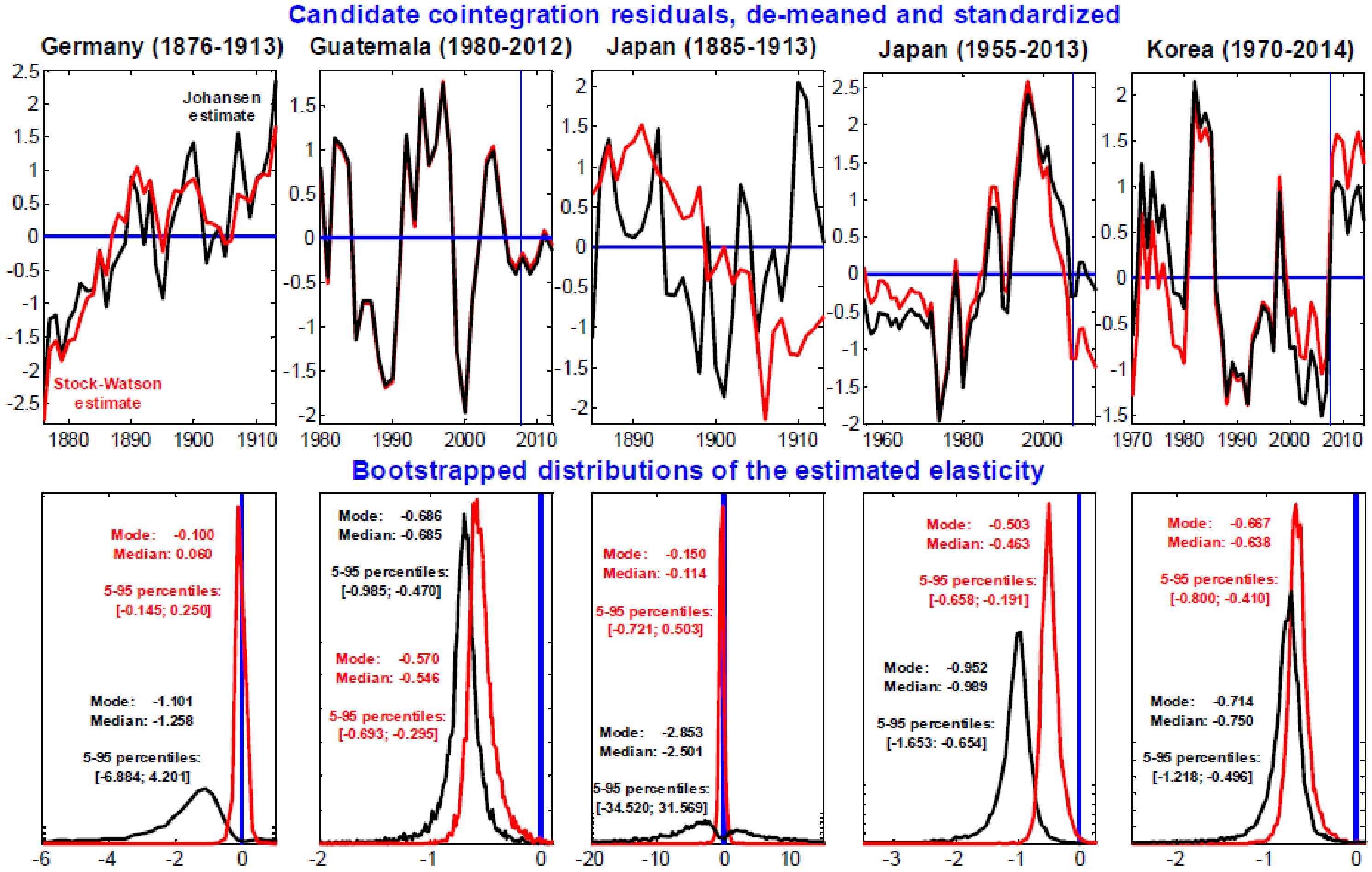


Figure LLCO.3 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

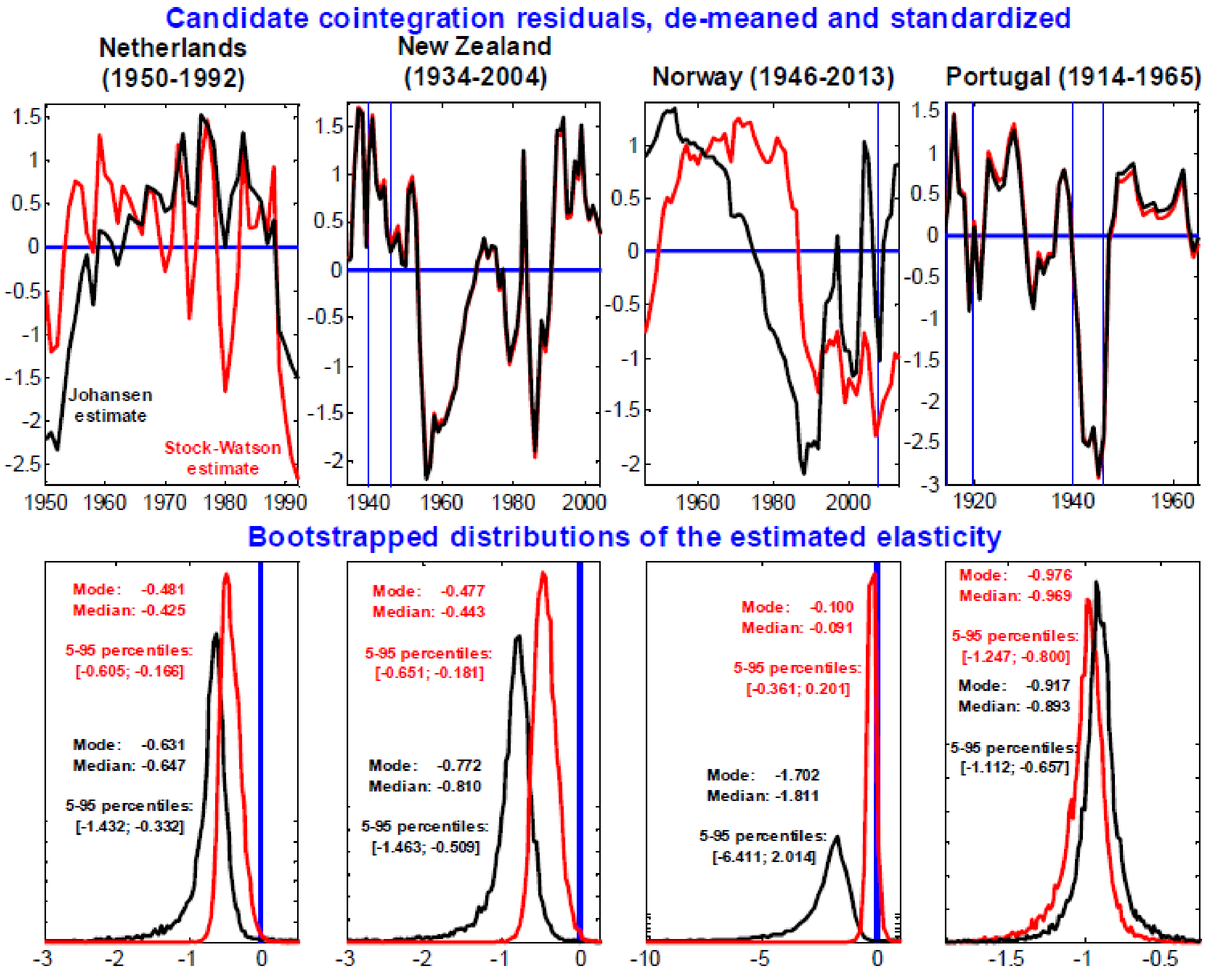


Figure LLCO.4 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

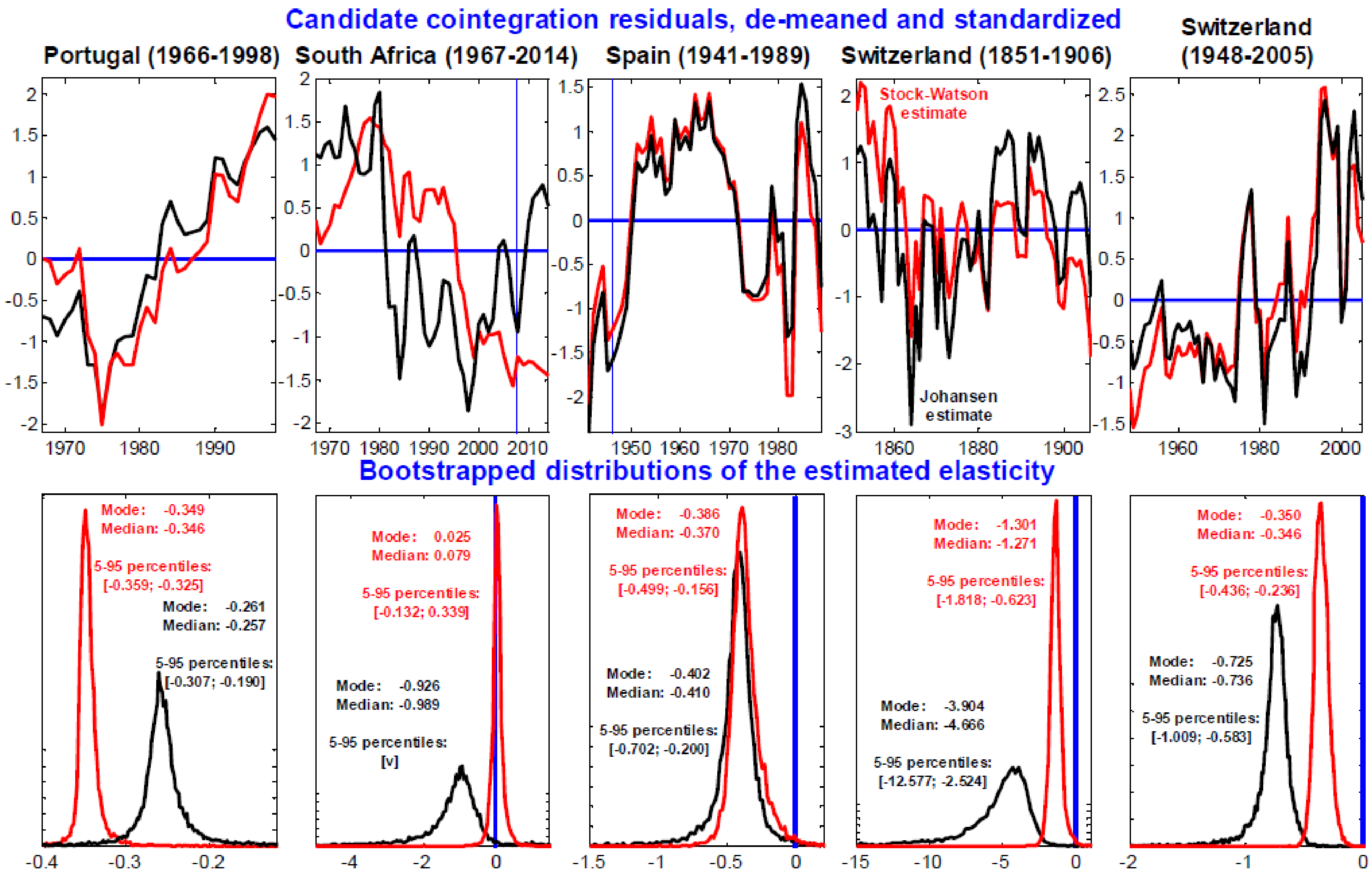


Figure LLCO.5 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

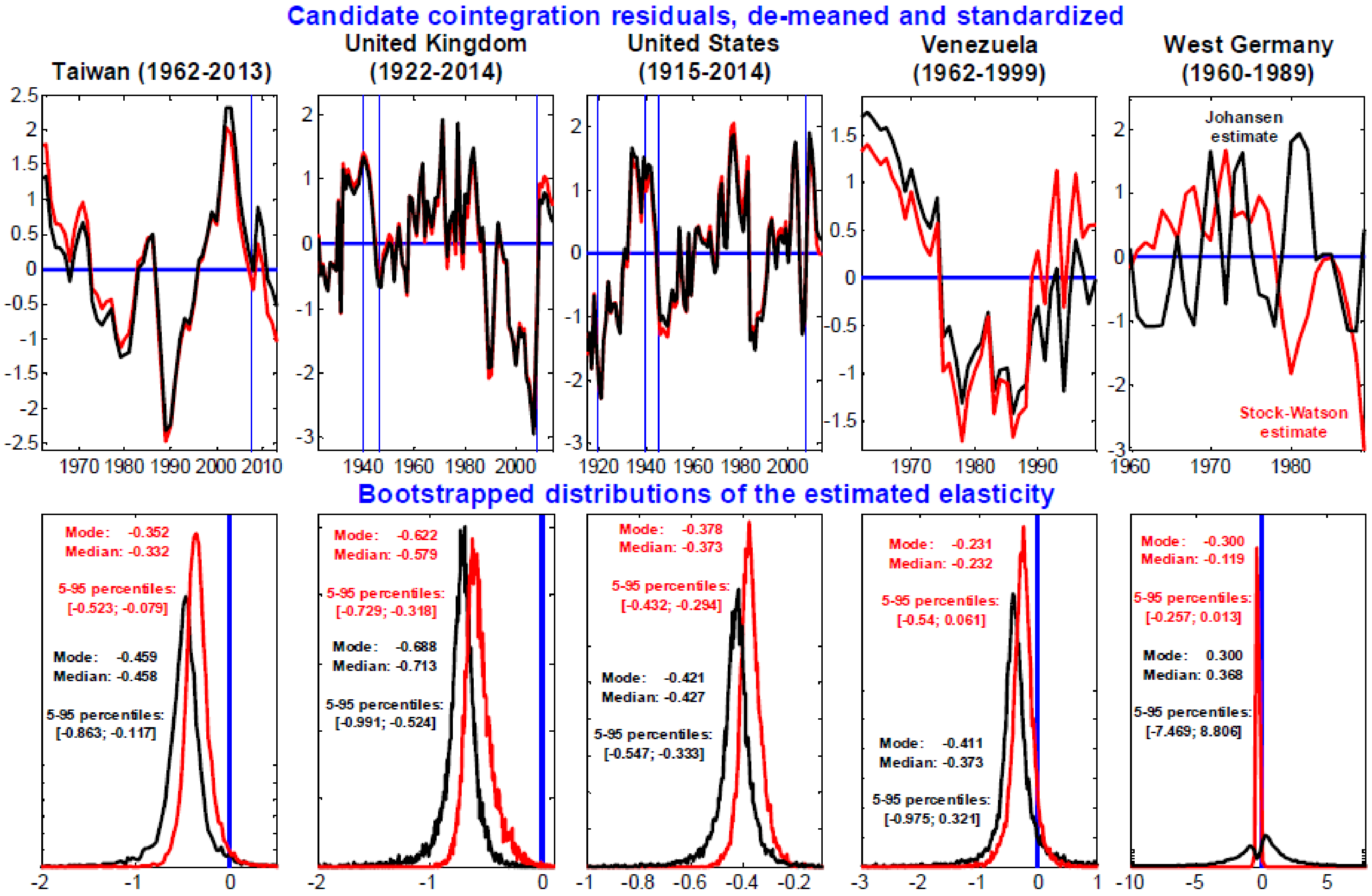


Figure LLCO.6 Log-log specification, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate