Storable Votes and Judicial Nominations in the U.S. Senate
Online Appendix

May 19, 2014

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Online Appendix

1 General Correlation Pattern

Section 4 in the main text discusses simulations results for a small number of intra- and inter-party correlation values ρ and ρ_{inter}. Extending the analysis to a larger set of correlations is more demanding computationally but presents no logical difficulty. We present here the same results exposed in the main text, obtained by increasing both ρ and ρ_{inter}, separately, from 0 to 0.9, in steps of 0.1. Without strong grounds to assume higher correlation among members of one or the other party, we maintain the assumption that intra-party correlation is equal in the two parties. In all graphs presented below, we plot intra-party correlation on the horizontal axis, and inter-party correlation on the vertical axis. We use a color scale such that lighter shades correspond to higher values for the quantities of interest. Because of the high computational demand, we limited ourselves to 200 simulations for each correlation pair[1].

The first question we ask is how the concentration of votes at the mutual best response rules is affected by changes in the correlations. Given the focus on concentration, we use the Gini coefficient of the best response voting rules as a summary statistic. We report it in Figure 1, in panel 1a for the majority and panel 1b for the minority. A lighter shade stands for higher concentration.[2]

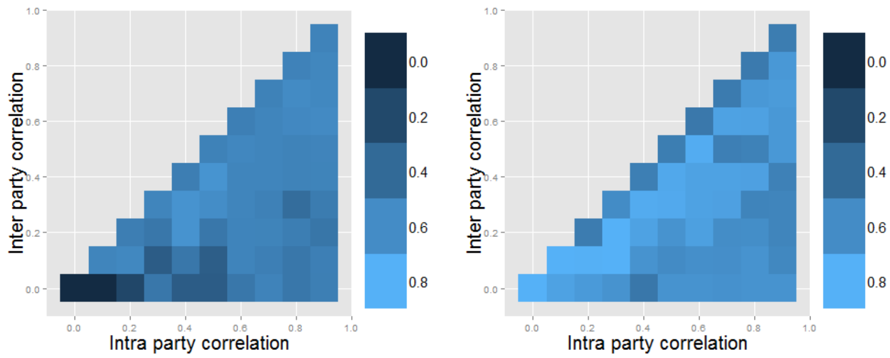
The figure has three main messages, confirming the results discussed in the text. First, the majority concentrates votes less than the minority–this is expected, is clear in the figure, and holds for most correlation values. Second, the majority's voting behavior is relatively insensitive to correlation values: most of the squares in Panel 1a indicate a Gini coefficient in the neighbourhood of 0.6. Third, the voting pattern is more variable for the minority party–there is more variability in the shading of the squares in Panel 1b. In particular, for given intraparty correlation, votes' concentration first increases and then falls as interparty correlation increases. We remarked on the same pattern in the cases considered in the text.

The expected number of minority blocks, calculated at the mutual best response rules, ranges from less than 1 to more than 2, depending on the correlation values. It is plotted in Figure 2. As predicted, the number of blocks is higher the higher is intra-party correlation, and the lower is inter-party correlation.

Figure 3 reports the two parties' welfares, at different correlation values, as fraction of maximal possible welfare for each party. Note that to make both

1. In the results presented in the main text, we ran 1000 simulations for each correlation pair.

2. The figure shows some unexpected non-monotonicities: squares of contrasting colors, relative to the neighboring squares (for example in the minority's rules at inter-party correlation of 0 and intra-party correlation of 0.5). Typically these occur when the best response rule is not unique, the alternative rules have varying Gini coefficients, and the program has randomly selected one of the best response rules. Because the alternatives are equivalent from a welfare perspective, the apparent non-monotonicites do not appear when describing outcomes.



(a) Majority party

(b) Minority party

Figure 1: Vote concentration in the mutual best response rules, at different intraparty and interparty correlations. Gini coefficients.

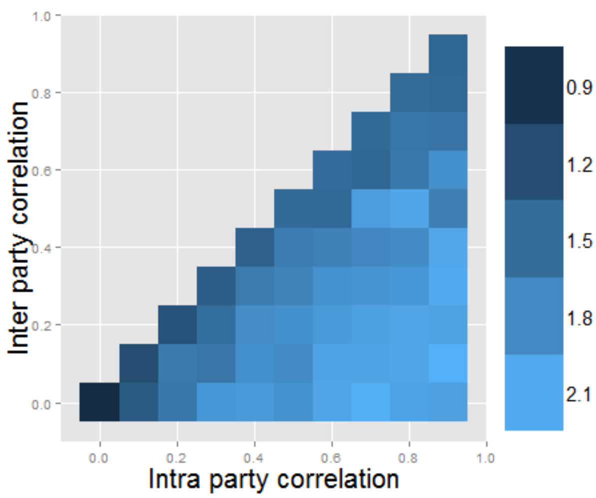
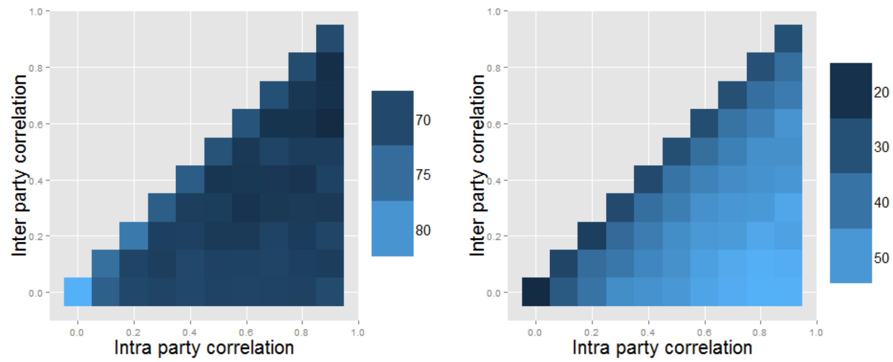


Figure 2: Number of minority blocks. Blocks are calculated at the mutual best response rules.



(a) Majority party

(b) Minority party

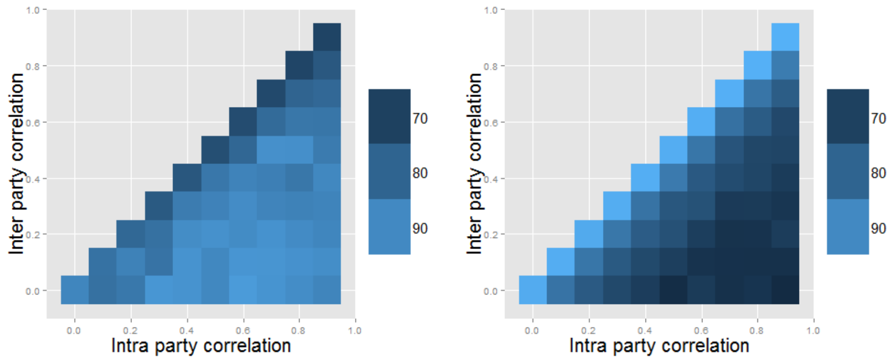
Figure 3: Welfare ratios for different correlation coefficients. Realized welfare as fraction of maximal possible welfare, calculated at the mutual best response rules.

panels readable, the scale differs: the majority appropriates between 66 and 84 percent of possible welfare; the minority between 18 and 57 percent.

Expected welfare is always higher and less variable for the majority party. Some variability, however, is observed across different correlation coefficients, for the majority too, and reflects the power of the minority. For the minority party (panel 3b), welfare mirrors the fraction of minority blocks in Figure 2 almost exactly. For the majority (panel 3a), the figure shows that welfare is affected not only by the number of minority blocks, but also by the importance of the nominations the minority derails. Because the concentration of votes by the minority is the highest at intermediate interparty correlation, it is in this range that it succeeds, occasionally, in stopping nominations that the majority also considers important. The result is that majority welfare is lowest at intermediate interparty correlation. At higher correlation, the minority shifts at least partially away from direct competition with the majority and minority blocks are not only less frequent but also less salient.

How do these patterns affect social welfare, and do storable votes improve efficiency, relative to simple majority voting? Figure 4 reports the share of maximal total welfare realized with storable votes (in panel4a), and with majority voting (panel 4b) in our simulations, at different correlation coefficients.

The figure is very instructive. Both panels are almost perfectly monotonic in the two correlation parameters, and in opposite direction. In particular, storable votes are closer to efficiency at high intraparty correlation and low interparty correlation and deliver progressively less social welfare (relative to the potential maximum) as intraparty correlation falls and interparty correlation increases, in the direction of the diagonal line. (Recall that the diagonal can be understood as the highest possible interparty correlation consistent with a



(a) Storable Votes *(b) Majority Voting*

Figure 4: Comparison of the share of efficient welfare realized for different correlation coefficients, by voting rule.

given intraparty correlation coefficient). On the other hand, the performance of majority voting display the opposite pattern: it performs closer to efficiency at the highest admissible inter party correlation and falls monotonically as the lower-right corner is approached, where intra party correlation is strong and interparty correlation is weak – i.e. when parties are cohesive but they do not compete on nominees. We discussed the reason in the main text of the paper: the more similar are the values draws across the two parties, the more does size alone determine which party should win, from an efficiency perspective. It is when interparty correlation is low and at the same time values are correlated within parties that the probability of efficient minority victories–larger total values on the minority size–is highest. An important result suggested by the graphs and confirmed by the simulation results is that majority voting yields more variable outcomes than storable votes. Majority voting performs the worst at an intraparty correlation of .9 and no interparty correlation, where it achieves 63% of efficiency, it performs the best when both correlations are .9, where it almost achieves perfect efficiency (99.97%). Storable votes, instead, reaches a minimum of 71% of efficiency when both correlations are .9 and a maximum of 94% when intra party correlation is .6 and interparty correlation is nil.

The relative performance of the two voting rules, in welfare terms, is summarized in Figure 5. The figure simply shows the difference in efficiency shares between storable votes and majority voting at different levels of correlation: a higher number, represented by a lighter shade, implies that storable votes performs all the better compared to majority voting. As anticipated, using our efficiency criterion, majority voting outperforms storable votes along and in the immediate vicinity of the diagonal; storable votes outperform majority in the remaining areas of the graph, and particularly so in the lower right corner. We

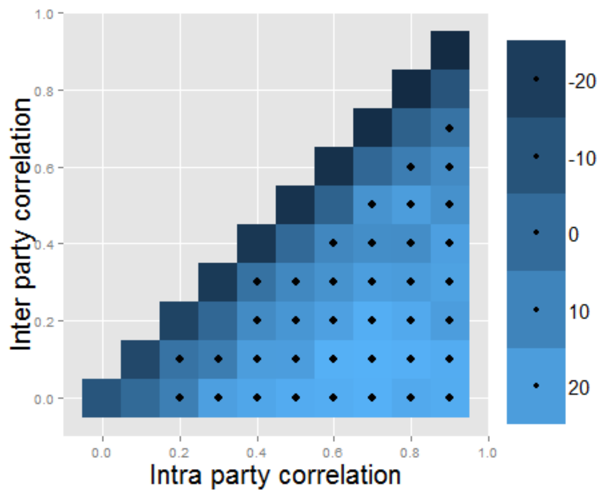


Figure 5: Difference, in percentage point, of the share of efficiency achieved by storable votes and the share of efficiency achieved by majority voting. Cases where storable votes achieves a higher share of efficiency are marked with a black dot.

marked with a dot the cases where storable votes performs better than majority voting, i.e. when the difference in efficiency shares is positive. The dot patterns makes it clear that storable votes usually does better on this metric unless the interparty correlation is at the boundary of the admissible values given an intra party correlation.

2 Simulating intensities

This section describes how intensities are generated in our simulations.

Consider the committee of N members with M majority members and m minority members who have to vote on a slate of K nominees. The simulations assume that the marginal distribution of intensities for each member on a given nominee, Γ', is a uniform: $\Gamma' \equiv \mathcal{U}[0,1]$. Moreover we also assume that the intensities are correlated: the intra party (linear) correlation in the majority party is ρ_M, it is ρ_m in the minority. Finally, the inter-party correlation is given by ρ_{inter}.

Standard statistical packages allow us to draw multivariate normals with a given mean and covariance matrix. We use *R* and the *mvtnorm* package. Hence, we can draw a multivariate normal of dimension N for each nominee, with covariance matrix Σ given by

$$\Sigma = \begin{array}{cl} \left(\begin{array}{cccc|cccc} 1 & \rho'_M & & \rho'_M & & & & \\ & \ddots & & & & \rho'_{inter} & & \\ & & \ddots & & & & & \\ \rho'_M & & \rho'_M & 1 & & & & \\ \hline & & & & 1 & & & \rho'_m \\ & \rho'_{inter} & & & & \ddots & & \\ & & & & \rho'_m & & & 1 \end{array}\right) & \begin{array}{l} \text{majority} \\ \\ \\ \\ \text{minority} \end{array} \end{array} \tag{1}$$

Consider a draw X from the multivariate normal with covariance matrix Σ and mean 0. The marginal distribution of each element of the vector is then a standard normal distribution. Using the uniform transformation and $\Phi(.)$ being the standard normal cumulative distribution function, for all element X_i of vector X, $\Phi(X_i) \sim \mathcal{U}[0,1]$.

The linear correlation between the variables $\Phi(X_i)$ is the Spearman (rank- or fractile-) correlation between the variables X_i. The rank correlation $\tilde{r}$ of the standard normal variables X, Y with correlation $\tilde{\rho}$ is given by[3]:

$$\tilde{\rho} = 2sin\left(\frac{\pi}{6}\tilde{r}\right) \tag{2}$$

Therefore, the draws of the valuations are done as follows:

- For each target uniform correlation profile $\tilde{r} \in \{\rho_M, \rho_m, \rho_{inter}\}$, compute the corresponding normal linear $\tilde{\rho}$ correlation via Equation 2
- For instance, $\rho'_{inter} = 2sin\left(\frac{\pi}{6}\rho_{inter}\right)$

3. See Hotelling and Pabst (1936) or Phoon, Quek, and Huang (2004)

- For each nominee, draw a multivariate normal of size N, mean 0 and correlation matrix Σ.
- For each nominee, apply the standard normal cdf $\Phi(.)$ to each element of the drawn vector

3 Constraints on the correlations

In order to be able to compute the intensity draws, one requires the matrix Σ defined in (1) to be invertible. In the following, the prime notations are omitted for simplicity.

Define

$$\begin{aligned} b &= (2 + (m-1)\rho_m + (M-1)\rho_M) \\ c &= (1 + (M-1)\rho_M)(1 + (m-1)\rho_m) - Mm\rho_{inter}^2 \end{aligned}$$

The eigenvalues of Σ are given by

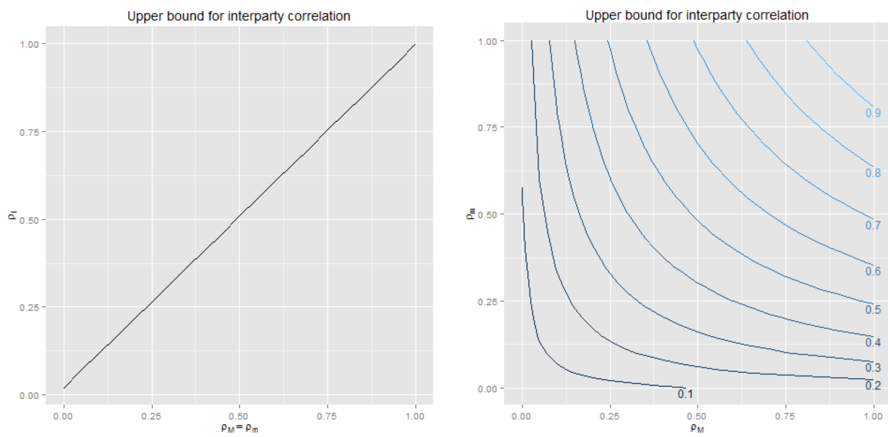
$$\begin{aligned} \lambda_1 &= 1 - \rho_M \\ \lambda_2 &= 1 - \rho_m \\ \lambda_3 &= \frac{1}{2}\left[b - \sqrt{b^2 - 4c}\right] \\ \lambda_4 &= \frac{1}{2}\left[b + \sqrt{b^2 - 4c}\right] \end{aligned}$$

Note that we always have $b^2 \geq 4c$ since $c \leq (1+(M-1)\rho_M)(1+(m-1)\rho_m) = \bar{c}$ and we can write $b = x + y$ and $\bar{c} = x \cdot y$. Finally, $(x + y)^2 \geq 4xy$ for any x, y. Hence, the necessary and sufficient condition for $\Sigma > 0$ is given by $\frac{1}{2}\left[b - \sqrt{b^2 - 4c}\right] > 0$ assuming all correlations are in $[0, 1]$. This is equivalent to testing that $c > 0$.

The condition $c > 0$ gives us that

$$\rho_{inter}^2 \leq \frac{(1 + (M-1)\rho_M)(1 + (m-1)\rho_m)}{Mm}$$

The constraint on the interparty correlation in the uniform case as a function of the uniform, intraparty correlations can be easily obtained by inverting the relation in equation 2. Note that, because the *arcsin* function is increasing on $[-1, 1]$, we have that if $\rho = 2sin\left(\frac{\pi}{6}r\right)$ Then $\rho \leq \bar{\rho} \Leftrightarrow r \leq \frac{6}{\pi}arcsin\left(\frac{\bar{\rho}}{2}\right)$. The upper bound on the interparty linear correlation when the values are drawn from a uniform $[0, 1]$ is shown in Figure 6. Figure 6a assumes the same intraparty correlation in both parties ($\rho_M = \rho_m$) while Figure 6b allows those correlations to differ. In the latter figure, we display the levels of the upper bound on the interparty correlation as a function of the two intra party correlations.



(a) Same intra party correlation *(b) Different intra party correlation*

Figure 6: Upper bound on interparty correlation with $M = 55$ and $m = 45$ as a function of the intra party correlations – Linear correlations for uniformly distributed intensities.

Importantly for our simulations and the results shown in the paper and in the appendix, the constraint on ρ_{inter} can be simplified for large groups. Notice that as M and m diverge to infinity, we have that $1 + (M - 1)\rho_M \sim M\rho_M$ and $1 + (m - 1)\rho_m \sim m\rho_m$ for $\rho_M, \rho_m > 0$[4]. Hence, we obtain that the upper bound for ρ^2_{inter} is equivalent to $\rho_M\rho_m$. If the intra party correlations are identical, $\rho_M = \rho_m = \rho$, we obtain that for large M and m, the constraint is close to $\rho_{inter} \leq \rho$. This explains why in the various tiled figures used in the main paper and in this appendix, the admissible cases of intra- and interparty correlation are below the diagonal of the unit square.

References

Hotelling, Harold, and Margaret Richards Pabst. 1936. "Rank Correlation and Tests of Significance Involving No Assumption of Normality." *The Annals of Mathematical Statistics* 7, no. 1 (March): 29–43.

Phoon, Kok-Kwang, Ser-Tong Quek, and Hongwei Huang. 2004. "Simulation of non-Gaussian processes using fractile correlation." *Probabilistic Engineering Mechanics* 19, no. 4 (October): 287–292.

4. Remember that the subscript M and m for the intra party correlation simply distinguish the two parties, and do not reflect a dependence of the correlation on party size.