A APPENDIX: Proofs

A.1 Expositional Model

Here, we detail some of the calculations of the expositional model in the text which is based on exogenous nominal spending. We have the following key equations

$$\Delta\hat{D}_t = \rho\Delta\hat{D}_{t-1} + \epsilon_t^D$$

$$\pi_t = \kappa\hat{Y}_t + \beta E_t\pi_{t+1}$$

$$D_t = P_tY_t.$$

The latter implies

$$\Delta\hat{D}_t = \pi_t + \hat{Y}_t - \hat{Y}_{t-1}.$$

The system has a solution of the following form

$$\hat{Y}_t = Y_d\Delta\hat{D}_t + Y_y\hat{Y}_{t-1}$$

$$\pi_t = \pi_d\Delta\hat{D}_t + \pi_y\hat{Y}_{t-1}.$$

Some algebra using the method of undetermined coefficients implies that

$$\hat{Y}_t = \frac{(1-\rho\beta)}{(1+\kappa-\rho\beta+\beta\left(1-Y_y\right))}\Delta\hat{D}_t + Y_y\hat{Y}_{t-1}$$

where Y_y is the root less than one of the quadratic equation

$$Y_y^2 - \frac{(1+\kappa+\beta)}{\beta}Y_y + \frac{1}{\beta} = 0.$$

Focussing on $\rho = 0$ for simplicity, this directly implies

$$VAR(Y_t) = \frac{\left[\frac{1}{(1+\kappa+\beta(1-Y_y))}\right]^2}{[1-{Y_Y}^2]}VAR\left(\Delta\hat{D}_t\right) = \Gamma * VAR\left(\Delta\hat{D}_t\right).$$

We have verified analytically that Γ is decreasing in κ and numerically that the same holds true for any value of ρ between 0 and 1.

A.2 Proofs of Propositions 1-5

In the following, to keep clutter to a minimum, we only keep track of A_t and ψ_t since η_t shows up in the same way as $\sigma^{-1}(\psi_t - E_t\psi_{t+1})$ and μ_t and τ_t^w as A_t. Then, under endogenous nominal demand and Taylor rule, the following equations hold:

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t\pi_{t+1}) + \psi_t - E_t\psi_{t+1}$$

$$\pi_t = \kappa\hat{Y}_t - \kappa\gamma_A A_t + \beta E_t\pi_{t+1}$$

$$\hat{i}_t = \phi_\pi\pi_t + \phi_y\hat{Y}_t + \eta_t$$

where demand and technology shocks evolve first-order auto-regressively as $\psi_t = \rho_\psi\psi_{t-1} + \epsilon_t^\psi$ and $A_t = \rho_A A_{t-1} + \epsilon_t^\psi$, and $\gamma_A = \frac{1+\phi}{\sigma^{-1}+\phi}$.

First, only consider demand shocks ψ_t. The system has a solution of the following form: $Y_t = Y_\psi\psi_t$, $\pi_t = \pi_\psi\psi_t$ and $i_t = i_\psi\psi_t$ which implies that $E_tY_{t+1} = Y_\psi\rho_\psi\psi_t$ and $E_t\pi_{t+1} = \pi_\psi\rho_\psi\psi_t$.

Matching coefficients yields the following expressions:

$$Y_\psi = \left[\frac{\sigma(1-\beta\rho_\psi)\,(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma\,(\phi_\pi-\rho_\psi)}\right]$$

$$\pi_\psi = \left[\frac{\kappa\sigma\,(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma\,(\phi_\pi-\rho_\psi)}\right]$$

This implies the following expression for the variance of output and inflation:

$$var(\hat{Y}_t/\psi_t) = \left(\frac{\sigma(1-\beta\rho_\psi)(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\sigma\kappa[\phi_\pi-\rho_\psi]}\right)^2 var(\psi_t)$$

$$var(\pi_t/\psi_t) = \left(\frac{\kappa\sigma\,(1-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma\,(\phi_\pi-\rho_\psi)}\right)^2 var(\psi_t)$$

Then, the derivative of the variance of output with respect to κ is:

$$\frac{\partial VAR(\hat{Y}_t/\psi_t)}{\partial k} = -2\sigma^2\frac{\sigma(\phi_\pi-\rho_\psi)(1-\beta\rho_\psi)^2(1-\rho_\psi)^2}{((1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\sigma\kappa[\phi_\pi-\rho_\psi])^3}var(\psi_t)$$

If $(\phi_\pi - \rho_\psi) > 0$, then this derivative is always negative. The sign of the derivative flips iff $(\phi_\pi - \rho_\psi) < 0$. Note that the denominator is always positive which follows from the bounds implied by the determinacy condition.

Next, consider a demand shock at the ZLB. The expression for the derivative of output with respect to κ is given by:

$$\frac{\partial \hat{Y}_S}{\partial \kappa} = -\frac{(1-\rho_\psi)(1-\beta\mu)\mu\sigma}{[(1-\mu)(1-\beta\mu)-\mu\sigma\kappa]^2} < 0$$

Now, only consider technology shocks A_t. The system has a solution of the following form: $Y_t = Y_A A_t$, $\pi_t = \pi_A A_t$ and $i_t = i_A A_t$ which implies that $E_t Y_{t+1} = Y_A \rho_A A_t$ and $E_t \pi_{t+1} = \pi_A \rho_A A_t$.Matching coefficients yields the following expressions:

$$Y_A = \frac{\kappa\sigma[\phi_\pi - \rho_A]}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma[\phi_\pi-\rho_A]]}\gamma_A$$

$$\pi_A = \frac{\kappa\gamma_A(1-\rho_A+\sigma\phi_y)}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma(\phi_\pi-\rho_A]]}\gamma_A$$

This implies the following expression for the variance of output and inflation:

$$var(\hat{Y}_t/A_t) = \left(\frac{\kappa\sigma\gamma_A[\phi_\pi-\rho_A]}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma[\phi_\pi-\rho_A]]}\gamma_A\right)^2 var(A_t)$$
$$var(\pi_t/A_t) = \left(\frac{-\kappa\sigma\gamma_A(1-\rho_A+\sigma\phi_y)}{[(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma(\phi_\pi-\rho_A]]}\gamma_A\right)^2 var(A_t)$$

Then, the derivative of the variance of output with respect to κ is:

$$\begin{aligned}\frac{\partial var(Y_t)}{\partial \kappa} &= 2Y_A\frac{\partial Y_A}{\partial \kappa}\\ &= 2\gamma_A\left[\frac{\kappa\sigma\left(\phi_\pi-\rho_A\right)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)}\right]\\ &\gamma_A\left[\frac{\sigma\left(\phi_\pi-\rho_A\right)(1-\beta\rho_A)(1-\rho_A+\sigma\phi_y)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)^2}\right]\\ &> 0\end{aligned}$$

since the denominator is always positive which follows from the bounds implied by the determinacy condition.

Next, notice that the shock η_t appears exactly in the same way as $\psi_t - E_t\psi_{t+1}$. Hence, the derivative of output has the same sign with respect to κ, depending on $\phi_\eta - \rho_\eta$: $\frac{\partial var(Y_t/\eta_t)}{\partial \kappa} < 0$ if $\phi_\eta - \rho_\eta < 0$ and $\frac{\partial var(Y_t/\eta_t)}{\partial \kappa} > 0$ if $\phi_\eta - \rho_\eta > 0$. The coefficients in the case of an idiosyncratic monetary policy shock η_t are:

$$Y_\eta = \frac{-\sigma(1-\beta\rho_\eta)}{1-\rho_\eta+\sigma\phi_y+\sigma\kappa(\phi_\pi-\rho_\eta)}$$
$$\pi_\eta = \frac{\kappa}{1-\beta\rho_\eta}\frac{-\sigma(1-\beta\rho_\eta)}{1-\rho_\eta+\sigma\phi_y+\sigma\kappa(\phi_\pi-\rho_\eta)}$$

Finally, consider a markup shock $\hat{\mu}_t$ – shocks to labor taxes $\hat{\tau}_t^w$ have isomorphic derivations. First, note that we have $Y_t^n = Y_t^n - Y_t^e = -\frac{1}{\sigma^{-1}+\phi}\hat{\mu}_t$. Then, applying the method of undetermined coefficients in a setup analogous to the above yields the following coefficients:

$$Y_\mu = -\left(\frac{1}{\sigma^{-1}+\phi}\right)\left[\frac{\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}\right]$$
$$\pi_\mu = -\frac{\kappa}{(1-\beta\rho_\mu)}\left(\frac{1}{\sigma^{-1}+\phi}\right)\left[\frac{-\left(1-\beta\rho_\mu\right)\left(1-\rho_\mu+\sigma\phi_y\right)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}\right]$$

This directly implies that the variance of output is

$$var\left(Y_t\right) = \left[\left(\frac{1}{\sigma^{-1}+\phi}\right)\left[\frac{\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}\right]\right]^2 var(\mu_t)$$

Taking derivatives of the variance of output with respect to kappa yields:

$$\begin{aligned}
\frac{\partial var(Y_t)}{\partial\kappa} &= 2Y_\mu\frac{\partial Y_\mu}{\partial\kappa} \\
&= 2\left(\frac{1}{\sigma^{-1}+\phi}\right)\left[\frac{\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)}\right] \\
&\left(\frac{1}{\sigma^{-1}+\phi}\right)\frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)\sigma(\phi_\pi-\rho_\mu)}{\left((1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)\right)^2} \\
&= 2\frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\left((1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma\left(\phi_\pi-\rho_\mu\right)\right)}var\left(Y_t\right) \\
&> 0
\end{aligned}$$

Note that the denominator is always positive which follows from the bounds implied by the determinacy condition.

A.3 Welfare - Proofs of Propositions 6-11

Proof of Proposition 6 For the technology shock, first note the for $Y_t^e = \frac{1+\phi}{\sigma^{-1}+\phi}\hat{A}_t$, we have that

$$var\left(Y_t - Y_t^e\right) = \left[\left(\frac{1+\phi}{\sigma^{-1}+\phi}\right)\left[\frac{-\left(1-\beta\rho_A\right)\left(1-\rho_A+\sigma\phi_y\right)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)}\right]\right]^2$$

Second, we take derivatives of the weighted variance term:

$$\begin{aligned}
\frac{\partial \frac{\theta}{\kappa} var(\pi_t)}{\partial\kappa} &= -\theta\frac{1}{\kappa^2}var(\pi) + \theta\frac{1}{\kappa}\frac{\partial var(\pi_t)}{\partial\kappa} \\
&= \theta\left[\frac{1}{(1-\beta\rho_A)}\right]^2 var\left(Y_t - Y_t^e\right)\left(\frac{(1-\rho_A+\sigma\phi_Y)(1-\beta\rho_A)-\kappa\sigma\left(\phi_\pi-\rho_A\right)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)+\kappa\sigma\left(\phi_\pi-\rho_A\right)}\right) \\
&> 0 \text{ if } \phi_\pi-\rho_A < \Gamma_A = \frac{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)}{\kappa\sigma} \\
&< 0 \text{ if } \phi_\pi-\rho_A > \Gamma_A = \frac{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A)}{\kappa\sigma}
\end{aligned}$$

For the demand shock, $Y_t^e = Y_t$. Since $var(\pi_t) = \frac{\kappa^2}{(1-\beta\rho_\psi)^2}var(Y_t)$, some algebra directly implies that

$$\begin{aligned}
\frac{\partial \frac{\theta}{\kappa} var(\pi_t)}{\partial\kappa} &= -\theta\frac{1}{\kappa^2}var(\pi_t) + \theta\frac{1}{\kappa}\frac{\partial var(\pi_t)}{\partial\kappa} \\
&= \frac{\theta}{\kappa^2}var\left(Y_t\right)\left[\frac{\kappa}{(1-\beta\rho_\psi)}\right]^2\left[-1+2\frac{\left(1-\rho_\psi+\sigma\phi_y\right)\left(1-\beta\rho_\psi\right)}{\left(1-\rho_\psi+\sigma\phi_y\right)\left(1-\beta\rho_\psi\right)+\sigma\kappa\left(\phi_\pi-\rho_\psi\right)}\right] \\
&= \frac{\theta}{\kappa^2}var\left(Y_t\right)\left[\frac{\kappa}{(1-\beta\rho_\psi)}\right]^2\left(\frac{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)-\kappa\sigma(\phi_\pi-\rho_\psi)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma(\phi_\pi-\rho_\psi)}\right) \\
&> 0 \text{ iff } \phi_\pi-\rho_\psi < \Gamma_\psi = \frac{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)}{\kappa\sigma} \\
&< 0 \text{ iff } \phi_\pi-\rho_\psi > \Gamma_\psi = \frac{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)}{\kappa\sigma}
\end{aligned}$$

For the markup shock, some algebra directly implies that

$$\begin{aligned}
\frac{\partial \frac{\theta}{\kappa} var(\pi_t)}{\partial \kappa} &= \frac{\theta}{\kappa^2}\left[\frac{(1-\rho_\mu+\sigma\phi_y)}{\sigma\left(\phi_\pi-\rho_\mu\right)}\right]^2 var(Y_t)\left(\frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)-\kappa\sigma(\phi_\pi-\rho_\mu)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma(\phi_\pi-\rho_\mu)}\right) \\
&< 0 \text{ iff } \phi_\pi-\rho_\mu > \Gamma_\mu = \frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\sigma} \\
&> 0 \text{ if } \phi_\pi-\rho_\mu < \Gamma_\mu = \frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\sigma}
\end{aligned}$$

Proof of Proposition 7 Noting that for the demand shock ψ_t it holds true that $Y_t^e = Y_t$, we take derivatives of W with respect to κ :

$$\begin{aligned}
\frac{\partial W}{\partial \kappa} &= -\left(\phi+\sigma^{-1}\right)\left[\frac{\partial \frac{\theta}{\kappa} var\left(\pi_t\right)}{\partial \kappa}+\frac{\partial var\left(Y_t\right)}{\partial \kappa}\right] \\
&= \frac{\left(\phi+\sigma^{-1}\right) var(Y_t)}{(1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)+\kappa\sigma(\phi_\pi-\rho_\psi)} \\
&(-\frac{\theta}{(1-\beta\rho_\psi)^2}\left((1-\rho_\psi+\sigma\phi_y)(1-\beta\rho_\psi)-\kappa\sigma(\phi_\pi-\rho_\psi)\right)+2\sigma(\phi_\pi-\rho_\psi)) \\
&< 0 \text{ iff } \ (\phi_\pi-\rho_\psi) < \Lambda_\psi = \frac{\theta(1-\beta\rho_\psi)(1-\rho_\psi+\sigma\phi_y)}{\sigma(2(1-\beta\rho_\psi)^2+\kappa\theta)} \\
&> 0 \text{ iff } \ (\phi_\pi-\rho_\psi) > \Lambda_\psi = \frac{\theta(1-\beta\rho_\psi)(1-\rho_\psi+\sigma\phi_y)}{\sigma(2(1-\beta\rho_\psi)^2+\kappa\theta)}
\end{aligned}$$

Proof of Proposition 8

First, consider the derivative of the weighted inflation term with respect to κ:

$$\begin{aligned}
\frac{\partial \frac{\theta}{\kappa} var(\pi_t)}{\partial \kappa} &= -\theta\frac{1}{\kappa^2} var(\pi)+\theta\frac{1}{\kappa}\frac{\partial var(\pi_t)}{\partial \kappa} \\
&= \frac{\theta}{\kappa^2}\left[\frac{(1-\rho_\mu+\sigma\phi_y)}{\sigma\left(\phi_\pi-\rho_\mu\right)}\right]^2 var(Y_t)\left(\frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)-\kappa\sigma(\phi_\pi-\rho_\mu)}{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)+\kappa\sigma(\phi_\pi-\rho_\mu)}\right) \\
&< 0 \text{ iff } \phi_\pi-\rho_\mu > \Gamma_\mu = \frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\sigma} \\
&> 0 \text{ iff } \phi_\pi-\rho_\mu < \Gamma_\mu = \frac{(1-\rho_\mu+\sigma\phi_y)(1-\beta\rho_\mu)}{\kappa\sigma}
\end{aligned}$$

Second, since $Y_t^e = 0$, taking derivatives of welfare with respect to κ yields after some algebra:

$$\begin{aligned}
\frac{\partial W}{\partial \kappa} &= -\left(\phi + \sigma^{-1}\right) \frac{1}{\kappa\left((1-\rho_\mu + \sigma\phi_y)(1-\beta\rho_\mu) + \kappa\sigma\left(\phi_\pi - \rho_\mu\right)\right)} var\left(Y_t\right) \\
&(\frac{\theta}{\kappa}\left[\frac{(1-\rho_\mu + \sigma\phi_y)}{\sigma\left(\phi_\pi - \rho_\mu\right)}\right]^2 (1-\rho_\mu + \sigma\phi_y)(1-\beta\rho_\mu) - \kappa\sigma(\phi_\pi - \rho_\mu) + 2(1-\rho_\mu + \sigma\phi_y)(1-\beta\rho_\mu)) \\
&< 0 \text{ if } \frac{\theta}{\kappa}\left[\frac{(1-\rho_\mu + \sigma\phi_y)}{\sigma\left(\phi_\pi - \rho_\mu\right)}\right]^2 (1-\rho_\mu + \sigma\phi_y)(1-\beta\rho_\mu) \\
&-\kappa\sigma(\phi_\pi - \rho_\mu) + 2(1-\rho_\mu + \sigma\phi_y)(1-\beta\rho_\mu) > 0 \\
&> 0 \text{ if } \frac{\theta}{\kappa}\left[\frac{(1-\rho_\mu + \sigma\phi_y)}{\sigma\left(\phi_\pi - \rho_\mu\right)}\right]^2 (1-\rho_\mu + \sigma\phi_y)(1-\beta\rho_\mu) \\
&- \kappa\sigma(\phi_\pi - \rho_\mu) + 2(1-\rho_\mu + \sigma\phi_y)(1-\beta\rho_\mu) < 0
\end{aligned}$$

A sufficient but not necessary condition for $\frac{\partial W}{\partial \kappa} < 0$ is $(\phi_\pi - \rho_\mu) < 0$.

Proof of Proposition 9

For technology shocks, $Y_t^e = \frac{1+\phi}{\sigma^{-1}+\phi}\hat{A}_t$. This implies that

$$var\left(Y_t - Y_t^e\right) = \left[\left(\frac{1+\phi}{\sigma^{-1}+\phi}\right)\left[\frac{-\left(1-\beta\rho_A\right)\left(1-\rho_A+\sigma\phi_y\right)}{(1-\rho_A+\sigma\phi_y)(1-\beta\rho_A) + \kappa\sigma\left(\phi_\pi - \rho_A\right)}\right]\right]^2$$

so that

$$\begin{aligned}
\frac{\partial var(Y_t - Y_t^e)}{\partial \kappa} &= -2\left(\frac{1+\phi}{\sigma^{-1}+\phi}\right)^2 \frac{\sigma(\phi_\pi - \rho_A)((1-\beta\rho_A)\left(1-\rho_A+\sigma\phi_y\right))^2}{((1-\rho_A+\sigma\phi_y)(1-\beta\rho_A) + \kappa\sigma\left(\phi_\pi - \rho_A\right))^3} \\
&= -2\frac{\sigma(\phi_\pi - \rho_A)}{((1-\rho_A+\sigma\phi_y)(1-\beta\rho_A) + \kappa\sigma\left(\phi_\pi - \rho_A\right))} var(Y_t - Y_t^e) \\
&> 0 \text{ iff } (\phi_\pi - \rho_A) < 0 \\
&< 0 \text{ iff } (\phi_\pi - \rho_A) > 0
\end{aligned}$$

Proof of Proposition 10

We combine results of Propositions 6 and 9, which directly yields after some algebra:

$$\frac{\partial W}{\partial \kappa} = -\left(\phi + \sigma^{-1}\right)\left[\frac{\partial \frac{\theta}{\kappa} var\left(\pi_t\right)}{\partial \kappa} + \frac{\partial var\left(Y_t - Y_t^e\right)}{\partial \kappa}\right]$$

$$(-\frac{\theta}{\left(1-\beta\rho_A\right)^2}\left((1-\rho_A+\sigma\phi_y)(1-\beta\rho_A) - \kappa\sigma(\phi_\pi - \rho_A)\right) + 2\ \sigma(\phi_\pi - \rho_A))$$

$$< 0 \text{ if } \ (\phi_\pi - \rho_A) < \Lambda_A = \frac{\theta(1-\beta\rho_A)(1-\rho_A+\sigma\phi_Y)}{\sigma(2(1-\beta\rho_A)^2+\kappa\theta)}$$

$$> 0 \text{ if } \ (\phi_\pi - \rho_A) > \Lambda_A = \frac{\theta(1-\beta\rho_A)(1-\rho_A+\sigma\phi_Y)}{\sigma(2(1-\beta\rho_A)^2+\kappa\theta)}$$

ZLB

When ψ_t becomes negative enough, the ZLB binds. We assume, like in Eggertsson and Woodford (2003) and Eggertsson (2008) that the shock to $\psi_t = \psi_S < 0$ in period 0 and which reverts back to steady state $\psi_S = \bar{\psi} > 0$ with a fixed probability $1-\mu$ every period thereafter. Under discretion, out of the trap, optimal policy is able to achieve $Y_t - Y_t^e$, $\pi_t = 0$. At the ZLB, we have $i_t = \beta - 1$.

First, consider the Phillips curve (no shock to Y_t^n now) where we denote by S the time in the trap:

$$\pi_S = \kappa Y_S + \beta\mu\pi_S.$$

Next, consider the IS equation (no shock to Y_t^n now)

$$Y_S = \mu Y_S + \sigma\mu\pi_S + (1-\mu)\psi_S$$

Some algebra directly implies that

$$Y_S = \frac{(1-\beta\mu)}{(1-\mu)\left(1-\beta\mu\right) - \kappa\sigma\mu}\psi_S$$

and

$$\pi_S = \frac{\kappa}{(1-\mu)\left(1-\beta\mu\right) - \kappa\sigma\mu}\psi_S$$

Note that here $Y_t^e = 0$. Consider each derivative of the welfare function with respect to κ. First, some algebra directly implies that

$$\frac{\partial var(Y_S)}{\partial \kappa} = \frac{2\sigma\mu}{(1-\mu)\left(1-\beta\mu\right) - \kappa\sigma\mu} var\left(Y_S\right) > 0$$

Next, since $\pi_S = \frac{\kappa}{1-\beta\mu} Y_S$, we have that

$$\begin{aligned}\frac{\partial var(\pi_S)}{\partial \kappa} &= 2\left(\frac{\kappa}{1-\beta\mu}\right)\frac{1}{1-\beta\mu} var\left(Y_S\right) + \left(\frac{\kappa}{1-\beta\mu}\right)^2 \frac{\partial var(Y_S)}{\partial \kappa} \\ &= \left[\frac{\left(1-\mu\right)\left(1-\beta\mu\right)}{\left[\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu\right]}\right]\frac{2\kappa}{\left(1-\beta\mu\right)^2} var\left(Y_S\right) > 0\end{aligned}$$

This implies that the derivative of the weighted variance of inflation is

$$\begin{aligned}\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} &= -var\left(\pi_S\right)\frac{\theta}{\kappa^2} + \frac{\theta}{\kappa}\frac{\partial var(\pi_S)}{\partial \kappa} \\ &= \left[\frac{\left(1-\mu\right)\left(1-\beta\mu\right)+\kappa\sigma\mu}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu}\right]\frac{\theta}{\left(1-\beta\mu\right)^2} var\left(Y_S\right) > 0\end{aligned}$$

Therefore, the derivative of welfare with respect to κ is negative:

$$\frac{\partial W}{\partial \kappa} < 0.$$

since all loss components have a positive derivative with respect to κ and are multiplied by -1.

A.4 Optimal policy

Optimal policy under discretion can be characterized easily here since there are no state variables. The problem is just a static one of minimizing

$$L_t = \left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa}\pi_t^2 + \left(Y_t - Y_t^e\right)^2\right]$$

subject to

$$\pi_t = \kappa Y_t - \kappa Y_t^n + \beta E_t \pi_{t+1}.$$

Lets reformulate it as minimizing

$$L_t = \left[\frac{\theta}{\kappa}\pi_t^2 + \left(Y_t - Y_t^e\right)^2\right]$$

subject to

$$\pi_t = \kappa\left(Y_t - Y_t^e\right) + \kappa\left(Y_t^e - Y_t^n\right) + \beta E_t \pi_{t+1}.$$

The FOC of this problem leads to the simple, well-known targeting rule

$$\theta\pi_t + (Y_t - Y_t^e) = 0.$$

Now we have to work with two equations only to pin down the solution of the model

$$\theta\pi_t + (Y_t - Y_t^e) = 0$$

$$\pi_t = \kappa (Y_t - Y_t^e) + \kappa (Y_t^e - Y_t^n) + \beta E_t \pi_{t+1}$$

Replace the first into the second

$$\pi_t = -\kappa\theta\pi_t + \kappa (Y_t^e - Y_t^n) + \beta E_t \pi_{t+1}$$

Now replace for

$$Y_t^e - Y_t^n = \frac{1}{\sigma^{-1} + \phi}\hat{\mu}_t$$

Then get

$$\pi_t = -\kappa\theta\pi_t + \kappa\frac{1}{\sigma^{-1} + \phi}\hat{\mu}_t + \beta E_t \pi_{t+1}$$

This gives the following first-order forward looking difference equation in π_t

$$(1 + \kappa\theta)\,\pi_t = \beta E_t \pi_{t+1} + \frac{\kappa}{\sigma^{-1} + \phi}\hat{\mu}_t$$

Guess

$$\pi_t = \pi_\mu \hat{\mu}_t$$

which gives

$$E_t \pi_{t+1} = \pi_\mu \rho_\mu \hat{\mu}_t$$

Replace above and match coeffcients to get

$$\pi_\mu = \frac{\kappa}{(\sigma^{-1} + \phi)(1 + \kappa\theta - \beta\rho_\mu)}$$

Thus,

$$\pi_t = \frac{1}{(\sigma^{-1} + \phi)}\left(\frac{\kappa}{(1 + \kappa\theta - \beta\rho_\mu)}\right)\hat{\mu}_t.$$

This implies that

$$(Y_t - Y_t^e) = -\theta\pi_t = -\frac{\theta}{(\sigma^{-1}+\phi)}\left(\frac{\kappa}{(1+\kappa\theta-\beta\rho_\mu)}\right)\hat{\mu}_t.$$

We first start with establishing what happens to the variance of output when prices become more flexible. First note that two cases are particularly easy. For demand shocks, output does not respond at all as long as the ZLB does not bind. That is, in that case, we have

$$Y_t = 0.$$

So variance of output does not depend on price stickiness.

For technology shocks, output responds one-to-one since we have

$$\pi_t = (Y_t - Y_t^e) = 0.$$

This means

$$Y_t = Y_t^e = \frac{1+\phi}{\sigma^{-1}+\phi}\hat{A}_t.$$

Again, variance of output does not depend on price stickiness.

For markup shocks, we have as the solution for output (since $Y_t^e = 0$)

$$Y_t = -\frac{\theta}{(\sigma^{-1}+\phi)}\left(\frac{\kappa}{(1+\kappa\theta-\beta\rho_\mu)}\right)\hat{\mu}_t.$$

$$var\left(Y_t\right) = \theta^2\left(\frac{\kappa}{(1+\kappa\theta-\beta\rho_\mu)}\right)^2.$$

Then

$$\frac{\partial var\left(Y_t\right)}{\partial\kappa} = \frac{\theta^2\left(1-\beta\rho_\mu\right)2\kappa}{\left(1+\kappa\theta-\beta\rho_\mu\right)^3} > 0.$$

Now, lets look at the effects of increased price flexibility on welfare. As is well-known with technology shocks only, both π_t and $(Y_t - Y_t^e)$ can be put to zero and one gets to first-best. Thus, there is no interesting relationship between price flexibility and welfare. With mak-up shocks, there is a trade-off as can be seen above.

For mark-up shocks, we want to evaluate

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa} var\left(\pi_t\right) + var\left(Y_t - Y_t^e\right)\right]$$

We have as the targeting rule

$$\theta\pi_t + \left(Y_t - Y_t^e\right) = 0$$

which gives

$$\theta^2 var\left(\pi_t\right) = var\left(Y_t - Y_t^e\right).$$

Then, welfare is given by

$$W = -\left(\phi + \sigma^{-1}\right)\theta\left[\left(\frac{1}{\kappa} + \theta\right) var\left(\pi_t\right)\right].$$

We have as the solution of the model

$$\pi_t = \frac{1}{\left(\sigma^{-1} + \phi\right)}\left(\frac{\kappa}{\left(1 + \kappa\theta - \beta\rho_\mu\right)}\right)\hat{\mu}_t$$

or

$$var\left(\pi_t\right) = \left[\frac{1}{\left(\sigma^{-1} + \phi\right)}\left(\frac{\kappa}{\left(1 + \kappa\theta - \beta\rho_\mu\right)}\right)\right]^2.$$

We can then establish how variance of inflation depends on price flexibility.

$$\frac{\partial var(\pi_t)}{\partial\kappa} = \frac{\left(1 - \beta\rho_\mu\right)2\kappa}{\left(1 + \kappa\theta - \beta\rho_\mu\right)^3} > 0.$$

Then, we can establish how the welfare relevant variance of inflation depends on price flexibility

$$\frac{\partial\frac{\theta}{\kappa} var\left(\pi_t\right)}{\partial\kappa} = \frac{1 - \kappa\theta - \beta\rho_\mu}{\left(1 + \kappa\theta - \beta\rho_\mu\right)^3} > 0 \text{ if } 1 - \kappa\theta > \beta\rho_\mu.$$

Thus, while with a low ρ_μ this variance of welfare relevant inflation term is increasing with greater price flexibility, it can decrease for a high enough ρ_μ. Third, we can consider how the variance of welfare relevant output gap depends on increased price flexibility (this is basically the same as the variance of output since $Y_t^e = 0$)

$$\frac{\partial var\left(Y_t - Y_t^e\right)}{\partial\kappa} = \theta^2\frac{\left(1 - \beta\rho_\mu\right)2\kappa}{\left(1 + \kappa\theta - \beta\rho_\mu\right)^3} > 0.$$

Now, lets finally move to welfare. Replace the expression for inflation above, along with the relationship between to get π_t and $(Y_t - Y_t^e)$ to get

$$W = -\frac{1}{(\sigma^{-1} + \phi)}\theta \left[\frac{\kappa (1 + \theta\kappa)}{(1 + \kappa\theta - \beta\rho_\mu)^2}\right]$$

For simplicity, first consider $\rho_\mu = 0$. Then, we have

$$W = -\frac{1}{(\sigma^{-1} + \phi)}\theta \left[\frac{\kappa}{(1 + \kappa\theta)}\right].$$

It is easy to see that in such a case

$$\frac{\partial W}{\partial \kappa} < 0.$$

When we consider a general ρ_μ however, note that this is not always the case. In particular, for a high enough ρ_μ, it can be the case that increased price flexibility leads to higher welfare. Generally,

$$\frac{\partial W}{\partial \kappa} = -\frac{1 + \kappa\theta - \beta\rho_\mu (1 + 2\kappa\theta)}{(1 + \kappa\theta - \beta\rho_\mu)^3}$$

The denominator is always positive, but the numerator can take either positive or negative value. Thus,

$$\frac{\partial W}{\partial \kappa} < 0 \text{ if } \frac{1 + \kappa\theta}{1 + 2\kappa\theta} > \beta\rho_\mu.$$

There are two forces at work: the variance of the welfare relevant output gap is always increasing in price flexibility, but the variance of the welfare relevant inflation can decrease with higher flexibility if ρ_μ is big enough.

Finally, we take into account the zero lower bound on interest rates explicitly. This happens when r_t^n becomes negative enough so that the ZLB binds. We assume, like in Eggertsson and Woodford (2003) and Eggertsson (2008) that the shock to $r_t^n = r_S^n < 0$ in period 0 and which reverts back to steady state $r_S = \bar{r} > 0$ with a fixed probability $1-\mu$ every period thereafter.

Under discretion, out of the trap, optimal policy is able to achieve $Y_t - Y_t^e$, $\pi_t = 0$. At ZLB, we have $i_t = \beta - 1$.We have to consider two cases: when the economy is in a ZLB situation and when it is out of it. Out of the trap, as discussed above, in this simple model under discretion, both $Y_t - Y_t^e$ and π_t are equal to zero when the shock that hits the economy is a shock to r_t^n such as a preference shock. In the trap, $i_t = \beta - 1$.

Now, consider the Phillips curve (no Y_t^n shock now)

$$\pi_t = \kappa Y_t + \beta E_t \pi_{t+1}$$

which we rewrite as

$$\pi_S = \kappa Y_S + \beta \mu \pi_S.$$

Next, consider the IS equation (no Y_t^n shock now)

$$Y_t = E_t Y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n)$$

which we rewrite as

$$Y_S = \mu Y_S + \sigma \mu \pi_S + \sigma r_S^n$$

Lets manipulate these two expressions

$$\pi_S = \left(\frac{\kappa}{1 - \beta\mu} \right) Y_S$$

$$\pi_S = \frac{(1-\mu) Y_S - \sigma r_S^n}{\sigma \mu}$$

and combine them to get

$$\left(\frac{\kappa}{1 - \beta\mu} \right) Y_S = \frac{(1-\mu) Y_S - \sigma r_S^n}{\sigma \mu}$$

or

$$\left[\left(\frac{1-\mu}{\sigma\mu} \right) - \left(\frac{\kappa}{1-\beta\mu} \right) \right] Y_S = \frac{1}{\mu} r_S^n$$

$$\left[\frac{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu}{\sigma\mu(1-\beta\mu)} \right] Y_S = \frac{1}{\mu} r_S^n$$

$$Y_S = \frac{\sigma(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu} r_S^n$$

and then

$$\pi_S = \left(\frac{\kappa}{1 - \beta\mu} \right) Y_S$$

$$\pi_S = \left(\frac{\kappa}{1 - \beta\mu} \right) \frac{\sigma(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu} r_S^n$$

$$\pi_S = \frac{\sigma\kappa}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu} r_S^n.$$

For welfare, we want to evaluate

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa} var\left(\pi_t\right) + var\left(Y_t - Y_t^e\right)\right]$$

which here is

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa} var\left(\pi_t\right) + var\left(Y_t\right)\right]$$

First, we have

$$var\left(\pi_S\right) = \left(\frac{\kappa}{1-\beta\mu}\right)^2 var\left(Y_S\right)$$

and

$$var\left(Y_S\right) = \left[\frac{\sigma\left(1-\beta\mu\right)}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu}\right]^2$$

so

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa} var\left(\pi_S\right) + var\left(Y_S\right)\right]$$

$$W = -\left(\phi + \sigma^{-1}\right)\left[\frac{\theta}{\kappa}\left(\frac{\kappa}{1-\beta\mu}\right)^2 var\left(Y_S\right) + var\left(Y_S\right)\right]$$

$$W = -\left(\phi + \sigma^{-1}\right)\left[\left(\frac{\theta}{\kappa}\left(\frac{\kappa}{1-\beta\mu}\right)^2 + 1\right) var\left(Y_S\right)\right]$$

$$W = -\left(\phi + \sigma^{-1}\right)\left[\left(\frac{\theta\kappa}{\left(1-\beta\mu\right)^2} + 1\right) var\left(Y_S\right)\right].$$

Now

$$\frac{\partial var(Y_S)}{\partial\kappa} = 2\left[\frac{\sigma\left(1-\beta\mu\right)}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu}\right]\frac{\sigma\left(1-\beta\mu\right)}{\left((1-\mu)(1-\beta\mu) - \kappa\sigma\mu\right)^2}\sigma\mu > 0$$

which also gives

$$\frac{\partial var(Y_S)}{\partial\kappa} = \frac{2\sigma\mu}{(1-\mu)(1-\beta\mu) - \kappa\sigma\mu} var\left(Y_S\right).$$

Next

$$\frac{\partial var(\pi_S)}{\partial\kappa} = 2\left(\frac{\kappa}{1-\beta\mu}\right)\frac{1}{1-\beta\mu} var\left(Y_S\right) + \left(\frac{\kappa}{1-\beta\mu}\right)^2\frac{\partial var(Y_S)}{\partial\kappa}$$

$$\frac{\partial var(\pi_S)}{\partial \kappa} = 2\left(\frac{\kappa}{1-\beta\mu}\right)\frac{1}{1-\beta\mu}var\left(Y_S\right) + \left(\frac{\kappa}{1-\beta\mu}\right)^2 \frac{2\sigma\mu}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu}var\left(Y_S\right)$$

$$\frac{\partial var(\pi_S)}{\partial \kappa} = \left[\frac{1}{\kappa} + \frac{\sigma\mu}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu}\right] 2\left(\frac{\kappa}{1-\beta\mu}\right)^2 var\left(Y_S\right)$$

$$\frac{\partial var(\pi_S)}{\partial \kappa} = \left[\frac{\left(1-\mu\right)\left(1-\beta\mu\right)}{\left[\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu\right]}\right]\frac{2\kappa}{\left(1-\beta\mu\right)^2}var\left(Y_S\right) > 0$$

Finally, the weighted variance of inflation term

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = -var\left(\pi_S\right)\frac{\theta}{\kappa^2} + \frac{\theta}{\kappa}\frac{\partial var(\pi_S)}{\partial \kappa}$$

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = -\left(\frac{\kappa}{1-\beta\mu}\right)^2 var\left(Y_S\right)\frac{\theta}{\kappa^2} + \frac{\theta}{\kappa}\left[\frac{\left(1-\mu\right)\left(1-\beta\mu\right)}{\left[\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu\right]}\right]\frac{2\kappa}{\left(1-\beta\mu\right)^2}var\left(Y_S\right)$$

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = -\frac{\theta}{\left(1-\beta\mu\right)^2}var\left(Y_S\right) + \left[\frac{\left(1-\mu\right)\left(1-\beta\mu\right)}{\left[\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu\right]}\right]\frac{2\theta}{\left(1-\beta\mu\right)^2}var\left(Y_S\right)$$

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = \left[-1 + \left[\frac{2\left(1-\mu\right)\left(1-\beta\mu\right)}{\left[\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu\right]}\right]\right]\frac{\theta}{\left(1-\beta\mu\right)^2}var\left(Y_S\right)$$

$$\frac{\partial \frac{\theta}{\kappa} var\left(\pi_S\right)}{\partial \kappa} = \left[\frac{\left(1-\mu\right)\left(1-\beta\mu\right)+\kappa\sigma\mu}{\left(1-\mu\right)\left(1-\beta\mu\right)-\kappa\sigma\mu}\right]\frac{\theta}{\left(1-\beta\mu\right)^2}var\left(Y_S\right) > 0$$

So,

$$\frac{\partial W}{\partial \kappa} < 0.$$

We can also study optimal monetary policy under commitment, which means specifying a fully state-contingent path at $t = 0$ for the endgeonous variables to minimize the loss-function subject to

$$\pi_t = \kappa Y_t - \kappa Y_t^n + \beta E_t \pi_{t+1}.$$

Lets then define the Lagrangian

$$\begin{aligned}\mathcal{L} =& \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\theta}{\kappa} \pi_t^2 + (Y_t - Y_t^e)^2 \right] \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t q_{1,t} \left\{ \pi_t - \kappa \left(Y_t - Y_t^e \right) - \kappa \left(Y_t^e - Y_t^n \right) - \beta E_t \pi_{t+1}. \right\}\end{aligned}$$

where $\{q_{1,t}\}$ is the sequence of Lagrange multiplier.

First order conditions are given as:

$$\partial \pi_t : 0 = \frac{\theta}{\kappa} \pi_t + q_{1,t} - q_{1,t-1}$$

$$\partial \left(Y_t - Y_t^e \right) : 0 = \left(Y_t - Y_t^e \right) - \kappa q_{1,t}$$

Consequently, the equilibrium time path of

$$\left\{ \hat{Y}_t, \pi_t, q_{1,t} \right\}_{t=0}^{\infty}$$

is characterized by the following 3 equations

$$\begin{aligned}\pi_t &= \kappa \left(Y_t - Y_t^e \right) - \kappa \left(Y_t^e - Y_t^n \right) - \beta E_t \pi_{t+1} \\ 0 &= \frac{\theta}{\kappa} \pi_t + q_{1,t} - q_{1,t-1} \\ 0 &= \left(Y_t - Y_t^e \right) - \kappa q_{1,t}\end{aligned}$$

given exogenous processes and initial conditions. We assume that all the variables are in the steady state initially: $q_{-1} = 0$.

So assuming the "time-less perspective" we have as the "targeting rule"

$$\theta \pi_t + \left(Y_t - Y_t^e \right) - \left(Y_{t-1} - Y_{t-1}^e \right) = 0.$$

B APPENDIX: Wage and Price Flexibility

B.1 General model

Woodford (2003) presents a simple model with wage and price stickiness that can be summarized under a Taylor rule as

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t\pi^p_{t+1} - r^e_t) \tag{1}$$

$$\pi^p_t = \kappa_p(Y_t - Y^n_t) + \xi_p(\hat{w}_t - \hat{w}^n_t) + \beta E_t\pi^p_{t+1} \tag{2}$$

$$\pi^w_t = \kappa_w(Y_t - Y^n_t) - \xi_w(\hat{w}_t - \hat{w}^n_t) + \beta E_t\pi^w_{t+1} \tag{3}$$

$$\hat{\imath}_t = \phi_\pi\pi^p_t + \phi_y\hat{Y}_t \tag{4}$$

$$\hat{w}_t = \hat{w}_{t-1} + \pi^w_t - \pi^p_t \tag{5}$$

where we have that $\hat{w}^n_t = (1+\omega_p)a_t - \omega_p\hat{Y}^n_t$ and $\hat{Y}^n_t = \frac{1+\omega}{\sigma^{-1}+\omega}a_t - \frac{1}{\sigma^{-1}+\omega}\hat{\mu}_t + \frac{1}{\sigma^{-1}+\omega}\hat{\tau}^w_t$. Also, the welfare- objective around the efficient steady state is given by

$$L_t = \lambda_p(\pi^p_t)^2 + \lambda_w(\pi^w_t)^2 + \lambda_x(\hat{Y}_t - \hat{Y}^e_t)^2.$$

Here, we have $\xi_w = \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w(1+\nu\theta_w)}$, $\xi_p = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\omega_p\theta_p)}$, $\kappa_w = \xi_w\left(\omega_w + \sigma^{-1}\right)$, $\kappa_p = \xi_p\omega_p$, $\kappa_w = \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w}\frac{\left(\omega_w+\sigma^{-1}\right)}{(1+\nu\theta_w)}$, $\kappa_p = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p}\frac{\omega_p}{(1+\omega_p\theta_p)}$, $\lambda_p = \frac{\theta_p\xi_p^{-1}}{\theta_p\xi_p^{-1}+\theta_w\phi_h^{-1}\xi_w^{-1}} > 0$, $\lambda_w = \frac{\theta_w\phi_h^{-1}\xi_w^{-1}}{\theta_p\xi_p^{-1}+\theta_w\phi_h^{-1}\xi_w^{-1}} > 0$, and $\lambda_x = \frac{\sigma^{-1}+\omega}{\theta_p\xi_p^{-1}+\theta_w\phi_h^{-1}\xi_w^{-1}} > 0$. Moreover, $\nu \equiv \frac{v_{hh}h}{v_h}$, $\phi_h \equiv \frac{f(h)}{hf'(h)}$, $\omega_w = \nu\phi_h$, and $\omega = \omega_w + \omega_p$.

Assume the production function $y_t(i) = A_th_t(i)^\gamma$ to get $\phi_h = 1/\gamma$, $\omega_w = \nu/\gamma$, and $\omega_p = \frac{1-\gamma}{\gamma}$.

B.2 Simplified Approximate model

Next, we make the assumption that simplifies the model and leads to sharp insights. We assume that $\kappa_p = \kappa_w = \kappa$. After some manipulation and using that $\Delta\hat{w}_t = \pi^w_t - \pi^p_t$, we obtain

$$\Delta w_t = -(\xi_w + \xi_p)(w_t - w^n_t) + \beta E_t\Delta w_{t+1}$$

and the solution for w_t can then be written as

$$w_t = \Gamma_w w_{t-1} + \Gamma_n w_t^n$$

where Γ_w is the root less than 1 of the polynomial $\mu^2 - \frac{\left(\beta+\kappa\left(\frac{1}{\omega_w+\sigma^{-1}}+\frac{1}{\omega_p}\right)+1\right)}{\beta}\mu + \frac{1}{\beta} = 0$ and $\Gamma_n = \frac{\Gamma_w}{1-\beta\Gamma_w\rho_A}\kappa\left(\frac{1}{\omega_w+\sigma^{-1}} + \frac{1}{\omega_p}\right)$.

As our second result, in this simplified case, the rest of the model equations reduce to three equations as given by

$$\pi_t^p = \kappa(Y_t - Y_t^n) + \beta E_t \pi_{t+1}^p + \kappa \frac{1}{\omega_p}(w_t - w_t^n)$$

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma(i_t - E_t \pi_{t+1}^p - r_t^e)$$

$$i_t = \phi_\pi \pi_t^p + \phi_y \hat{Y}_t$$

This then implies that our previous result on demand shocks will go through fully in this case.

For productivity shocks, it is tedious to analytically show how the variance of output varies with κ. The solution of the model however can be shown in closed-form. For simplicity, assume log-utility ($\sigma = 1$) and i.i.d. technology shocks ($\rho_A = 0$). Then,

$$\hat{Y}_t = Y_A a_t + Y_w w_t$$

where

$$Y_A = \frac{\kappa\left(1 + \frac{1}{\omega_p}\right)}{\kappa + \frac{(\phi_y+1)}{\phi_\pi}}; Y_w = -\frac{\kappa\left(\phi_\pi - \Gamma_w\right)}{\omega_p\left[\kappa\left(\phi_\pi - \Gamma_w\right) - \left(\Gamma_w - 1 - \phi_y\right)\left(1 - \beta\Gamma_w\right)\right]}.$$

This together with

$$w_t = \Gamma_w w_{t-1} + \Gamma_n w_t^n$$

and

$$w_t^n = a_t$$

completes the solution.

B.3 Discretion

The objective function is given by the following:

$$
\begin{aligned}
L_t &= \lambda_p(\pi_t^p)^2 + \lambda_w(\pi_t^w)^2 + \lambda_x(\hat{Y}_t - \hat{Y}_t^e)^2 \\
\lambda_p &= \frac{\theta_p \xi_p^{-1}}{\theta_p \xi_p^{-1} + \theta_w \phi_h^{-1} \xi_w^{-1}} > 0, \lambda_w = \frac{\theta_w \phi_h^{-1} \xi_w^{-1}}{\theta_p \xi_p^{-1} + \theta_w \phi_h^{-1} \xi_w^{-1}} > 0, \lambda_x = \frac{\sigma^{-1} + \omega}{\theta_p \xi_p^{-1} + \theta_w \phi_h^{-1} \xi_w^{-1}} > 0 \\
Y_t^e &= \frac{1 + \omega}{\sigma^{-1} + \omega} a_t
\end{aligned}
$$

Given our specific assumptions, w_t is an exogenous process and hence there are no endogenous state variables in the model. This greatly simplifies things as the discretion problem just reduces to a period by period minimization problem.Also, note that our assumptions $\xi_w = \frac{1}{\omega_w + \sigma^{-1}} \kappa$, $\xi_p = \frac{1}{\omega_p} \kappa$ and $\Delta \hat{w}_t = \pi_t^w - \pi_t^p$ and the assumption of log utility (for expository reasons only) allow us to write the following Lagrangian after some manipulation:

$$
\begin{aligned}
\mathcal{L}_t = &\frac{1}{2} \left[\lambda_p(\pi_t^p)^2 + \lambda_w(\Delta \hat{w}_t + \pi_t^p)^2 + \lambda_x(\hat{Y}_t - \hat{Y}_t^e)^2 \right] \\
&+ q_{1,t} \left\{ \pi_t^p - \kappa(\hat{Y}_t - \hat{Y}_t^e) - \beta E_t \pi_{t+1}^p + \kappa \frac{1}{\omega_p} a_t - \kappa \frac{1}{\omega_p} w_t \right\}
\end{aligned}
$$

where the central bank will take expectation functions as given since there are no endogenous state variables and we use the fact that the IS equation is not binding. This yields the following FOCs:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi_t^p} &= \lambda_p \pi_t^p + \lambda_w \left(\Delta \hat{w}_t + \pi_t^p \right) + q_{1,t} = 0 \\
\frac{\partial \mathcal{L}}{\partial \left(\hat{Y}_t - \hat{Y}_t^e \right)} &= \lambda_x(\hat{Y}_t - \hat{Y}_t^e) - \kappa q_{1,t} = 0
\end{aligned}
$$

with the exogenous processes

$$
\begin{aligned}
w_t &= \Gamma_w w_{t-1} + \frac{\beta \Gamma_w}{1 - \beta \Gamma_w \rho_A} \kappa \left(\frac{1}{\omega_w + 1} + \frac{1}{\omega_p} \right) a_t \\
\hat{Y}_t^e &= a_t
\end{aligned}
$$

Now combine the two FOCs to get the targeting rule, which is our main result here

$$
(\lambda_p + \lambda_w) \pi_t^p + \lambda_w \Delta \hat{w}_t + \frac{\lambda_x}{\kappa} (\hat{Y}_t - \hat{Y}_t^e) = 0.
$$

Assuming i.i.d. shock for simplicity, one can derive the solution of the model in closed form:

$$\pi_t^p = \pi_{w1}\hat{w}_{t-1} + \pi_w \hat{w}_t + \pi_A a_t$$

$$\pi_{w1} = \frac{\frac{\kappa}{(1+\omega)}\theta_w \phi_h^{-1}\left(\omega_w + 1\right)}{\left(1 + \frac{\kappa\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\right)}$$

$$\pi_w = \left[\frac{1}{\left(1 + \frac{\kappa\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\right) - \beta\Gamma_w}\right]\left[\beta\frac{\frac{\kappa}{(1+\omega)}\theta_w\phi_h^{-1}\left(\omega_w+1\right)}{\left(1 + \frac{\kappa\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\right)} - \frac{\kappa}{(1+\omega)}\theta_w\phi_h^{-1}\left(\omega_w+1\right) + \kappa\frac{1}{\omega_p}\right]$$

$$\pi_A = \frac{1}{\left(1 + \frac{\kappa\left(\theta_p\omega_p + \theta_w\phi_h^{-1}(\omega_w+1)\right)}{(1+\omega)}\right)}\left[-\kappa\frac{1}{\omega_p}\right]$$

For the output gap, we have

$$\left(\theta_p\omega_p + \theta_w\phi_h^{-1}\left(\omega_w+1\right)\right)\pi_t^p + \theta_w\phi_h^{-1}\left(\omega_w+1\right)\Delta\hat{w}_t + (1+\omega)\,(\hat{Y}_t - \hat{Y}_t^e) = 0$$

which gives

$$(\hat{Y}_t - \hat{Y}_t^e) = -\frac{\left(\theta_p\omega_p + \theta_w\phi_h^{-1}\left(\omega_w+1\right)\right)}{(1+\omega)}\pi_t^p - \frac{\theta_w\phi_h^{-1}\left(\omega_w+1\right)}{(1+\omega)}\left(\hat{w}_t - \hat{w}_{t-1}\right)$$

and for output, since $\hat{Y}_t^e = a_t$

$$\hat{Y}_t = -\frac{\left(\theta_p\omega_p + \theta_w\phi_h^{-1}\left(\omega_w+1\right)\right)}{(1+\omega)}\pi_t^p - \frac{\theta_w\phi_h^{-1}\left(\omega_w+1\right)}{(1+\omega)}\left(\hat{w}_t - \hat{w}_{t-1}\right) + a_t$$

B.4 Commitment

In the case of commitment, we have:

$$\mathcal{L} = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\lambda_p(\pi_t^p)^2 + \lambda_w(\Delta\hat{w}_t + \pi_t^p)^2 + \lambda_x(\hat{Y}_t - \hat{Y}_t^e)^2\right]$$
$$+ E_0\sum_{t=0}^{\infty}\beta^t q_{1,t}\left\{\pi_t^p - \kappa(\hat{Y}_t - \hat{Y}_t^e) - \beta E_t\pi_{t+1}^p + \kappa\frac{1}{\omega_p}a_t - \kappa\frac{1}{\omega_p}w_t\right\}$$

where the central bank can commit and hence does not take $E_t\pi_{t+1}^p$ as given

$$\frac{\partial\mathcal{L}}{\partial\pi_t^p} = \lambda_p\pi_t^p + \lambda_w\left(\Delta\hat{w}_t + \pi_t^p\right) + q_{1,t} - q_{1,t-1} = 0$$
$$\frac{\partial\mathcal{L}}{\partial\left(\hat{Y}_t - \hat{Y}_t^e\right)} = \lambda_x(\hat{Y}_t - \hat{Y}_t^e) - \kappa q_{1,t} = 0$$

with the exogenous processes

$$w_t = \Gamma_w w_{t-1} + \frac{\beta\Gamma_w}{1-\beta\Gamma_w\rho_A}\kappa\left(\frac{1}{\omega_w+1} + \frac{1}{\omega_p}\right)a_t$$
$$\Delta\hat{w}_t = \left(\Gamma_w - 1\right)w_{t-1} + \frac{\beta\Gamma_w}{1-\beta\Gamma_w\rho_A}\kappa\left(\frac{1}{\omega_w+1} + \frac{1}{\omega_p}\right)a_t$$
$$\hat{Y}_t^e = a_t$$

Now combine the two FOCs to get the targeting rule

$$\left(\lambda_p + \lambda_w\right)\pi_t^p + \lambda_w\Delta\hat{w}_t + \frac{\lambda_x}{\kappa}(\hat{Y}_t - \hat{Y}_t^e) - \frac{\lambda_x}{\kappa}(\hat{Y}_{t-1} - \hat{Y}_{t-1}^e) = 0.$$

The closed-form solution of the model is not very instructive, although possible.

C APPENDIX: Smets-Wouters Model

We refer the reader to the original Smets and Wouters (2007) paper for a detailed description of the model. Here we present the log-linearized equilibrium conditions in line with the notation in their paper (for the expressions for the reduced-form parameters below as a function of the structural parameters, please see Smets and Wouters (2007) and its appendix).

$$\hat{c}_t = c_1 \hat{c}_{t-1} + (1 - c_1) E_t \hat{c}_{t+1} - c_2 \{\hat{r}_t - E_t \hat{\pi}_{t+1} + \varepsilon_t^b\} - c_3 \left(E_t \hat{n}_{t+1} - \hat{n}_t\right)$$

$$\hat{\imath}_t = i_1 \hat{\imath}_{t-1} + (1 - i_1) E_t \hat{\imath}_{t+1} + i_2 \hat{q}_t + \varepsilon_t^q$$

$$\hat{q}_t = -\left(\hat{r}_t - E_t \hat{\pi}_{t+1} + \varepsilon_t^b\right) + q_1 E_t \hat{r}_{t+1}^k + (1 - q_1) E_t \hat{q}_{t+1}$$

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{\imath}_t + \hat{g}_t + \upsilon_y \hat{\upsilon}_t$$

$$\hat{\pi}_t - \iota_p \hat{\pi}_{t-1} = \bar{\beta}\bar{\gamma} \left(E_t \hat{\pi}_{t+1} - \iota_p \hat{\pi}_t\right) - \pi_1 \left(-\left(\alpha \hat{r}_t^k + (1 - \alpha)\hat{w}_t - a_t\right) - \hat{\mu}_t^p\right)$$

$$\hat{\pi}_t^w - \iota_w \hat{\pi}_{t-1} = \bar{\beta}\bar{\gamma} \left(E_t \hat{\pi}_{t+1}^w - \iota_w \hat{\pi}_t\right) - \lambda_w \left(\hat{w}_t - \left(\left(\frac{1}{1 - h/\bar{\gamma}}\right) \hat{c}_t - \left(\frac{h/\bar{\gamma}}{1 - h/\bar{\gamma}}\right) \hat{c}_{t-1} + \sigma_l \hat{n}_t\right) - \hat{\mu}_t^w\right)$$

$$\hat{\bar{k}}_t = k_1 \hat{\bar{k}}_{t-1} + (1 - k_1)\, \hat{\imath}_t + k_2 \varepsilon_t^q$$

$$\hat{k}_t = \hat{\upsilon}_t + \hat{\bar{k}}_{t-1}$$

$$\hat{\upsilon}_t = \left(\frac{1}{(\psi/1 - \psi)}\right) \hat{r}_t^k$$

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{n}_t$$

$$r_t = \rho r_{t-1} + (1 - \rho) \left(r_\pi \pi_t + r_y \widehat{ygap}_t\right) + r_{\Delta y} \widehat{\Delta ygap}_t + \varepsilon_t^r$$

D APPENDIX: Solution and Estimation Method

We use a Bayesian framework for estimation. The first-order approximation to the equilibrium conditions of the model can be written as

$$\Gamma_0(\theta)\, s_t = \Gamma_1(\theta)\, s_{t-1} + \Gamma_\varepsilon(\theta)\varepsilon_t + \Gamma_\eta(\theta)\pi_t$$

where s_t is a vector of model variables and ε_t is a vector of shocks to the exogenous processes. π_t is a vector of rational expectations forecast errors, which implies $E_{t-1}\pi_t = 0$ for all $t,$ and θ contains the structural model parameters. The solution to this system is given by

$$s_t = \Omega_1(\theta)s_{t-1} + \Omega_\varepsilon(\theta)\varepsilon_t.$$

which can be obtained using standard methods in the literature. Finally, the model variables are related to the observables by the measurement equation

$$y_t = Bs_t$$

where y_t is the vector of observables.

Let $Y = \{y\}_{t=1}^{T}$ be the data. In a Bayesian framework, the likelihood function $L(Y \mid \theta)$ is combined with a prior density $p(\theta)$ to yield the posterior density

$$p(\theta \mid Y) \quad \propto \quad p(\theta)L(Y \mid \theta).$$

Assuming Gaussian shocks, it is straightforward to evaluate the likelihood function using the Kalman filter. A numerical optimization routine is used to maximize $p(\theta \mid Y)$ and find the posterior mode. Then, we can generate draws from $p(\theta \mid Y)$ using the Metropolis-Hastings algorithm where we use a Gaussian proposal density in the algorithm, using a inverse of a scaled Hessian computed at the posterior mode as the covariance matrix.

The Metropolis-Hastings algorithm works as follows. Let the posterior mode computed from the numerical optimization routine be $\tilde{\theta}$. Let the inverse of the Hessian computed at $\tilde{\theta}$ be $\tilde{\Sigma}$.

(a) Choose a starting value θ^0. Then use a loop over the following steps (b)-(d).

(b) For $d = 1, ..., D,$ draw a θ^* from the proposal distribution $N(\theta^{d-1},\ c\tilde{\Sigma})$.

(c) Accept $\theta^*,$ that is $\theta^d = \theta^*,$ with probability $\min\{1, r(\theta^{d-1},\ \theta^*)\}$. Reject θ^*, that is $\theta^d = \theta^{d-1},$ otherwise.

(d) $r(\theta^{d-1},\ \theta^*)$ is given by:

$$r(\theta^{d-1},\ \theta^*) = \frac{p(\theta^*)L(Y \mid \theta^*)}{p(\theta^{d-1})L(Y \mid \theta^{d-1})}$$

The scale parameter c is chosen to lead to acceptance rates of around 30%.

To settle on a model specification, we do Bayesian model comparison using the marginal data densities of the models. In comparing models A and B we are interested in the relative posterior probabilities of the models given the data. That is, $\frac{p(A|Y)}{p(B|Y)} = \frac{p(A)}{p(B)} \frac{p(Y|A)}{p(Y|B)}$ where $p(A)$ and $p(B)$ are the prior probabilities of the models A and B. Since we do not specifying different prior probabilities over the models, we just compare the marginal data densities given by $p(Y \mid A)$ and $p(Y \mid B)$. The marginal data density of a model is given by

$$p(Y) = \int p(\theta)L(Y \mid \theta)\ d\theta.$$

Note that this measure penalizes overparameterized models.

The marginal data density is approximated by the Geweke (1998) modified harmonic-mean estimator. First note that we can write

$$\frac{1}{p(Y)} = \int \frac{f(\theta)d\theta}{p(\theta)L(Y \mid \theta)} d\theta$$

where f is a probability density function such that $\int f(\theta)d\theta = 1$. Then, we can use the following estimator

$$\hat{p}(Y) = \left[\frac{1}{D} \sum_{d=1}^{D} \frac{f(\theta^d)}{p(\theta^d)L(Y \mid \theta^d)} \right]^{-1}$$

where d denotes the posterior draws obtained using the Metropolis-Hastings algorithm. For f , Geweke (1998) proposed a truncated multivariate normal distribution.

E APPENDIX: Gali-Smets-Wouters Model

E.1 Log-linearized equilibrium

We refer the reader to the original Gali, Smets, and Wouters (2012) paper for a detailed description of the model and its notation. Compared to the model in Smets and Wouters (2007), the model here features log-utility as well as no Kimball demand in wages. Moreover, the model features unemployment. Here we present the log-linearized equilibrium conditions in line with the notation in their paper and the priors and posterior estimates of the model parameters.

$$\hat{c}_t = c_1\hat{c}_{t-1} + (1-c_1)E_t\hat{c}_{t+1} - c_2^{'}\{\hat{r}_t - E_t\pi_{t+1} + \varepsilon_t^b\}$$

where $c_1 = (h/\tau)/(1+h/\tau)$ and $c_2^{'} = \frac{1-h/\tau}{1-h/\tau}$.

$$\hat{\imath}_t = i_1^{'}\hat{\imath}_{t-1} + (1-i_1^{'})E_t\hat{\imath}_{t+1} + i_2^{'}\hat{q}_t + \varepsilon_t^q$$

where $i_1^{'} = \frac{1}{1+\beta}$, $i_2^{'} = i_1^{'}/(\tau^2\Psi)$, and ε_t^q is the investment specific shock .

$$\hat{q}_t = -\left(\hat{r}_t - E_t\hat{\pi}_{t+1} + \varepsilon_t^b\right) + q_1E_t\hat{r}_{t+1}^k + (1-q_1)E_t\hat{q}_{t+1}$$

where $q_1 = r^k/\left(r^k + (1-\delta)\right)$.

$$\begin{aligned} y_t &= c_y\hat{c}_t + i_y\hat{\imath}_t + \hat{g}_t + v_y\hat{v}_t \\ &= M_p\left(\alpha\hat{k}_t + (1-\alpha)\hat{n}_t + \varepsilon_t^a\right) \end{aligned}$$

where $c_y = (C/Y)$, $i_y = (I/Y)$,and $v_y = R^kK/Y$. M_p is the price markup in steady state.

$$\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} = \beta\left(E_t\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) - \pi_1^{'}\left(-\left(\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - a_t\right) - \hat{\mu}_t^P\right)$$

where $\pi_1^{'} = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\frac{1}{\left[1+(M_p^{'}-1)\varsigma_p\right]}$.

$$\hat{\pi}_t^w - \gamma_w\hat{\pi}_{t-1} = \beta\left(E_t\hat{\pi}_{t+1}^w - \gamma_w\hat{\pi}_t\right) - \lambda_w\left(\hat{w}_t - (\hat{z}_t + \varepsilon_t^x + \phi\hat{n}_t) - \hat{\mu}_t^w\right)$$

where $\lambda_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}\frac{1}{[\theta_w(1+(M_w/M_w-1)\varphi))]}$ with M_w as the wage markup in steady state, ε_t^x is the labor supply shock, and

$$\hat{z}_t = (1-\nu)\hat{z}_{t-1} + v\left[\left(\frac{1}{1-h/\tau}\right)\hat{c}_t - \left(\frac{h/\tau}{1-h/\tau}\right)\hat{c}_{t-1}\right].$$

Note that

$$\hat{w}_t - (\hat{z}_t + \varepsilon_t^x + \phi\hat{n}_t) = \varphi\hat{u}_t$$

where $\hat{u}_t$ is unemployment. By using the unemployment rate $\hat{u}_t$ as an additional observable, Gali-Smets-Wouters can identify the labor supply shock and the usual labor markup shock separately:

$$\hat{l}_t = \hat{n}_t + \hat{u}_t$$

where $\hat{l}_t$ is the labor force.

$$\widehat{\bar{k}}_t = k_1 \widehat{\bar{k}}_{t-1} + (1 - k_1)\, \hat{\imath}_t + k_2^{'} \varepsilon_t^q$$

where $k_1 = 1 - \left(I/\bar{K}\right)$ and $k_2^{'} = \left(\frac{\bar{\imath}}{\bar{k}}\right)(1+\beta)\, \tau^2 \Psi$.

$$\hat{k}_t = \hat{v}_t + \widehat{\bar{k}}_{t-1}$$

$$\hat{v}_t = \left(\frac{1-\psi}{\psi}\right) \hat{r}_t^k$$

where $\bar{r}^k = 1 - \Psi$.

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{n}_t$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(r_\pi \pi_t + r_y \widehat{ygap}_t\right) + r_{\Delta y} \widehat{\Delta ygap}_t + \varepsilon_t^r$$

where $\widehat{ygap}_t$ is the deviation of actual output from the flexible price level in the absence of the price and wage markups shocks. Unless noted otherwise, the notation for the shocks is the same as reported in the main text of our paper for the Smets-Wouters model, except for the new labor supply shock, which follows

$$\varepsilon_t^x = 0.999 * \varepsilon_{t-1}^x + \varepsilon\varepsilon_t^x$$

and has measurement error component as described below.

E.2 Estimation

We use the same data as in Gali, Smets, and Wouters (2012) (including unemployment as an observable) as well as the same strategy as them regarding measurement errors on the wage series (with two wage series used: compensation per employee (refered to as wc below) and average weekly earnings (refered to as wE below)). Thus, we use the notation σ_{wC} for the standard deviation of the measurement error on one wage series and σ_{wE} for the standard deviation of the measurement error on the other. The average weekly earnings series is allowed to have a different trend growth from the common economy-wide trend, given by τ_{wE}. Also, the price and wage markup shocks are normalized by a factor of 100 during the estimation as in Gali, Smets, and Wouters (2012).

E.3 Prior distribution

Table 1: Parametrization of Priors in Gali-Smets-Wouters, Structural Parameters

Parameters	**Density**	**Prior Mean**	**Prior Standard Deviation**
Ψ	Normal	4.00	1.00
h	Beta	0.70	0.10
θ_w	Beta	0.50	0.15
φ	Normal	2.00	1.0
ν	Beta	0.50	0.20
θ_p	Beta	0.50	0.15
γ_w	Beta	0.50	0.15
γ_p	Beta	0.50	0.15
ψ	Beta	0.50	0.15
M_p	Normal	1.25	0.12
r_π	Normal	1.50	0.25
ρ_r	Beta	0.75	0.10
r_y	Normal	0.12	0.05
$r_{\Delta y}$	Normal	0.12	0.05
$\overline{\pi}$	Gamma	0.62	0.10
$100(\beta^{-1} - 1)$	Gamma	0.25	0.10
$\bar{l}$	Normal	0.00	2.00
τ	Normal	0.40	0.10
τ_{WE}	Normal	0.20	0.10
M_w	Normal	1.5	0.25
α	Normal	0.30	0.05

Table 2: Parametrization of Priors in Gali-Smets-Wouters, Shock Processes

Parameters	**Density**	**Prior Mean**	**Prior Standard Deviation**
ρ_a	Beta	0.5	0.2
ρ_b	Beta	0.5	0.2
ρ_g	Beta	0.5	0.2
ρ_q	Beta	0.5	0.2
ρ_r	Beta	0.5	0.2
ρ_p	Beta	0.5	0.2
ρ_w	Beta	0.5	0.2
ρ_{ga}	Normal	0.5	0.25
μ_p	Beta	0.5	0.2
μ_w	Beta	0.5	0.2
σ_a	Uniform	2.5	1.44
σ_b	Uniform	2.5	1.44
σ_g	Uniform	2.5	1.44
σr	Uniform	2.5	1.44
σ_r	Uniform	2.5	1.44
σ_p	Uniform	2.5	1.44
σ_w	Uniform	2.5	1.44
σ_x	Uniform	2.5	1.44
σ_{wc}	Uniform	2.5	1.44
σ_{wE}	Uniform	2.5	1.44

E.4 Posterior estimates

Our posterior estimates are basically the same as in Gali, Smets, and Wouters (2012), with some minor differences from the different number of draws.

Table 3: Posterior Estimates of Gali-Smets-Wouters, Structural Parameters

Parameters	**Posterior Mean**	**Probability Interval** (90%)
Ψ	3.8895	[2.0728 5.6435]
h	0.7437	[0.6258 0.8468]
θ_w	0.5563	[0.4402 0.6738]
φ	4.3831	[3.3576 5.3685]
ν	0.0244	[0.0096 0.0381]
θ_p	0.6310	[0.5349 0.7339]
γ_w	0.1812	[0.0723 0.2876]
γ_p	0.4856	[0.1938 0.7738]
ψ	0.5554	[0.3605 0.7506]
M_p	1.7452	[1.6121 1.8785]
r_π	1.8863	[1.6162 2.1517]
ρ_r	0.8573	[0.8259 0.8894]
r_y	0.1675	[0.1090 0.2264]
$r_{\Delta y}$	0.2553	[0.2013 0.3081]
$\overline{\pi}$	0.6642	[0.4917 0.8340]
$100(\beta^{-1} - 1)$	0.3054	[0.1734 0.4382]
$\bar{l}$	-1.6492	[-3.9587 0.6353]
τ	0.3347	[0.2957 0.3745]
τ_{WE}	0.0748	[0.0256 0.1210]
M_w	1.2224	[1.1479 1.2968]
α	0.1712	[0.1416 0.2004]

F APPENDIX: Two-Sector Model

We present in detail below a two-sector model in the spirit of de Walque, Smets and Wouters (2006).

Table 4: Posterior Estimates of Gali-Smets-Wouters, Shock Processes

Parameters	**Posterior Mean**	**Probability Interval** (90%)
ρ_a	0.9782	[0.9676 0.9893]
ρ_b	0.4305	[0.1934 0.7053]
ρ_g	0.9723	[0.9556 0.9905]
ρ_q	0.7521	[0.6202 0.8861]
ρ_r	0.1000	[0.0211 0.1742]
ρ_p	0.4519	[0.0879 0.8123]
ρ_w	0.9831	[0.9654 0.9991]
ρ_{ga}	0.6916	[0.5532 0.8314]
μ_p	0.5969	[0.2569 0.9668]
μ_w	0.6393	[0.3585 0.9295]
σ_a	0.4156	[0.3709 0.4583]
σ_b	1.5743	[0.4540 2.4475]
σ_g	0.4780	[0.4341 0.5216]
σ_q	0.4207	[0.3397 0.4944]
σ_r	0.2159	[0.1946 0.2366]
σ_p	0.1214	[0.0328 0.2221]
σ_w	0.0679	[0.0131 0.1307]
σ_x	1.1823	[0.8896 1.4640]
σ_{wC}	0.4559	[0.4073 0.5040]
σ_{wE}	0.3625	[0.3137 0.4114]

F.1 Households

There is a continuum of households on the unit interval. Each household specializes in the supply of a particular type of labor. A household that supplies labor of type-j maximizes the utility function:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \delta_t \left[\log \left(C_t^j - \eta C_{t-1} \right) - \varpi \frac{\left(H_t^j \right)^{1+\varphi}}{1+\varphi} \right] \right\},$$

where H_t^j denotes the hours of type-j labor services, C_t is aggregate consumption, and C_t^j is consumption of household j. The parameters β, φ, and η are, respectively, the discount factor, the inverse of the (Frisch) elasticity of labor supply, and the degree of external habit formation, while δ_t represents an intertemporal preference shock that follows:

$$\delta_t = \delta_{t-1}^{\rho_\delta} \exp(\varepsilon_{\delta,t}),$$

where $\varepsilon_{\delta,t} \sim$ i.i.d. $N\left(0, \sigma_\delta^2\right)$.

Household j's flow budget constraint is:

$$\begin{aligned} &P_t C_t^j + P_t I_t^j + B_t^j + E_t \left[Q_{t,t+1} V_{t+1}^j \right] \\ &= W_t(j) H_t^j + V_t^j + R_{t-1} B_{t-1}^j - P_t a(u_t) \bar{K}_{t-1}^j + \Pi_t, \end{aligned}$$

where P_t is the price level, B_t^j is the amount of one-period risk-less nominal government bond held by household j, R_t is the interest rate on the bond, $W_t(j)$ is the nominal wage rate for type-j labor, Π_t denotes profits of intermediate firms. In addition to the government bond, households trade at time t one-period state-contingent nominal securities V_{t+1}^jat price $Q_{t,t+1}$, and hence fully insure against idiosyncratic risk.

Moreover, I_t^j is investment, R_t^K is the rental rate of effective capital $K_t^j = u_t \bar{K}_{t-1}^j$ where u_t is the variable capacity utilization rate, and $a(u_t)$ is the cost of capital utilization. In steady-state, $u = 1$ and $a(1) = 0$. Moreover, in the first-order approximation of the model, the only parameter that matters for the dynamic solution of the model is the curvature $\chi \equiv \frac{a''(1)}{a'(1)}$. The capital accumulation equation is then given by:

$$\bar{K}_t^j = (1-d)\, \bar{K}_{t-1}^j + \mu_t \left(1 - S\left(\frac{I_t^j}{I_{t-1}^j} \right) \right) I_t^j,$$

where d is the depreciation rate and $S(.)$ is the adjustment cost function. In steady-state, $S = S' = 0$ and $S'' > 0$. μ_t represents an investment shock that follows:

$$\mu_t = \mu_{t-1}^{\rho_\mu} \exp(\varepsilon_{\mu,t}),$$

where $\varepsilon_{\mu,t} \sim$ i.i.d. $N\left(0, \sigma_\mu^2\right)$.

Each household monopolistically provides differentiated labor. There are competitive employment agencies that assemble these differentiated labor into a homogenous labor input that is sold to intermediate goods firms. The assembling technology is a Dixit-Stiglitz production technology $H_t = \left(\int_0^1 \left(H_t^j\right)^{\frac{\theta_{W,t}-1}{\theta_{W,t}}} dj\right)^{\frac{\theta_{W,t}}{\theta_{W,t}-1}}$, where $\theta_{W,t}$ denotes the time-varying elasticity of substitution between differentiated labor. The corresponding wage index for the homogenous labor input is $W_t = \left(\int_0^1 W_t(j)^{1-\theta_{W,t}} dj\right)^{\frac{1}{1-\theta_{W,t}}}$ and the optimal demand for H_t^j is given by $H_t^j = \left(W_t(j)/W_t\right)^{-\theta_{W,t}} H_t$. The elasticity of substitution $\theta_{W,t}$ follows:

$$\left(\frac{\theta_{W,t}}{\theta_{W,t}-1}\right) = \left(\frac{\bar{\theta}_W}{\bar{\theta}_W - 1}\right)^{1-\rho_H} \left(\frac{\theta_{W,t-1}}{\theta_{W,t-1}-1}\right)^{\rho_H} \exp(\varepsilon_{W,t} - \upsilon_W \varepsilon_{W,t-1})$$

where $\varepsilon_{W,t} \sim$ i.i.d. $N\left(0, \sigma_W^2\right)$.

Each household resets its nominal wage optimally with probability $1 - \alpha_W$ every period. Households that do not optimize adjust their wages according to the simple partial dynamic indexation rule:

$$W_t(j) = W_{t-1}(j) \left[\pi_{t-1} a_{t-1}\right]^{\gamma_W} \left[\bar{\pi}\bar{a}\right]^{1-\gamma_W},$$

where γ_W measures the extent of indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$. All optimizing households choose a common wage W_t^* to maximize the present discounted value of future utility:

$$E_t \sum_{k=0}^{\infty} \alpha_W^k \beta^k \left[-\delta_{t+k} \varpi \frac{\left(H_{t+k}^j\right)^{1+\varphi}}{1+\varphi} + \Lambda_{t+k} \left(1 - \tau_{t+k}^H\right) W_t^* X_{W,t,k} H_{t+k}^j\right],$$

where Λ_{t+k} is the marginal utility of nominal income and

$$X_{W,t,k} \equiv \begin{cases} \left(\pi_t \pi_{t+1} \cdots \pi_{t+k-1} a_t a_{t+1} \cdots a_{t+k-1}\right)^{\gamma_W} \left[\bar{\pi}\bar{a}\right]^{(1-\gamma_W)k}, & k \geq 1 \\ 1, & k = 0 \end{cases}.$$

Maximization is subject to the sequence of labor demand function effective while W_t^* remains in place:

$$H_{t+k}^{j} = \left(\frac{W_t^* X_{W,t,k}}{W_{t+k}}\right)^{-\theta_{W,t+k}} H_{t+k}$$

Finally, due to the wage rigidity assumption, the nominal aggregate wage evolves according to:

$$W_t = \left[(1-\alpha_W)\, W_t^{*1-\theta_{W,t}} + \alpha_W \left\{W_{t-1}\left[\pi_{t-1}a_{t-1}\right]^{\gamma_W}\left[\bar{\pi}\bar{a}\right]^{1-\gamma_W}\right\}^{1-\theta_{W,t}}\right]^{\frac{1}{1-\theta_{W,t}}}.$$

F.2 Firms

The final good Y_t, which is consumed by the government and households as well as used to invest, is a Cobb-Douglas aggregate of the flexible price and sticky price sectoral goods ($Y_{F,t}$ and $Y_{S,t}$ respectively)

$$Y_t = (Y_{S,t})^{\xi}\,(Y_{F,t})^{1-\xi}$$

which gives the following relative demand and price index expressions

$$\frac{Y_{S,t}}{Y_{F,t}} = \left(\frac{\xi}{1-\xi}\right)\frac{P_{F,t}}{P_{S,t}}$$

$$\frac{Y_{S,t}}{Y_t} = \xi\frac{P_t}{P_{S,t}}$$

$$P_t = \frac{1}{\xi^{\xi}\,(1-\xi)^{1-\xi}}\,(P_{S,t})^{\xi}\,(P_{F,t})^{1-\xi}.$$

These sectoral goods $Y_{F,t}$ and $Y_{S,t}$ are produced by perfectly competitive firms assembling intermediate goods, $Y_{F,t}(i)$ and $Y_{S,t}(i)$, with a Dixit-Stiglitz production technology $Y_{F,t} = \left(\int_0^1 Y_{F,t}(i)^{\frac{\theta_P-1}{\theta_P}}di\right)^{\frac{\theta_P}{\theta_P-1}}$ and $Y_{S,t} = \left(\int_0^1 Y_{S,t}(i)^{\frac{\theta_P-1}{\theta_P}}di\right)^{\frac{\theta_P}{\theta_P-1}}$, where θ_P denotes the elasticity of substitution between intermediate goods. The corresponding price indices for the sectoral consumption goods are $P_{F,t} = \left(\int_0^1 P_{F,t}(i)^{1-\theta_P}di\right)^{\frac{1}{1-\theta_P}}$ and $P_{S,t} = \left(\int_0^1 P_{S,t}(i)^{1-\theta_P}di\right)^{\frac{1}{1-\theta_P}}$, where $P_{F,t}(i)$ and $P_{S,t}(i)$ are the prices of the intermediate goods i. The optimal demand for $Y_{F,t}(i)$ and $Y_{S,t}(i)$ are given by $Y_{F,t}(i) = \left(P_{F,t}(i)/P_{F,t}\right)^{-\theta_P} Y_{F,t}$ and $Y_{S,t}(i) = \left(P_{S,t}(i)/P_{S,t}\right)^{-\theta_P} Y_{S,t}$ respectively.

Monopolistically competitive firms produce intermediate goods using the production function in the sticky price sector:

$$Y_{S,t}(i) = \left(A_t H_{S,t}(i)\right)^{1-\lambda} K_{S,t}(i)^{\lambda},$$

where $H_{S,t}(i)$ and $K_{S,t}(i)$ denote the homogenous labor and capital employed by firm i and A_t represents exogenous economy-wide technological progress. The gross growth rate of technology $a_t \equiv A_t/A_{t-1}$ follows:

$$a_t = \bar{a}^{1-\rho_a} a_{t-1}^{\rho_a} \exp(\varepsilon_{a,t}),$$

where $\bar{a}$ is the steady-state value of a_t and $\varepsilon_{a,t} \sim$ i.i.d. $N\left(0, \sigma_a^2\right)$.

Similarly, monopolistically competitive firms produce intermediate goods using the production function in the flexible price sector:

$$Y_{F,t}(i) = A_{F,t}\left(A_t H_{F,t}(i)\right)^{1-\lambda} K_{F,t}(i)^{\lambda},$$

where $H_{F,t}(i)$ and $K_{F,t}(i)$ denote the homogenous labor and capital employed by firm i and A_t represents exogenous economy-wide technological progress. The gross growth rate of technology $a_t \equiv A_t/A_{t-1}$ follows:

$$a_t = \bar{a}^{1-\rho_a} a_{t-1}^{\rho_a} \exp(\varepsilon_{a,t}),$$

where $\bar{a}$ is the steady-state value of a_t and $\varepsilon_{a,t} \sim$ i.i.d. $N\left(0, \sigma_a^2\right)$. In addition, there is a flexible price sector specific technology shock $A_{F,t}$ that follows:

$$A_{F,t} = A_{F,tt-1}^{\rho_{A,F}} \exp(\varepsilon_{AF,t}),$$

where $\varepsilon_{AF,t} \sim$ i.i.d. $N\left(0, \sigma_{AF}^2\right)$.

In the sticky price sector, a firm resets its price optimally with probability $1-\alpha_P$ every period. Firms that do not optimize adjust their price according to the simple partial dynamic indexation rule:

$$P_{S,t}(i) = P_{S,t-1}(i)\pi_{S,t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P},$$

where γ_P measures the extent of indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_{S,t} \equiv P_{S,t}/P_{S,t-1}$. All optimizing firms choose a common price $P_{S,t}^{*}$ to maximize the present discounted value of future profits:

$$E_t \sum_{k=0}^{\infty} \alpha_P^k \beta^k \frac{\Lambda_{t+k}}{\Lambda_t} \left[P_{S,t}^* X_{P,t,k} Y_{S,t+k}(i) - W_{t+k} H_{S,t+k}(i) - R_{t+k}^K K_{S,t+k}(i) \right],$$

where

$$X_{P,t,k} \equiv \begin{cases} \left(\pi_{S,t} \pi_{S,t+1} \cdots \pi_{S,t+k-1} \right)^{\gamma_P} \bar{\pi}^{(1-\gamma_P)k}, & k \geq 1 \\ 1, \quad k = 0 \end{cases}.$$

In the flexible price sector, all firms choose a common price $P_{F,t}^*$ to maximize profits:

$$P_{F,t}^* Y_{F,t}(i) - W_{t+k} H_{F,t}(i) - R_{t+k}^K K_{F,t}(i).$$

F.3 Government

F.3.1 Monetary Policy

The central bank sets the nominal interest rate according to a Taylor-type rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\phi_Y} \right]^{1-\rho_R} \left(\frac{Y_t / Y_{t-1}}{Y_t^* / Y_{t-1}^*} \right)^{d\phi_Y} \exp\left(\varepsilon_{R,t} \right),$$

which features interest rate smoothing and systematic responses to deviation of GDP from its target level Y_t^* and deviation of inflation from steady-state $\pi^* = \bar{\pi}$. As in Smets and Wouters (2007) there is also a dependence of the interest rate on the growth rate of deviation of output from the target level. We set the target level of output Y_t^* equal to the flex-price output as in Smets and Wouters (2007) (here, the flex-price output is defined as the output that would prevail in the presence of flexible prices and absence of the wage markup shock and the flex-price sector-specific shock). $\bar{R}$ is the steady-state value of R_t and the non-systematic monetary policy shock $\varepsilon_{R,t}$ is assumed to follow an i.i.d. $N\left(0, \sigma_R^2\right)$.

F.3.2 Fiscal Policy

Government spending follows an exogenous AR(1) process

$$\tilde{G}_t - \overline{\tilde{G}} = \rho_G \left(\tilde{G}_{t-1} - \overline{\tilde{G}} \right) + \varepsilon_{G,t},$$

where $\tilde{G}_t$ is government spending-to-output ratio, $\overline{\tilde{G}}$ is its steady-state value, and $\varepsilon_{G,t} \sim$ i.i.d. $N\left(0, \sigma_G^2\right)$. Lump-sum taxes are available and hence we abstract from government debt dynamics completely.

F.4 Equilibrium

Equilibrium is characterized by the prices and quantities that satisfy the households' and firms' optimality conditions, the government budget constraint, monetary and fiscal policy rules, and the clearing conditions for the product, labor, capital rental and asset markets:

$$
\begin{aligned}
\int_0^1 C_t^j dj + G_t + \int_0^1 I_t^j dj + a(u_t) \int_0^1 \bar{K}_{t-1}^j dj &= Y_t, \\
\int_0^1 H_{F,t}(i) di + \int_0^1 H_{S,t}(i) di &= H_t, \\
\int_0^1 V_t^j dj &= 0, \\
\int_0^1 K_{F,t}(i) di + \int_0^1 K_{S,t}(i) di &= K_t, \\
\int_0^1 B_t^j dj &= 0.
\end{aligned}
$$

Note that $C_t^j = C_t$, $I_t^j = I_t$, and $\bar{K}_{t-1}^j = \bar{K}_{t-1}$ due to the complete market assumption and the separability between consumption and leisure. The capital accumulation equation in the aggregate is then given by:

$$
\bar{K}_t = (1-d)\,\bar{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.
$$

and the aggregate resource constraint then takes the form:

$$
C_t + I_t + G_t + a(u_t)\bar{K}_{t-1} = Y_t.
$$

F.5 Stationary equilibrium

F.5.1 De-trending

The technology process A_t induces a common trend in output, consumption, real wage, government purchases, investment, and capital. In addition, introducing non-zero steady state inflation also creates a trend in nominal prices. The trend in nominal prices is the same across sectors and in the aggregate. We will also define a relative price term. We detrend the variables as follows:

$$\tilde{Y}_t \equiv \frac{Y_t}{A_t},\ \tilde{C}_t \equiv \frac{C_t}{A_t},\quad \tilde{I}_t \equiv \frac{I_t}{A_t},\quad \tilde{K}_t \equiv \frac{K_t}{A_t},\quad \tilde{\bar{K}}_t \equiv \frac{\bar{K}_t}{A_t},\ \tilde{Y}_{F,t} \equiv \frac{Y_{F,t}}{A_t}, \tilde{Y}_{S,t} \equiv \frac{Y_{S,t}}{A_t}$$
$$\tilde{w}_t \equiv \frac{W_t}{P_t A_t},\quad r_t^K \equiv \frac{R_t^K}{P_t},\quad p_{F,t}^* = \frac{P_{F,t}^*}{P_{F,t}},\quad p_{S,t}^* = \frac{P_{S,t}^*}{P_{S,t}},\ mc_{F,t} \equiv \frac{MC_{F,t}}{P_{F,t}},\ mc_{S,t} \equiv \frac{MC_{S,t}}{P_{S,t}},$$
$$p_{R,t} = \frac{P_{F,t}}{P_{S,t}},\ \tilde{w}_t^S \equiv \frac{W_t}{P_{S,t}A_t},\ \tilde{w}_t^F \equiv \frac{W_t}{P_{F,t}A_t},\ r_t^{K,S} \equiv \frac{R_t^K}{P_{S,t}},\ r_t^{K,F} \equiv \frac{R_t^K}{P_{F,t}}$$
$$\tilde{G}_t \equiv \frac{G_t}{Y_t}$$
$$\tilde{\Lambda}_t \equiv A_t P_t \Lambda_t,\quad \tilde{\Phi}_t \equiv A_t \Phi_t$$

F.5.2 Equilibrium conditions

Aggregator of the sectoral outputs is given by

$$\tilde{Y}_t = \left(\tilde{Y}_{S,t}\right)^{\xi} \left(\tilde{Y}_{F,t}\right)^{1-\xi} \tag{6}$$

while the sector specific production functions are given by

$$\tilde{Y}_{S,t}(i) = H_{S,t}(i)^{1-\lambda} \tilde{K}_{S,t}(i)^{\lambda} \tag{7}$$

$$\tilde{Y}_{F,t}(i) = A_{F,t} H_{F,t}(i)^{1-\lambda} \tilde{K}_{F,t}(i)^{\lambda}. \tag{8}$$

The relative demand equations are given by

$$\frac{\tilde{Y}_{S,t}}{\tilde{Y}_{F,t}} = \left(\frac{\xi}{1-\xi}\right) p_{R,t}$$

where

$$p_{R,t} = \frac{P_{F,t}}{P_{S,t}}. \tag{9}$$

Moreover,

$$\frac{P_t}{P_{t-1}} = \left(\frac{P_{S,t}}{P_{S,t-1}}\right)^{\xi} \left(\frac{P_{F,t}}{P_{F,t-1}}\right)^{1-\xi}$$

or

$$\pi_t = \left(\pi_{S,t}\right)^{\xi} \left(\pi_{F,t}\right)^{1-\xi}.$$

Also note the definition that gives the law of motion for $p_{R,t}$ is

$$\frac{p_{R,t}}{p_{R,t-1}} = \frac{\pi_{F,t}}{\pi_{S,t}}.$$

The capital-labor ratios evolves as

$$\frac{\tilde{K}_{S,t}(i)}{H_{S,t}(i)} = \frac{\tilde{w}_t}{r_t^K}\frac{\lambda}{1-\lambda} \tag{10}$$

$$\frac{\tilde{K}_{F,t}(i)}{H_{F,t}(i)} = \frac{\tilde{w}_t}{r_t^K}\frac{\lambda}{1-\lambda} \tag{11}$$

while the nominal marginal costs evolev as

$$mc_{S,t} = \lambda^{-\lambda}\left(1-\lambda\right)^{-(1-\lambda)}\left(r_t^{K,S}\right)^{\lambda}\tilde{w}_t^{S1-\lambda} \tag{12}$$

$$mc_{F,t} = \lambda^{-\lambda}\left(1-\lambda\right)^{-(1-\lambda)}\left(r_t^{K,F}\right)^{\lambda}\tilde{w}_t^{F1-\lambda}\frac{1}{A_{F,t}} \tag{13}$$

with the following relationships holding by definition

$$\frac{\tilde{w}_t^S}{\tilde{w}_t^F} \equiv p_{R,t},\ \ \frac{r_t^{K,S}}{r_t^{K,F}} = \ p_{R,t}$$

$$\tilde{w}_t = \frac{1}{\xi^{\xi}\left(1-\xi\right)^{1-\xi}}\left(\tilde{w}_t^S\right)^{\xi}\left(\tilde{w}_t^F\right)^{1-\xi}$$

$$r_t^K = \frac{1}{\xi^{\xi}\left(1-\xi\right)^{1-\xi}}\left(r_t^{K,S}\right)^{\xi}\left(r_t^{K,F}\right)^{1-\xi}.$$

Optimal price setting equation for sticky price firms is given by

$$0 = E_t\sum_{k=0}^{\infty}\alpha_P^k\beta^k\tilde{\Lambda}_{t+k}\tilde{Y}_{St,t+k}\left(p_{S,t}^*\frac{X_{P,t,k}}{\prod_{s=1}^k\pi_{S,t+s}} - x_P mc_{S,t+k}\right). \tag{14}$$

where

$$\tilde{Y}_{St,t+k} = \left[p_{S,t}^*\frac{X_{P,t,k}}{\prod_{s=1}^k\pi_{S,t+s}}\right]^{-\theta_P}\tilde{Y}_{t+k}.$$

Optimal price setting equation for flexible price firms is given by

$$0 = \left(1 - x_P mc_{F,t}\right). \tag{15}$$

Relatedly, aggregate price index.for the sticky price sector is given by

$$1=\left[\left(1-\alpha_P\right)p_{S,t}^{*1-\theta_P}+\alpha_P\left(\frac{\bar{\pi}}{\pi_{S,t}}\left(\frac{\pi_{S,t-1}}{\bar{\pi}}\right)^{\gamma_P}\right)^{1-\theta_P}\right]^{\frac{1}{1-\theta_P}} \tag{16}$$

while the aggregate price index for the flexible price sector is given by

$$p_{F,t}^{*}=1. \tag{17}$$

The marginal utility of income can be expressed as

$$\tilde{\Lambda}_t=\frac{a_t\delta_t}{\left(a_t\tilde{C}_t-\eta\tilde{C}_{t-1}\right)} \tag{18}$$

which implies a euler equation of the form

$$\tilde{\Lambda}_t=\beta R_t E_t\left[\frac{\tilde{\Lambda}_{t+1}}{a_{t+1}\pi_{t+1}}\right]. \tag{19}$$

Optimal condition for capital utilization is given by

$$r_t^K=a^{'}(u_t) \tag{20}$$

while the optimality condition for capital is given by

$$\tilde{\Phi}_t=\beta E_t\left[\frac{1}{a_{t+1}}\tilde{\Lambda}_{t+1}\left\{r_{t+1}^K u_{t+1}-a(u_{t+1})\right\}\right]+\left(1-d\right)\beta E_t\left[\frac{1}{a_{t+1}}\tilde{\Phi}_{t+1}\right]. \tag{21}$$

Moreover, the optimal choice of investment is governed by

$$\begin{aligned}\tilde{\Lambda}_t=\tilde{\Phi}_t\mu_t&\left[1-S\left(a_t\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right)-a_t\frac{\tilde{I}_t}{\tilde{I}_{t-1}}S^{'}\left(a_t\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right)\right]\\&+\beta E_t\left[\frac{1}{a_{t+1}}\tilde{\Phi}_{t+1}\mu_{t+1}\left(a_{t+1}\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\right)^2S^{'}\left(a_{t+1}\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\right)\right].\end{aligned} \tag{22}$$

Definition of effective capital is then

$$\tilde{K}_t=\frac{1}{a_t}u_t\tilde{\bar{K}}_{t-1} \tag{23}$$

while the law of motion of capital is

$$\tilde{\bar{\bar{K}}}_t = (1-d)\frac{1}{a_t}\tilde{\bar{\bar{K}}}_{t-1} + \mu_t \left(1 - S\left(a_t \frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right)\right)\tilde{I}_t. \tag{24}$$

Optimal wage setting is governed by

$$0 = E_t \sum_{k=0}^{\infty} \alpha_W^k \beta^k \tilde{\Lambda}_{t+k} H_{t,t+k} \left[\tilde{w}_t^* \frac{X_{W,t,k}}{\prod_{s=1}^k \pi_{t+s} a_{t+s}} - x_{W,t+k}\delta_{t+k}\varpi \frac{H_{t,t+k}^{\varphi}}{\tilde{\Lambda}_{t+k}}\right], \tag{25}$$

where

$$H_{t,t+k} = \left(\tilde{w}_t^* \frac{X_{W,t,k}}{\prod_{s=1}^k \pi_{t+s} a_{t+s}} \frac{1}{\tilde{w}_{t+k}}\right)^{-\theta_{W,t+k}} H_{t+k}.$$

Aggregate wage then evolves as

$$\tilde{w}_t = \left[(1-\alpha_W)\,\tilde{w}_t^{*1-\theta_{W,t}} + \alpha_W \left\{\tilde{w}_{t-1}\left(\frac{\bar{\pi}\bar{a}}{\pi_t a_t}\right)\left(\frac{\pi_{t-1}a_{t-1}}{\bar{\pi}\bar{a}}\right)^{\gamma_W}\right\}^{1-\theta_{W,t}}\right]^{\frac{1}{1-\theta_{W,t}}}. \tag{26}$$

The aggregate resource constraint takes the form

$$\tilde{C}_t + \tilde{I}_t + \tilde{G}_t\tilde{Y}_t + a(u_t)\frac{1}{a_t}\tilde{\bar{\bar{K}}}_{t-1} = \tilde{Y}_t. \tag{27}$$

Finally, the monetary policy rule is

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}_t^*}\right)^{\phi_Y}\right]^{1-\rho_R}\left(\frac{\tilde{Y}_t/\tilde{Y}_{t-1}}{\tilde{Y}_t^*/\tilde{Y}_{t-1}^*}\right)^{d\phi_Y}\exp\left(\varepsilon_{R,t}\right) \tag{28}$$

and the law of motion of government spending is

$$\tilde{G}_t - \overline{\tilde{G}} = \rho_G\left(\tilde{G}_{t-1} - \overline{\tilde{G}}\right) + \varepsilon_{G,t}. \tag{29}$$

F.5.3 Steady state

Recall that in steady-state, we assume

$$\bar{u} = 1,\ a(1) = 0, \text{and } \bar{S} = \bar{S}' = 0.$$

From (21) and (22), we get

$$\bar{r}^K = \bar{a}/\beta - (1-d)\,. \tag{30}$$

Given $\bar{r}^K$, (12), (13), (14), and (15) imply

$$\bar{\tilde{w}} = \left[\frac{1}{1+\bar{x}_P}\lambda^{\lambda}(1-\lambda)^{(1-\lambda)}\left(\bar{r}^K\right)^{-\lambda}\right]^{\frac{1}{1-\lambda}} \qquad (\text{note } \overline{mc_S} = \overline{mc_F} = \frac{1}{1+\bar{x}_P} = \frac{\bar{\theta}_P - 1}{\bar{\theta}_P}). \tag{31}$$

Given $\bar{r}^K$ and $\bar{\tilde{w}}$, (10) and (11) imply

$$\frac{\bar{\tilde{K}}}{\bar{H}} = \frac{\overline{\tilde{K}_S}}{\bar{H}_S} = \frac{\overline{\tilde{K}_F}}{\bar{H}_F} = \frac{\bar{\tilde{w}}}{\bar{r}^K}\frac{\lambda}{1-\lambda} \tag{32}$$

From the production functions, we can obtain

$$\frac{\bar{\tilde{Y}}_S}{\bar{H}_S} = \frac{\bar{\tilde{Y}}_F}{\bar{H}_F} = \left(\frac{\bar{\tilde{K}}}{\bar{H}}\right)^{\lambda}. \tag{33}$$

From (24) and (27), we can compute

$$\frac{\bar{\tilde{I}}}{\bar{H}} = \frac{\bar{\tilde{K}}}{\bar{H}}\left[\bar{a} - (1-d)\right]. \tag{34}$$

$$\frac{\bar{\tilde{C}}}{\bar{H}} = \left(1 - \bar{\tilde{G}}\right)\frac{\bar{\tilde{Y}}}{\bar{H}} - \frac{\bar{\tilde{I}}}{\bar{H}}. \tag{35}$$

Importantly, from (32)-(35), we can easily obtain (note $\bar{H} = 1$)

$$\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}, \quad \frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}, \quad \frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}.$$

The nominal interest rate is obtained from Euler equation

$$\bar{R} = \frac{\bar{\pi}\bar{a}}{\beta}. \tag{36}$$

We can obtain an expression for $\bar{H}$ from (25) and (18)

$$\bar{H} = \left[\frac{\bar{\tilde{w}}\bar{a}}{\bar{x}_W\left(\bar{a} - \eta\right)\frac{\bar{\tilde{C}}}{\bar{H}}}\frac{1}{\varpi}\right]^{\frac{1}{1+\varphi}}. \tag{37}$$

The steady-state hours $\bar{H}$ depend on the preference parameter ϖ. We normalize $\bar{H}$ to unity by calibrating ϖ appropriately.

Now, we need to determine the sector specific steady states. From (6) we get

$$\frac{\bar{Y}_S}{\bar{Y}_F} = \frac{\xi}{1-\xi} \tag{38}$$

while from (F.5.2) we get

$$\bar{p}_R = 1. \tag{39}$$

Finally, from (7) and (8) we derive

$$\frac{H_S}{H_F} = \frac{K_S}{K_F} = \frac{\xi}{1-\xi}. \tag{40}$$

F.6 Linear Model and Estimation

F.6.1 Model

$$
\begin{aligned}
\hat{\tilde{Y}}_t &= \xi\hat{\tilde{Y}}_{S,t} + (1-\xi)\,\hat{\tilde{Y}}_{F,t} \\
\hat{\tilde{Y}}_{S,t} &= \lambda\hat{\tilde{K}}_{S,t} + (1-\lambda)\,\widehat{H}_{S,t} \\
\hat{\tilde{Y}}_{F,t} &= \hat{A}_{F,t} + \lambda\hat{\tilde{K}}_{F,t} + (1-\lambda)\,\widehat{H}_{F,t} \\
\hat{\tilde{Y}}_{S,t} - \hat{\tilde{Y}}_{F,t} &= \hat{p}_{R,t} \\
\hat{\pi}_t &= \xi\hat{\pi}_{s,t} + (1-\xi)\hat{\pi}_{F,t} \\
\hat{p}_{R,t} - \hat{p}_{R,t-1} &= \hat{\pi}_{F,t} - \hat{\pi}_{S,t} \\
\widehat{r}^K_t &= \hat{\tilde{w}}_t - \hat{\tilde{K}}_{S,t} + \widehat{H}_{S,t} \\
\widehat{r}^K_t &= \hat{\tilde{w}}_t - \hat{\tilde{K}}_{F,t} + \widehat{H}_{F,t} \\
\widehat{mc}_{S,t} &= \lambda\widehat{r}^{K,S}_t + (1-\lambda)\,\widehat{\tilde{w}^S_t} \\
\widehat{mc}_{F,t} &= \lambda\widehat{r}^{K,F}_t + (1-\lambda)\,\widehat{\tilde{w}^F_t} - \hat{A}_{F,t} \\
\widehat{\tilde{w}^S_t} - \widehat{\tilde{w}^F_t} &\equiv \hat{p}_{R,t} \\
\widehat{r}^{K,S}_t - \widehat{r}^{K,F}_t &= \hat{p}_{R,t} \\
\hat{\tilde{w}}_t &= (\xi)\,\widehat{\tilde{w}^S_t} + (1-\xi)\,\widehat{\tilde{w}^F_t} \\
\widehat{r}^K_t &= (\xi)\,\widehat{r}^{K,S}_t + (1-\xi)\,\widehat{r}^{K,F}_t \\
\hat{\pi}_{s,t} &= \frac{\beta}{1+\beta\gamma_P}E_t\hat{\pi}_{S,t+1} + \frac{\gamma_P}{1+\beta\gamma_P}\hat{\pi}_{S,t-1} + \kappa_P\widehat{mc}_{S,t} \\
0 &= \widehat{mc}_{F,t} \\
\hat{\tilde{\Lambda}}_t &= -\frac{\bar{a}}{\bar{a}-\eta}\hat{\tilde{C}}_t + \frac{\eta}{\bar{a}-\eta}\hat{\tilde{C}}_{t-1} - \frac{\eta}{\bar{a}-\eta}\widehat{a}_t + \frac{\bar{a}+\eta}{(1-\rho_\delta)\,(\bar{a}-\eta)}\widehat{\delta}^*_t \\
\hat{\tilde{\Lambda}}_t &= \widehat{R}_t + E_t\left[\hat{\tilde{\Lambda}}_{t+1} - \widehat{\pi}_{t+1} - \widehat{a}_{t+1}\right] \\
\chi\widehat{u}_t &= \widehat{r}^K_t \\
\hat{\tilde{\Phi}}_t &= \frac{(1-d)\,\beta}{\bar{a}}E_t\left[\hat{\tilde{\Phi}}_{t+1}\right] + \left(1 - \frac{(1-d)\,\beta}{\bar{a}}\right)E_t\left[\tilde{\Lambda}_{t+1} + \widehat{r}^K_{t+1}\right] - E_t\left[\widehat{a}_{t+1}\right] \\
\hat{\tilde{\Lambda}}_t &= \hat{\tilde{\Phi}}_t + \widehat{\mu}_t - \bar{a}^2S''\left(\hat{\tilde{I}}_t - \hat{\tilde{I}}_{t-1} + \hat{a}_t\right) + \beta\bar{a}^2S''E_t\left[\hat{\tilde{I}}_{t+1} - \hat{\tilde{I}}_t + \hat{a}_{t+1}\right] \\
\hat{\tilde{K}}_t &= \widehat{u}_t + \hat{\bar{\tilde{K}}}_{t-1} - \widehat{a}_t \\
\hat{\bar{\tilde{K}}}_t &= \frac{1-d}{\bar{a}}\left(\hat{\bar{\tilde{K}}}_{t-1} - \widehat{a}_t\right) + \left(1 - \frac{1-d}{\bar{a}}\right)\left(\widehat{\mu}_t + \hat{\tilde{I}}_t\right) \\
\hat{\tilde{w}}_t &= \frac{1}{1+\beta}\hat{\tilde{w}}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\tilde{w}}_{t+1} - \kappa_W\left[\hat{\tilde{w}}_t - \left(\varphi\widehat{H}_t - \hat{\tilde{\Lambda}}_t + \frac{\bar{a}+\eta}{(1-\rho_\delta)\,(\bar{a}-\eta)}\widehat{\delta}^*_t\right)\right] \\
&\quad + \frac{\gamma_W}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\gamma_W}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{\gamma_W}{1+\beta}\hat{a}_{t-1} - \frac{1+\beta\gamma_W-\beta\rho_a}{1+\beta}\hat{a}_t + \kappa_W\widehat{x}^*_{W,t} \\
\left(1-\overline{\tilde{G}}\right)\hat{\tilde{Y}}_t &= \frac{\overline{\tilde{C}}}{\overline{\tilde{Y}}}\hat{\tilde{C}}_t + \frac{\overline{\tilde{I}}}{\overline{\tilde{Y}}}\hat{\tilde{I}}_t + \hat{\tilde{G}}_t + \bar{r}^K\frac{\overline{\tilde{K}}}{\overline{\tilde{Y}}}\widehat{u}_t
\end{aligned}
$$

$$\widehat{H}_t = \xi \widehat{H}_{S,t} + (1-\xi)\,\widehat{H}_{F,t}$$
$$\widehat{\tilde{K}}_t = \xi \widehat{\tilde{K}}_{S,t} + (1-\xi)\,\widehat{\tilde{K}}_{F,t}$$
$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\phi_\pi \hat{\pi}_{s,t} + \phi_Y\left(\widehat{\tilde{Y}}_t - \widehat{\tilde{Y}}^*_t\right)\right] + \phi_{dY}(\widehat{\tilde{Y}}_t - \widehat{\tilde{Y}}_{t-1}) + \varepsilon_{R,t}$$

Finally, exogenous variables evolve as follows:

$$\hat{\delta}^*_t = \rho_\delta \hat{\delta}^*_{t-1} + \varepsilon_{\delta,t}$$
$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \varepsilon_{\mu,t}$$
$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t}$$
$$\hat{A}_{F,t} = \rho_{a,F} \hat{A}_{F,t} + \varepsilon_{aF,t}$$
$$\widehat{x}_{W,t} = \rho_W \hat{x}_{W,t-1} + \varepsilon_{W,t} - \upsilon_w \varepsilon_{w,t-1}$$
$$\widehat{\tilde{G}}_t = \rho_G \widehat{\tilde{G}}_{t-1} + \varepsilon_{G,t}$$

F.7 Estimation

F.7.1 Data

We use the same data as in Smets and Wouters (2007) from 1966:I to 2007:IV.

F.7.2 Prior distribution

We will calibrate $d, \tilde{G}$, and $1/\left(\bar{\theta}_W - 1\right)$ at the same values as in Smets and Wouters (2007) and ξ and $\rho_{a,F}$ at the same values as in deWalque, Smets, and Wouters (2006). We also normalize the wage markup and the flex-price technology shock by a factor of 100.

Table 5: Parametrization of Priors Two-Sector Model, Structural Parameters

Parameter	**Density**	**Prior Mean**	**Prior Standard Deviation**
d	calibrated	0.025	
$\tilde{G}$	calibrated	0.18	
ξ	calibrated	0.85	
λ	Normal	0.30	0.05
γ_P	Beta	0.50	0.15
γ_W	Beta	0.50	0.15
η	Beta	0.50	0.1
α_P	Beta	0.50	0.1
α_W	Beta	0.50	0.1
$1/\left(\bar{\theta}_P - 1\right)$	Normal	0.15	0.05
$1/\left(\bar{\theta}_W - 1\right)$	calibrated	0.5	
φ	Gamma	2.00	0.75
χ	Gamma	5.00	1.00
$S^{''}$	Gamma	4.00	1.00
$100(\bar{a} - 1)$	Normal	0.54	0.1
$100(\bar{\pi} - 1)$	Normal	0.62	0.1
$100(\beta^{-1} - 1)$	Gamma	0.25	0.1
ϕ_π	Normal	1.5	0.15
ϕ_Y	Normal	0.13	0.03
ϕ_{dY}	Normal	0.13	0.03

Table 6: Parametrization of Priors Two-Sector Model, Shock Processes

Parameters	**Density**	**Prior Mean**	**Prior Standard Deviation**
ρ_δ	Beta	0.60	0.20
ρ_μ	Beta	0.60	0.20
ρ_a	Beta	0.60	0.20
$\rho_{a,F}$	calibrated	0.99	
ρ_W	Beta	0.60	0.20
υ_W	Beta	0.50	0.20
ρ_R	Beta	0.60	0.20
$\tilde{\rho}_G$	Beta	0.60	0.20
$100\sigma_\delta$	InvG	0.50	1.00
$100\sigma_\mu$	InvG	0.50	1.00
$100\sigma_a$	InvG	0.50	1.00
$100\sigma_{a,F}$	InvG	0.5	1.00
$100\sigma_W$	InvG	0.5	1.00
$100\sigma_R$	InvG	0.10	1.00
$100\sigma_G$	InvG	0.50	1.00

F.7.3 Posterior estimates

Table 7: Posterior Estimates Two-Sector Model, Structural Parameters

Parameter	**Posterior Mean**	**Probability Interval (90%)**
λ	0.1166	[0.0814 0.1534]
γ_P	0.1030	[0.0370 0.1667]
γ_W	0.1821	[0.0987 0.2650]
η	0.6766	[0.5790 0.7782]
α_P	0.7886	[0.7865 0.7902]
α_W	0.5617	[0.5078 0.6172]
$1/\left(\bar{\theta}_P - 1\right)$	0.1435	[0.0654 0.2198]
φ	3.9326	[2.5851 5.2326]
χ	5.6442	[3.9603 7.2807]
$S^{''}$	6.0680	[4.0670 7.9204]
$100(\bar{a} - 1)$	0.5226	[0.4313 0.6133]
$100(\bar{\pi} - 1)$	0.6388	[0.4788 0.8001]
$100(\beta^{-1} - 1)$	0.1754	[0.0813 0.2668]
ϕ_π	1.6356	[1.4402 1.8298]
ϕ_Y	0.0512	[0.0212 0.0796]
ϕ_{dY}	0.1786	[0.1421 0.2162]

Table 8: Posterior Estimates Two-Sector Model, Shock Processes

Parameter	**Posterior Mean**	**Probability Interval (90%)**
ρ_δ	0.6136	[0.4748 0.7558]
ρ_μ	0.7892	[0.7198 0.8571]
ρ_a	0.0747	[0.0184 0.1276]
ρ_W	0.8134	[0.7624 0.8663]
υ_W	0.3188	[0.1386 0.4954]
ρ_R	0.8008	[0.7642 0.8374]
$\tilde{\rho}_G$	0.9815	[0.9675 0.9965]
$100\sigma_\delta$	0.1685	[0.1255 0.2097]
$100\sigma_\mu$	5.0681	[3.3722 6.7231]
$100\sigma_a$	0.9340	[0.8385 1.0260]
$100\sigma_{a,F}$	0.0652	[0.0645 0.0662]
$100\sigma_W$	0.1943	[0.1276 0.2575]
$100\sigma_R$	0.2596	[0.2345 0.2843]
$100\sigma_G$	0.4949	[0.4500 0.5393]