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Technical Appendix: Capital Taxation During the U.S. Great Depression[†]

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[†] The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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In this appendix, I provide details on the data and computational methods used in "Capital Taxation During the U.S. Great Depression" (Staff Report 451). I also conduct sensitivity analyses of the main results.

1. U.S. Data

The main source for the data used in this study is the U.S. Department of Commerce, Bureau of Economic Analysis (BEA), which publishes the U.S. national accounts and fixed asset tables in the *Survey of Current Business* (available online at www.bea.gov), SCB hereafter. NIPA tables referenced below are the main tables of the National Income and Product Accounts. FA tables referenced below are the main tables of the Fixed Assets. Annual supplements of the SCB are the main source of market values for U.S. corporations. For information on taxes not available through the SCB, I use the U.S. Treasury's *Statistics of Income*. Auxiliary references are discussed where relevant.

1.1. National Accounts and Fixed Assets

Gross domestic product (GDP), components of GDP, and capital stocks are all divided by population midyear (NIPA Table 2.1), which is plotted in Figure 1.

Several adjustments are made to GDP to make it consistent with theory. Specifically, adjusted GDP is GDP (NIPA Table 1.1.5) less sales taxes (NIPA Table 3.5) plus imputed capital rents (equal to 4.1 percent of the stock of consumer durables in FA Table 1.1 plus fixed government capital in FA Table 1.1) plus depreciation of consumer durables (FA Table 1.3). To convert to real dollars, the adjusted GDP series is then divided by the GDP deflator (NIPA Table 1.1.9). In Figure 2, I plot this adjusted series after dividing it by the population in Figure 1 and 1.019^{t-1929}, $t = 1929, \ldots, 1939$. The latter is an estimate of the growth factor for labor-augmenting technical change. To convert the series into an index, I then divide the series by the 1929 value and multiply by 100.

Next, consider the main components of GDP. Consumption is defined to be personal consumption expenditures (NIPA Table 1.1.5) less PCE durables (NIPA Table 1.1.5) plus imputed capital rents (equal to 4.1 percent of the stock of consumer durables in FA Table 1.1 plus fixed government capital in FA Table 1.1) plus depreciation of consumer durables (FA Table 1.3) less a prorated portion of sales taxes on nondurables and services (NIPA Table 3.5). To convert to real dollars, the consumption series—along with all components of income and product—is divided by the GDP deflator. Finally, I divide the consumption series by the population times 1.019^{t-1929} times the 1929 level of adjusted GDP and then multiply by 100. The result is shown in Figure 3. Note that the 1929 value shown in the figure is the share of nondurables plus services (adjusted for taxes and capital services) in output of that year. This share, which is equal to 68 percent, is used later when parameterizing the models.

Investment is defined to be gross private domestic investment (NIPA Table 1.1.5) plus net exports (NIPA Table 1.1.5) plus government investment (NIPA Table 3.1) plus PCE durables (NIPA Table 1.1.5) less a prorated portion of sales taxes on durables (NIPA Table 3.5). To deflate and detrend the series, I use the same procedure as with consumption. Detrended real investment is plotted in Figure 4.

Government spending is defined to be government consumption (NIPA Table 3.1). To deflate and detrend this series, I again use the same procedure as with consumption.

Figure 5 plots the actual detrended series along with a smooth trend. All exogenous inputs are first filtered using the Hodrick and Prescott (1997) filter (with smoothing parameter equal to 1) before being fed into the models described later. I do this because these values are the basis of expectations of future spending and tax rates. In the case of the extended model, the computation is easier if expected future paths are smooth.

For the extended model, GDP and investment are subdivided into components for business and nonbusiness. Business GDP is the sum of corporate profits (NIPA Table 1.10) plus nonfarm proprietors' income (NIPA Table 1.12) plus compensation, net interest, and consumption of fixed capital of corporate business (NIPA Table 1.14) plus compensation and net interest of of sole proprietorships and partnerships (available prior to 2002 in NIPA Table 1.15) plus nonfarm proprietors' consumption of fixed capital (NIPA Table 7.5) plus taxes on imports and production (NIPA Table 1.10) less taxes on imports and production of the housing sector (NIPA Table 7.4.5) and farm sector (NIPA Table 7.3.5) and less sales taxes (NIPA Table 3.5). Nonbusiness GDP is GDP as defined above less business GDP.

Business investment is the sum of fixed investment of corporations plus nonfarm proprietors (FA Table 6.7) plus a change in inventories (NIPA Table 1.1.5) less a change in farm inventories (NIPA Table 7.3.5). Nonbusiness investment is investment as defined above less business investment.

The nonbusiness subcomponents of GDP and investment—after the series have been deflated and detrended—are plotted in Figures 6 and 7. In addition, I plot the smoothed series after applying the Hodrick and Prescott (1997) filter.

1.2. Hours Per Capita

Hours used in the study are total manhours from Kendrick (1961). The fraction of time at work is total manhours divided by time available for work, which is assumed to be 5,000 hours per year times the number of persons over 16. The population over 16 is from the U.S. Department of Commerce (1975, Series A39). In Figure 8, I plot per capita hours as a fraction of time at work. In 1929, 29 percent of available time was devoted to work.

For the extended model, I need business and nonbusiness hours. For nonbusiness hours, I use Kendrick's (1961) manhours for farm plus government. Nonbusiness hours per capita are plotted in Figure 9 along with the filtered series, which is used as an input for the numerical simulations. Business hours are then found residually by taking total hours less nonbusiness hours.

1.3. Market Value

The total value of all U.S. corporations is not available for this period, but an estimate for the index plotted in Figure 10 is constructed using the New York Stock Exchange market capitalization.

1.4. Tax Rates

I turn next to estimates for tax rates.

In Figures 11 and 12, I plot the tax rates on capital and labor from Joines (1981). I include filtered series, which are used as inputs in the computer simulations. (Later, I show that the results for the basic growth model are robust to whether I use filtered inputs or unfiltered inputs.)

For the extended model, I use Joines' tax rate on labor but not capital. For capital, I instead decompose capital taxation into parts. In other words, for the tax rate on business profits, I do not use Joines' comprehensive capital tax measure but instead use the rate on corporate income taxes.

In Figure 13, I plot the statutory corporate income tax rate from the U.S. Treasury's *Statistics of Income*. This is the rate that firms paying taxes on profits faced.

In Figure 14, I plot the effective tax rate on dividends of McGrattan and Prescott (2003). They calculate this rate as tax paid on an additional dollar of dividend income using data from the U.S. Treasury's *Statistics of Income*.

Figure 15 shows the effective tax rate on property (NIPA Table 3.5). To construct taxes paid on property for the business sector, I sum taxes on imports and production plus business current transfer payments and subtract off these taxes and transfers for the farm (NIPA Table 7.3.5) and housing sectors (NIPA Table 7.4.5). To construct the tax rate, I then divide taxes paid by business fixed capital (FA Table 6.1) plus land and inventories from Goldsmith (1962).

Finally, the effective tax rate on consumption is found by dividing sales taxes (NIPA Table 3.5) by consumption (as defined above). The rate is plotted in Figure 16.

2. Basic Growth Model

The one-sector neoclassical growth model analyzed by Cole and Ohanian (1999) serves as the baseline for the conventional view described in the main text.

2.1. Household Problem

I'll start with the household's problem. The household chooses consumption c, investment x, and hours of work h to solve the following maximization problem:

$$\max_{\{c_t,x_t,h_t\}} E\sum_{t=0}^{\infty}\beta^t\left[\log\left(c_t\right)+\psi\left(\left(1-h_t\right)^{\phi}-1\right)/\phi\right]N_t$$

$$\text{subject to}\;\; c_t+x_t=r_tk_t+w_th_t+\kappa_t$$

$$- \tau_{ht} w_t h_t - \tau_{pt} \{ (r_t - \delta - \tau_{kt}) \, k_t \}$$

$$k_{t+1} = [(1-\delta) \, k_t + x_t] \, / \, (1+\eta)$$

$$x_t \geq 0 \quad \text{in all states}$$

with processes for factor prices (r_t, w_t), taxes (τ_{ht}, τ_{pt}), and transfers (κ_t) given. Quantities are in per capita terms. N_t is the number of family members. Growth in N_t is η.

I next derive the necessary first-order conditions that I use in the computer code. The Lagrangian for the optimization problem is

$$\begin{aligned}
\mathcal{L} = E \sum_t \left[\beta \left(1+\eta\right)\right]^t \Bigg\{ & \log\left(\hat{c}_t\right) + \psi \left(\left(1-h_t\right)^{\phi} - 1 \right) / \phi + \frac{\zeta}{3} \min\left(\hat{x}_t, 0\right)^3 \\
& + \mu_t \Big\{ \left(r_t - \tau_{kt}\right) \hat{k}_t + \left(1-\tau_{ht}\right) \hat{w}_t h_t + \hat{\kappa}_t \\
& \qquad - \hat{c}_t - \hat{x}_t \\
& \qquad - \tau_{pt} \{ \left(r_t - \delta - \tau_{kt}\right) \hat{k}_t \} \\
& + \lambda_t \Big\{ \left(1-\delta\right) \hat{k}_t + \hat{x}_t - \left(1+\eta\right)\left(1+\gamma\right) \hat{k}_{t+1} \Big\} \Bigg\},
\end{aligned}$$

where ζ is a penalty parameter used to deal with the constraint $x_t \geq 0$. Variables that grow over time with increasing technology are detrended, e.g., $\hat{c}_t = c_t/(1+\gamma)^t$.

Taking derivatives with respect to all decision variables yields the following first-order conditions:

$$1/\hat{c}_t = \mu_t \tag{2.1}$$

$$\psi \left(1-h_t\right)^{\phi-1} = \mu_t \left(1-\tau_{ht}\right) \hat{w}_t \tag{2.2}$$

$$\zeta \min\left(\hat{x}_t, 0\right)^2 + \lambda_t = \mu_t \tag{2.3}$$

$$\begin{aligned}
\left(1+\eta\right)\left(1+\gamma\right) \lambda_t = \tilde{\beta} E_t \Big[& \lambda_{t+1} \left(1-\delta\right) \\
& + \mu_{t+1} \big\{ r_{t+1} - \tau_{pt+1} \left(r_{t+1} - \delta\right) \big\} \Big].
\end{aligned} \tag{2.4}$$

If I simplify these equations, I have

$$\psi \hat{c}_t (1 - h_t)^{\phi - 1} = (1 - \tau_{ht}) \hat{w}_t \tag{2.5}$$

$$\frac{1}{\hat{c}_t} - \zeta \min(\hat{x}_t, 0)^2 = \hat{\beta} E\left[\frac{1}{\hat{c}_{t+1}} \left\{ 1 + (1 - \tau_{pt+1})(r_{t+1} - \delta) \right\} - (1 - \delta) \zeta \min(\hat{x}_{t+1}, 0)^2 \,\middle|\, k_t, s_t \right], \tag{2.6}$$

where $\hat{\beta} = \beta/(1+\gamma)$.

2.2. Factor Prices

Factor prices are derived from the first-order conditions of

$$\max_{\{K_t, L_t\}} K_t^{\theta} (Z_t L_t)^{1-\theta} - r_t K_t - w_t H_t,$$

which implies

$$r_t = \theta \left(\hat{k}_t \right)^{\theta - 1} (z_t h_t)^{1-\theta}$$

$$\hat{w}_t = (1 - \theta) \left(\hat{k}_t \right)^{\theta} z_t^{1-\theta} h_t^{-\theta}$$

when variables are normalized.

2.3. Government Budget Constraint

The government's budget constraint, written in per capita and detrended terms, is given by

$$\hat{g}_t + \hat{\kappa}_t = \tau_{ht} \hat{w}_t h_t + \tau_{pt} (r_t - \delta - \tau_{kt}) \hat{k}_t.$$

2.4. Resource Constraint

The original resource constraint of the economy is given by

$$N_t \left(c_t + x_t + g_t\right) = \left(N_t k_t\right)^{\theta} \left(Z_t N_t h_t\right)^{1-\theta},$$

where g_t is per capita spending of the government. Once I divide by population and account for growth in technology, I have a normalized resource constraint given by

$$\hat{c}_t + \hat{x}_t + \hat{g}_t = \hat{y}_t = \hat{k}_t^{\theta} \left(z_t h_t\right)^{1-\theta}.$$

2.5. Exogenous Processes

I next specify exogenous processes for $\{\hat{g}, \tau_h, \tau_p, z\}$. Let s index the state, where s is determined by an nth-order Markov chain. Then at time t if the state is s, $g_t = g(s)$, $\tau_{ht} = \tau_h(s)$, etc. The process for s is intended to capture different states of the world. Note that the state vector for the economy is $\hat{k}$, s.

2.6. Computation

The first step is to find α, which is used to represent the consumption function,

$$\hat{c}\left(\hat{k}, s\right) = \sum_{j=1}^{nnodes} \alpha_j^s \Phi_j\left(\hat{k}\right),$$

where the functions $\Phi_j(\hat{k})$ are known basis functions. For the finite element method, the $\Phi_j(\hat{k})$'s are low-order polynomials that are nonzero on small subdomains and the vector α satisfies

$$R\left(\hat{k}, s; \alpha\right) = 1 - \zeta \hat{c} \min\left(\hat{x}, 0\right)^2 + \hat{\beta}\left(1-\delta\right)\zeta\hat{c}\sum_{s'} \pi_{s,s'} \min\left(\hat{x}', 0\right)^2$$
$$- \hat{\beta}\sum_{s'} \pi_{s,s'} \left(\frac{\hat{c}}{\hat{c}'}\right)\left\{1 + \left(1-\tau_p\left(s'\right)\right)\left(\theta\left(\hat{k}'\right)^{\theta-1}\left(z\left(s'\right)h'\right)^{1-\theta} - \delta\right)\right\}.$$

To speed up the computation, I will need the derivatives of R with respect to coefficients on current consumption and the coefficients on next period consumption. I'll start with current consumption:

$$\begin{aligned}
\frac{\partial R\left(\hat{k}, s; \alpha\right)}{\partial \alpha_j^s} = & -\zeta[\min\left(\hat{x}, 0\right)^2 + 2\hat{c}\min\left(\hat{x}, 0\right)\frac{d\hat{x}}{d\hat{c}} - \hat{\beta}\left(1-\delta\right)\sum_{s'}\pi_{s,s'}\min\left(\hat{x}', 0\right)^2]\frac{d\hat{c}}{d\alpha_j^s} \\
& + 2\zeta\hat{\beta}\left(1-\delta\right)\hat{c}\sum_{s'}\pi_{s,s'}\min\left(\hat{x}', 0\right)\frac{d\hat{x}'}{d\alpha_j^s} \\
& - \hat{\beta}\sum_{s'}\pi_{s,s'}\left\{1 + \left(1-\tau_p\left(s'\right)\right)\left(\theta\left(\hat{k}'\right)^{\theta-1}\left(z\left(s'\right)h'\right)^{1-\theta} - \delta\right)\right\}\left(\frac{\hat{c}}{\hat{c}'}\right) \\
& \qquad \cdot\left[\frac{1}{\hat{c}}\frac{d\hat{c}}{d\alpha_j^s} - \frac{1}{\hat{c}'}\frac{d\hat{c}'}{d\alpha_j^s}\right] \\
& - \hat{\beta}\sum_{s'}\pi_{s,s'}\left(\frac{\hat{c}}{\hat{c}'}\right)\left(1-\tau_p\left(s'\right)\right)\theta\left(\theta-1\right)\left(\hat{k}'\right)^{\theta-1}\left(z\left(s'\right)h'\right)^{1-\theta} \\
& \qquad \cdot\left[\frac{1}{\hat{k}'}\frac{d\hat{k}'}{d\alpha_j^s} - \frac{1}{h'}\frac{dh'}{d\alpha_j^s}\right]\Bigg\}
\end{aligned}$$

and, then, next period consumption:

$$\begin{aligned}
\frac{\partial R\left(\hat{k}, s; \alpha\right)}{\partial \alpha_j^{s'}} = & 2\zeta\hat{\beta}\left(1-\delta\right)\hat{c}\sum_{s'}\pi_{s,s'}\min\left(\hat{x}', 0\right)\frac{d\hat{x}'}{d\alpha_j^{s'}} \\
& - \hat{\beta}\sum_{s'}\pi_{s,s'}\left\{1 + \left(1-\tau_p\left(s'\right)\right)\left(\theta\left(\hat{k}'\right)^{\theta-1}\left(z\left(s'\right)h'\right)^{1-\theta} - \delta\right)\right\}\left(\frac{\hat{c}}{\hat{c}'}\right) \\
& \qquad \cdot\left[-\frac{1}{\hat{c}'}\frac{d\hat{c}'}{d\alpha_j^{s'}}\right] \\
& - \hat{\beta}\sum_{s'}\pi_{s,s'}\left(\frac{\hat{c}}{\hat{c}'}\right)\left(1-\tau_p\left(s'\right)\right)\theta\left(\theta-1\right)\left(\hat{k}'\right)^{\theta-1}\left(z\left(s'\right)h'\right)^{1-\theta} \\
& \qquad \cdot\left[-\frac{1}{h'}\frac{dh'}{d\alpha_j^{s'}}\right].
\end{aligned}$$

To compute these expressions, I need formulas for the derivatives in these equations.

I'll start with the next period capital, which has the derivative

$$
\begin{aligned}
d\hat{k}' &= d\hat{x}/[(1+\eta)(1+\gamma)] \\
&= [(1-\theta)\,\hat{y}/hdh - d\hat{c}]/[(1+\eta)(1+\gamma)].
\end{aligned}
$$

Next, I'll derive dh, which involves differentiating the intratemporal first-order condition (2.5):

$$
\begin{aligned}
0 &= d[(1-h)^{\phi-1}\,h^{\theta}\,\hat{c}] \\
&= (1-h)^{\phi-1}\,h^{\theta}\,\hat{c}\left\{\left[-\frac{\phi-1}{1-h}+\frac{\theta}{h}\right]dh+\frac{1}{\hat{c}}d\hat{c}\right\}.
\end{aligned}
$$

This result shows that dh can be written as a function of dc. Next consider dh', which is slightly different since it depends on dk' as well:

$$
\begin{aligned}
0 &= d[(1-h')^{\phi-1}\,(h')^{\theta}\,\hat{c}'\left(\hat{k}'\right)^{-\theta}] \\
&= (1-h')^{\phi-1}\,(h')^{\theta}\,\hat{c}'\left(\hat{k}'\right)^{-\theta}\left\{\left[-\frac{\phi-1}{1-h'}+\frac{\theta}{h'}\right]dh'+\frac{1}{\hat{c}'}d\hat{c}'-\frac{\theta}{\hat{k}'}d\hat{k}'\right\}.
\end{aligned}
$$

Finally I need

$$
\frac{d\hat{c}'}{d\alpha_j^s} = \left(\sum_l \alpha_l^s \frac{\partial N_l\left(\hat{k}'\right)}{\partial \hat{k}'}\right)\frac{d\hat{k}'}{d\alpha_j^s}
$$

$$
\frac{d\hat{c}'}{d\alpha_j^{s'}} = N_j\left(\hat{k}'\right),
$$

and all other derivatives are explicit or implicit functions of these.

2.7. Sensitivity Analysis

For the basic growth model, I now show that using the smoothed time series for government spending in Figure and the smoothed tax series in Figures 11 and 12 does not affect the results (shown in Figures 1-4 in the main text). In Figures 17–20, I compare predictions of the model (along with U.S. data) with both filtered and unfiltered spending and tax rate inputs. The results are nearly indistinguishable.

3. Extended Growth Model

Here, I consider an extension of the stochastic growth model that has both tangible and intangible capital. As in the home production model, this extension requires me to compute two-dimensional decision functions.

3.1. Household Problem

I'll start with the household's problem. The problem is to choose consumption c_t, hours h_t, and investments x_{Tt}, x_{It} to maximize

$$\max \ E\sum_{t=0}^{\infty}\beta^t[\log(c_t)+\psi\left((1-h_t)^{\phi}-1\right)/\phi]N_t$$

subject to

$$\begin{aligned}
c_t + x_{Tt} + q_t x_{It} \leq r_{Tt}k_{Tt} &+ r_{It}k_{It} + w_t h_t + \kappa_t \\
&- \tau_{ct}c_t - \tau_{ht}w_t h_t + \tau_{bt}(1-\chi)\,q_t x_{It} \\
&- \tau_{xt}x_{Tt} - \tau_{kt}k_{Tt} \\
&- \tau_{pt}\left[r_{Tt}k_{Tt} + r_{It}k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{kt}k_{Tt}\right] \\
&- \tau_{ut}\left[(1+\eta)\,k_{Tt+1} - k_{Tt}\right] \\
&- \tau_{dt}\big[r_{Tt}k_{Tt} + r_{It}k_{It} - x_{Tt} - \chi q_t x_{It} - \tau_{kt}k_{Tt} - \tau_{xt}x_{Tt} \\
&\qquad - \tau_{pt}\big(r_{Tt}k_{Tt} + r_{It}k_{It} - \delta_T k_{Tt} \\
&\qquad\qquad - \chi q_t x_{It} - \tau_{kt}k_{Tt}\big) \\
&\qquad\qquad - \tau_{ut}\left((1+\eta)\,k_{Tt+1} - k_{Tt}\right)\big],
\end{aligned} \tag{3.1}$$

taking as given the initial capital stocks, factor prices (r_{Tt}, r_{It}, w_t), other incomes (κ), and exogenous shocks. Hours are the sum of business hours $h_{bt} = h_{bt}^1 + h_{bt}^2$ and nonbusiness hours h_{nt}.

Constraints that must be satisfied in addition to the budget constraint are the capital accumulation equations:

$$k_{Tt+1} = [(1-\delta_T)\, k_{Tt} + x_{Tt}]/\,(1+\eta)$$
$$k_{It+1} = [(1-\delta_I)\, k_{It} + x_{It}]/\,(1+\eta)$$

and nonnegativity constraints on investment: $x_{Tt} \geq 0$ and $x_{It} \geq 0$.

Note that I am assuming that households own the capital stocks and pay all taxes directly. Separating the problems of households and firms will not affect the equations to which I apply the numerical algorithm.

Also, I will assume, as in McGrattan and Prescott (2010), that nonbusiness income, investment, and hours are given exogenously. Here, they will be indexed by the state s.

Before deriving first-order conditions for the problem, I first modify the objective of the household to incorporate penalty functions for the nonnegativity constraints:

$$E \max \sum_{t=0}^{\infty} \sum_{s^t} [\beta\,(1+\eta)]^t \Big\{ \log(\hat{c}_t) + \psi \Big((1-h_t)^{\phi} - 1 \Big) / \phi$$
$$+ \zeta/3 \big(\min(\hat{x}_{Tt}, 0)^3 + \min(\hat{x}_{It}, 0)^3 \big) \Big\}.$$

If $\zeta = 0$, this is the utility defined above.

The Lagrangian for the optimization problem is

$$\mathcal{L} = E\sum_t \left[\beta\left(1+\eta\right)\right]^t \Bigg\{\log\left(\hat{c}_t\right) + \psi\left(\left(1-h_t\right)^{\phi} - 1\right)/\phi$$

$$+\frac{\zeta}{3}\left[\min\left(\hat{x}_{Tt},0\right)^3 + \min\left(\hat{x}_{It},0\right)^3\right]$$

$$+\mu_t\Big\{\left(r_{Tt}-\tau_{kt}\right)\hat{k}_{Tt} + r_{It}\hat{k}_{It} + \left(1-\tau_{ht}\right)\hat{w}_t h_t + \hat{\kappa}_t$$

$$-\tau_{pt}\big\{\left(r_{Tt}-\delta_T-\tau_{kt}\right)\hat{k}_{Tt} + r_{It}\hat{k}_{It} - \chi q_t\hat{x}_{It}\big\}$$

$$-\tau_{ut}\big\{\left(1+\eta\right)\left(1+\gamma\right)\hat{k}_{Tt+1} - \hat{k}_{Tt}\big\}$$

$$-\tau_{dt}\big\{\left(r_{Tt}-\tau_{kt}\right)\hat{k}_{Tt} + r_{It}\hat{k}_{It} - \chi q_t\hat{x}_{It} - \left(1+\tau_{xt}\right)\hat{x}_{Tt}$$

$$-\tau_{pt}\{\left(r_{Tt}-\delta_T-\tau_{kt}\right)\hat{k}_{Tt} + r_{It}\hat{k}_{It} - \chi q_t\hat{x}_{It}\}$$

$$-\tau_{ut}\{\left(1+\eta\right)\left(1+\gamma\right)\hat{k}_{Tt+1} - \hat{k}_{Tt}\}\big\}$$

$$-\left(1+\tau_{ct}\right)\hat{c}_t - \left(1+\tau_{xt}\right)\hat{x}_{Tt} - \left(1-\left(1-\chi\right)\tau_{bt}\right)q_t\hat{x}_{It}$$

$$+\lambda_{Tt}\Big\{\left(1-\delta_T\right)\hat{k}_{Tt} + \hat{x}_{Tt} - \left(1+\eta\right)\left(1+\gamma\right)\hat{k}_{Tt+1}\Big\}$$

$$+\lambda_{It}\Big\{\left(1-\delta_I\right)\hat{k}_{It} + \hat{x}_{It} - \left(1+\eta\right)\left(1+\gamma\right)\hat{k}_{It+1}\Big\}\Bigg\}.$$

Consider the first-order conditions with respect to consumption, labor, and next period capital stocks. They are as follows:

$$1/\hat{c}_t = \left(1+\tau_{ct}\right)\mu_t$$

$$\psi\left(1-h_t\right)^{\phi-1} = \mu_t\left(1-\tau_{ht}\right)\hat{w}_t$$

$$\zeta\min\left(\hat{x}_{Tt},0\right)^2 + \lambda_{Tt} = \mu_t\left(1+\tau_{xt}\right)\left(1-\tau_{dt}\right)$$

$$\zeta\min\left(\hat{x}_{It},0\right)^2 + \lambda_{It} = \mu_t q_t[\left(1-\chi\right)\left(1-\tau_{ht}\right) + \chi\left(1-\tau_{pt}\right)\left(1-\tau_{dt}\right)]$$

$$\left(1+\eta\right)\left(1+\gamma\right)\left(\lambda_{Tt} + \mu_t\tau_{ut}\left(1-\tau_{dt}\right)\right) = \tilde{\beta}E_t\Big[\lambda_{Tt+1}\left(1-\delta_T\right)$$

$$+\mu_{t+1}\left(1-\tau_{dt+1}\right)\left\{r_{Tt+1} - \tau_{kt+1} - \tau_{pt+1}\left(r_{Tt+1}-\delta_T-\tau_{kt+1}\right) + \tau_{ut+1}\right\}\Big]$$

$$\left(1+\eta\right)\left(1+\gamma\right)\lambda_{It} = \tilde{\beta}E_t\Big[\lambda_{It+1}\left(1-\delta_I\right)$$

$$+ \mu_{t+1} (1 - \tau_{pt+1}) (1 - \tau_{dt+1}) r_{It+1} \Big].$$

Rewriting the dynamic first-order conditions, I get

$$\frac{(1 + \tau_{xt} + \tau_{ut}) (1 - \tau_{dt})}{(1 + \tau_{ct}) \hat{c}_t} - \zeta \min (\hat{x}_{Tt}, 0)^2$$
$$= \hat{\beta} E \Big[\frac{(1 - \tau_{dt+1})}{(1 + \tau_{ct+1}) \hat{c}_{t+1}} \{ (1 - \tau_{pt+1}) (r_{Tt+1} - \tau_{kt+1})$$
$$+ (1 - \delta_T) (1 + \tau_{xt+1}) + \delta_T \tau_{pt+1} + \tau_{ut+1} \}$$
$$- \zeta (1 - \delta_T) \min (\hat{x}_{Tt+1}, 0)^2 \Big]$$

$$\frac{q_t [(1 - \chi) (1 - \tau_{ht}) + \chi (1 - \tau_{pt}) (1 - \tau_{dt})]}{(1 + \tau_{ct}) \hat{c}_t} - \zeta \min (\hat{x}_{It}, 0)^2$$
$$= \hat{\beta} E \Big[\frac{1}{(1 + \tau_{ct+1}) \hat{c}_{t+1}} \{ (1 - \tau_{dt+1}) (1 - \tau_{pt+1}) r_{It+1}$$
$$+ (1 - \delta_I) q_{t+1} [(1 - \chi) (1 - \tau_{ht+1})$$
$$+ \chi (1 - \tau_{pt+1}) (1 - \tau_{dt+1})] \}$$
$$- \zeta (1 - \delta_I) \min (\hat{x}_{It+1}, 0)^2 \Big].$$

3.2. Factor Prices

Assume that the technologies are

$$\hat{y}_{bt} = \left(\hat{k}_{Tt}^1 \right)^{\theta_1} \left(\hat{k}_{It} \right)^{\alpha_1} \left(z_t^1 h_t^1 \right)^{1 - \theta_1 - \alpha_1}$$
$$\hat{x}_{It} = \left(\hat{k}_{Tt}^2 \right)^{\theta_2} \left(\hat{k}_{It} \right)^{\alpha_2} \left(z_t^2 h_t^2 \right)^{1 - \theta_2 - \alpha_2},$$

where the total tangible capital (in business) is $\hat{k}_{Tt} = \hat{k}_{Tt}^1 + \hat{k}_{Tt}^2$ and total hours is $h_t = h_t^1 + h_t^2 + h_{nt}$.

The factor prices are

$$r_{Tt} = \theta_1 \hat{y}_{bt}/\hat{k}^1_{Tt} = \theta_2 q_t \hat{x}_{It}/\hat{k}^2_{Tt}$$

$$r_{It} = (\alpha_1 \hat{y}_{bt} + \alpha_2 q_t x_{It}) / \hat{k}_{It}$$

$$\hat{w}_t = (1 - \theta_1 - \alpha_1) \hat{y}_{bt}/h^1_t = (1 - \theta_2 - \alpha_2) q_t \hat{x}_{It}/h^2_t.$$

3.3. Government Budget Constraint

The government's budget constraint, written in per capita and detrended terms, is given by

$$\begin{aligned} \hat{g}_t + \tilde{\kappa}_t = \tau_{ct}\hat{c}_t + \tau_{ht}\hat{w}_t h_t + \tau_{bt}(1-\chi) q_t x_{It} + \tau_{xt}\hat{x}_{Tt} + \tau_{kt}\hat{k}_{Tt} \\ + \tau_{pt}\{(r_{Tt} - \delta_T - \tau_{kt}) \hat{k}_{Tt} + r_{It}\hat{k}_{It} - \chi q_t \hat{x}_{It}\} \\ + \tau_{ut}\{\hat{x}_{Tt} - \delta_T \hat{k}_{Tt}\} \\ + \tau_{dt}\{(r_{Tt} - \tau_{kt}) \hat{k}_{Tt} - (1 + \tau_{xt}) \hat{x}_{Tt} + r_{It}\hat{k}_{It} - \chi q_t \hat{x}_{It} \\ - \tau_{pt}[(r_{Tt} - \delta_T - \tau_{kt}) \hat{k}_{Tt} + r_{It}\hat{k}_{It} - \chi q_t \hat{x}_{It}]\}, \end{aligned}$$

where $\tilde{\kappa} = \hat{\kappa} + \hat{w} h_n + \hat{x}_n - \hat{y}_n$.

3.4. Resource Constraint

GDP in this economy is (after normalizing)

$$\hat{y}_{bt} + \hat{y}_{nt} = \hat{c}_t + \hat{x}_{Tt} + \hat{x}_{nt} + \hat{g}_t.$$

Total output is $\hat{y}_{bt} + \hat{y}_{nt} + q_t \hat{x}_{It}$.

3.5. Exogenous Processes

The exogenous variables for this model are $\{\hat{g}, \tau_c, \tau_h, \tau_x, \tau_k, \tau_p, \tau_d, z^1, z^2, h_n, \hat{x}_n, \hat{y}_n\}$. The state is indexed by s. Therefore, the full state vector for the economy is $[\hat{k}_T,\ \hat{k}_I,\ s]$.

3.6. Computation

I am going to compute two decision functions which are represented as sums of known basis functions $\Phi_j(\hat{k}_T, \hat{k}_I)$:

$$\hat{c}\left(\hat{k}_T, \hat{k}_I, s\right) = \sum_{j=1}^{nnodes} \alpha^s_{cj} \Phi_j\left(\hat{k}_T, \hat{k}_I\right) \tag{3.2}$$

$$\hat{x}_T\left(\hat{k}_T, \hat{k}_I, s\right) = \sum_{j=1}^{nnodes} \alpha^s_{xj} \Phi_j\left(\hat{k}_T, \hat{k}_I\right). \tag{3.3}$$

The static first-order conditions can be used to determine the current period variables, given guesses for the decision variables c and x_T and the state variables $\hat{k}_T$, $\hat{k}_I$, and all exogenous variables. Start by guessing a value for h^1 given c, $\hat{k}_T$, and $\hat{k}_I$. Then I have (in order):

$$\begin{aligned}
\hat{y}_b &= \hat{c} + \hat{x}_T + \hat{g} + (\hat{x}_n - \hat{y}_n) \\
\hat{w} &= (1-\theta_1-\alpha_1)\,\hat{y}_b/h^1 \\
h &= 1 - \left[(1-\tau_h)\,\hat{w}/\left(\psi\,(1+\tau_c)\,\hat{c}\right)\right]^{1/(\phi-1)} \\
h^2 &= h - h^1 - h_n \\
\xi &= (1-\theta_2-\alpha_2)\,h^1/\left[(1-\theta_1-\alpha_1)\,h^2\right] \\
\hat{k}^1_T &= \theta_1\xi/\left(\theta_2+\theta_1\xi\right)\hat{k}_T,
\end{aligned}$$

where $\xi = \hat{y}_b/(q\hat{x}_I)$. If the guess for h^1 is correct, then the following should hold exactly:

$$\hat{y}_b = \hat{A}^1\left(\hat{k}^1_T\right)^{\theta_1}\left(\hat{k}_I\right)^{\alpha_1}\left(h^1\right)^{1-\theta_1-\alpha_1}.$$

If it does not, then I update the guess for h^1 and continue until convergence.

With values for $\hat{k}_1^T$, $\hat{w}$, and h^2, I can back out

$$\begin{aligned}
\hat{k}_T^2 &= \hat{k}_T - \hat{k}_T^1 \\
q\hat{x}_I &= \hat{w}h^2 / \left(1 - \theta_2 - \alpha_2\right) \\
\hat{x}_I &= \hat{A}^2 \left(\hat{k}_T^2\right)^{\theta_2} \left(\hat{k}_I\right)^{\alpha_2} \left(h^2\right)^{1-\theta_2-\alpha_2} \\
q &= \left(q\hat{x}_I\right) / \hat{x}_I .
\end{aligned}$$

Then, capital stocks can be updated given current period investments x_T and x_I.

The unknowns coefficients in (3.2) and (3.3), which I can stack into the vector $\vec{\alpha}$, are set so that the residuals of the two dynamic Euler equations are approximately zero. These residuals can be written as follows:

$$\begin{aligned}
R_1\left(\vec{k}, s; \vec{\alpha}\right) = {} & \left(1 + \tau_x(s) + \tau_u(s)\right)\left(1 - \tau_d(s)\right) / \left(1 + \tau_c(s)\right) \\
& - \zeta \hat{c} \min\left(\hat{x}_T, 0\right)^2 + \hat{\beta}\zeta\hat{c}\left(1 - \delta_T\right) \sum_{s'} \pi_{s,s'} \min\left(\hat{x}_T', 0\right)^2 \\
& - \hat{\beta} \sum_{s'} \pi_{s,s'} \Big\{ \frac{\left(1 - \tau_d(s')\right)}{\left(1 + \tau_c(s')\right)} \frac{\hat{c}}{\hat{c}'} \big[\left(1 - \tau_p(s')\right)\left(r_T' - \tau_k(s')\right) \\
& + \left(1 - \delta_T\right)\left(1 + \tau_x(s')\right) + \delta_T \tau_p(s') + \tau_u(s')\big] \Big\}
\end{aligned}$$

$$\begin{aligned}
R_2\left(\vec{k}, s; \vec{\alpha}\right) = {} & q[\left(1 - \chi\right)\left(1 - \tau_b(s)\right) + \chi\left(1 - \tau_d(s)\right)\left(1 - \tau_p(s)\right)] / \left(1 + \tau_c(s)\right) \\
& - \zeta \hat{c} \min\left(\hat{x}_I, 0\right)^2 + \hat{\beta}\zeta\hat{c}\left(1 - \delta_I\right) \sum_{s'} \pi_{s,s'} \min\left(\hat{x}_I', 0\right)^2 \\
& - \hat{\beta} \sum_{s'} \pi_{s,s'} \Big\{ \frac{1}{\left(1 + \tau_c(s')\right)} \frac{\hat{c}}{\hat{c}'} \big[\left(1 - \tau_d(s')\right)\left(1 - \tau_p(s')\right) r_I' \\
& + \left(1 - \delta_I\right) q' [\left(1 - \chi\right)\left(1 - \tau_b(s')\right) \\
& + \chi\left(1 - \tau_d(s')\right)\left(1 - \tau_p(s')\right)] \big] \Big\},
\end{aligned}$$

where $\hat{\beta} = \beta(1+\gamma)^{-\sigma}$ and $\vec{k} = (\hat{k}_T, \hat{k}_I)$. If I apply a standard finite element method, I find $\vec{\alpha}$ to ensure that weighted sums of residuals R_1 and R_2 are equal to zero.

When computing $\vec{\alpha}$, I take derivatives of the residuals with respect to these unknown coefficients; this speeds up the numerical algorithm considerably, especially if the number of unknowns is large.

I'll start by totally differentiating the static first-order conditions in order to write the derivative of h^1, h^2, y_b, w, and k_T^1 in terms of the derivatives of decision functions and states:

$$\begin{aligned}
d\hat{y}_b &= d\hat{c} + d\hat{x}_T \\
d\hat{w} &= \hat{w}[d\hat{y}_b/\hat{y}_b - dh^1/h^1] \\
dh &= (1-h)\,/\,(1-\phi)\,[d\hat{w}/\hat{w} - d\hat{c}/\hat{c}] \\
dh^2 &= dh - dh^1 \\
d\xi &= \xi\left[dh^1/h^1 - dh^2/h^2\right] \\
d\hat{k}_T^1 &= \hat{k}_T^1\left[d\hat{k}_T/\hat{k}_T + \left(1 - \hat{k}_T^1/\hat{k}_T\right)d\xi/\xi\right] \\
d\hat{y}_b &= \hat{y}_b\left[\theta_1 d\hat{k}_T^1/\hat{k}_T^1 + \alpha_1 d\hat{k}_I/\hat{k}_I + (1-\theta_1-\alpha_1)\,dh^1/h^1\right].
\end{aligned}$$

This can be summarized as

$$Adh^1 = Bd\hat{c} + Cd\hat{x}_T + Dd\hat{k}_T + Ed\hat{k}_I,$$

where the coefficients are

$$\begin{aligned}
A &= (1-\theta_1-\alpha_1)\,\hat{y}_b/h^1 + \theta_1\hat{y}_b\left(1-\hat{k}_T^1/\hat{k}_T\right)\left(h^2 + h^1 + (1-h)\,/\,(1-\phi)\right)/\left(h^1h^2\right) \\
B &= 1 + \theta_1\left(1-\hat{k}_T^1/\hat{k}_T\right)(1-h)\,(1-\hat{y}_b/\hat{c})\,/\left(h^2 - \phi h^2\right) \\
C &= 1 + \theta_1\left(1-\hat{k}_T^1/\hat{k}_T\right)(1-h)\,/\left(h^2 - \phi h^2\right)
\end{aligned}$$

$$D = -\theta_1 \hat{y}_b / \hat{k}_T$$

$$E = -\alpha_1 \hat{y}_b / \hat{k}_I.$$

3.7. Sensitivity Analysis

Here, I show that the results in the main text are not affected by parameters governing the size and time series of intangible capital. In Figures 21 through 27, I compare the main equilibrium paths for three different parameterizations of the extended model.[1]

The benchmark parameterization assumes that half of intangible investments are expensed by shareholders and that the depreciation rate on intangible capital is zero. The alternative parameterizations have a higher fraction of expensing done by shareholders and equal depreciation rates for intangible and tangible investment.

As Figures 21 through 27 make clear, these alternatives generate similar model predictions. The main finding that capital taxation had a significant impact on economic activity in the 1930s is not overturned by these alternative numerical experiments.

[1] In each case, parameters of the model are recalibrated so that the model generates 1929 levels of consumption, tangible investment, tangible capital, GDP, NIPA compensation, and hours comparable to the United States.

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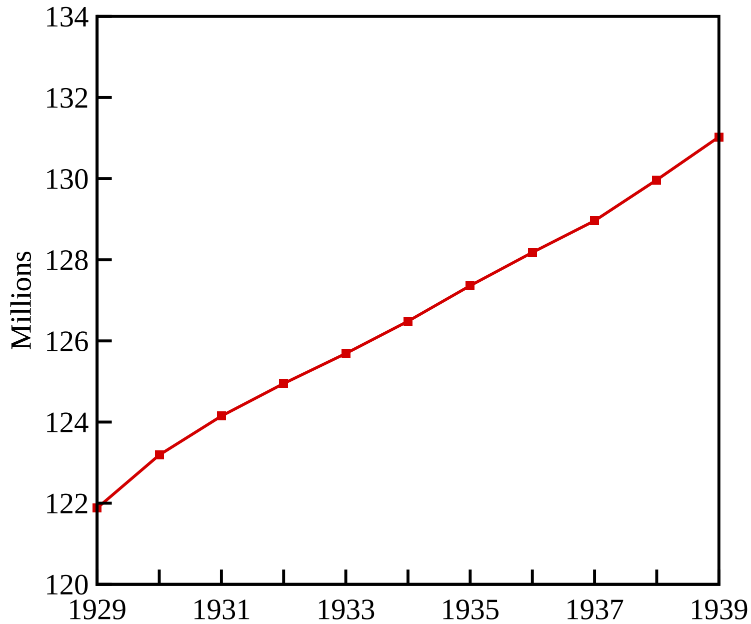

Figure 1. Population at Midyear

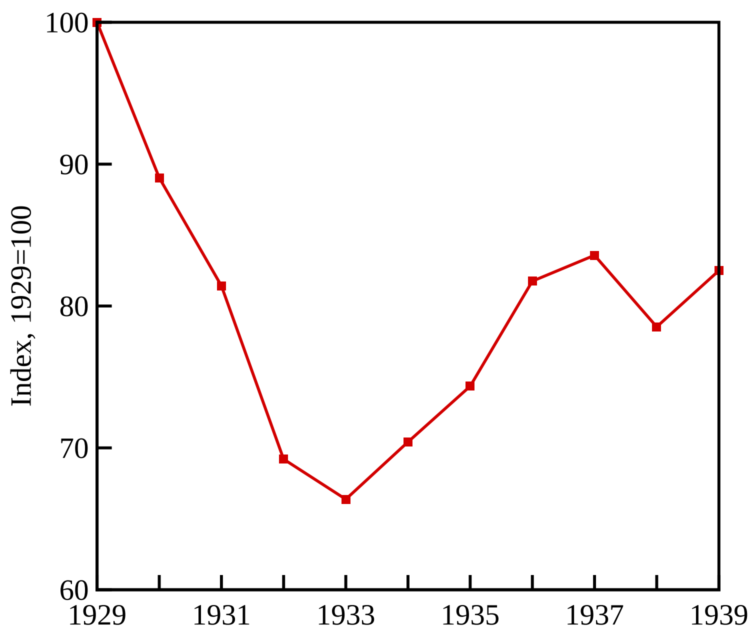

Figure 2. Detrended Real GDP

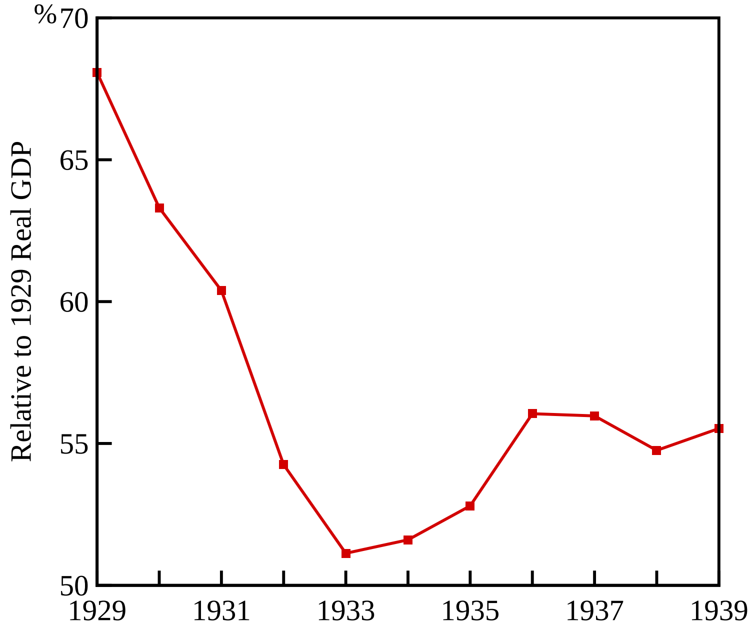

Figure 3. Detrended Real Consumption

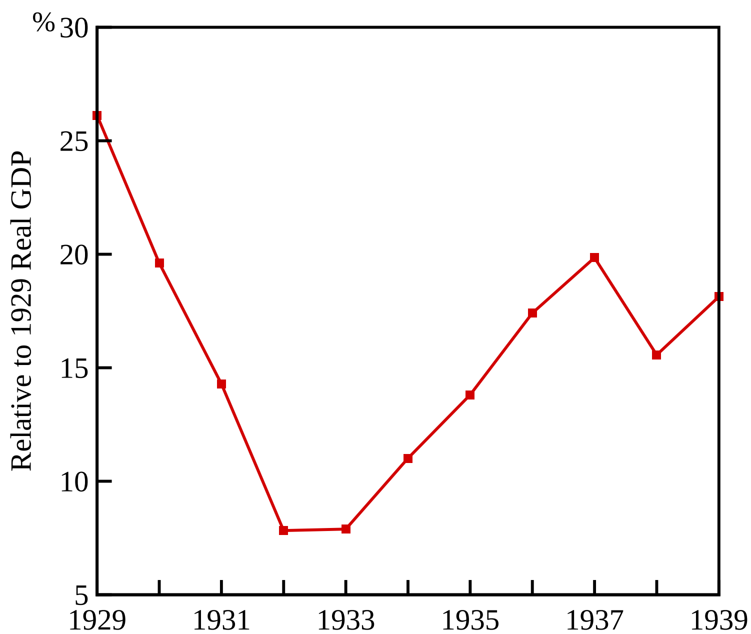

Figure 4. Detrended Real Investment

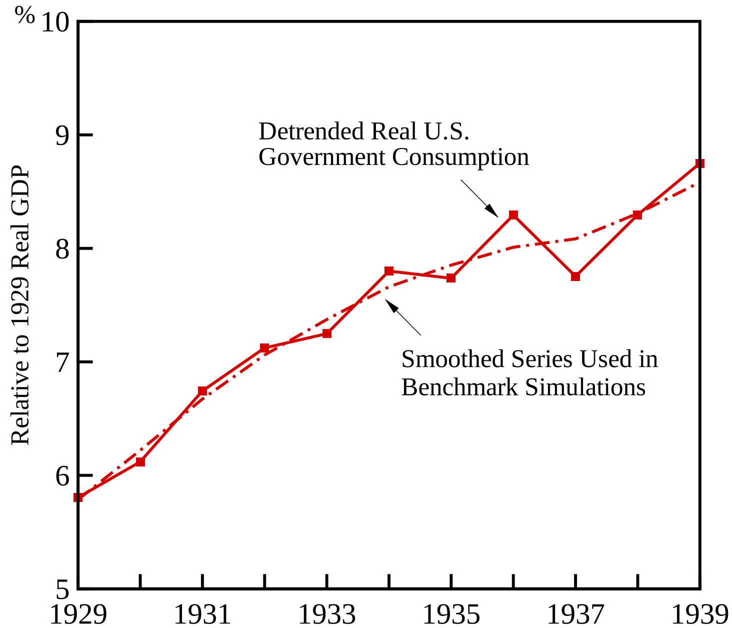

Figure 5. Detrended Government Spending

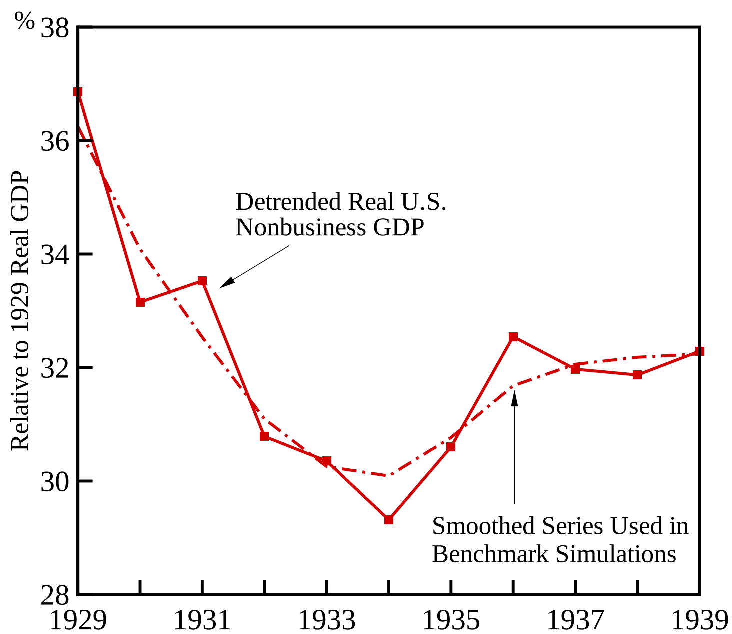

Figure 6. Detrended Nonbusiness Output

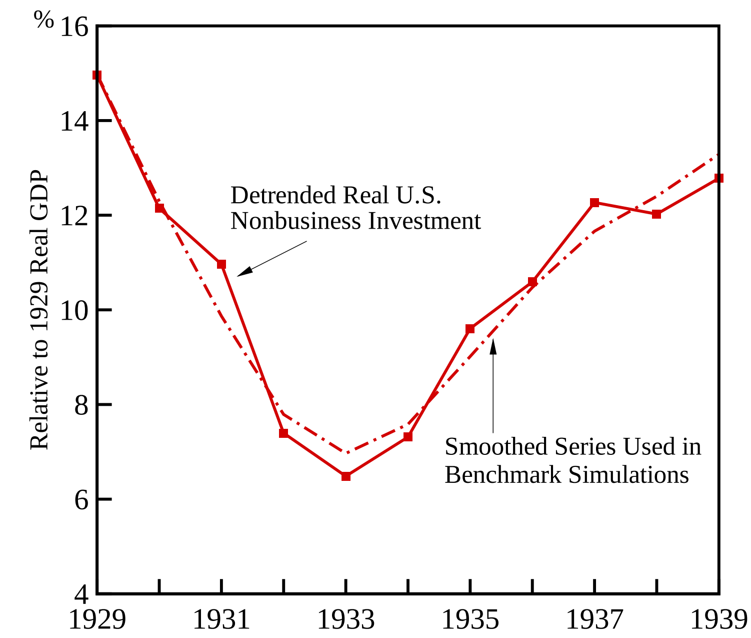

Figure 7. Detrended Nonbusiness Investment

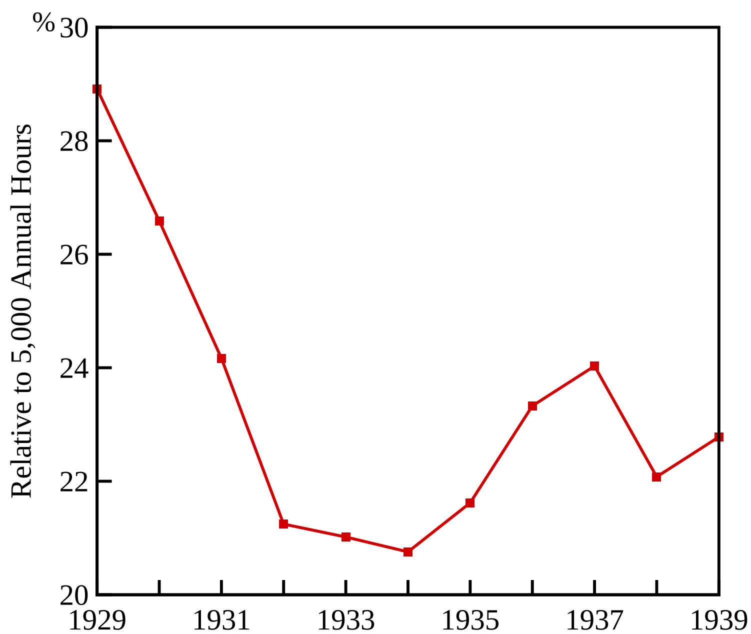

Figure 8. Per Capita Hours

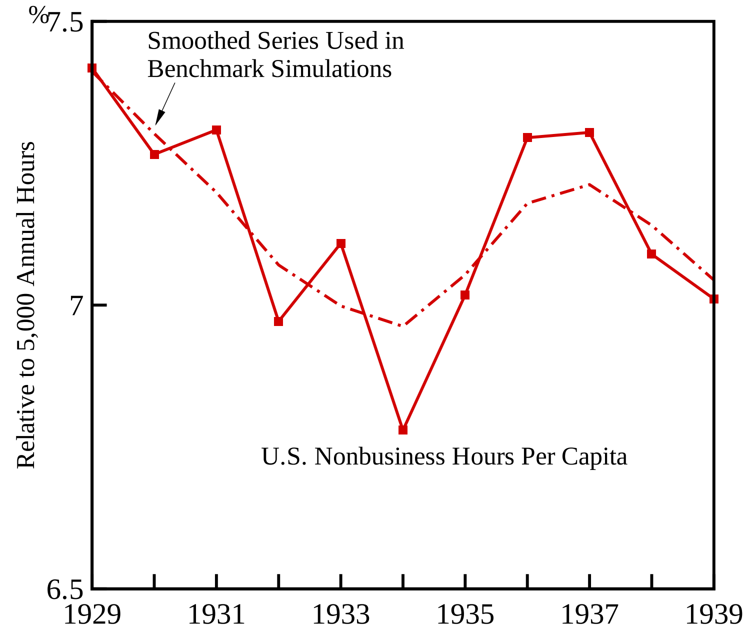

Figure 9. Per Capita Nonbusiness Hours

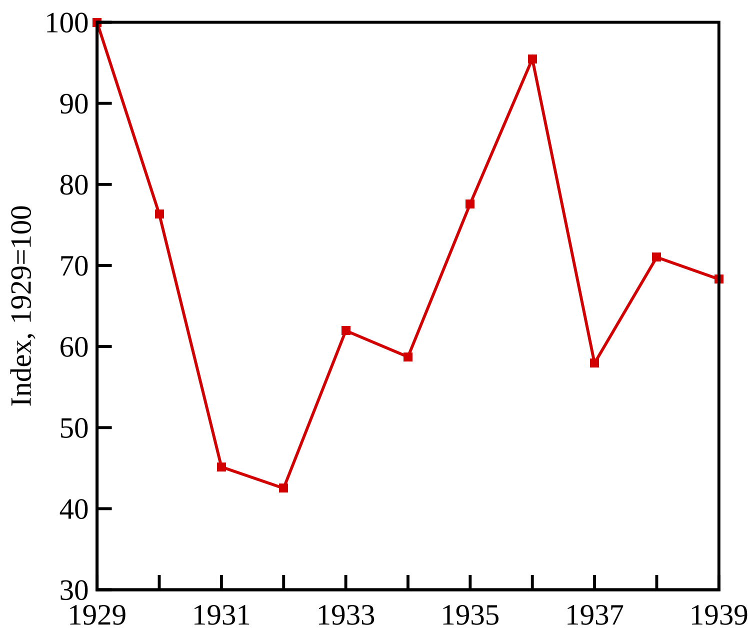

Figure 10. NYSE Market Value

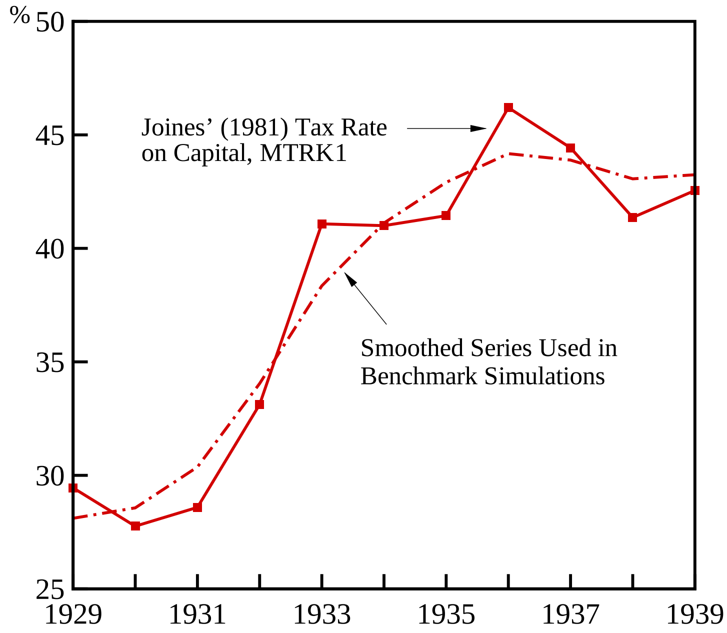

Figure 11. Joines' (1981) Tax Rate on Capital

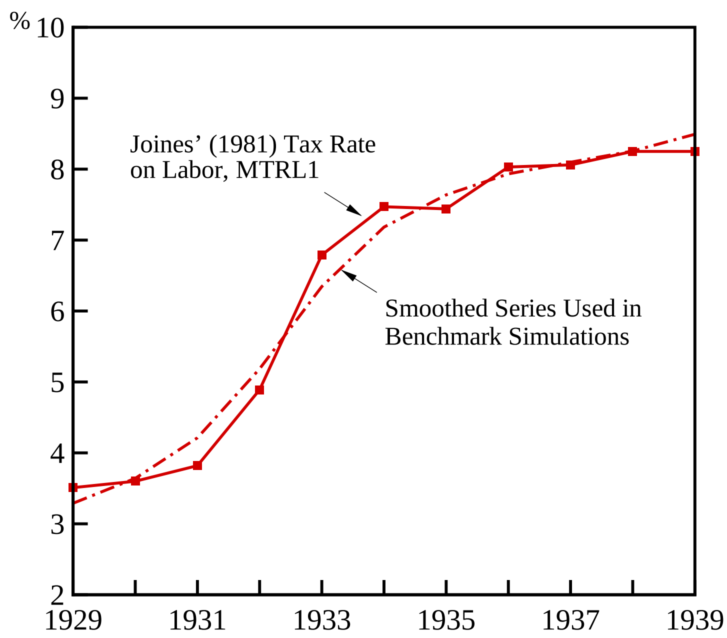

Figure 12. Joines' (1981) Tax Rate on Labor

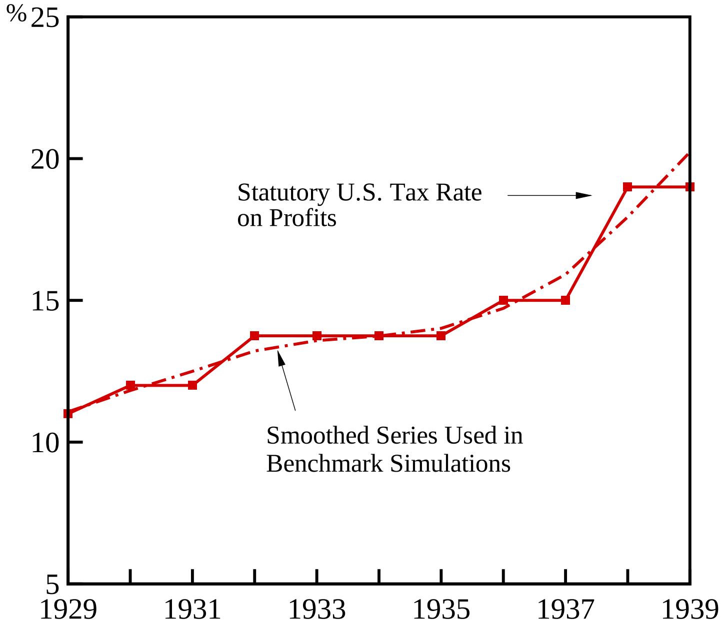

Figure 13. Statutory Tax Rate on Profits

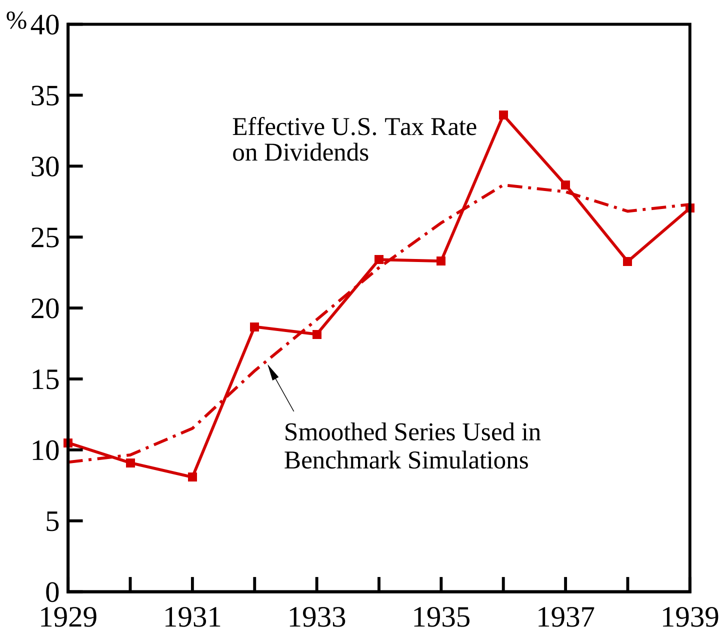

Figure 14. Effective Tax Rate on Dividends

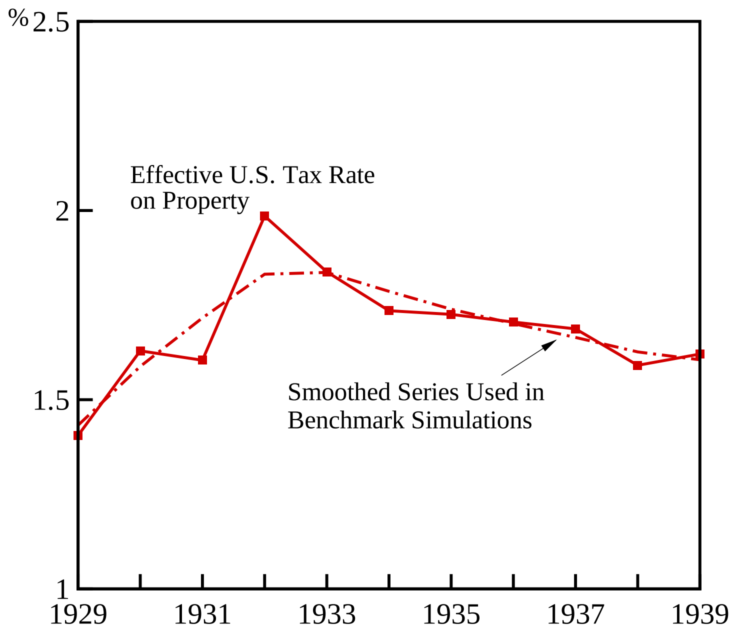

Figure 15. Effective Tax Rate on Property

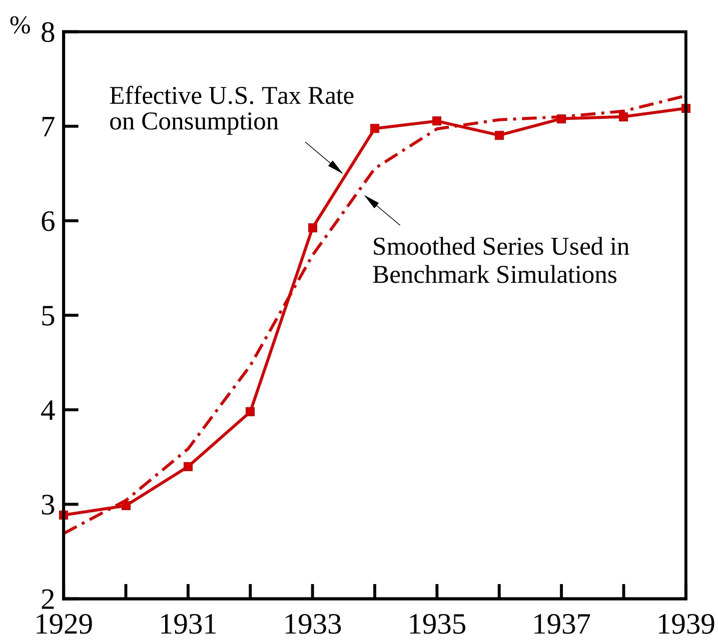

Figure 16. Effective Tax Rate on Consumption

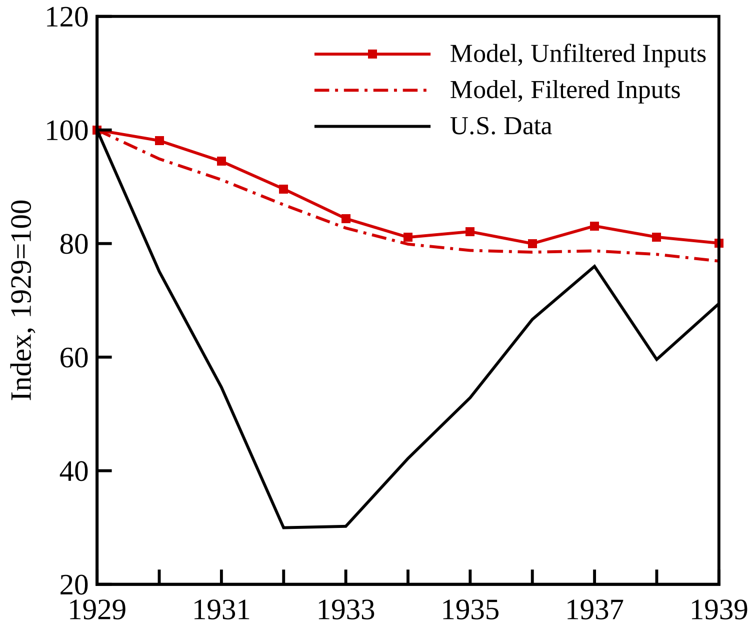

Figure 17. Detrended Real Investment in the United States and Predictions of the Basic Growth Model, 1929–1939

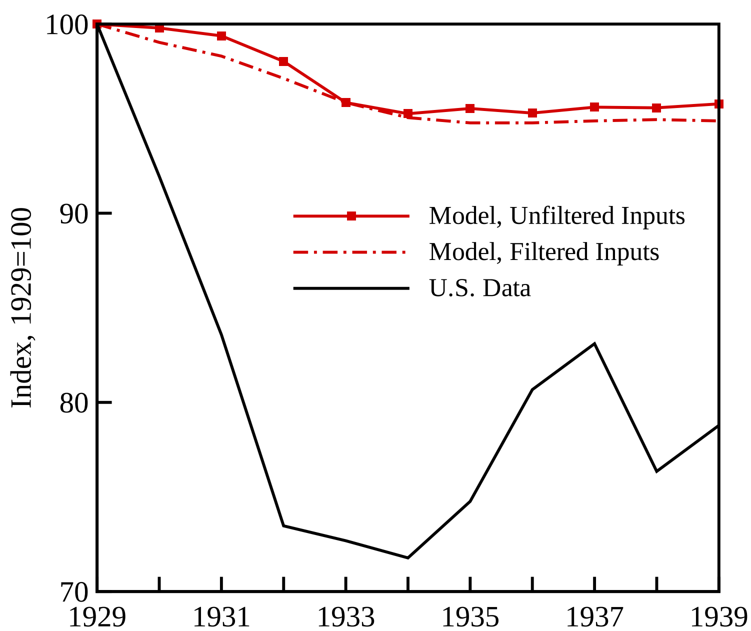

Figure 18. Hours Per Capita in the United States and Predictions of the Basic Growth Model, 1929–1939

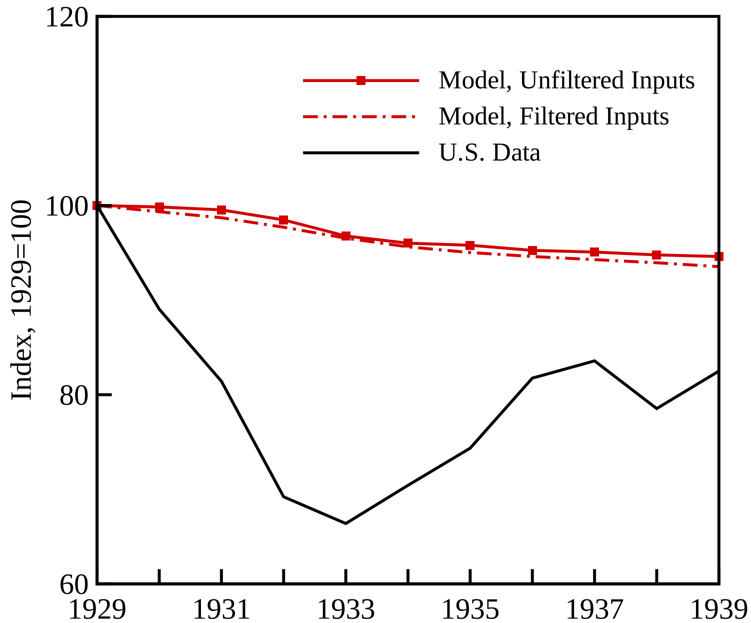

Figure 19. Detrended Real GDP in the United States
and Predictions of the Basic Growth Model, 1929–1939

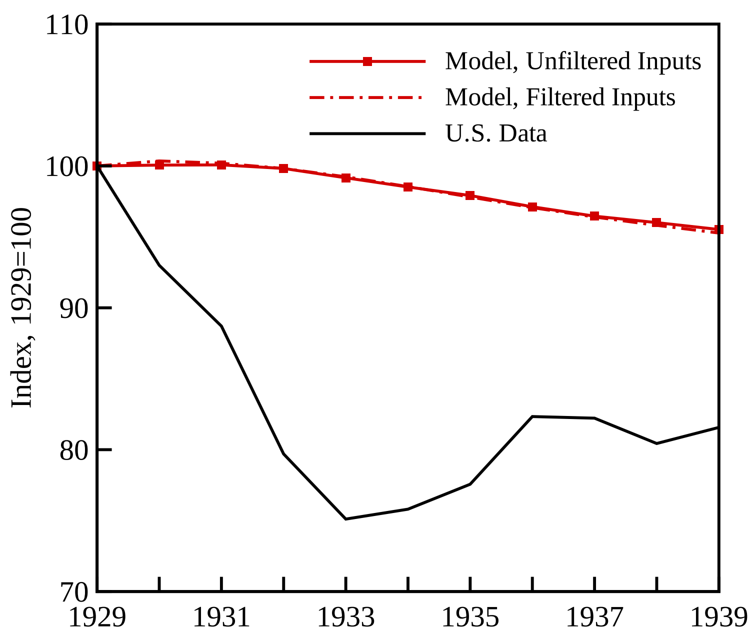

Figure 20. Detrended Real Consumption in the United States
and Predictions of the Basic Growth Model, 1929–1939

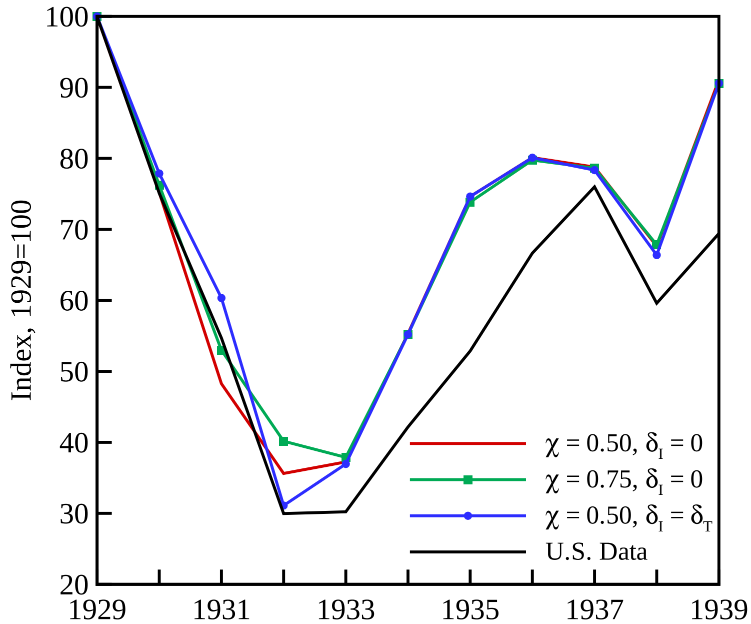

Figure 21. Detrended Real Investment in the United States
and Predictions of the Extended Model, 1929–1939

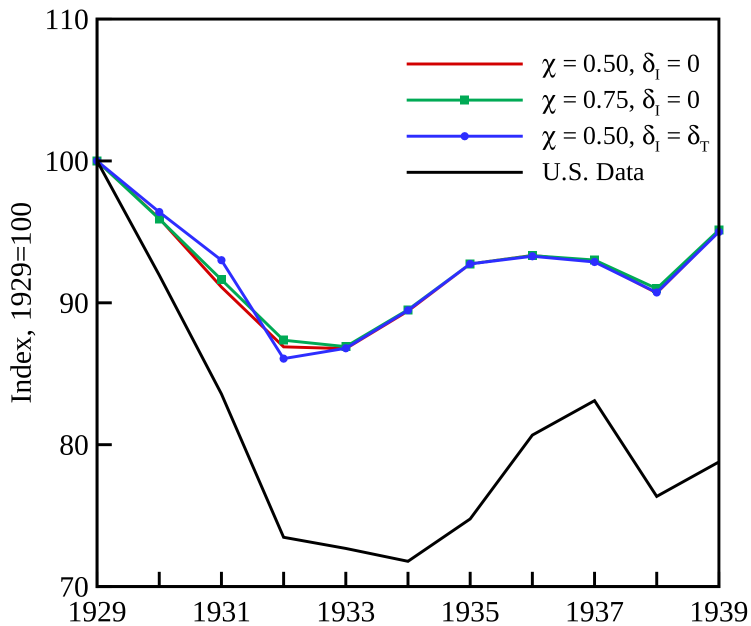

Figure 22. Hours Per Capita in the United States
and Predictions of the Extended Model, 1929–1939

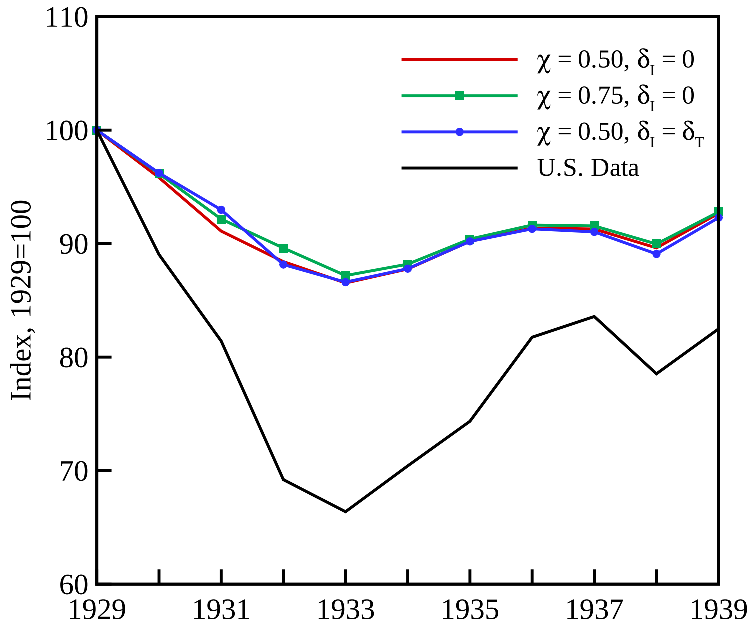

Figure 23. Detrended Real GDP in the United States and Predictions of the Extended Model, 1929–1939

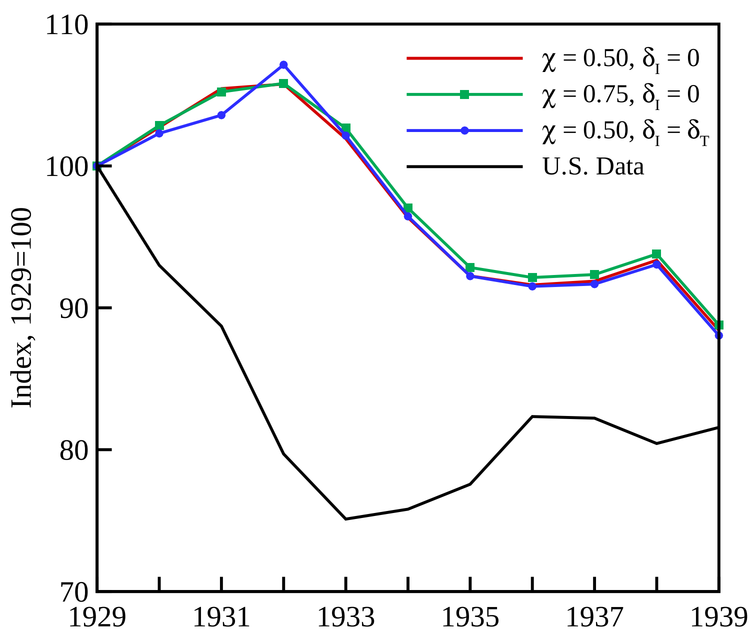

Figure 24. Detrended Real Consumption in the United States and Predictions of the Extended Model, 1929–1939

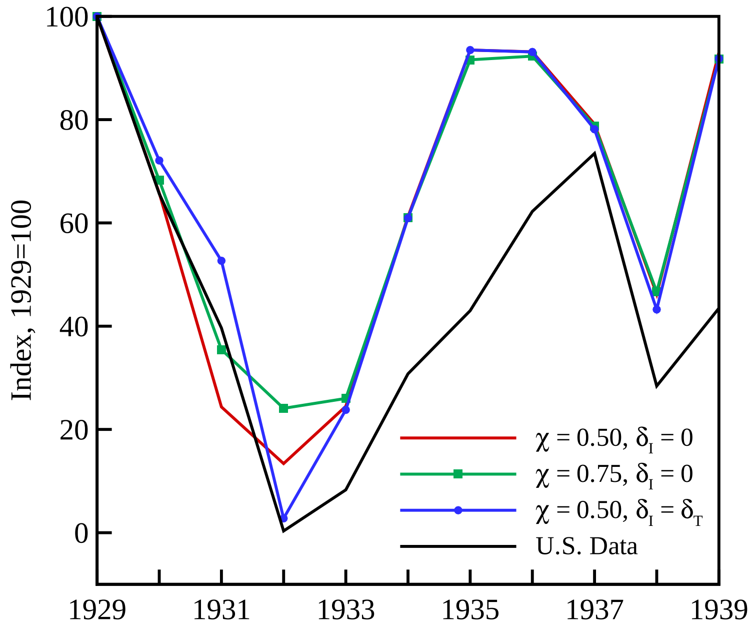

Figure 25. Detrended Tangible Investment in the United States and Predictions of the Extended Model, 1929–1939

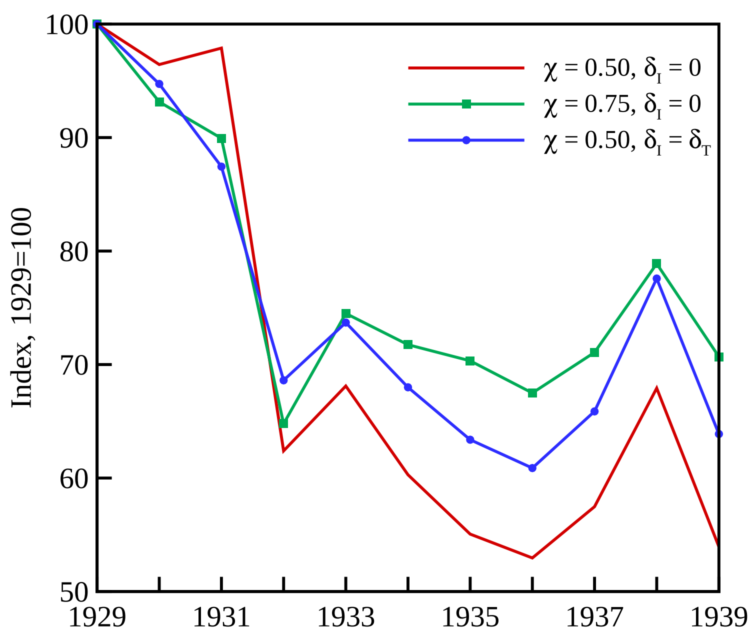

Figure 26. Detrended Intangible Investment, Predictions of the Extended Model, 1929–1939

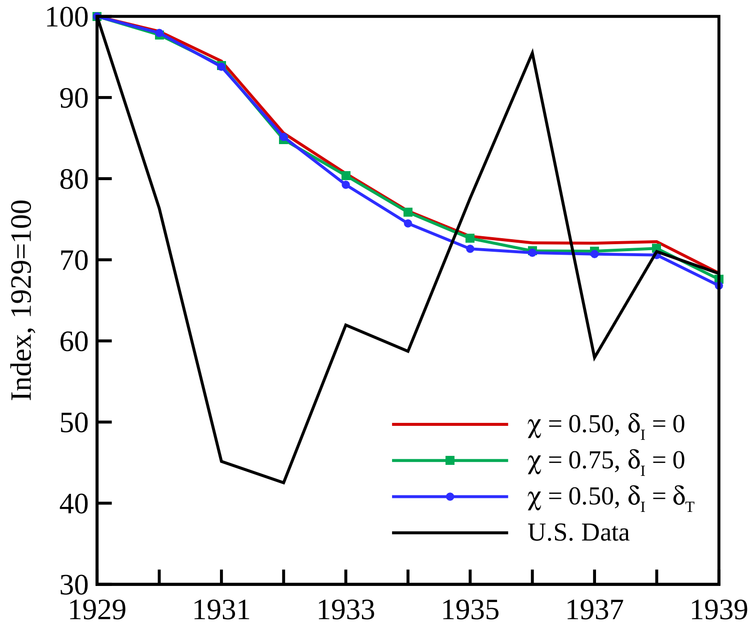

Figure 27. Detrended Real Market Value in the United States and Predictions of the Extended Model, 1929–1939