Supplementary Appendix for "Candidates, Character, and Corruption"

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This Supplementary Appendix studies how term limits and incumbency affect the character of self-selected candidates.

1 A Two-Period Model

Assuming character is at least partially revealed during a governor's first term, reelection opportunities can promote better governance through two channels. The first is mechanical: the electorate gains opportunities to reelect desirable incumbents. The second operates through self-selection effects: the benefits of running for office in the first place rise for high-quality candidates (for whom the odds of re-election are high) relative to low-quality candidates (for whom the odds are low). Both effects argue for longer term limits. This section explores these effects and also identifies why, perhaps surprisingly, longer term limits can also produce detrimental selection effects. Throughout this Supplementary Appendix, to avoid uninteresting cases, we assume that all equilibria of the baseline model with vanishing running costs involve multiple candidates with a candidate pool consisting of both Sell-Outs and Scoundrels.[1]

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[1] Formally, this requires $Y(1,1) \geq \max\left\{\frac{u^G(1,0|\sigma,s)}{2}, u^G(0,0 \mid \sigma, s)\right\}$ and $Y(0,0|\sigma) < y^*(\sigma) < Y(1,0|\sigma)$.

1.1 Self-Selection Benefits of Reelection Opportunities

The impact of incumbency on candidate selection is most easily illustrated through a simple "reduced form" extension of our basic model to two periods; subsequently we will discuss how to enrich it. Assume for simplicity that a governor can run for reelection at no personal cost (an advantage reflecting the benefits of incumbency), and that a governor of quality y is re-elected with an exogenous probability $\Pi(y)$ that is non-decreasing in y, so that higher quality incumbents are re-elected (weakly) more frequently. Further assume that the net gains from holding office for a second term are $\lambda > 1$ times those from holding office for a first term. Thus, when the probability of winning conditional on running is ρ and the alternative quality of governance is y', a candidate with characteristics (a, h) will be willing to run if and only if

$$\rho(1 + \Pi(Y(a, h|\sigma))\lambda) \left[u^G (a, h \mid \sigma, s) - (1 + a)y'\right] \geq k.$$

The following result shows that if Sell-Outs are re-elected with strictly higher probability than near-Scoundrels, then for small running costs, the expected quality of governance in the first period of the two-period model is strictly higher than $y^*(\sigma)$, the expected quality of governance in the original model.

Theorem 7. *Suppose $\Pi (Y(1, 0 \mid \sigma)) > \Pi (y)$ for all y within some neighborhood of $Y(0, 0 \mid \sigma)$. Then for some $\varepsilon > 0$, there exists $k' > 0$ such that when $k < k'$, any multiple-candidate equilibrium of the extended model, $(\mathcal{N}, \mu)$, has $y^{\mathcal{N}}(\mu, \sigma) \geq y^*(\sigma) + \varepsilon$.*

(All proofs are collected in Section 3.[2])

It follows that the ability to re-elect better governors has a beneficial selection effect on the candidate pool in non-incumbent elections, in addition to any direct benefit of re-electing good governors. The logic of this result is straightforward. With $\lambda = 0$ (in effect, the one-period model), the set of insiders with the greatest incentives to run consists of Sell-Outs alone when the average quality of governance, call it y, is less than $y^*(\sigma)$, and both Sell-Outs and Scoundrels when $y = y^*(\sigma)$. Thus, with strictly positive λ and $y \leq y^*(\sigma)$, Sell-Outs have strictly greater incentives to run than any lower quality candidate. Consequently, $y \leq y^*(\sigma)$ rules out the possibility that, with vanishingly small entry costs, any candidate of quality $y^*(\sigma)$ or lower would run. It follows that $y \leq y^*(\sigma)$ is not sustainable in equilibrium.

[2]Note that the numbering of theorems continues from the main paper.

So far we have imposed transparent but exogenous assumptions concerning re-election bids. That is both a virtue and a limitation. It is not hard, however, to see that similar results hold when the second-period election is modeled explicitly. Assume for simplicity that a governor's character is necessary revealed while in office. Because the second period of the two-period model closely resembles the single-period model, the most natural continuation equilibrium has the property that the average quality of challengers (if any run) is $y^*(\sigma)$; the incumbent runs for re-election if and only if his quality is at least $y^*(\sigma)$, and he wins when he runs.[3] Thus, $\Pi(\cdot)$ endogenously satisfies the assumption in Theorem 7. Though the benefits from holding office for two terms is no longer a fixed multiple of the benefits from holding office for a single term,[4] the main insight developed in the context of our simple reduced-form model — that re-election opportunities improve expected candidate quality in the first-period non-incumbent election — carries over, for essentially the same reasons. In some cases (e.g., when citizens heavily discount future payoffs), the first-period candidate pool still consists of only Sell-Outs and Scoundrels, but a higher fraction are Sell-Outs than in the one-period model. The fact that Sell-Outs seek and win re-election (whereas Scoundrels do not) bears out the adage that voters prefer a known crook to an unknown crook.

A two-period model is somewhat artificial because a non-incumbent candidate in the second period has no opportunity to seek re-election. This can be remedied by considering an infinite-horizon model but maintaining a two-term limit; Section 2 shows that similar equilibria also exist in such a model, but other types of equilibria also emerge, some with even higher governance quality (even restricting attention to Markovian strategies).

Before turning to an infinite-horizon model, however, we first address the question of whether re-election possibilities always improve the expected quality of (first-period) governance within the context of the two-period model.

[3]To describe an equilibrium, one must specify voters' beliefs about the average quality of non-incumbent candidates for out-of-equilibrium realizations (i.e., ones in which the number of candidates falls outside the support of the equilibrium distribution). Unless one introduces belief restrictions, the set of equilibria is large, and many equilibria have implausible properties. We opted for the simple reduced-form model presented in the text to avoid a lengthy treatment of these technical and ultimately unenlightening complications.

[4]This is because, in equilibrium, the expected quality of the non-incumbent candidate pools in the first and second periods will differ.

1.2 Self-Selection Costs of Reelection Opportunities

The possibility of re-election can also have pernicious selection effects if lower-quality candidates benefit more from re-election than higher-quality candidates. Such effects can emerge if, as many have suggested, more senior politicians are able to extract greater pork and/or rents from holding office, e.g. by cultivating relationships with large contributors or obtaining appointments to powerful committees. To capture that possibility, we adopt the same simplifying framework (with exogenous re-election probabilities) and make the same assumptions as in Theorem 7, with the following exception: the fraction of lobbying surplus extracted by an incumbent governor, α_2, exceeds α, the fraction extracted by a first-term governor. With this modification, we obtain:

Theorem 8. *Assume $\alpha_2 > \alpha$. There exist $\varepsilon, \eta > 0$ and $\hat{k} > 0$ such that if $\Pi(Y(0,0 \mid \sigma)) + \varepsilon > \Pi(Y(1,1 \mid \sigma)) > 0$ and $k < \hat{k}$, any multiple-candidate equilibrium of the extended model, $(\mathcal{N}, \mu)$, has $y^{\mathcal{N}}(\mu, \sigma) \leq y^*(\sigma) - \eta$.*

Thus, if incumbency confers additional bargaining power with special interests, then unless the electorate can differentiate sufficiently well between governors of good and bad character, the possibility of re-election causes adverse self-selection in non-incumbent elections. Intuitively, an increase in the governor's ability to extract rents from the lobby group resembles a decrease in anti-corruption policy: while it generally increases the benefits to holding office (fixing the quality of opponents), the effect on entry incentives is greatest for Scoundrels because they accept special interest transfers more often than all other types. Taking the boundary case where $\Pi(\cdot)$ is constant and $\alpha_2 = \alpha$, we know that the set of insiders with the greatest incentives to run consists of Scoundrels alone when the average quality of governance, call it y, is greater than $y^*(\sigma)$, and both Scoundrels and Sell-Outs when $y = y^*(\sigma)$. Thus, with $\alpha_2 > \alpha$ and $y \geq y^*(\sigma)$, Scoundrels have strictly greater incentives to run than any candidate of higher quality. Consequently, $y \geq y^*(\sigma)$ rules out the possibility that, with vanishingly small entry costs, any candidate of quality $y^*(\sigma)$ or higher would run. It follows that $y \geq y^*(\sigma)$ is not sustainable in equilibrium.

2 An Infinitely-Repeated Game

Now, we study an infinitely repeated version of the incumbency model, in which insiders are infinite-lived and discount the future at the rate $\delta \in (0, 1)$.[5] We assume infinite repetitions for two reasons. First, the impact of a future opportunity to seek re-election on a prospective candidate's decision to run for office in the first place depends on the expected quality of future opponents, which should in turn depend on their opportunities to run as incumbents. Assuming a fixed horizon amounts to imposing a single-term limit as of some point in the future. Second, the infinite horizon case turns out to be simpler in some important ways,[6] and yields some sharper results.

We will assume that governors can hold office for a maximum of two terms, postponing a discussion of longer terms until later. We will also assume that, when an incumbent is eligible to run, she must announce her candidacy before other insiders make their decisions. This sequencing of entry choices is not material but simplifies some of the analysis. Finally, as mentioned earlier, we assume that the electorate knows the character of an incumbent governor because it is unavoidably revealed in office. Though somewhat stark, the latter assumption captures the essence of the phenomenon we seek to study.

Our analysis will allow for the possibility that a governor's compensation differs across her first and second terms. Back-loading compensation into the second term can reinforce the incentives for advantageous selection that arise with opportunities for re-election. Henceforth, we will use s_t to denote the governor's compensation in her t-th term of office.

Artificial results can arise in such settings through two separate channels: history dependence and out-of-equilibrium beliefs. With respect to history dependence, the infinite horizon creates scope for sustaining a wide variety of outcomes. We therefore focus on equilibria that employ Markovian strategies and beliefs. That is, we allow the entry choice of an incumbent to depend only on her own characteristics, and the entry choice of a non-incumbent insider to depend only on the presence and characteristics of an incumbent candidate. For voters, we allow beliefs about the composition of the non-incumbent candidate pool to depend only on the presence and characteristics of an incumbent candidate, and on the number of non-incumbent candidates. We also insist that the Markovian strategies con-

[5]Because we focus below on Markov-Perfect Equilibria, our analysis would be essentially unchanged if we assumed that each insider was finite-lived. The assumption of infinite-lived agents is for convenience only.

[6]For example, the existence of pure-strategy equilibria is problematic with a finite horizon, but not with an infinite horizon. An infinite horizon also allows us to exploit the stationary structure of the problem.

stitute equilibria in every proper subgame, and refer to such equilibrium as *Markov-perfect*.[7]

The freedom to specify out-of-equilibrium beliefs also create scope for perverse equilibria. For example, one can construct an equilibrium wherein an incumbent Scoundrel wins while an incumbent of higher quality loses, both with probability one.[8] To avoid such possibilities, we will impose a simple and plausible belief restriction that we now describe.

In a Markov-perfect equilibrium, both choices and voters' beliefs about the composition of the candidate pool can depend on state variables. What follows should be interpreted as being conditional upon a particular state, but for simplicity we suppress that conditionality in the notation. Given the incumbent's quality and entry decision as well as continuation strategies, there is some set of non-incumbent candidates, $\mathcal{N}$, who enter with positive probability, of whom members of some set $\mathcal{N}_1$ enter with probability one. Thus, the realized number of non-incumbent candidates can be any integer between $|\mathcal{N}_1|$ and $|\mathcal{N}|$ (which may be infinite); realizations smaller than $|\mathcal{N}_1|$ and larger than $|\mathcal{N}|$ lie off the equilibrium path. For all $i \in \mathcal{N}_1$, let ρ_W^i be the probability of winning that renders i indifferent between entering and not entering, given the continuation equilibria that would follow each choice. Also, for all types $(a, h) \in [0, 1]^2$, let $\rho_U(a, h)$ be the probability of winning that would render one additional insider of type (a, h) indifferent between entering and not entering, once again given the continuation equilibria.

Belief Restriction: For any realized number of candidates $N < |\mathcal{N}_1|$, voters assume that the $|\mathcal{N}_1| - N$ members of $\mathcal{N}_1$ with the largest values of ρ_W^i did not enter. For any realized number of candidates $N > |\mathcal{N}|$, voters assume that all members of $\mathcal{N}$ entered along with $N - |\mathcal{N}|$ additional insiders whose characteristics minimize $\rho_U(a, h)$ on $[0, 1]^2$.

The above restriction attributes zero-probability events to deviations by the smallest possible number of insiders, and to the particular insiders with the greatest incentives (or smallest disincentives) to make those deviations. Thus, it is in the spirit of other common belief restrictions (e.g. Cho and Kreps, 1987; Bagwell and Ramey, 1991). We do not mean to suggest that all equilibria that violate this restriction are *necessarily* problematic. Rather, by imposing this restriction, we are insuring against results that are driven by contrived or otherwise unrealistic out-of-equilibrium beliefs.

[7] Our usage of this term is slightly non-standard because Markov-perfection is typically used with reference to games of complete information. Here, voters have incomplete information about candidates' characteristics.

[8] An incumbent Scoundrel can win with probability one if voters assume that all who enter against him are also Scoundrels.

Our main result for this infinitely-repeated game is:

Theorem 9. *For any $k > 0$, provided s_2 is sufficiently large, for every $y' \in [Y(1,0 \mid \sigma), Y(1,1 \mid \sigma)]$, there exists a pure-strategy Markov-perfect equilibrium satisfying the Belief Restriction for which the quality of governance is y' in all periods.*

The theorem establishes several important principles. Allowing governors to run for a second term can attract high-quality candidates and eliminate the variability of governance. If second-term compensation is sufficiently high, there are equilibria in which only citizens of the highest character run for office. However, even incumbency with highly back-loaded compensation does not *guarantee* the highest levels of quality. There are also equilibria in which governors of much lower quality are reelected (though average quality is still higher than with one-term limits).[9]

The equilibria that give rise to the outcomes described in Theorem 9 share a simple structure: all candidates are (and are believed to be) of quality y', and incumbent candidates win if and only if their quality is no lower than y'. Intuitively, supposing candidates win re-election if and only if they are of quality y' or greater (with $y' \geq Y(1,0 \mid \sigma)$), then the insiders with the greatest incentive to run for office in the first place have quality y' (candidates of lower quality benefit from only a single term, while candidates of higher quality forego a portion of the benefits potentially received from special interests).[10] Thus, any re-election quality threshold y' between $Y(1,0 \mid \sigma)$ and $Y(1,1 \mid \sigma)$ bootstraps itself into an equilibrium. Notice also that backloading pay to a greater degree does not discriminate in favor of those with higher quality among the set of insiders whose quality exceeds y'; hence, it cannot improve selection, and only serves to reinforce the disincentives to enter among insiders whose quality falls below any given reelection threshold, y'.

Theorem 9 pertains to a two-term limit. With a one-term limit, a Markov-perfect equilibrium simply repeats the equilibria of the one-period model. Thus, for small k, switching from a two-term limit to a one-term limit reduces the quality of governance. In contrast, a two-term limit does not inherently reduce the quality of governance relative to limits of three

[9] Furthermore, there are likely other equilibria satisfying the Belief Restriction that do not belong to the class we examine in proving the Theorem; we have not ruled out the possibility that some involve even lower quality or substantial variability in the quality of governance. However, Theorem 7 implies that any equilibrium yielding governance quality at or below the level achieved in the one-period model would have to be rather strange, e.g., involving probabilities of reelection that are decreasing in quality.

[10] For $y < Y(1,0 \mid \sigma)$, Sell-Outs may have the greatest incentive to run for office, so the argument breaks down.

or more terms. In this model, extending the limit beyond two terms simply increases the reward for those of sufficient quality to win re-election. The same end can be accomplished with a two-term limit through higher second-term compensation.[11]

3 Proofs

Proof of Theorem 7. Define

$$\Delta^{\Pi}(a,h,y) := \left[1 + \lambda \Pi\left(Y\left(a,h \mid \sigma\right)\right)\right] \Delta(a,h,y).$$

We know from the arguments given in the paper (see Lemma 3 therein) that $\Delta(1,0,y) \geq \Delta(a,h,y)$ for all $(a,h) \neq (1,0)$ and $y \leq y^*(\sigma)$, with strict equality except for $(a,h,y) = (0,0,y^*(\sigma))$. As long as $\Delta(1,0,y) > 0$, given our assumption on Π, we have $\Delta^{\Pi}(1,0,y) > \Delta^{\Pi}(a,h,y)$ for all $(a,h) \neq (1,0)$ with $Y(a,h \mid \sigma) \leq Y(1,0 \mid \sigma)$ and $y \leq y^*(\sigma)$. By continuity of Δ^{Π} in its third argument, for any $\eta_1 > 0$ and some small $\eta_2 > 0$, the same statement holds for $Y(a,h \mid \sigma) \leq Y(1,0 \mid \sigma) - \eta_1$ and $y \leq \overline{y}(\sigma) + \eta_2$.

Now assume the theorem is false. Then it must be possible to select some sequence of entry costs $k_m \to 0$ for which there is a corresponding sequence of multi-candidate equilibria, $(\mathcal{N}_m, \mu_m)$ such that $\lim_{m\to\infty} y^{\mathcal{N}_m}(\mathcal{N}_m, \mu_m) \leq y^*(\sigma)$. By the argument in the preceding paragraph, for sufficiently large m, Sell-Outs would have strictly greater incentives to enter than any other type (a,h) with $Y(a,h \mid \sigma) \leq Y(1,0 \mid \sigma) - \eta_1$. Through an argument paralleling the one given in the proof of Theorem 5 from the paper, one can then show that, in the limit, the quality of the worst candidate must converge to a limit no less than $Y(1,0 \mid \sigma)$. But that implication contradicts the assumption that average quality converges to a limit no greater than $y^*(\sigma)$. □

Proof of Theorem 8. For this proof we will augment the arguments of Δ to including α, writing $\Delta(a,h,y,\alpha)$. It is easily verified that Δ is weakly decreasing in α, and strictly so for any a,h such that $\overline{v} - v^*(a,h,\sigma) > 0$, which is the case for any a and $h = 0$.

Fix $\alpha_2 > \alpha$. Define

$$\Delta^{\Pi,\alpha_2}(a,h,y) := \Delta(a,h,y,\alpha) + \lambda\Pi(Y(a,h \mid \sigma))\Delta(a,h,y,\alpha_2).$$

[11] Plainly, if there is a binding upper bound on the level of compensation that can be given in any one period, longer term limits may be useful.

Define C to be the set of character types of quality strictly less than $(1/2)(Y(0,0 \mid \sigma)+y^*(\sigma))$. From Lemma 3 in the paper we know that $\Delta(a,h,y,\alpha) - \Delta(0,0,y,\alpha) \leq 0$ for all $y \geq y^*(\sigma)$ and $(a,h) \neq (0,0)$, with strict inequality when $y > y^*(\sigma)$ or $(a,h) \neq (1,0)$. Thus, if $\Pi(Y(1,1)) = \Pi(Y(0,0|\sigma))$, then for all $y \geq y^*(\sigma)$,

$$\sup_{(a,h)\notin C} \left(\Delta^{\Pi,\alpha_2}(a,h,y) - \Delta^{\Pi,\alpha_2}(0,0,y)\right) < 0. \tag{1}$$

By the continuity of Δ, there exist $\varepsilon, \eta > 0$ with $y^*(\sigma) - \eta > (1/2)(Y(0,0 \mid \sigma) + y^*(\sigma))$ such that (1) holds for all $y \geq y^*(\sigma) - \eta$ provided $\Pi(Y(1,1|\sigma)) < \Pi(Y(0,0|\sigma)) + \varepsilon$.

We claim that the theorem holds for the ε and η defined in the previous paragraph. Assume not. Then there is some non-decreasing $\Pi(\cdot)$ satisfying $\Pi(Y(1,1|\sigma)) < \Pi(Y(0,0|\sigma))+\varepsilon$ such that it is possible to select a sequence of entry costs $k_m \to 0$ for which there is a corresponding sequence of multi-candidate equilibria, $(\mathcal{N}_m, \mu_m)$, such that $y^{\mathcal{N}_m}(\mathcal{N}_m, \mu_m) > y^*(\sigma) - \eta$. From the preceding paragraph, we know that for all m, Scoundrels have a strictly greater incentive to enter than any type with quality exceeding $(1/2)(Y(0,0 \mid \sigma) + y^*(\sigma))$. Through an argument paralleling the one given in the proof of Theorem 5 in the paper, one can then show that, in the limit as $m \to 0$, the quality of the best candidate cannot exceed $(1/2)(Y(0,0 \mid \sigma) + y^*(\sigma)) < y^*(\sigma) - \eta$. But that contradicts the assumption that $y_{\mathcal{N}_m}(\mathcal{N}_m, \mu_m) > y^*(\sigma) - \eta$ for all m. □

Proof of Theorem 9. The proof proceeds via a few steps.

Step 1: We begin by constructing a class of equilibria; subsequently we verify that these equilibria have the desired properties.

Select some $y' \in [Y(1,0 \mid \sigma), Y(1,1 \mid \sigma)]$. Construct insiders' strategies as follows:

(s-i) If there is an incumbent, she runs for re-election if and only if $y^I \geq y'$.

(s-ii) If there is an incumbent with $y^I > y'$ who runs, no non-incumbent candidates enter.

(s-iii) If no incumbent runs (possibly because there is no incumbent), or if there is an incumbent with $y^I < y'$ who runs, $N' \geq 2$ non-incumbent candidates enter, each with characteristics (a', h') and quality y' (where we define N', a', and h' below).

Construct voters beliefs about non-incumbent candidates as follows:

(b-i) If there is an incumbent with $y^I \geq y'$ who runs opposed, voters believe that all non-incumbent candidates have characteristics $\left(\widehat{a}(y^I), \widehat{h}(y^I)\right)$ and quality y' (where we define the functions $\left(\widehat{a}, \widehat{h}\right)$ below).

(b-ii) If there is no incumbent, an incumbent who does not run, or an incumbent with $y^I < y'$ who does run, then voters believe that any and all non-incumbent candidates have characteristics (a', h') and quality y'.

Observe that the beliefs are consistent with the strategies on the equilibrium path. If $y^I \geq y'$ then the incumbent runs (by (s-i)) and no non-incumbents enter (by (s-ii)); hence beliefs about non-incumbent candidates are not relevant. If $y^I < y'$ then the incumbent does not run (by (s-i)) and N' non-incumbents enter (by (s-iii)), each with characteristics (a', h') and quality y', which is consistent with beliefs (according to (b-ii)).

Next observe that with these strategies and beliefs, elections play out as follows:

(e-i) If $y^I \geq y'$ and the incumbent runs for re-relection, whether opposed or unopposed, she wins. (When opposed, voters believe that the non-incumbent candidates are of quality y' by (b-i), so the incumbent prevails.)

(e-ii) If $y^I < y'$ and the incumbent runs for re-election, she is opposed (by s-iii) and loses (because voters believe that the non-incumbent candidates are of quality y' by (b-ii)). Notice that she loses even if the number of opposing candidates is not N'.

(e-iii) If the incumbent does not run for re-election, a non-incumbent candidate with characteristics (a', h') and quality y' wins (by (s-iii)).

Now we define N', (a', h'), and $\left(\widehat{a}, \widehat{h}\right)$. Consider the locus C^* defined by the condition $(a, h) \in C^*$ if and only if $Y(a, h \mid \sigma) = y'$. Also define

$$\Delta^I(a, h, y) := u^G\left(a, h \mid \sigma, s_1\right) + \delta u^G\left(a, h \mid \sigma, s_2\right) - a\left(y + \delta y'\right).$$

Let

$$\left(\widehat{a}(y), \widehat{h}(y)\right) := \arg \max_{(a,h)\in C^*} \Delta^I(a, h, y),$$

taking an arbitrary selection if there are multiple maximizers. Let

$$(a', h') := \left(\widehat{a}(y'), \widehat{h}(y')\right) \text{ and } N' := \left\lfloor \frac{\Delta^I(a', h', y')}{k}, \right\rfloor$$

(where $\lfloor x \rfloor$ denotes the "floor" function, which returns the largest integer less than or equal to x). Notice that for s_2 sufficiently large, $N' \geq 2$ as required.

Step 2: Next we verify that no insider has an incentive to deviate from the strategies identified in Step 1. There are a number of cases to consider.

(i) An incumbent with characteristics (a^I, h^I) and quality $y^I \geq y'$. If she does not run, the quality of governance is y' in the current period (by (e-iii)) and all subsequent periods (by (e-i) through (e-iii)). If she runs, she wins re-election with probability one (by (e-i)), and the quality of governance is y' in all subsequent periods. Hence, she is willing to run if and only if $u^G\left(a^I, h^I \mid \sigma, s_2\right) - a^I y' \geq 0$, which plainly holds for s_2 sufficiently large.

(ii) An incumbent with characteristics (a^I, h^I) and quality $y^I < y'$. If she runs, she looses; whether she runs or not, the quality of governance is y' in all periods (by (e-i) through (e-iii)). Therefore, with $k > 0$, she prefers not to run.

(iii) A non-incumbent non-candidate when there is an incumbent with $y^I \geq y'$, who runs. If the non-incumbent non-insider runs, she loses; whether she runs or not, the quality of governance is y' in all periods. Therefore, with $k > 0$ she prefers not to run.

(iv) A non-incumbent non-candidate when there is no incumbent, there is an incumbent who does not run, or there is an incumbent with $y^I < y'$ who runs. First consider a non-incumbent non-candidate with characteristics (a', h') and quality $y' \geq y'$. If such a candidate runs and wins, she will also win re-election, so her payoff is

$$u^G\left(a', h' \mid \sigma, s_1\right) + \delta u^G\left(a', h' \mid \sigma, s_2\right) + (1 + a')\frac{\delta^2}{1-\delta}y'.$$

If she does not run, her payoff is $(1 + a')\frac{1}{1-\delta}y'$. With $N' + 1$ candidates, she therefore prefers not to run when

$$\frac{1}{N'+1}\Delta^I(a', h', y') \leq k. \tag{2}$$

As we have previously noted, u^G (and hence Δ^I) is strictly decreasing in h. Thus, within the set of candidate characteristics $\{(a', h') \mid Y(a', h' \mid \sigma) \geq y'\}$ (for which C^* is the lower boundary), the left-hand side of (2) is maximized for $(a', h') = (a', h')$ (which by construction maximizes that expression on C^*). It follows that (2) is satisfied within $\{(a', h') \mid Y(a', h' \mid \sigma) \geq y'\}$ provided $\frac{1}{N'+1}\Delta^I(a', h', y') \leq k$. But by construction, $N' + 1 > \frac{\Delta^I(a',h',y')}{k}$, so the preceding inequality holds.

Next consider a non-incumbent non-candidate with characteristics (a', h') and quality $y' < y'$. If such a candidate runs and wins, she will not win re-election, so her payoff is $u^G\left(a', h' \mid \sigma, s_1\right) + (1 + a')\frac{\delta}{1-\delta}y'$. If she does not run, her payoff is $(1 + a')\frac{1}{1-\delta}y'$. With $N' + 1$

candidates, she therefore prefers not to run when

$$\frac{1}{N'+1}\left[u^G\left(a',h'\mid\sigma,s_1\right)-(1+a')y'\right]\leq k. \tag{3}$$

But with s_2 sufficiently large, $\Delta^I(a',h',y') > u^G\left(a',h'\mid\sigma,s_1\right)-(1+a')y'$. Thus, (3) follows from $\frac{1}{N'+1}\Delta^I(a',h',y')\leq k$.

(v) A non-incumbent candidate when there is no incumbent, there is an incumbent who does not run, or there is an incumbent with $y^I < y'$ who runs. By (s-iii), all such candidates have characteristics (a',h') and are of quality y'. If one such candidate runs and wins, she will also win re-election, so her payoff is

$$u^G\left(a',h'\mid\sigma,s_1\right)+\delta u^G\left(a',h'\mid\sigma,s_2\right)+(1+a')\frac{\delta^2}{1-\delta}y'.$$

If she does not run, her payoff is $(1+a')\frac{1}{1-\delta}y'$. With N' candidates, she therefore prefers to run when $\frac{1}{N'}\Delta^I(a',h',y')\geq k$. But by construction, $N'\leq\frac{\Delta^I(a',h',y')}{k}$, so the preceding inequality holds.

Step 3: Finally we verify that beliefs satisfy the Belief Restriction.

We begin with (b-i). Suppose there is an incumbent with $y^I\geq y'$ who runs. Here, $|\mathcal{N}|=0$, so we are concerned with beliefs when entry unexpectedly occurs. In the continuation (with no entry against the incumbent), the quality of governance would be y^I for a single period, followed by y' in all subsequent periods. For an insider with characteristics (a,h) such that $Y(a,h\mid\sigma)\geq y'$, we therefore have $\rho_U(a,h)=\frac{k}{\Delta^I(a,h,y^I)}$, because such an insider, if victorious, would be re-elected. Notice that $\left(\widehat{a}(y^I),\widehat{h}(y^I)\right)$ by construction maximizes $\Delta^I(a,h,y^I)$ and therefore minimizes $\rho_U(a,h)$ for (a,h) satisfying $Y(a,h\mid\sigma)\geq y'$. Moreover, for an insider with characteristics (a,h) such that $Y(a,h\mid\sigma)<y'$,

$$\rho_W(a,h)=\frac{k}{u^G\left(a,h\mid\sigma,s_1\right)-(1+a)y^I},$$

because such an insider, if victorious, would not be re-elected. But with s_2 sufficiently large, $\Delta^I(\widehat{a}(y^I),\widehat{h}(y^I),y^I)>u^G\left(a,h\mid\sigma,s_1\right)-(1+a)y'$. Thus, $\left(\widehat{a}(y^I),\widehat{h}(y^I)\right)$ minimizes $\rho_W^n(a,h)$ for $(a,h)\in[0,1]^2$, as required.

Now consider (b-ii). Suppose there is no incumbent, an incumbent who does not run, or an incumbent with $y^I<y'$ who does run. Here $|\mathcal{N}|=|\mathcal{N}_1|=N'$, so we are concerned with

beliefs when the realized number of firms, N, is both greater than and less than N'. The case of $N < N'$ is simple because the pool of equilibrium entrants is homogeneous, consisting of insiders with characteristics (a', h'); hence, any subset must also consist of insiders with characteristics (a', h'). For the case of $N > N'$, the argument parallels the one given for (b-i), except that the quality of governance in the continuation (conditional on the presence of an incumbent as well as the incumbent's type and entry choice) is y' for the current period, rather than y^I, so that y' replaces y^I throughout. $\square$

References

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