**Appendix 1: Proof of Lemma**

From the definition of the price index:

$$\begin{aligned} 1 &= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di \\ &= \int_0^1 \exp\{(1-\epsilon)(p_t(i)-p_t)\}\ di \\ &\simeq 1+(1-\epsilon)\int_0^1 (p_t(i)-p_t)\ di + \frac{(1-\epsilon)^2}{2}\int_0^1 (p_t(i)-p_t)^2\ di \end{aligned}$$

where the approximation results from a second-order Taylor expansion around the zero inflation steady state. Thus, and up to second order, we have

$$p_t \simeq E_i\{p_t(i)\} + \frac{(1-\epsilon)}{2}\int_0^1 (p_t(i)-p_t)^2\ di$$

where $E_i\{p_t(i)\} \equiv \int_0^1 p_t(i)\ di$ is the cross-sectional mean of (log) prices.

In addition,

$$\begin{aligned} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di &= \int_0^1 \exp\left\{-\epsilon\ (p_t(i)-p_t)\right\}\ di \\ &\simeq 1-\epsilon\int_0^1 (p_t(i)-p_t)\ di + \frac{\epsilon^2}{2}\int_0^1 (p_t(i)-p_t)^2\ di \\ &\simeq 1+\frac{\epsilon}{2}\int_0^1 (p_t(i)-p_t)^2\ di \\ &\simeq 1+\frac{\epsilon}{2}\ var_i\{p_t(i)\} \geq 1 \end{aligned}$$

where the last equality follows from the observation that, up to second order,

$$\begin{aligned} \int_0^1 (p_t(i)-p_t)^2\ di &\simeq \int_0^1 (p_t(i)-E_i\{p_t(i)\})^2\ di \\ &\equiv var_i\{p_t(i)\} \end{aligned}$$

Finally, using the definition of $d_t^p$ we obtain

$$d_t^p \simeq \frac{\epsilon}{2}\ var_i\{p_t(i)\} \geq 0$$

On the other hand,

$$\begin{aligned}\int_0^1 \left(\frac{N_t(j)}{N_t}\right)^{1-\alpha} dj &= \int_0^1 \exp\left\{(1-\alpha)\ (n_t(j)-n_t)\right\}\ dj \\ &\simeq 1+(1-\alpha)\int_0^1 (n_t(j)-n_t)\ dj + \frac{(1-\alpha)^2}{2}\int_0^1 (n_t(j)-n_t)^2\ dj \\ &\simeq 1-\frac{\alpha(1-\alpha)}{2}\int_0^1 (n_t(j)-n_t)^2\ dj \leq 1\end{aligned}$$

where the third equality follows from the fact that $\int_0^1 (n_t(j)-n_t)\ dj \simeq -\frac{1}{2}\int_0^1 (n_t(j)-n_t)^2\ dj$ (using a second order approximation of the identity $1 \equiv \int_0^1 \frac{N_t(j)}{N_t}\ dj$).

Log-linearizing the optimal hiring condition (11) around a symmetric equilibrium we have

$$n_t(j)-n_t \simeq -\frac{1-\Phi}{\alpha}\ (w_t(j)-w_t)$$

Thus

$$\int_0^1 \left(\frac{N_t(j)}{N_t}\right)^{1-\alpha}\ dj \simeq 1-\frac{(1-\Phi)^2(1-\alpha)}{2\alpha}\int_0^1 (w_t(j)-w_t)^2\ dj$$

implying

$$d_t^w \equiv -\log\int_0^1 \left(\frac{N_t(j)}{N_t}\right)^{1-\alpha} \simeq \frac{(1-\Phi)^2(1-\alpha)}{2\alpha}\ var_j\{w_t(j)\} \geq 0$$

**Appendix 2: Linearization of Participation Condition**

*Lemma.* Define $Q_t \equiv \int_0^1 \left(\frac{H_t(z)}{H_t}\right) \ \mathcal{S}_t^H(z) \ dz$. Then, around a zero inflation deterministic steady state we have

$$\widehat{q}_t \simeq \widehat{g}_t - \Xi \ \pi_t^w$$

where $\Xi \equiv \frac{\xi(W/P)}{(1-\xi)G} \frac{\theta_w}{(1-\theta_w)(1-\beta(1-\delta)\theta_w)}$.

*Proof of Lemma:*

$$\begin{aligned} Q_t &\simeq \int_0^1 \mathcal{S}_t^H(z) \ dz \\ &= (1-\theta_w) \sum_{q=0}^{\infty} \theta_w^q \ \mathcal{S}_{t|t-q}^H \\ &= (1-\theta_w) \sum_{q=0}^{\infty} \theta_w^q \ (\mathcal{S}_{t|t}^H + \mathcal{S}_{t|t-q}^H - \mathcal{S}_{t|t}^H) \end{aligned}$$

where the first equality holds up to a first order approximation in a neighborhood of a symmetric steady state.

Using the Nash bargaining condition (31) we have:

$$\xi \ Q_t = (1-\xi) \ G_t + \xi(1-\theta_w) \sum_{q=0}^{\infty} \theta_w^q \ (\mathcal{S}_{t|t-q}^H - \mathcal{S}_{t|t}^H)$$

Note however that

$$\begin{aligned} \mathcal{S}_{t|t-q}^H - \mathcal{S}_{t|t}^H &= E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k} \ \left( \frac{W_{t-q}^*}{P_{t+k}} - \frac{W_t^*}{P_{t+k}} \right) \right\} \\ &= \left( \frac{W_{t-q}^* - W_t^*}{P_t} \right) \ E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k} \ \left( \frac{P_t}{P_{t+k}} \right) \right\} \end{aligned}$$

Using the law of motion for the aggregate wage,

$$
\begin{aligned}
(1-\theta_w)\sum_{q=0}^{\infty}\theta_w^q\ (\mathcal{S}^H_{t|t-q}-\mathcal{S}^H_{t|t}) &= \left(\frac{W_t-W_t^*}{P_t}\right)\ E_t\left\{\sum_{k=0}^{\infty}((1-\delta)\theta_w)^k\Lambda_{t,t+k}\ \left(\frac{P_t}{P_{t+k}}\right)\right\} \\
&= -\pi_t^w\ \left(\frac{\theta_w}{1-\theta_w}\right)\frac{W_{t-1}}{P_t}\ E_t\left\{\sum_{k=0}^{\infty}((1-\delta)\theta_w)^k\Lambda_{t,t+k}\ \left(\frac{P_t}{P_{t+k}}\right)\right\} \\
&\simeq -\pi_t^w\ \left(\frac{\theta_w}{(1-\theta_w)(1-\beta(1-\delta)\theta_w)}\right)\left(\frac{W}{P}\right)
\end{aligned}
$$

where the approximation holds in a neighborhood of the zero inflation steady state. It follows that

$$
\xi\ Q_t \simeq (1-\xi)\ G_t - \xi\ \left(\frac{\theta_w}{(1-\theta_w)(1-\beta(1-\delta)\theta_w)}\right)\left(\frac{W}{P}\right)\ \pi_t^w
$$

or, equivalently, in (log) deviations from steady state values:

$$
\widehat{q}_t \simeq \widehat{g}_t - \Xi\ \pi_t^w
$$

where $\Xi \equiv \frac{\xi(W/P)}{(1-\xi)G}\frac{\theta_w}{(1-\theta_w)(1-\beta(1-\delta)\theta_w)}$.

**Appendix 3: Log-linearized Equilibrium Conditions**

- Technology, Resource Constraints and Miscellaneous Identities

*Goods market clearing (44)*

$$\widehat{y}_t = (1-\Theta)\ \widehat{c}_t + \Theta\ (\widehat{g}_t + \widehat{h}_t)$$

where $\Theta \equiv \frac{\delta N G}{Y}$.

*Aggregate production function*

$$\widehat{y}_t = a_t + (1-\alpha)\ \widehat{n}_t$$

*Aggregate hiring and employment*

$$\delta\ \widehat{h}_t = \widehat{n}_t - (1-\delta)\ \widehat{n}_{t-1}$$

*Hiring cost*

$$\widehat{g}_t = \gamma\ \widehat{x}_t$$

*Job finding rate*

$$\widehat{x}_t = \widehat{h}_t - \widehat{u}_t^o$$

*Effective Market Effort*

$$\widehat{l}_t = \left(\frac{N}{L}\right)\ \widehat{n}_t + \left(\frac{\psi U}{L}\right)\ \widehat{u}_t$$

*Labor force*

$$\widehat{f}_t = \left(\frac{N}{F}\right)\ \widehat{n}_t + \left(\frac{U}{F}\right)\ \widehat{u}_t$$

*Unemployment:*

$$\widehat{u}_t = \widehat{u}_t^o - \frac{x}{1-x}\ \widehat{x}_t$$

*Unemployment rate*

$$\widehat{ur}_t = \widehat{f}_t - \widehat{n}_t$$

- Decentralized Economy: Other Equilibrium Conditions

*Euler equation*

$$\widehat{c}_t = E_t\{\widehat{c}_{t+1}\} - \widehat{r}_t$$

*Fisherian equation*

$$\widehat{r}_t = \widehat{i}_t - E_t\{\pi_{t+1}\}$$

*Inflation equation*

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} - \lambda_p \ \widehat{\mu}_t^p$$

*Optimal hiring condition*

$$\alpha \ \widehat{n}_t = a_t - [(1-\Phi) \ \widehat{\omega}_t + \Phi \ \widehat{b}_t] - \widehat{\mu}_t^p$$

$$\widehat{b}_t = \frac{1}{1-\beta(1-\delta)} \ \widehat{g}_t - \frac{\beta(1-\delta)}{1-\beta(1-\delta)} \ (E_t\{\widehat{g}_{t+1}\} - \widehat{r}_t)$$

*Optimal participation condition* (only when $\psi > 0$)

$$\widehat{c}_t + \varphi \widehat{l}_t = \frac{1}{1-x} \ \widehat{x}_t + \widehat{g}_t - \Xi \ \pi_t^w$$

where $\Xi \equiv \frac{\xi(W/P)}{(1-\xi)G} \frac{\theta_w}{(1-\theta_w)(1-\beta(1-\delta)\theta_w)}$ (note $\Xi = 0$ under flexible wages). When $\psi = 0$, $\widehat{l}_t = \widehat{n}_t$ and $\widehat{f}_t = 0$ hold instead.

*Interest rate rule*

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

- Wage Setting Block: Flexible Wages

*Nash wage equation*

$$\widehat{\omega}_t = (1-\Upsilon) \ (\widehat{c}_t + \varphi \widehat{l}_t) + \Upsilon \ (-\widehat{\mu}_t^p + a_t - \alpha \ \widehat{n}_t)$$

where $\Upsilon \equiv \frac{(1-\xi)MRPN}{W/P}$

- Wage Setting Block: Sticky Wages

$$\widehat{\omega}_t = \widehat{\omega}_{t-1} + \pi_t^w - \pi_t^p$$

$$\pi_t^w = \beta(1-\delta)\ E_t\{\pi_{t+1}^w\} - \lambda_w\ (\widehat{\omega}_t - \widehat{\omega}_t^{tar})$$

$$\widehat{\omega}_t^{tar} = (1-\Upsilon)\ (\widehat{c}_t + \varphi\widehat{l_t}) + \Upsilon\ (-\widehat{\mu}_t^p + a_t - \alpha\ \widehat{n}_t)$$

- Social Planner's Problem: Efficiency Conditions

$$a_t - \alpha\ \widehat{n}_t = (1-\Omega)\ (\widehat{c}_t + \varphi\widehat{l_t}) + \Omega\ \widehat{b}_t$$

$$\widehat{c}_t + \varphi\widehat{l_t} = \frac{1}{1-x}\ \widehat{x}_t + \widehat{g}_t$$

where $\Omega \equiv \frac{(1+\gamma)B}{MPN}$ .

**Appendix 4: Sketch of the Derivation of Loss Function**

Combining a second order expansion of the utility of the representative household and the resource constraint around the constrained-efficient allocation yields

$$E_0 \sum_{t=0}^{\infty} \beta^t \ \widetilde{U}_t \simeq - \ E_0 \sum_{t=0}^{\infty} \beta^t \ \left( \frac{1}{1-\Theta}(d_t^p + d_t^w) + \frac{1}{2}(1+\varphi)\chi L^{1+\varphi} \ \widetilde{l}_t^2 \right)$$

As shown in appendix 1 $d_t^p \simeq \frac{\epsilon}{2} \ var_i(p_t(i))$.and $d_t^w \simeq \frac{(1-\Phi)^2(1-\alpha)}{2\alpha} \ var_j\{w_t(j)\}$. I make use of the following property of the Calvo price and wage setting environment:

*Lemma*:

$$\sum_{t=0}^{\infty} \beta^t \ var_i\{p_t(i)\} = \frac{\theta_p}{(1-\theta_p)(1-\beta\theta_p)} \sum_{t=0}^{\infty} \beta^t \ (\pi_t^p)^2$$

$$\sum_{t=0}^{\infty} \beta^t \ var_j\{w_t(j)\} = \frac{\theta_w}{(1-\theta_w)(1-\beta\theta_w)} \sum_{t=0}^{\infty} \beta^t \ (\pi_t^w)^2$$

Proof: Woodford (2003, chapter 6).

Combining the previous results and letting $\mathbb{L} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \ \widetilde{U}_t(C/Y)$ denote the utility losses expressed as a share of steady state GDP we can write

$$\mathbb{L} \equiv \frac{1}{2} \ E_0 \sum_{t=0}^{\infty} \beta^t \ \left[ \frac{\epsilon}{\lambda_p}(\pi_t^p)^2 + \frac{(1-\Phi)^2(1-\alpha)}{\alpha\lambda_w^*}(\pi_t^w)^2 + (1+\varphi)(\chi C L^{1+\varphi}/Y)\widetilde{l}_t^2 \right]$$

where $\lambda_w^* \equiv (1-\theta_w)(1-\beta\theta_w)/\theta_w$ .

Next note that, up to first order,

$$\begin{aligned} \widetilde{l}_t &= \left( \frac{N}{L(1-\alpha)} \right) \widetilde{y}_t + \left( \frac{\psi U}{L} \right) \widetilde{u}_t \\ &= \left( \frac{N}{L(1-\alpha)} \right) \left( \widetilde{y}_t + \frac{(1-\alpha)\psi U}{N} \widetilde{u}_t \right) \end{aligned}$$

Thus we have:

$$\mathbb{L} \equiv \frac{1}{2}\, E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\epsilon}{\lambda_p} (\pi_t^p)^2 + \frac{(1-\Phi)^2(1-\alpha)}{\alpha \lambda_w^*} (\pi_t^w)^2 + \frac{(1+\varphi)(1-\Omega)N}{(1-\alpha)L} \left( \widetilde{y}_t + \frac{(1-\alpha)\psi U}{N} \widetilde{u}_t \right)^2 \right]$$

where $1 - \Omega \equiv \frac{MRS}{MPN} = 1 - \frac{B(1+\gamma)}{MPN}$ is the steady state gap between the marginal rate of substitution and the marginal product of labor resulting from the existence of labor market frictions.