Risk, Returns, and Multinational Production*

José L. Fillat†
Federal Reserve Bank of Boston

Stefania Garetto‡
Boston University

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Abstract

This paper starts by unveiling a new empirical regularity: multinational corporations tend to exhibit systematically higher returns and earnings yields than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings yields and returns than firms selling only in their domestic market. To explain this pattern, we develop a real option value model where firms are heterogeneous in productivity, and have to decide whether and how to sell in a foreign market where demand is risky. Firms can serve the foreign market through trade or foreign direct investment, thus becoming multinationals. Multinational firms are more exposed to risk: following a negative shock, they are reluctant to exit the foreign market because they would forgo the option premium (sunk cost) that they paid to become multinationals. The theory provides a complementary explanation for the cross section of returns by exploiting the production side from an international point of view. We calibrate the model to match U.S. export and FDI dynamics, and use it to explain cross-sectional differences in earnings yields and returns.

Keywords: Multinational firms, option value, cross-sectional returns

JEL Classification : F12, F23, G12

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†Federal Reserve Bank of Boston, 600 Atlantic Avenue, Boston MA 02210. E-mail: Jose.Fillat@bos.frb.org. The views expressed in this paper are those of the authors and not necessarily those of the Federal Reserve Bank of Boston or Federal Reserve System.

‡Department of Economics, Boston University, 270 Bay State Road, Boston MA 02215. E-mail: garettos@bu.edu.

1 Introduction

Multinational firms tend to have higher returns and earnings yields than non-multinational firms. Among non-multinationals, exporters tend to have higher returns and earnings yields than firms selling only in their domestic market. Many studies in the new trade literature have documented features determining selection into foreign markets: exporters and multinational firms tend to be larger, more productive, to employ more workers, and sell more products than firms selling only domestically.[1] However, none of this literature has studied whether the international status of the firm matters for its investors. Similarly, in the financial literature, explanations of the cross section of returns disregarded the role of the international status of the firm.

In this paper we attempt to fill this gap in the literature. We develop a real option value model where firms' heterogeneity, aggregate uncertainty and sunk costs provide the missing link between firms' international status and their returns on the stock market.

In our framework, firms are heterogeneous in productivity, and choose optimally whether to produce only domestically, export, or set up an offshore affiliate to serve a foreign market where aggregate demand is uncertain. By selling in their domestic market, firms effectively purchase an option that allows them to enter into the foreign market through exports or direct investment (henceforth, FDI). Exercising the option (*i.e.* entering the market) entails a premium, in the form of a sunk cost. Firm-level productivity and prospects of growth of the foreign demand for goods determine the equilibrium choice. The presence of uncertainty generates larger persistence in a status with respect to a deterministic world: when the foreign market is hit by a negative shock, firms are reluctant to exit even if their profits fall below zero, since by exiting they would forgo the option premium they paid to enter. It may be optimal for them to bear losses for a while, and wait for a possible reversal of the shock. Firms' optimal strategies depend on their underlying productivities. As a result, different firms will differ in the covariance of their earnings yields with the aggregate uncertainty. The model endogenously determines cross-sectional differences in earnings-to-price ratios and returns, and provides a complementary explanation for the cross section of returns exploiting the production side from an international point of view.

The selection mechanism is modeled following Helpman, Melitz, and Yeaple (2004). Exports are characterized by low sunk costs and high variable costs, due to the necessity of shipping goods every period, while FDI entails high sunk costs of setting up a plant and starting production abroad,

[1] See, among others, Bernard, Jensen, and Schott (2009).

but low variable costs, since there is no physical separation between production and sales. The standard Helpman, Melitz, and Yeaple (2004) model is static, hence the value of a firm coincides with its profits, and earnings yields and returns are constant across firms. A dynamic but deterministic model shares the same feature, with earnings-to-price ratios and returns simply given by the discount rate. To generate variation in these variables across firms, we extend the basic Helpman, Melitz, and Yeaple (2004) framework to a dynamic and stochastic environment, using Dixit (1989) as a benchmark to model entry decision under uncertainty. Since foreign investment is associated to larger sunk costs compared to trade, it generates more persistence in the firm's strategies over time. This implies larger losses if the foreign economy is hit by negative shocks. As a result, multinationals are more exposed to aggregate risk since – in case of a negative shock – they are more reluctant to exit. Pulling back is a more expensive strategy for multinationals than for exporters, who paid a lower option premium to serve the foreign market. For this riskiness to appear in higher stock market returns, we model risk-averse consumers, discounting future consumption streams with a stochastic discount factor dependent on the aggregate shocks.

The dynamic model generates an imperfect sorting of firms' productivities into international statuses and is able to explain the observed yields' variation across firms. More productive firms display less persistent dynamics, because they are on average more likely to have positive profits, but have more incentives to become multinationals, and hence self-select into the status that is associated with more persistence. The imperfect sorting of productivities into status, together with the possibility of changing status (the option value) create a wedge between per-period profits and the firm's valuation over time. This systematic difference between profits and firm valuation generates heterogeneity in the financial variables we are interested in: earnings yields and returns depend on the choice of whether and how to serve foreign markets, therefore vary across firms with different international status.

The solution of the model delivers a series of predictions relating firms' productivity and the realization of the shocks to the pattern of status changes. First, more productive firms need smaller positive shocks to enter a foreign market, and larger negative shocks to exit. Second, a larger positive shock is needed to induce a domestic firm to become multinational with respect to the one needed to become an exporter. Third, a larger negative shock is needed to induce a multinational to exit the market with respect to the one needed to induce an exporter to exit. These predictions find support in the pattern of status changes in the data. The model is also consistent with other qualitative features of the data on trade and FDI dynamics, like the imperfect sorting of productivity into statuses, and the fact that also relatively large firms exhibit changes of status. We calibrate the theory to match these facts also quantitatively, and with the parameterized model we compute moments of the financial variables from simulated data. We show that the model is able to match features of the financial data which were not targeted in the calibration.

Why are we interested in the cross section of returns and earnings yields? Historically, average returns vary across stocks. Fama and French (1996) is a comprehensive description of the cross-sectional picture of returns. In this paper we address the risk-return trade-off concerning multinational and non-multinational firms. We focus on cash flow dynamics of the firm and how these are determined by endogenous decisions and exogenous risk. Multinational firms are exposed to foreign demand risk for longer due to the higher persistence in their status. This risk must be rewarded by a higher asset returns in equilibrium. Investors will be willing to hold these companies if the returns are high enough to compensate for the risk. We find that this risk is not fully captured by the multifactor model in Fama and French (1993).

The existing financial literature that focuses on cross-sectional differences in earnings-to-price ratios and returns abstracts from the international organization of the firm. Various attempts to explain cross sectional differences in expected returns are based either on different specifications of preferences, or on the presence of persistent shocks to the endowments, or both.[2] We contribute to the financial literature by endogenizing the exposure of cash-flows to these types of shocks. Exposure is directly linked to the decision of when and how to serve the foreign market, which is ultimately driven by the interaction between productivity and cost structure.

To our knowledge, Rob and Vettas (2003) is the only other paper that developed a model of trade and FDI with uncertain demand growth. In their framework FDI is irreversible, so it can generate excess capacity, but has lower marginal cost compared to export. The authors show that uncertainty implies existence of an interior equilibrium where export and FDI coexist. Our work generalizes their model to one with many heterogeneous firms and a more general process for demand growth. Russ (2007) also formulates a problem of foreign investment under uncertainty to study the response of FDI to exchange rate fluctuations. Her model features firm heterogeneity, but does not allow trade as a way to serve foreign markets. Ramondo and Rappoport (2008) introduce idiosyncratic and aggregate shocks in a model where firms can locate plants both domestically and abroad. Multinational production allows firms to match domestic productivity and foreign

[2] Yogo (2006) and Piazzesi, Schneider, and Tuzel (2007) are examples of non-separable goods in the utility function. Campbell and Cochrane (1999) use internal habits specifications. Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008) use recursive preferences with persistent shocks to the endowment.

shocks, and works as a mechanism for risk sharing. Our framework allows for risk sharing and diversification in addition to the risk exposure driven by the combination of aggregate shocks and sunk costs. We allow for country-specific shocks with various correlation patterns. Moreover, we model both trade and multinational production as different modes of dealing with uncertainty in foreign markets.

Our work is related to the literature on trade dynamics with sunk costs. Particularly, Alessandria and Choi (2007) and Irrarazabal and Opromolla (2009) model entry and exit into the export market in a world with idiosyncratic productivity shocks and sunk costs. Our model is closer to the framework in Irrarazabal and Opromolla (2009) for the use of the real option value analogy in solving the firm's optimization problem. While Irrarazabal and Opromolla (2009) concentrate their attention on the impact of idiosyncratic productivity shocks for firm dynamics, we model aggregate demand shocks that affect firms differently only through their endogenous choice of international status. Alessandria and Choi (2007) study the impact of firms' shocks and sunk costs on the business cycle. While the focus of our exercise is different from their paper, we follow their calibration methodology. Both papers analyze the decision to export, but do not consider the possibility of FDI sales. Roberts and Tybout (1997) and Das, Roberts, and Tybout (2007) address empirically the issue of market participation for export. Our model has similar predictions for both exports and FDI sales, and can be estimated by using information from both trade/FDI flows and stock market prices. In general, we contribute to the trade dynamics literature both empirically and theoretically: we document features of trade and FDI dynamics for large firms, and we incorporate in the model the mode of entry (*i.e.*, the decision between export and FDI sales).

While individual elements of our framework are found in other work, this paper is the first to propose a dynamic industry equilibrium model where risk affects firms' international strategies and their financial variables in the stock market. The remainder of the paper is organized as follows. Section 2 presents empirical evidence establishing the ranking in earnings-to-price ratios and returns according to the firms' international status. Sections 3 and 4 develop the model and illustrate its analytical properties. Section 5 brings the model to the data: we report qualitative results on trade dynamics and quantitative results on the earnings yields and returns predicted by a calibrated version of the model. Section 6 is devoted to robustness checks, and Section 7 concludes.

2 Motivating Evidence

In this section we document a new fact distinguishing firms that sell only in their domestic market from exporters and multinational firms. The data display a precise sorting in the financial variables of these firms: multinational firms tend to exhibit significantly higher earnings yields and returns than exporters, and exporters in turn have higher earnings yields and returns than firms selling only in their domestic market.

Our sample consists of US-based manufacturing firms in the Compustat Segments database, and tracks about 5300 firms from 1979 to 2006. We define a firm to be a multinational (MN) if it reports the existence of a foreign geographical segment associated with positive sales. Similarly, we define a firm to be an exporter if it reports a positive level of export sales.[3,4] According to this definition, on average, 34.81% of firms sell only in the U.S. market, 29.2% also export to foreign countries, and the remaining 35.99% have positive levels of FDI sales.[5] Table 1 reports descriptive statistics of the sample we use.

In line with the numbers reported by other papers, exporters and multinationals have a size advantage with respect to domestic firms, both in terms of sales and number of employees. Particularly, the size advantage of multinationals is extremely large: on average, multinational firms hire about five times more workers than exporters, and have sales about eight times larger. Consistently with previous evidence, export sales are a small percentage (10%) of exporters' total sales.[6] The novel facts that Table 1 highlights are that the same ordered differences hold for financial variables like book value, earnings, share prices, and returns.

[3]Multinational and exporter dummies are constructed based on Compustat geographic and operating segments data. According to the Statement of Financial Accounting Standards (FAS) No. 131, *"segments are components of an enterprise about which separate financial information is available"*. Firms must report information about profits, revenues and assets for each segment. For geographical segments, this information includes revenues perceived and assets hold in foreign countries. The FAS is not explicit in defining an ownership threshold for reporting, but the existence of accounting standards for the segments themselves leads us to think that the parent (U.S.-based) firm must have a control stake in the foreign entity. Moreover, one of the Financial Accounting Standards Board (FASB)'s roles is to *"require significant disclosures about the separate operating segments of an entity's business so that investors can evaluate the differing risks in the diverse operations"*. Appendix A contains a summary of the FAS Statement, ans more details about the construction of the sample.

[4]6% of firms in the sample report both positive exports and FDI sales. We classify these firms as multinationals, based on the criterium that the existence of FDI sales is sufficient to make them subject to the risk of owning a plant in a foreign market. For robustness, however, we run all the regressions contained in this section also excluding those firms from the sample. The results are unchanged.

[5]Notice that the shares of firms belonging to each group are very different from what reported in other papers that use different data. Particularly, the share of multinational firms is disproportionately large. This is due to the fact that Compustat collects data for publicly listed firms only, which tend to be the largest firms in the population.

[6]Bernard et al. (2003) report an average ratio of export over total sales of 10% for a sample of manufacturing firms in the OECD countries.

Table 1: **Summary Statistics**.

	Domestic	Exporters	Multinationals
domestic sales (millions $)	274.18	301.03	1562.39
export sales (millions $)	0	32.25	127.36
FDI sales (millions $)	0	0	937.52
number of employees (thousands)	1.71	2.43	12.27
capital/labor ratio (millons $ per worker)	0.11	0.05	0.06
book value per share ($)	5.26	6.78	9.43
market capitalization (billions $)	0.16	0.16	1.82
book/market	0.68	0.73	0.61
earnings per share ($)	0.23	0.5	0.88
share price ($)	8.38	10.31	17.47
annual return	0.06	0.1	0.11
number of firms	2580	2164	2667

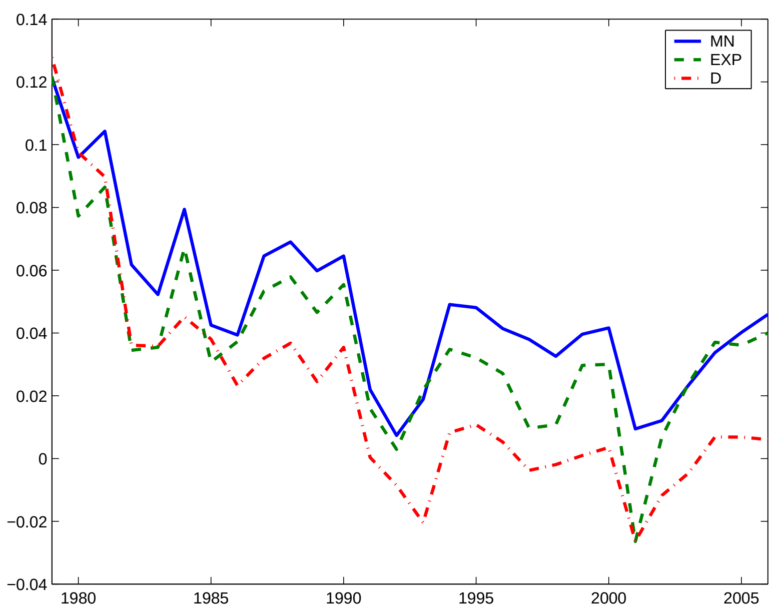

Figure 1: Earnings-to-price ratios, portfolios of firms in each group.

Differences in earnings and market prices do not cancel out: at the contrary, also earnings yields (or earnings-to-price ratios) are ordered. Figure 1 shows earnings-to-price ratios over time for three portfolios of firms. Each portfolio is composed by firms with the same international status (only domestic sales, exporters, multinationals).[7] The solid line represents earnings-to-price ratios

[7]The portfolios are constructed as follows. For each firm i, determine its status S ($S = D, EXP, MN$) at the end

of multinational firms, the dashed line the ones of exporters, and the dash-dotted line the ones of firms selling only domestically. Multinational firms exhibit higher earnings-to-price ratios than non-multinational firms, consistently over the entire time period. Similarly, exporters exhibit higher earnings-to-price ratios than firms selling only in their domestic market. The average e/p ratios for the three groups are 4.85% for multinational firms, 4.1% for exporters, and 2.25% for firms selling only domestically.

Figure 1 plots the raw data, but the ordering is robust to controlling for the effect of other variables related to size and industry. To separate the effects of the international status from other firm characteristics, Table 2 displays the results of the following firm-level regression:

$$(e/p)_{it} = \alpha + \gamma_1 D_{it}^{MN} + \gamma_2 D_{it}^{EXP} + \gamma_3 \beta_i^{MKT} + \gamma_4 X_{it} + \delta_{NAICS} + \delta_t + \varepsilon_{it}. \tag{1}$$

The dependent variable $(e/p)_{it}$ is the earnings-to-price ratio of firm i in year t. D_{it}^{MN} and D_{it}^{EXP} are multinational and exporter dummies, respectively, β_i^{MKT} is the market *beta* of the primary security of firm i,[8] X_{it} is a set of controls, including capital/labor ratio, sales per employee (our measure of productivity), book-to-market ratio, total revenues and market capitalization (measures of size). δ_{NAICS} and δ_t are 4-digit industry and year fixed effects, respectively, and ε_{it} is an orthogonal error term.

The coefficients associated to exporters and MN dummies are positive and significant. Moreover, the coefficient associated to multinationality appears to be significantly larger than the one associated to export status, identifying a further difference between these two groups. The Wald test of the null hypothesis that the two dummies' coefficients are the same confirms the difference in earnings yields of multinational firms *versus* exporters. While Table 2 only contains sales per employee and total revenues as additional controls, we run the regression also adding capital intensity and market capitalization.[9] The purpose of adding the market *betas* is to control for ag-

of year $t-1$, and collect data on earnings (e_t^i) and market capitalization (p_t^i) in year t. Portfolio earnings E_t^S and portfolio value P_t^S are constructed as equally weighted averages of individual values:

$$E_t^S = \frac{1}{N_t^S} \sum_{i \in S} e_t^i \quad , \quad P_t^S = \frac{1}{N_t^S} \sum_{i \in S} p_t^i \ , \ \ \forall\ S$$

where N_t^S denotes the number of firms in status S at time t. Portfolio earnings yields are given by E_t^S / P_t^S.

[8] The market *betas* have been computed by running a regression of individual security returns on the market aggregate returns (NYSE, AMEX, and Nasdaq) for the entire sample period. Adding the market *betas* in the regression captures the exposure of the security to aggregate market risk.

[9] In the data capital intensity appears to be highly correlated with sales per employee, while market capitalization is highly correlated with total revenues. For this reason, we run the regression with all the combinations of the two controls that are not strongly correlated with each other. The results are basically identical across specifications.

Table 2: **Earnings-to-Price Regressions.** Firm-level regressions of earnings-to-price ratios on multinational and exporter dummies, market *betas* and other controls, with year and industry fixed effects. Standard errors clustered by firm and status. (Top and bottom one percent of sample excluded, all dollar values expressed in billions).

	(1)	(2)	(3)
MN dummy	0.092	0.093	0.098
	(0.005)***	(0.005)***	(0.005)***
EXP dummy	0.061	0.062	0.062
	(0.006)***	(0.006)***	(0.006)***
β^{MKT}		-0.004	-0.004
		(0.001)***	(0.001)***
sales per emp.	5.23e-05	5.24e-05	5.24e-05
	(2.92e-05)*	(2.99e-05)*	(2.99e-05)*
total revenue	8.68e-07	8.38e-07	8.02e-07
	(4.04e-07)**	(4.07e-07)**	(4.03e-07)**
book/market			-0.001
			(0.007)
constant	-0.095	-0.094	-0.093
	(0.005)***	(0.006)***	(-0.007)***
Prob > F:			0
H_0: MN=EXP	0	0	0
No. of obs.	47691	45979	45979
R^2	0.073	0.073	0.072

Table 3: **Returns Regressions.** Firm-level regressions of returns on multinational and exporter dummies, market *betas* and other controls, with year and industry fixed effects. Standard errors clustered by firm and status.(Top and bottom five percent of sample excluded, all dollar values expressed in billions).

	(1)	(2)	(3)
MN dummy	0.05	0.05	0.046
	(0.005)***	(0.005)***	(0.005)***
EXP dummy	0.035	0.035	0.037
	(0.006)***	(0.006)***	(0.006)***
β^{MKT}		0.008	0.005
		(0.002)***	(0.002)***
sales per emp.	5.24e-05	4.95e-05	4.77e-05
	(3.9e-05)*	(2.97e-05)*	(2.88e-05)
total revenue	4.77e-07	6.07e-07	3.36e-07
	(3.07e-07)	(3.27e-07)*	(2.65e-07)
book/market			-0.104
			(0.011)***
constant	0.021	0.007	0.081
	(0.005)***	(0.005)	(0.01)***
Prob $> F$:			
H_0: MN=EXP	0	0	0.06
Obs.	47547	45975	45858
R^2	0.134	0.134	0.161

gregate market risk and to highlight the contribution of the international status to the magnitude of earnings yields once market risk is accounted for. Similarly, measures of size (revenues, market capitalization) and book-to-market ratio are meant to control for other potential sources of risk.

Earnings-to-price ratios carry information about returns on the firms' stocks. Table 3 reports the results of a regression analogous to (1), but with annual firm-level returns as the dependent variable.[10]

The coefficients on the multinational and exporter dummies are positive and significant, confirming that firms selling in foreign markets tend to have higher returns than firms selling only domestically. The coefficient on the multinational dummy is significantly higher than the one on the exporter dummy, indicating even larger excess returns for multinational firms. The ranking

[10]We identify firm-level returns with the returns of the firm's common equity. Data on returns are taken from CRSP.

holds controlling for market risk, sales per employee, total revenues, and book-to-market.[11] Any cross-sectional differences in returns generated by exposure to aggregate risk is captured by cross sectional differences in their market *betas*. Hence, the significant coefficients on the multinational and exporter dummies identify a separate source of risk.

After an exploration of earnings-to-price and returns across the three groups of firms, it seems natural to explore the source of higher returns. Higher returns do not constitute by themselves a puzzle; the fact simply indicates that multinational firms are riskier. From a CAPM point of view, higher returns must be explained by higher *betas*, or co-movements with the aggregate risks. Beyond the one-factor CAPM model, Fama and French (1993) introduced a multifactor extension to the original CAPM. Fama and French (1993) argue that a unique source of risk is not able to explain the cross section of returns. Instead, a three-factor model explains a higher fraction of the variation in expected returns. Higher returns must be explained by higher exposure to either of these three factors: market excess returns, high-minus-low book-to-market, or small-minus-big portfolio.[12] Each of the three factors is assumed to mimic a macroeconomic aggregate risk. Therefore, any asset is represented as a linear combination of the three Fama-French factors.

Columns 1 and 2 in Table 4 show the results of running one time-series Fama-French regression for each firm in the sample. We report averages and standard deviations of the sample of estimated coefficients. The first coefficient is the excess return on the U.S. market only. In Columns 3 and 4 we enlarge the set of factors by considering the excess returns on an "international market" portfolio that serves as a market benchmark for firms with foreign operations. Data on the excess returns on this global market portfolio are obtained from Kenneth French's data library on international indexes.[13] While no clear pattern emerges from the first specification, we notice that in the second one multinationals feature higher pricing errors on average than exporters and domestic firms, while exporters display higher pricing errors than domestic firms.[14] An explanation for this *anomaly* is that the risk incurred by a firm which decides to serve a foreign market is not fully captured by the

[11] The discussion about the correlations across controls in footnote 9 still applies.

[12] The small-minus-big (SMB) and high-minus-low (HML) factors are constructed upon 6 portfolios formed on size and book-to-market. The portfolios are the intersection of 2 portfolios formed on size (small and big) and 3 portfolios formed on book equity to market equity (from higher to lower: value, neutral, and growth.) This generates 6 portfolios: small-value, small-neutral, small-growth, big-value, big-neutral, and big-growth. SMB is the average returns on the three small portfolios minus the average returns on the three big portfolios. HML is the average return on the two value portfolios minus the average return on the two growth portfolios. The third factor, the excess return on the market, is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month Treasury bill rate. For more details see Fama and French (1993).

[13] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/int_index_port_formed.html.

[14] We refer to the estimated constant coefficients α as the pricing errors, as they reflect returns that are not priced by one of the three factors.

Table 4: **Firm-level Regressions**. Summary statistics of the firm-level time series coefficient estimates of Fama-French 3 factor regressions, by international status. Firm level annual excess returns are regressed on the three Fama-French factors (market excess return, Small Minus Big, High Minus Low), plus an international market portfolio. The α coefficients capture the pricing errors of the three-factor model. There are 1,387 regressions for multinational firms (MN), 923 for exporting firms (X), and 893 for domestic firms (D).

	(1) Mean	(2) Std. Dev.	(3) Mean	(4) Std. Dev.
α_{MN}	-0.018	0.227	-0.007	0.336
α_X	-0.029	0.269	-0.018	0.321
α_D	-0.071	0.28	-0.055	0.441
β_{MN}^{mkt}	-0.658	0.999	0.617	2.316
β_X^{mkt}	0.688	1.499	0.693	2.019
β_D^{mkt}	0.715	1.66	0.768	2.937
β_{MN}^{SMB}	0.6	1.542	0.623	2.078
β_X^{SMB}	0.74	2.57	0.748	2.767
β_D^{SMB}	0.643	2.498	0.608	3.197
β_{MN}^{HML}	0.19	1.521	0.134	2.36
β_X^{HML}	0.07	1.955	-0.058	2.59
β_D^{HML}	0.313	2.048	0.121	3.493
β_{MN}^{INT}			0.006	2.87
β_X^{INT}			-0.138	2.311
β_D^{INT}			-0.282	3.942

three Fama-French factors, or by the global market portfolio. The production-based model that we use to explain firms' behavior identifies indeed a "new" source of risk, being *new* defined as a risk factor beyond value or growth. It is beyond the scope of this paper to answer which macroeconomic variable, or portfolio of assets, would mimic the risk source that we identify in the structural model.

Firm level regressions are subject to substantial idiosyncratic noise. Next, we compute three portfolios composed by firms according to their international status, and we run the same multifactor regressions at the portfolio level.[15] We present the results in Table 5. It is worthy to mention that the risk to which multinationals and exporters are exposed, and the corresponding higher returns that they provide to investors, are not captured by the three existing Fama-French factors. On the contrary, we find that the market *betas* are lower than those of domestic firms. Multinationals and exporters' exposure to the HML factor, related to the value premium, is also significantly lower than the exposure of domestic firms to the same factor. Columns (4)-(6) add the excess returns on an international market portfolio to the standard Fama-French specification. The coefficients on the international market *betas* are not significant. Evidently, if the exposure to the three factors does not explain the higher reward that multinational stocks provide, it must be reflected in the pricing errors of the model. Effectively, the *alpha* of the portfolio of multinational firms is significantly higher than the one of the exporters' portfolio, which in turn is higher than the *alpha* of the portfolio of domestic firms. The results are unchanged when we include among the regressors the returns on the global market portfolio. GRS tests on the null hypothesis that the *alphas* are jointly equal to zero strongly reject the hypothesis.

Tables 2-5 consistently convey the message that the three groups of firms differ significantly in their financial variables, and that these differences cannot be accounted for either by differences in productivity and size or by traditional risk-factor explanations. The exposure to a global index does not explain the differences in returns either. The excess return of multinational firms is not explained by the common factors widely used in the finance literature. Our goal is to understand what is the underlying risk driver that generates multinationals' returns in excess of the exporters or domestic firms. In the next section we develop a dynamic structural model in which productivity differences determine the selection of firms into the three statuses, and the presence of sunk costs and aggregate risk gives rise to the observed pattern in their financial variables.

[15] Every year portfolios are formed by equally-weighting firms belonging to each of the three categories. See footnote 7.

Table 5: **Portfolio Regressions**. Time-series coefficient estimates of Fama-French 3 factor regressions for the three equally-weighted portfolios based on international status. Portfolio annual excess returns are regressed on the three Fama-French factors (market excess return, Small Minus Big, and High Minus Low) for specification (1)-(3). In Columns (4)-(6) we add excess returns from an international index portfolio. The α coefficients capture the pricing errors of the three-factor model.

	(1) DOM	(2) EXP	(3) MN	(4) DOM	(5) EXP	(6) MN
R^{mkt}	0.695	0.611	0.671	0.693	0.636	0.66
	(0.07)***	(0.075)***	(0.071)***	(0.084)***	(0.09)***	(0.086)***
R^{SMB}	0.735	0.744	0.529	0.735	0.74	0.531
	(0.081)***	(0.087)***	(0.083)***	(0.083)***	(0.089)***	(0.085)***
R^{HML}	0.267	0.172	0.188	0.267	0.171	0.189
	(0.069)***	(0.074)**	(0.07)**	(0.07)***	(0.075)**	(0.071)**
R^{INT}				0.002	-0.031	0.015
				(0.056)	(0.06)	(0.058)
α	-0.079	-0.029	-0.02	-0.079	-0.028	-0.02
	(0.013)***	(0.014)**	(0.013)	(0.013)***	(0.0144)*	(0.013)
Prob > F:						
H_0: $\alpha_{DOM} = \alpha_{EXP} = \alpha_{MN} = 0$	0			0		
Obs.	28	28	28	28	28	28
R^2	0.903	0.878	0.87	0.903	0.879	0.87

3 Model

3.1 Preferences and Technology

The economy is composed of two countries, Home and Foreign. Variables related to consumers and firms from the foreign country are going to be denoted with an asterisk (*). In both countries, agents are infinitely lived, and have preferences defined by:

$$U = \int_0^{\infty} e^{-\vartheta t} \left[bH(t) + \frac{Q(t)^{1-\gamma}}{1-\gamma} \right] dt$$

where $b > 0$, $\theta > 0$ is the subjective discount factor, and $\gamma > 1$ denotes risk aversion. H is a homogeneous good, and Q is a CES aggregate of differentiated varieties:

$$Q(t) = \left(\int q_i(t)^{1-1/\eta} di \right)^{\eta/(\eta-1)}$$

where $\eta > 1$ denotes the elasticity of substitution across varieties.

Agents maximize U subject to their budget constraint. Labor supply is perfectly inelastic. Income is given by the wage plus the profit shares deriving from ownership of firms incorporated in the country where the agents live. Quasi-linearity of the intratemporal utility function implies that consumption of $Q(t)$ is independent of income, while variations in income get reflected into variations in the consumption of H. In each country, aggregate demand of the differentiated good is hit by random shocks. Precisely, Q and Q^* evolve according to a geometric Brownian motion:

$$\frac{dQ}{Q} = \mu dt + \sigma dz^* \tag{2}$$

$$\frac{dQ^*}{Q^*} = \mu^* dt + \sigma^* dz \tag{3}$$

where $\mu, \mu^* \geq 0$, $\sigma, \sigma^* > 0$ and dz, dz^* are the increments of two standard Wiener processes with correlation ρ:

$$dz = dW \tag{4}$$

$$dz^* = \rho dW + \sqrt{1-\rho^2} dW^2 \tag{5}$$

where dW is the increment of a standard Wiener process, hence $E(dz) = E(dz^*) = 0$, and $E(dz^2) =$

$E(d(z^*)^2) = dt$, $E(dzdz^*) = \rho$.

Agents are risk averse, and discount future utility with a stochastic discount factor described by the following geometric Brownian motion:

$$\frac{dM}{M} = -rdt - \sigma_M dz \tag{6}$$

$$\frac{dM^*}{M^*} = -r^*dt - \sigma_M^* dz^* \tag{7}$$

where $r = \vartheta + \gamma\mu$ ($r^* = \vartheta + \gamma\mu^*$) is the risk-free rate, $\sigma_M = \gamma\sigma$ ($\sigma_M^* = \gamma\sigma^*$) and dz, dz^* are the increments of the Brownian motions ruling the evolution of Q and Q^*.[16] For convergence of the risk-adjusted expected values, we will assume that $r > \mu + \sigma\sigma_M$ and $r > \mu^* + \sigma^*\sigma_M$.

Labor is the only factor of production. In each country, the homogeneous good is produced by perfectly competitive firms with a one-to-one linear technology, and is assumed to be non-tradeable. Conversely, in each country the differentiated sector is populated by a continuum of firms of total mass n (n^*), which operate under a monopolistically competitive market structure. Each firm produces a differentiated variety with a linear technology defined by a unit labor requirement a, which is a random draw from a distribution $G(a)$ ($G^*(a)$). a indicates the number of units of labor that a firm needs in order to produce one unit of a differentiated variety. Differentiated varieties q_i are tradeable, hence a firm may sell its own variety only in its domestic market or in the foreign market as well.

Let's now turn to the description of the production costs in the differentiated sector. We assume that there are no fixed costs associated to production for the domestic market, so every firm makes positive profits from domestic sales, and always sells in its domestic market.[17] Besides producing for its domestic market, firms can produce also for the foreign country one. Production in the foreign market involves fixed operating costs, to be paid every period, and sunk costs of entry. If a firm decides to sell in the foreign market, it can do so either via exports or via foreign direct

[16] The stochastic discount factor is equal to the intertemporal marginal rate of substitution. Marginal utility of consumption is: $M = e^{-\vartheta t}Q(t)^{-\gamma}$. Hence:

$$\frac{dM}{M} = -\vartheta dt - \gamma\frac{dQ}{Q} = -(\vartheta + \gamma\mu)dt - \gamma\sigma dz.$$

[17] We could have introduced positive fixed costs of domestic production, and modeled the initial decision of entry in the domestic market, like in Helpman, Melitz, and Yeaple (2004). This would have introduced additional complications in solving for firms' optimal dynamics, without any gains for our empirical analysis. Compustat includes only publicly listed firms, so when a firm enters or exits Compustat we do not have any information about whether the firm is infact entering or exiting the market, or whether it just started or stopped being listed.

investment. We call multinationals those firms that decide to serve the foreign market through FDI sales.

We model the choice between trade and FDI along the lines of Helpman, Melitz, and Yeaple (2004): exports entail a relatively small sunk cost of entry F_X, but a per-unit iceberg transportation cost τ to be paid every period,[18] while FDI is associated to a larger sunk cost F_I ($F_I > F_X$), but there are no transportation costs to be covered every period, as both production and sales happen in the foreign market:[19]

Assumption 1. $F_I > F_X$.

Both exports and FDI also entail fixed operating costs to be paid every period, that we denote with f_I and f_X for FDI and exports, respectively. Once entered the foreign market, a firm can exit at no cost. However, if it decides to re-enter, it will have to pay the sunk cost again.[20] Sunk costs and stochastic demand imply that firms decide to enter when their expected profits are well above zero, and – once entered – are reluctant to exit even in case of losses due to negative shocks. We show that this dynamic behavior, labeled "hysteresis" by the literature (see Dixit and Pindyck (1994)), is more severe for multinational firms than for exporters, due to the larger sunk costs of FDI. Notice also that the cost structure and the nature of uncertainty imply that if a firm decides to enter the foreign market, it will do so either as an exporter or as a multinational firm, but it will never adopt the two strategies at the same time.[21]

[18] $\tau > 1$ units of good need to be shipped for one unit of good to arrive to the destination country.

[19] Assumption 1 is key for our results on hysteresis and risk exposure of profit flows. It seems intuitive to us that the costs of starting operations in a new production facility are higher than the costs of establishing an export channel. FDI entails a series of one-time costly activities, like acquiring licenses, dealing with local institutions (often in a foreign language), searching for qualified local labor, arrange relationships with suppliers, and so on. When the investment is greenfield, these costs are added to the cost of actually building a foreign plant. When the investment takes the form of a merger, the firm has to pay the initial acquisition cost. Clearly in both cases foreign plants can be sold, so part of the initial cost may not be sunk. However, all those activities related to starting production in a foreign country are.

[20] Roberts and Tybout (1997) report evidence on the fact that previous exporting experience matters as long as firms do not exit the foreign market. They find that sunk costs of entry for first-time exporters are not statistically different from sunk-costs for second-time exporters, *i.e.* firms that were once selling in the foreign market, exited, and decided to re-enter. Our assumptions on the structure of sunk costs reflect these findings.

[21] This feature of the model is the same as in Helpman, Melitz, and Yeaple (2004). Rob and Vettas (2003) obtain the existence of an equilibrium where firms can optimally choose to adopt simultaneously the two strategies because in their model firms choose the amount of the foreign investment, and given the structure of demand there may be the possibility of overinvestment. In their framework, FDI can be adopted to cover certain demand, while exports are used to serve the additional random excess demand without incurring the cost of a larger investment that could be underutilized. In the data we do observe firms that both export and have FDI sales (about 6% of the total). This fact could be rationalized within our framework by having multiproduct firms that choose different strategies for different product lines, or in a multi-country model where firms choose different strategies to enter different countries. Unfortunately, there is not enough information in the Compustat Segments data to check whether any of these is

Hence for a given realization of (Q, Q^*), a firm with productivity $1/a$ must choose its optimal status S ($S \in \{D, X, I\}$, *i.e.* domestic, exporter, or multinational), the current selling prices $p_S(a)$ charged in the two markets, and an updating rule (how to change the optimal price and status following changes in aggregate demand). The state of the economy is described by the realizations of aggregate demand (Q, Q^*).

The CES aggregation over individual varieties implies that individual pricing rules are independent on (Q, Q^*). However, marginal costs of production and optimal pricing rules vary with the status of the firm. Let w, w^* denote the wages in the home and foreign countries, respectively. We describe here the pricing problem of firms from the Home country. Prices charged by firms from the Foreign country are determined in the same way.

The marginal cost of domestic production is given by the labor requirement times the domestic wage, $MC_D = aw$. The marginal cost of exporting is augmented by the iceberg transportation cost: $MC_X = \tau aw$. When the firm serves the foreign market through FDI, firm-specific productivity is transferred to the foreign country and the firm employs foreign labor: $MC_I = aw^*$. CES preferences across varieties of the differentiated good imply that the optimal prices are $p_S(a) = \dfrac{\eta}{\eta - 1} MC_S(a)$.

Let $\pi_D(a; Q)$, $\pi_X(a; Q^*)$ and $\pi_I(a; Q^*)$ denote the per-period profits from domestic sales, from exports and from FDI sales abroad, respectively, for a firm from the Home country with productivity $1/a$, given a realization of the aggregate quantity demanded (Q, Q^*):

$$\pi_D(a; Q) = B(aw)^{1-\eta} P^\eta Q \tag{8}$$

$$\pi_X(a; Q^*) = B(\tau aw)^{1-\eta} (P^*)^\eta Q^* - f_X \tag{9}$$

$$\pi_I(a; Q^*) = B(aw^*)^{1-\eta} (P^*)^\eta Q^* - f_I \tag{10}$$

where $B \equiv \frac{1}{\eta-1} \left(\frac{\eta}{\eta-1}\right)^{-\eta}$, and P (P^*) is the aggregate price of the differentiated good in the Home (Foreign) country, that firms take as given while solving their maximization problem. In order to assure the existence of exporters in equilibrium, we assume that the cost parameters satisfy the following inequality:[22]

the case. Explaining the choice of firms to adopt both entry strategies would probably need a differently tailored framework, and is beyond the scope of this paper.

[22] Condition (11) is the "present discounted value equivalent" of the analogous assumption in Helpman, Melitz, and Yeaple (2004). It is derived by imposing that the profit functions of an exporter and of a multinational firm – expressed as functions of the productivity level $1/a$ – cross at a point associated with positive profits.

Assumption 2.

$$\left(\frac{w^*}{w}\right)^{\eta-1}(f_I + rF_I) > \tau^{\eta-1}(f_X + rF_X). \tag{11}$$

3.2 Value Functions

We solve the model along the lines of Dixit (1989). Let $\mathcal{V}_S(a, Q, Q^*)$ denote the expected net present value of a firm from the Home country whose productivity is $1/a$, starting in status S ($S = D, X, I$) when the realization of aggregate demand is (Q, Q^*), and following optimal policy. As we assume the absence of fixed costs to sell domestically, all firms are active in their domestic market and make positive profits $\pi_D(a; Q)$ from domestic sales. Domestic activities are not affected by the realization of foreign demand Q^*. Similarly, the decision of whether to sell in the foreign market is not affected by the realization of domestic demand Q. For this reason, we can express the value function as:

$$\mathcal{V}_S(a, Q, Q^*) = \mathcal{S}(a, Q) + V_S(a, Q^*) \tag{12}$$

where $\mathcal{S}(a, Q)$ is the expected present discounted value of profits from domestic sales, which is independent on firm status, and $V_S(a, Q^*)$ is the expected present discounted value of profits from foreign sales for a firm in status S.

Over a generic time interval Δt, the two components of the value function for a firm that is currently selling only in its domestic market can be expressed as:

$$\begin{aligned} \mathcal{S}(a, Q) &= \pi_D(a, Q) M\Delta t + E[M\Delta t \cdot \mathcal{S}(a, Q')|Q]. && (13) \\ V_D(a, Q^*) &= \max\left\{E[M\Delta t \cdot V_D(a, (Q^*)')|Q^*]\ ;\ V_X(a, Q^*) - F_X\ ;\ V_I(a, Q^*) - F_I\right\}. && (14) \end{aligned}$$

Notice that while (13) simply tracks the evolution of domestic profits, the right hand side of (14) expresses the firm's possibilities. If it sells only domestically, it gets the continuation value from not changing status, equal to the expected discounted value of the firm conditional on the current realization of foreign demand Q^*. If it decides to switch to export (FDI) it gets the value of being an exporter, V_X (multinational, V_I) minus the sunk cost of entry F_X (F_I). Similarly, the p.d.v. of profits from foreign sales for an exporter is:

$$V_X(a,Q^*) = \max\left\{\pi_X(a,Q^*)M\Delta t + E[M\Delta t \cdot V_X(a,(Q^*)')|Q^*]\ ;\ V_D(a,Q^*)\ ;\ V_I(a,Q^*) - F_I\right\} \tag{15}$$

and for a multinational:

$$V_I(a,Q^*) = \max\left\{\pi_I(a,Q^*)M\Delta t + E[M\Delta t \cdot V_I(a,(Q^*)')|Q^*]\ ;\ V_D(a,Q^*)\ ;\ V_X(a,Q^*) - F_X\right\}. \tag{16}$$

Notice that the continuation value of an exporter (a multinational) also includes the flow profits from sales in the foreign market $\pi_X(a,Q^*)M\Delta t$ ($\pi_I(a,Q^*)M\Delta t$). There are no costs of exiting the foreign market: if a firm decides to exit, its value is simply the one of a domestic firm: $V_D(a,Q^*)$.

We start by solving for the value function of a firm that is currently selling only in its domestic market (a "domestic" firm). In the continuation region:

$$\pi_D(a,Q)M\Delta t + E[M\Delta t \cdot \mathcal{S}(a,Q')|Q] - \mathcal{S}(a,Q) + E[M\Delta t \cdot V_D(a,(Q^*)')|Q^*] - V_D(a,Q^*) = 0.$$

For $\Delta t \to 0$:

$$\pi_D(a,Q)Mdt + E[d(M \cdot \mathcal{S}(a,Q))] + E[d(M \cdot V_D(a,Q^*))] = 0. \tag{17}$$

Using (2), (3) and (6), applying Ito's Lemma, and rearranging, we obtain:[23]

$$\begin{aligned}&\pi_D(a,Q) - r\mathcal{S}(a,Q) + (\mu + \sigma\sigma_M)Q\mathcal{S}'(a,Q) + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''(a,Q) - ...\\ &...\ rV_D(a,Q^*) + (\mu^* + \rho\sigma^*\sigma_M)Q^*V_D'(a,Q^*) + \frac{1}{2}V_D''(a,Q^*)(\sigma^*)^2(Q^*)^2 = 0.\end{aligned} \tag{18}$$

A solution to this equation is given by:

$$\pi_D(a,Q) - r\mathcal{S}(a,Q) + (\mu + \sigma\sigma_M)Q\mathcal{S}'(a,Q) + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''(a,Q) = 0 \tag{19}$$

$$rV_D(a,Q^*) + (\mu^* + \rho\sigma^*\sigma_M)Q^*V_D'(a,Q^*) + \frac{1}{2}V_D''(a,Q^*)(\sigma^*)^2(Q^*)^2 = 0. \tag{20}$$

In Appendix B we show that the value functions solution of (19) and (20) take the form:

$$S(a,Q) = \frac{\pi_D(a,Q)}{r - (\mu + \sigma\sigma_M)} \tag{21}$$

$$V_D(a,Q^*) = A_D(a)(Q^*)^\alpha + B_D(a)(Q^*)^\beta \tag{22}$$

[23] Details on the derivation of equations (18), (19), and (20) are contained in Appendix B.

where α and β are the negative and positive values of ξ:[24]

$$\xi = \frac{(1-m) \pm \sqrt{(1-m)^2 + 4\bar{r}}}{2}$$

and $m = \frac{2(\mu^* + \rho\sigma^*\sigma_M)}{(\sigma^*)^2}$, $\bar{r} = \frac{2r}{(\sigma^*)^2}$.[25] $A_D(a)$ and $B_D(a)$ are firm-specific parameters to be determined.

Notice that since there are no fixed or sunk costs associated to domestic production, there is no option value associated to domestic profits. Conversely, the present discounted value of foreign profits is affected by the option value of changing status. Particularly, the option value is the only component of the present discounted value of foreign profits for domestic firms. For exporters and multinationals, the value is given by the foreign profit flow and by the option value of changing status.

By following the same procedure, in Appendix B we show that the present discounted value of foreign profits for exporters and multinationals can be written as:

$$V_X(a, Q^*) = A_X(a)(Q^*)^\alpha + B_X(a)(Q^*)^\beta + \frac{B(\tau a w)^{1-\eta}(P^*)^\eta Q^*}{r - (\mu^* + \rho\sigma^*\sigma_M)} - \frac{f_X}{r} \tag{23}$$

$$V_I(a, Q^*) = A_I(a)(Q^*)^\alpha + B_I(a)(Q^*)^\beta + \frac{B(a w^*)^{1-\eta}(P^*)^\eta Q^*}{r - (\mu^* + \rho\sigma^*\sigma_M)} - \frac{f_I}{r} \tag{24}$$

where $A_X(a)$, $B_X(a)$, $A_I(a)$ and $B_I(a)$ are parameters to be determined.

Equations (22), (23), and (24) describe the value of foreign profits in the firmst' continuation regions. We still need to solve for the updating rule, which in this case consists of thresholds in the realizations of the state Q^* that induce firms to switch status. Let $Q^*_{SR}(a)$ denote the quantity threshold at which a firm with productivity $1/a$ switches from status S to status R, for $S, R \in \{D, X, I\}$. In order to find the six quantity thresholds $Q^*_{SR}(a)$ and the six value function parameters $A_S(a)$, $B_S(a)$, for $S \in \{D, X, I\}$, we impose the following value-matching and smooth-pasting conditions:

[24] $\alpha < 0$, $\beta > 1$.

[25] The terms $\hat{\mu} \equiv \mu + \sigma\sigma_M$, $\tilde{\mu} \equiv \mu^* + \sigma^*\sigma_M$ represent the risk-adjusted drifts, result of taking expectations of the value function under the risk-neutral measure.

$$
\begin{aligned}
V_D(a, Q^*_{DX}(a)) &= V_X(a, Q^*_{DX}(a)) - F_X && (25)\\
V_D(a, Q^*_{DI}(a)) &= V_I(a, Q^*_{DI}(a)) - F_I && (26)\\
V_X(a, Q^*_{XD}(a)) &= V_D(a, Q^*_{XD}(a)) && (27)\\
V_X(a, Q^*_{XI}(a)) &= V_I(a, Q^*_{XI}(a)) - F_I && (28)\\
V_I(a, Q^*_{ID}(a)) &= V_D(a, Q^*_{ID}(a)) && (29)\\
V_I(a, Q^*_{IX}(a)) &= V_X(a, Q^*_{IX}(a)) - F_X && (30)\\
V'_R(a, Q^*_{RS}(a)) &= V'_S(a, Q^*_{RS}(a)) \ , \ for \ S, R \in \{D, X, I\}. && (31)
\end{aligned}
$$

For each a, equations (25)-(31) are a system of twelve equations in the twelve unknowns given by the six quantity thresholds $Q^*_{SR}(a)$ and by the value functions parameters $A_S(a)$, $B_S(a)$, for $S, R \in \{D, X, I\}$. The system is highly nonlinear, and as such is associated to multiple solutions. To get an economically sensible solution, we follow Dixit (1989) and impose $A_D(a) = 0$, $\forall a$: the option of entering a foreign market is nearly worthless for a domestic firm experiencing a very low Q^*. Consistently, it must be that $B_D(a) \geq 0$ to insure non-negativity of $V_D(a, Q^*)$. The option of quitting FDI for another strategy is nearly worthless for a multinational firm experiencing an extremely high Q^*, hence $B_I(a) = 0$. Similarly, a multinational firm has expected value $\frac{B(aw^*)^{1-\eta}(P^*)^\eta Q^*}{r-\tilde{\mu}} - \frac{f_I}{r}$ from the strategy of never changing status, hence the optimal strategy must yield a no lesser value: $A_I(a) \geq 0$. Finally, an exporter has expected value $\frac{B(\tau aw)^{1-\eta}(P^*)^\eta Q^*}{r-\tilde{\mu}} - \frac{f_X}{r}$ from the strategy of never changing status, hence its optimal strategy must yield a no lesser value for any realization of Q^*: $A_X(a), B_X(a) \geq 0$. Consequently, the value function of a domestic firm V_D is increasing on the entire domain, indicating the fact that, as the realized aggregate demand in the foreign market Q^* increases, the value of the option of entering the foreign market (either through trade or FDI) increases. The value functions of an exporter and of a multinational (V_X and V_I respectively) are U-shaped: for low levels of Q^*, the term with the negative exponent α dominates, and the value is high thanks to the option of leaving the market. Conversely, for high levels of Q^* the value is high due to the profit stream that the firm derives from staying in the market and – for exporters – due to the additional option value of becoming a multinational firm (the term with the positive exponent β).

Value functions and quantity thresholds for firms from the Foreign country are derived in an analogous manner.

3.3 Equilibrium

The price indexes in the two countries are the solution of the following system of two equations, where each price index is an aggregate of prices of domestic sales, prices of imports, and prices of FDI sales of multinational firms from the other country:

$$\begin{aligned} P^{1-\eta} = &\quad n \int \left(\frac{\eta a w}{\eta - 1}\right)^{1-\eta} dG(a) + n^* \left[\int_{\Omega_X^*(Q)} \left(\frac{\eta \tau a w^*}{\eta - 1}\right)^{1-\eta} dG^*(a) + \int_{\Omega_I^*(Q)} \left(\frac{\eta a w}{\eta - 1}\right)^{1-\eta} dG^*(a)\right] \\ (P^*)^{1-\eta} = &\quad n^* \int \left(\frac{\eta a w^*}{\eta - 1}\right)^{1-\eta} dG^*(a) + n \left[\int_{\Omega_X(Q^*)} \left(\frac{\eta \tau a w}{\eta - 1}\right)^{1-\eta} dG(a) + \int_{\Omega_I(Q^*)} \left(\frac{\eta a w^*}{\eta - 1}\right)^{1-\eta} dG(a)\right]. \end{aligned}$$

where n (n^*) is the mass of firms from the Home (Foreign) country selling differentiated varieties, and $\Omega_X(Q)$, $\Omega_I(Q)$ ($\Omega_X(Q^*)$, $\Omega_I(Q^*)$) are the subsets of these firms that export or have multinational sales when the realization of the state is (Q, Q^*).

Since we abstract from the problem of entry in the domestic market, we take the mass of firms n (n^*) as given[26]. The intial values of the processes ruling the evolution of the state, $Q(0)$ and $Q^*(0)$, are also taken as given.

3.4 Earnings-to-Price Ratios and Returns

In Section 2 we showed data on earnings-to-price ratios and returns, displaying a ranking across firms with different international status. We need to compute earnings yields and returns in the model to be able to compare the predictions of the theory with the data.

Our earnings yields measure in the model is given by the ratio $\pi_t/\mathcal{V}_t$, where π_t represents per-period profits and $\mathcal{V}_t$ is the market value of the firm. In a static model, or in a dynamic but deterministic model, $\pi_t/\mathcal{V}_t$ is constant and independent on the firm's status, since per-period profits and value of the firm coincide. Dynamics and uncertainty introduce a wedge between these two magnitudes, which reflects the option value.

Let $ep_S(a, Q, Q^*)$ denote the earnings yield of a firm with productivity $1/a$ in status S when

[26] Since we do not impose free-entry, we set the masses of firms to $n = n^* = 1$, and present the results in terms of shares of total firms.

the realization of aggregate demand is (Q, Q^*). Earnings yields in the model are given by:

$$
\begin{aligned}
ep_D(a,Q,Q^*) &= \frac{\pi_D(a,Q)}{\mathcal{V}_D(a,Q,Q^*)} && (32)\\
ep_X(a,Q,Q^*) &= \frac{\pi_D(a,Q)+\pi_X(a,Q^*)}{\mathcal{V}_X(a,Q,Q^*)} && (33)\\
ep_I(a,Q,Q^*) &= \frac{\pi_D(a,Q)+\pi_I(a,Q^*)}{\mathcal{V}_I(a,Q,Q^*)}. && (34)
\end{aligned}
$$

Similarly, our model-based measure of returns $r_S(a, Q, Q^*)$ is given by:

$$
ret_S(a,Q,Q^*) = ep_S(a,Q,Q^*) + \frac{E[d\mathcal{V}_S(a,Q,Q^*)]}{\mathcal{V}_S(a,Q,Q^*)}\ ,\ for\ S \in \{D,X,I\}. \quad (35)
$$

Returns are given by the earnings yield plus the expected change in the valuation of the firm.

The empirical evidence presented in Section 2 suggests the following ordering in aggregate earnings yields and returns across groups:

$$
\int_{\Omega_D(Q^*)} ep_D(a,Q,Q^*)dG(a) < \int_{\Omega_X(Q^*)} ep_X(a,Q,Q^*)dG(a) < \int_{\Omega_I(Q^*)} ep_I(a,Q,Q^*)dG(a) \quad (36)
$$

$$
\int_{\Omega_D(Q^*)} ret_D(a,Q,Q^*)dG(a) < \int_{\Omega_X(Q^*)} ret_X(a,Q,Q^*)dG(a) < \int_{\Omega_I(Q^*)} ret_I(a,Q,Q^*)dG(a) \quad (37)
$$

where $\Omega_D(Q^*)$, $\Omega_X(Q^*)$, and $\Omega_I(Q^*)$ are the sets of firms that have domestic sales only, exporters, and multinationals, respectively.

In Section 5 we will parameterize the model to match quantitatively features of trade and multinational production dynamics that we observe in the data. With this parameterization, we will show that the model is also able to generate the observed ranking in average earnings-to-price ratios and returns. Before these results, in the next section we show a series of qualitative properties of the model that illustrate its amenability to reproduce features of the trade dynamics data.

4 Qualitative Properties of the Solution

In this section we illustrate the workings of the model with a series of analytical properties of the solution. These properties illustrate the potential of the model to account for the export and FDI dynamics of heterogeneous firms, and the departure from the deterministic model which allows to match the ranking in the financial variables.

4.1 Ordering of the Quantity Thresholds

The relationship between the sunk costs of exporting and FDI, $F_I > F_X$, implies a precise ordering of the quantity thresholds that are solution of (25)-(31). Theorem 1 contains this result.

Theorem 1. *If $F_I > F_X$, the quantity thresholds $Q^*_{RS}(a)$, for $R, S \in \{D, X, I\}$, solution of system (25)-(31), satisfy the following ordering:*

$$Q^*_{IX}(a) < Q^*_{ID}(a) < Q^*_{XD}(a) < Q^*_{DX}(a) < Q^*_{DI}(a) < Q^*_{XI}(a)$$

for each given productivity level $1/a$.

Proof: See Appendix B.

Like in Dixit (1989), the pure presence of sunk costs implies that "entry" thresholds are higher than "exit" thresholds: $Q_{DX} > Q_{XD}$, $Q_{DI} > Q_{ID}$, and $Q_{XI} > Q_{IX}$.[27] A higher quantity demanded Q^* is needed to induce a firm to enter a foreign market through FDI with respect to the quantity necessary to induce the firm to export: $Q_{DI} > Q_{DX}$. An even larger positive shock is needed to induce and exporter to become a multinational, since he is already serving the foreign market with exports: $Q_{XI} > Q_{DI}$. Similarly, a larger negative shock is needed to induce a multinational to exit the foreign market with respect to the shock needed to induce an exporter to exit: $Q_{ID} < Q_{XD}$. Finally, an even larger negative shock is needed to induce a multinational to divest but serve the foreign market as an exporter: $Q_{IX} < Q_{ID}$.

The result of Theorem 1 applies for any given productivity level $1/a$. The following subsection illustrates the behavior of the value functions and quantity thresholds while letting vary firm-level productivity.

4.2 Comparative Statics: Value and Productivity

System (25)-(31) makes clear that both the quantity thresholds and the parameters of the value functions depend on the productivity level $1/a$. Figure 2 shows the value function of a domestic firm as a function of the aggregate quantity demanded in the foreign market Q^* and of productivity $1/a$. As observed in the previous section, V_D is increasing in Q^*, as the option value of entering the

[27] Results about hysteresis, together with the analytical proof of the ordering of the thresholds Q_{RS}, for $R, S, \in \{D, X, I\}$, are contained in Appendix B.

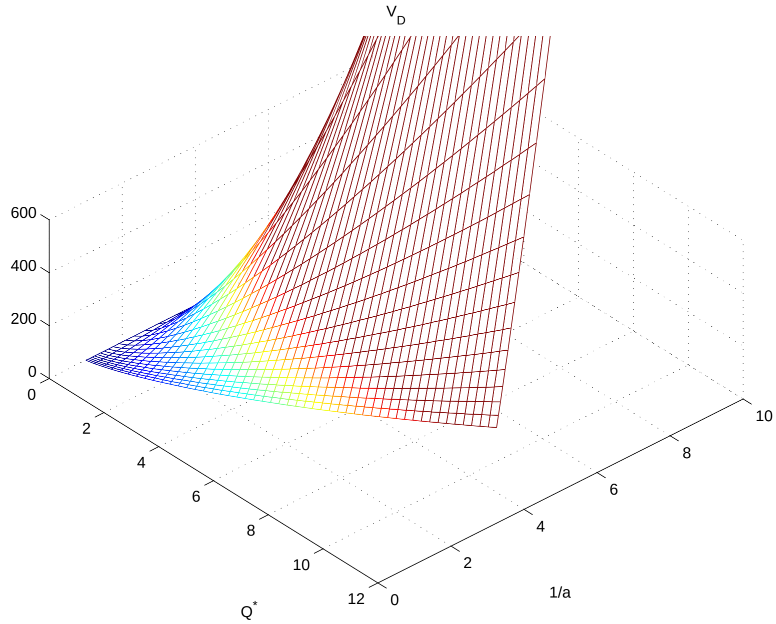

Figure 2: Value function of a domestic firm.

foreign market is increasing in the quantity demanded. V_D is also increasing in firm's productivity, as more productive firms can get higher profits from entering the foreign market.

Figure 3 shows the value functions of an exporter and of a multinational firm as functions of Q^* and $1/a$. As previously observed, V_X and V_I are U-shaped functions of Q^*, indicating the high option value of exiting for low realizations of Q^* and the high option value of not changing status for high realizations of Q^*.[28] The behavior of the value functions for $Q^* \to 0$ does not vary much across the productivity dimension: when Q^* is low, the value is high as firms of all productivity levels associate a high value to the option of exiting. The behavior of the value functions when Q^* is "large", conversely, varies with individual productivity: the value function is steeper for higher productivity firms, indicating that more productive firms obtain higher returns from staying in the foreign market when the realized aggregate demand is high.

From Figure 3, the qualitative behavior of V_X and V_I appears very similar. Figure 4 plots the difference between the value functions of firms serving the foreign market and firms selling only domestically, $V_X - V_D$ and $V_I - V_D$. For each productivity level $1/a$, each plot has two stationary points, a local maximum and a local minimum. The value matching and smooth pasting conditions

[28] Notice that for $Q^* \to \infty$, the value funcion of an exporter is steeper than the one of a multinational, because the exporter gets high value both from staying in the market as an exporter and from the option value of becoming a multinational.

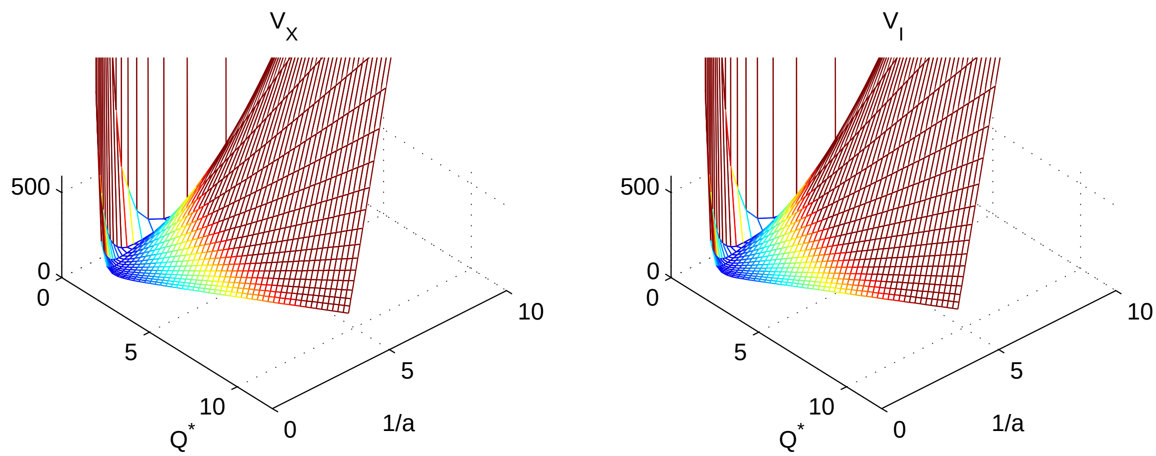

Figure 3: Value functions of an exporter and of a multinational firm.

imply that the local maxima correspond to the "entry" thresholds (Q^*_{DX} and Q^*_{DI} in the left and right plot respectively), while the local minima correspond to the "exit" thresholds (Q^*_{XD} and Q^*_{ID}). The picture shows that both entry and exit thresholds are decreasing in $1/a$, indicating that more productive firms enter the foreign market for lower realizations of aggregate demand Q^* with respect to less productive firms. Similarly, more productive firms need larger negative shocks to demand to be induced to exit the foreign market with respect to less productive firms.

Notice that for $Q^* \to 0$, $V_X - V_D$ and $V_I - V_D$ tend to infinity, because the option value of exiting the foreign market is extremely high for very low realizations of Q^* (and irrespective of firm's productivity). Conversely, for $Q^* \to \infty$, $V_X - V_D$ and $V_I - V_D$ tend to negative infinity, because the domestic firms' option value of entering the foreign market is exremely high, compared to the flow profits of staying for firms that are already serving that market.

Figure 5 plots the difference between the value functions of a multinational firm and of an exporter, $V_I - V_X$, as a function of Q^* and $1/a$. In this picture, for each value of $1/a$, the peak of the surface represents the quantity threshold where the firm switches from being an exporter to being a multinational, Q_{XI}. The figure shows that also the threshold Q_{XI} is decreasing in $1/a$, consistent with the prediction of Helpman, Melitz, and Yeaple (2004), according to which the most productive firms are more likely to become multinationals than exporters. Notice also that, for constant Q^*, the excess value $V_I - V_X$ decreases as productivity $1/a$ increases: for the same level of Q^*, a more productive exporter has a higher option value of switching to FDI compared to a less productive one.

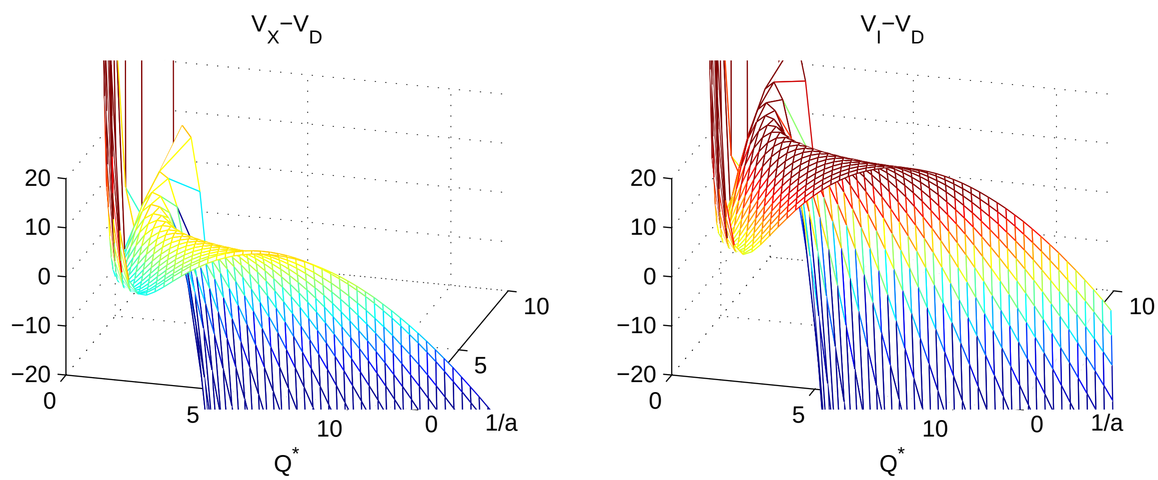

Figure 4: Difference between the value functions of exporters and multinationals and the value function of domestic firms.

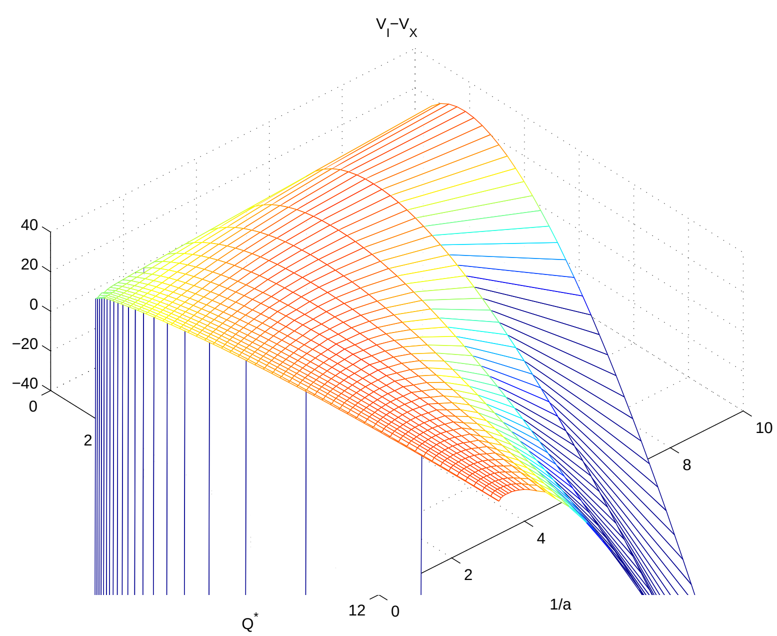

Figure 5: Difference between the value functions of an exporter and of a multinational firm.

Figures 4 and 5 suggest a systematic relationship between the quantity thresholds Q^*_{RS} and the firm productivity level $1/a$. Theorem 2 establishes this result.

Theorem 2.

$$\frac{\partial Q^*_{RS}(a)}{\partial a} > 0, \quad for \quad R, S \in \{D, X, I\}, \ \forall a. \tag{38}$$

Proof: See Appendix B.

Theorem 2 establishes that the six thresholds $Q^*_{RS}(a)$ are decreasing in productivity $1/a$, indicating that more productive firms need smaller positive shocks to demand to enter the foreign market, and larger negative shocks to exit. The one-to-one correspondence between productivities and quantity thresholds established by Theorem 2 implies that the functions $Q^*_{RS}(a)$ are invertible, hence for each realization of aggregate foreign demand Q^* we can compute six productivity thresholds $a_{RS}(Q^*)$, for $R, S \in \{D, X, I\}$, that determine the selection of heterogeneous firms into the three statuses and their likelihood of switching across statuses. This redefinition of the thresholds in terms of productivity is extremely helpful to compute the model numerically. The sets of firms belonging to each status can be written as functions of the productivity thresholds a_{RS}. At time $t = 0$:

$$\Omega_{D0} = [a_{DX}, \infty) \tag{39}$$

$$\Omega_{X0} = [a_{XI}, a_{XD}] \tag{40}$$

$$\Omega_{I0} = (0, a_{ID}] \tag{41}$$

and for the following periods:

$$\Omega_{Dt+1} = \left\{ \left\{ \Omega_{Dt} \bigcap [a_{DX}, \infty) \right\} \bigcup \left\{ \Omega_{Xt} \bigcap [a_{XD}, \infty) \right\} \bigcup \left\{ \Omega_{It} \bigcap [a_{ID}, a_{IX}] \right\} \right\} \tag{42}$$

$$\Omega_{Xt+1} = \left\{ \left\{ \Omega_{Dt} \bigcap [a_{DI}, a_{DX}] \right\} \bigcup \left\{ \Omega_{Xt} \bigcap [a_{XI}, a_{XD}] \right\} \bigcup \left\{ \Omega_{It} \bigcap [a_{IX}, \infty) \right\} \right\} \tag{43}$$

$$\Omega_{It+1} = \left\{ \left\{ \Omega_{Dt} \bigcap (0, a_{DI}] \right\} \bigcup \left\{ \Omega_{Xt} \bigcap (0, a_{XI}] \right\} \bigcup \left\{ \Omega_{It} \bigcap (0, a_{ID}] \right\} \right\} \tag{44}$$

where we omitted the dependence of Ω_{St} and a_{RS} on Q^* to ease the notation. Notice that the sets Ω_S vary with the realization of Q^*, as firms may switch status, but only depend on the firms' status in the previous period, due to the Markov property of Brownian motions.

Figure 6 illustrates Theorem 2. For an arbitrary parameterization, we plot the six quantity thresholds as functions of firm-level productivity. The picture also shows an additional property of

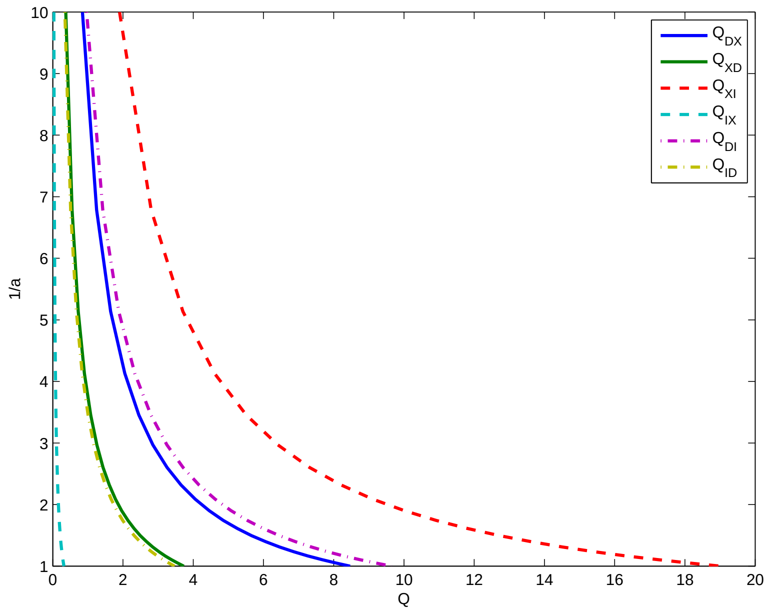

Figure 6: Quantity thresholds as functions of firm's productivity.

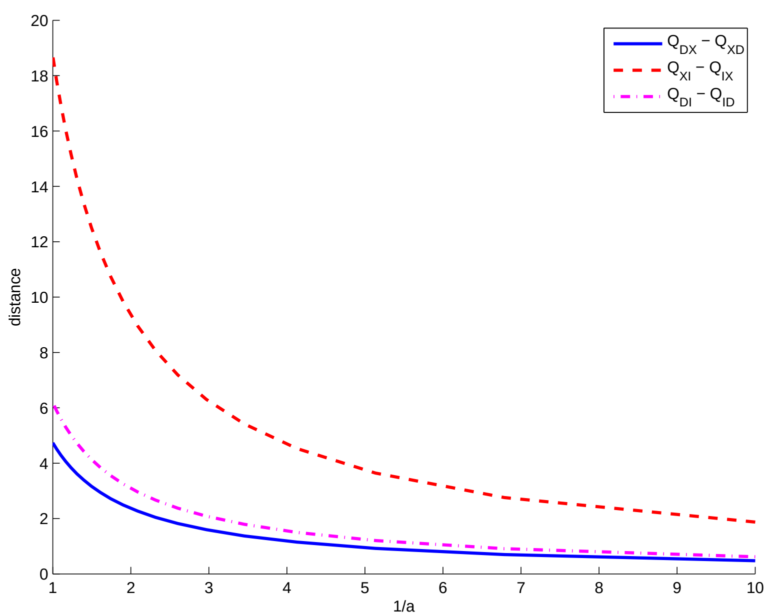

Figure 7: Distances between entry and exit thresholds as functions of firm's productivity.

the thresholds: hysteresis, defined as the horizontal distance between "entry" and "exit" thresholds, is also decreasing in productivity. Figure 7 plots the three distances $Q_{DX} - Q_{XD}$, $Q_{DI} - Q_{ID}$, and $Q_{XI} - Q_{IX}$ as functions of firm productivity: the distance between entry and exit thresholds is decreasing in productivity, indicating that more productive firms suffer less from being locked into a market by the presence of sunk entry costs. This result is established by Theorem 3:

Theorem 3.

$$\frac{\partial[|Q^*_{RS}(a) - Q^*_{SR}(a)|]}{\partial a} \geq 0, \quad for \quad R, S \in \{D, X, I\}, \ \forall a. \tag{45}$$

Proof: See Appendix B.

Theorem 3 is useful to investigate how hysteresis combines with productivity. On the one hand, for the same choice of status, the theorem implies that more productive firms exhibit less hysteresis. On the other hand, more productive firms self-select into the status (I) that is associated with more hysteresis.[29] This generates an imperfect sorting of productivities into status, which is a well-documented feature of the data.[30]

5 Empirical Results

Objective of this section is an evaluation of the performance of the model in matching qualitatively and quantitatively features of the data on trade and FDI dynamics, and of the pattern of earnings yields and returns across firms. In Section 5.1 we show that the ordering of thresholds established by Theorem 1 is consistent with the pattern of firms switching status in the data. In Section 5.2 we calibrate the parameters of the model to match quantitatively moments related to the switching pattern, and to relative size and presence of the three types of firms in the data. In Section 5.3 we use the calibrated version of the model to compute earnings yields and returns, which are not targeted in the calibration. We show that the quantitative theory is able to replicate the observed ranking in financial variables that we documented in Section 2.

[29] Dixit and Pindyck (1994) show that hysteresis is increasing in the sunk cost of investment.
[30] See **?**).

Table 6: **Switching Behavior.** Average percentages of firms switching status each year (Source: Compustat).

t \ $t+1$	D		X		I
D			1.02	$>$	0.86
					$\wedge$
X	1.05				1.66
	$\vee$				
I	0.39	$>$	0.37		

5.1 Qualitative Results: Switching Pattern in the Data

The ordering in the quantity thresholds that is the result of Theorem 1, associated with the Markov properties of Brownian motions, has implications for the expected switching pattern of firms across statuses. Since $Q^*_{DX} < Q^*_{DI} < Q^*_{XI}$, on average, it should be more likely for a domestic firm to start exporting than to do FDI, and it should be more likely for an exporter to start FDI than for a domestic firm. Similarly, $Q^*_{IX} < Q^*_{ID} < Q^*_{XD}$ implies that it should be more likely that multinationals exit completely the foreign market instead than switching to exports following a negative shock, and it should be more likely for an exporter to exit the foreign market than for a multinational firm.

We find that the average shares of firms switching across statuses in the data are consistent with these preditions. In Compustat, each year on average 5.36% of firms switch status. Of those, 1.02% switch from domestic to exporters, more than the 0.86% that switch from domestic to multinational. 1.66% of firms switch from exporters to multinational, more than the 0.86% that switch from domestic to multinational. 0.39% of firms switch back from being multinational to domestic only, more than the 0.37% that switch back from being multinational to being an exporter. Finally, 1.05% of firms switch back from exports to domestic sales only, more than the 0.39% that switch back from being a multinational to selling only domestically. Table 6 summarizes the average percentages of firms switching status in the data.

This qualitative evidence raises our confidence on the fact that the model can be successfully calibrated to match quantitatively the switching pattern and other firm characteristics. This will be the object of the next section.

5.2 Calibration

The calibration exercise that we present in this section is designed to match a series of facts on exports and FDI sales dynamics. We do not make any use of financial variables in this exercise. In the next section we show that the model calibrated by targeting trade facts only performs well also in matching non-targeted moments like earnings yields and returns.

We present a bilateral calibration exercise, that describes export and FDI activity between the U.S. and an aggregate set of trading partners. Due to data availability,[31] we impose a series of symmetry assumptions. In particular, we assume that preferences and productivity distributions are identical in the U.S. and in the other countries, and that the cost structure (the values of τ, f_X, F_X, f_I, F_I) is also the same across countries.

To calibrate the model, we need to choose a functional form for the cost distribution $G(a)$, and assign values to its parameters. We need to parameterize the Brownian motions, and to choose values for preference parameters and parameters describing trade and FDI costs. We refer to the literature to assign parameters to the preferences and to the firms' productivity distributions. The parameters ruling the Brownian motions are chosen to match data on standard deviations and correlation of GDP growth between the U.S. and its major trading partners. We choose the remaining parameters such that the model matches a series of moments computed from the data. We start describing the calibration with the parameters we adopt from the literature.

Several studies document that the tail of the empirical firm size distribution is well approximated by a Pareto distribution (see for example Luttmer (2007)). As in the model firm's size (sales) is linked to the productivity distribution, we assume that firms' productivities $1/a$ are distributed according to a Pareto law with location parameter b and shape parameter k.[32] We normalize $b = 1$, and choose k to match the coefficient of the empirical sales distribution: if productivity is Pareto-distributed with shape parameter k, sales in the model are also Pareto-distributed with shape parameter $k/(\eta-1)$. By regressing firm rank on firm size, Luttmer (2007) finds that $k/(\eta-1) = 1.06$. We then choose k accordingly, given a value for η. There is little agreement in the literature on the

[31] Compustat records data of firms with activities in the U.S., among which there are both U.S. based firms and foreign firms. However, only data of foreign firms with activities in the U.S. are reported (in other words, we have no data about foreign firms with activities only in their domestic market), which implies that we cannot construct shares of firms in each status or switching behavior for those firms.

[32] The Pareto distribution is also a convenient choice for computational reasons, since it allows to solve explicitly for the aggregate prices P, P^* as functions of the productivity thresholds a_{RS} and of the other parameters of the model.

value to attribute to the elasticity of substitution across differentiated varieties, η. Many papers focusing on long-run macroeconomic predictions use a standard value of 2. Other papers, focusing on matching data at business cycle frequencies, choose much higher values. Alessandria and Choi (2007), for example, set $\eta \approx 10$ (to match markups of about 11%). We set $\eta = 2.6$, equal to the median value in Broda and Weinstein (2006) SITC 5-digit estimates.[33] This choice implies $k = 1.7$. Finally, we set the risk aversion parameter to $\gamma = 20$, as an intermediate value among those proposed by the literature.[34]

We abstract from labor cost and market size differences, and set both wages to $w = w^* = 1$ and the mass of firms in each country to $n = n^* = 1$.

We impose that the drifts of the Brownian motions ruling the evolution of Q, Q^* have value $\mu = \mu^* = 0$. The need to impose zero expected demand growth arises from the fact that we abstract from firms' productivity growth.[35] We compute σ and $sigma^*$ as the standard deviations of consumption growth for the U.S. and for a set of OECD countries:

$$\begin{aligned} \sigma &= st.dev.(g_t^{us}) = 0.031 \\ \sigma^* &= st.dev.(g_t^{oecd}) = 0.023 \end{aligned}$$

where g_t^{oecd} (g_t^{us}) is the average growth rate of consumption across OECD countries (in the U.S.) in year t (t=1979-2006).[36] We set the risk-free rate in both economies (r and r^*) to match a long-run average of 3-month T-bills rate of 2%. For the calibration of σ_M and σ_M^*, notice that CRRA preferences with risk-aversion coefficient γ imply that $\sigma_M = \gamma\sigma$ ($\sigma_M^* = \gamma\sigma^*$)[37]. Hence $\sigma_M = 0.46$ and $\sigma_M^* = 0.62$[38] The coefficient of correlation between the two Brownian motions, ρ, is a key

[33] Broda and Weinstein (2006) estimate sectoral elasticities of substitution from price and volume data on U.S. consumption of imported goods. By using data at the 10-digit Harmonized System, they estimate how much demand shifts between 10-digit varieties when relative prices vary, within each 3-digit SITC sector.

[34] Assigning a value to γ is a difficult choice to make in this setting. In their seminal contribution, Mehra and Prescott (1985) report evidence from several micro and macro studies suggesting a value of γ between 1 and 4. They also show that a model with CRRA preferences and such a low value of γ can match the risk-free rate, but generates returns that are too low compared with the data (the equity premium puzzle). Hansen and Singleton (1982) estimate the value of γ that matches returns, obtaining a value of $\gamma \approx 200$. This high value is implusible based on the empirical evidence, and generates too high a risk-free rate. Our model features CRRA preferences for the the differentiated good, and is hence subject to the same problem: we are not able to match quantitatively both the risk-free rate and the returns. For this reason, we choose an "intermediate" value in our baseline calibration.

[35] If $\mu > 0$, $E(Q)$ would be increasing over time and for $t \to \infty$ all firms would become multinationals. By setting $\mu = 0$, we are implicitly assuming that Q, Q^* and b grow at a rate such that the distribution of firms in the three groups does not degenerate over time.

[36] Data source: OECD Statistics.

[37] See footnote 16.

[38] The choice of $\gamma = 20$ is consistent with empirical estimates of the Sharpe ratio: $S \equiv \frac{E(ret-r)}{\sigma_{ret}}$. It is possible to

parameter in this exercise. To select a value for this parameters, we computed correlations in GDP growth rates between the U.S. and a set of countries that include the U.S. major trading partners (Canada, Mexico, France, Germany, the U.K., Spain, Japan, Hong Kong, Korea, Malaysia and Taiwan) and the largest developing economies (China, Brazil, India and Russia). For the sample period (1979-2006) correlation vary from a minimum of -0.0428 (between U.S. and Brazil, which is the only country in the sample displaying a negative correlation) to a maximum of 0.8062 (between U.S. and Canada). The mean correlation is 0.19, and the median is 0.12. Based on this numbers, for our baseline calibration we choose a value of $\rho = 0.12$.

It remains to calibrate the variable trade cost τ, fixed operating costs f_X, f_I, sunk costs F_X, F_I (that we assume to be equal in the U.S. and in its aggregate trading partners), and the initial values for the aggregate demand levels, $Q(0)$ and $Q^*(0)$. We follow the methodology of the calibration in Alessandria and Choi (2007), and select values for these parameters to match a set of moments related to trade and FDI dynamics. We target data on firms' persistence in the same status and on the relative presence of the three types of firms in the data.

We compute all the moments from Compustat data. In terms of status persistence, on average, 93.11% of domestic firms remain domestic the following year, while 3.73% of them become exporters, and the remaining become multinationals. 90.27% of exporters continue exporting the following year, while 5.96% of them become multinationals, and the remaining exit the foreign market to sell domestically only. Multinational firms exhibit even higher persistence, with 98.31% of them continuing being multinationals the following year, and only 0.82% of them becoming exporters the following year. Domestic firms' and exporters' persistence moments are close to the ones reported in Alessandria and Choi (2007), but we are unaware of other papers computing moments related to persistence in multinational activity.

Next, we look at the average share of firms in each status. In Compustat, the average share of firms selling only domestically is 34.81% of the sample, while exporters are 29.2% of the sample,

prove that $S \leq (1+r)(\gamma\sigma_{\Delta C})$: let $R = 1 + ret$, $R^f = 1 + r$; expected returns can be expressed as:

$$\begin{aligned} E(R) &= R^f + R^f \cdot Cov(M, R) \\ E(R) &= R^f + R^f \gamma \cdot Cov(\Delta C, R) \\ E(R - R^f) &= R^f \gamma \cdot \rho(\Delta C, R) \cdot \sigma_{\Delta C} \cdot \sigma_R \\ \frac{E(R - R^f)}{\sigma_R} &\leq R^f \gamma \cdot \sigma_{\Delta C}. \end{aligned}$$

Estimates of the Sharpe ratio for the U.S. found $S \approx 0.4$. Our chosen value of γ is in line with these estimates, generating a Sharpe ratio of 0.47.

and multinational firms account for the remaining 35.99% of the sample. As previously noted, also these numbers reflect selection of the largest firms in the data set.[39] Despite the divergence of our moments with the ones reported from other papers, to be internally consistent we decided to match Compustat data in this exercise. While not representative of the entire population of U.S. firms, Compustat offers a detailed portrait of the largest firms in the economy, which are the major actors when talking about volumes of trade and FDI.[40]

Table 7 summarizes the calibrated parameters. The calibrated iceberg cost is 22%, consistent with a low-range estimate in Eaton and Kortum (2002). Sunk costs of export and FDI equal to 0.95 and 3.5, respectively, indicate that a domestic firm must spend on average about 1.74 times its per-period revenue to enter the foreign market as an exporter, and about 6.4 times its per-period revenue to start FDI operations there. Fixed costs of export and FDI equal to 0.33 and 0.55, respectively, indicate that an exporter must spend on average about 38% of its per-period revenue in operating costs, and that a multinational firms must spend on average about 5% of its per-period revenue in operating costs. Aggregate demand parameters are set at $Q = Q^*(0) = 3.5$.

Table 8 displays jointly the moments computed from the data and the moments generated by the calibrated model. The model does a good job in matching the persistence parameters. The shares of exporters and multinational are slightly smaller than in the data.

5.3 Quantitative Results: Earnings Yields and Returns

With the calibrated version of the model, we compute earnings yields and returns across the three groups of firms. In our calculations, we follow the construction of the portfolios we used in the data analysis presented in Section 2. We run 20 Monte Carlo simulations of an artificial dataset of 100 firms (with productivities drawn from a Pareto distribution) for 10 years.[41] For each firm, year, and Monte Carlo simulation we compute earnings, equilibrium value (our model-based measure of the market price), and variation in the equilibrium value of the firm. For each year and Monte Carlo simulation, we create the three portfolios of domestic firms, exporters, and multinationals, and we

[39] Bernard et al. (2007), among others, report that the average share of manufacturing firms that export is about 18%, while Bernard and Jensen (2007) report that multinational firms represent only 1% of manufacturing firms.

[40] The choice of a Pareto distribution for firms' productivity is robust to the selection problem associated with dealing with Compustat data. The Pareto distribution is invariant to lower truncations, hence if we assume that the entire productivity distribution is Pareto with parameters (b, k), the distribution of firms in Compustat will also be Pareto, with parameters (b', k), $b' > b$. As the lower bound of the distribution does not enter the computation of the moments, we normalize it to one.

[41] The small number of Monte Carlo simulations is justified by the fact that for $\mu = 0$ there are small differences across simulations.

Table 7: **Summary of Calibrated Parameters.**

Parameter	Definition	Value	Source
Brownian motions			
μ, μ^*	drift of Q, Q^*	0	no productivity growth
σ	variance of Q	0.023	st.dev. of U.S. cons. growth
σ^*	variance of Q^*	0.031	st.dev. of OECD cons. growth
ρ	correlation of Q, Q^*	0.12	GDP growth correlations
r, r^*	risk-free rate	0.02	3-month T-bills rate
σ_M	variance of s.d.f.	0.46	model restriction ($\sigma_M = \gamma\sigma$)
σ_M^*	variance of s.d.f.	0.62	model restriction ($\sigma_M^* = \gamma\sigma^*$)
Pareto distribution			
b	lower bound	1	normalization
k	shape parameter	1.7	Luttmer (2007)
Preferences			
η	el. of substitution	2.6	Broda and Weinstein (2006)
γ	risk aversion	20	Sharpe ratio
Trade and FDI costs			
τ	iceberg export cost	1.22	to match data
f_X	fixed export cost	0.33	to match data
F_X	sunk export cost	0.95	to match data
f_I	fixed FDI cost	0.55	to match data
F_I	sunk FDI cost	3.5	to match data
Aggregate demand			
$Q(0)$	initial domestic demand	3.5	to match data
$Q^*(0)$	initial foreign demand	3.5	to match data

Table 8: **Moments.** Comparison of the moments, model *versus* data. (Source: Compustat)

	Data	Model		Data	Model
$D \rightarrow D$ (%)	93.11	96.97	X (%)	29.2	26.79
$D \rightarrow X$ (%)	3.73	2.78	I (%)	35.99	33.15
$X \rightarrow X$ (%)	90.27	90.83			
$X \rightarrow I$ (%)	5.96	1.59			
$I \rightarrow I$ (%)	98.31	94.35			
$I \rightarrow X$ (%)	0.82	5.65			

Table 9: **Earnings Yields**. Summary statistics of earnings yields computed from simulated data, and comparison with real data. All values are in percentage terms.

	Mean (model)	Std. Dev. (model)	Mean (data)	Std. Dev. (data)
DOM	0.22	0.0008	2.12	3.54
EXP	0.71	0.0009	3.7	2.85
MN	1.63	0.0034	4.85	2.76

compute portfolio earnings, prices, value changes, earnings yields (earnings-to-price ratios), and returns. For each simulation, we compute the mean and standard deviation of earning yields and returns over time. Finally, we average our results across simulations. Table 9 reports the results for earnings yields.

The model generates average earnings yields of 1.63% for multinational firms, 0.71% for exporters, and 0.22% for firms selling only domestically, which are consistent with the ordering we found in the data. Table 10 reports the results for the returns. The model generates average returns of 4.56% for multinational firms, 4.25% for exporters, and -1.35% for firms selling only domestically, which are also consistent with the ordering we found in the data. While computed returns exhibit the expected ranking, there is no ranking in their volatility.[42]

6 Robustness

To be added.

[42] The small predicted volatilities are a consequence of the Monte-Carlo simulation procedure, as they are infact averages of standard deviations over simulations.

Table 10: **Returns**. Summary statistics of returns computed from simulated data, and comparison with real data. All values are in percentage terms.

	Mean (model)	Std. Dev. (model)	Mean (data)	Std. Dev. (data)
DOM	-1.35	0.1434	7.37	15.03
EXP	4.25	0.2173	10.69	15.34
MN	4.56	0.1981	11.62	12.86

7 Conclusions

This paper started by presenting a novel fact distinguishing multinational firms from exporters and from firms selling only domestically. Multinational corporations tend to have higher earnings yields and returns than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings yields than firms selling only in their domestic market. To explain this fact, we presented a real option value model where firms choose optimally whether to produce only domestically, export, or serve the foreign market through FDI. In equilibrium, firms imperfectly sort into the three statuses according to their productivity and to the realization of the shocks, and the option value – or the option of changing status – introduces a wedge between the firm's profits and its valuation, which generates the sought ranking in the financial variables.

While being consistent with a number of facts about trade and FDI dynamics, the model provides a complementary explanation for the cross section of returns by exploiting the production side from an international point of view. Firms selling in foreign markets are more exposed to aggregate risk since – in case of a negative shock – by exiting they would forgo the sunk cost that they paid to enter. This generates status persistence, and the risk of negative profits. Moreover, exiting is a more expensive strategy for multinationals than for exporters, who paid a lower premium and are hence less exposed and less risky: the difference in sunk costs generates a difference in exposure and in excess reutns.

The solution of the model delivers a series of predictions relating firms' productivity and the realization of the shocks to the pattern of status changes. These predictions find support in the switching pattern in the data. We calibrate the model to match quantitatively the persistence moments and other relevant moments on firm size and selection into export/FDI. With the parameterized model we compute moments of the financial variables from simulated data. We show that while matching fairly well the overall aggregate dynamics of trade and FDI, the model is also able to reproduce the ordering in earnings yields and returns we observe in the data.

We see this paper as the first step in a novel research agenda linking trade and FDI dynamics to asset pricing. Interesting extensions could include a careful analysis of the evolution of financial variables at the times of status switches, and the study of differential exit patterns of exporters *versus* multinational firms. We think this is a promising avenue for research in finance and international trade, that we plan to pursue in future work.

Appendix

A Accounting Standards and Data Selection

The empirical analysis contained in this paper is based on annual, firm-level data. We limit the present study to the universe of publicly traded, US based, manufacturing firms[1] included in the Standard & Poors Compustat Segments Database. COMPUSTAT data is comprised of key components from annual regulatory filings. Information on firms' foreign operations is included in the Segments Data. Segments data categorize a firm's operations along a particular business division[2] and report sales, asset, and other information based on these groups.

The Financial Accounting Standards Board (FASB), in its Statement No. 131, sets the standards for the way in which public businesses report information about operating segments in their annual financial statements. Operating segments are defined by the FASB as *"components of an enterprise about which separate financial information is available that is evaluated regularly by the chief operating decision maker in deciding how to allocate resources and in assessing performance"*.

FAS 131 establishes standards for the way firms should disclose data about products and services, geographic areas, and major customers. The FAS 131 determines that firms should report data about revenues derived from the firm's *products or services*, *countries* in which they earn revenues and hold assets, and about major *customers* regardless of whether that information is used in making operating decisions. However, the statement does not require firms to disclose the information on all the different segment types if it is not prepared for internal use and reporting would be impracticable. Therefore, the firms decide how to report the data, disaggregated in several different

[1]The NAICS code for manufacturing firms contain the 2 digit prefix 31, 32, or 33.
[2]The four segment classifications are business, geographic, operating, and state.

ways: either by product, geography, legal entity, or by customer, but they do not necessarily have to report all of them. This method is referred to as the management approach. The statement establishes a minimum threshold to report separately information about an operating segment: either revenues of the segment are 10% or more of the combined revenue of all operating segments, or profits or losses are 10% or more of the combined reported profit or losses, or its assets are 10% or more of the combined assets of all operating segments. Hence, if a given firm considers best practice to aggregate the information upstream to the management level by customer, it may elect not to disclose geographical segments information. That contrasts with the previous FAS No. 14, superseded by FAS No. 131 in 1998, in which firms were required that the financial statements of a business include information about the enterprise's operations in different industries, its foreign operations by geographical area and export sales, and its major customers.

According to the new FAS 131, when an enterprise reports the existence of a geographical segment, it must report revenues and holdings of long-lived assets. This information may or may not be disaggregated by individual foreign country. In a sense, the new regulation goes towards a major disaggregation of the information, provided that it does not contrast with the normal management of the firm.

Faced with the potential measurements problems associated with the loose reporting requirements of the Compustat Segments, we had two options to select our dataset: 1) include in the dataset only those firms that reported the existence of operating segments and drop all the others, or 2) include all firms in Compustat and impute as Domestic the status for those firms that did not report the existence of operating segments. The data analysis reported in Section 2 corresponds to the first selection criterium, which we prefer, because it generates a cleaner, albeit smaller, dataset. For robustness, we run all the regressions also using the dataset constructed with the second selection criterium, and the results on the ranking of earnings yields and returns are unchanged.

The relevant segment for our classification of firms by status division is the geographic segment. 96% of the firms that reported the existence of operating segments also report the existence of a geographical segment. Segments that report only domestic sales are classified as domestic, those that report positive export sales are classified as exports are exporters, and those that report positive foreign sales are classified as multinational. For the remaining firms, we aggregate data from the business segment and assume the firm's operations are wholely domestic. However, due to reporting errors, missclassifications, and multiple classifications, a few notes are required.

As is typical when a data point is unreliable, unreported, or otherwise a break from the traditional definition the provider will report codes in place of an interpretable value. Compustat employs a similar methodology. In these instances we assume the segment to be of negligible importance and consider associated sales and exports to be null. As mentioned above our implementation of segment data relies entirely on the classification provided in the data. However, in a few instances sales for the non-domestic segments indicate the market of operation as the United States. In these cases we assume the reported classification was in error and re-classify the segment as domestic.

The data is then aggregated by firm-year. For many firms this aggregation requires combining multiple segments and may result in competing classifications for a firm in a particular year. In these instances we classify the firm by the most "advanced" reported segment (for example domestic firms with exports are classified as exporters, while exporters with a foreign sales are considered multinational). Once firms have been appropriately classified by their international status we are able to observe the dynamics of this classification.

Examining a firm's international classification over time reveals what we believe to be reporting errors. These cases are characterized by a one-year "downward" status change, which results in a return to the original status. We believe this transient status change is a result of inaccurate segment reporting. As such we re-classify the observation to ensure continuity in the series. However, the opposite is not true: when a firm enters into an international market only to return to a less advanced geographic segment the following year, that firm retains the reported classification. The logic for this is evident - it is far easier to omit classification in a given year than to report segment details that were nonexistent.

Another dimension of selecting the data involves which criteria to use to establish the unit of observation and to eliminate outliers. The data span 28 years, from 1979 to 2006,[3] but many firms have observations only for a part of this time interval. We impose a lower bound of 6 yearly observation for a firm to be included in our sample. Additionally, we remove extreme observations in each international classification by dropping the top and bottom 1% of earnings-to-price ratios by group and the top and bottom 5% of returns by group.

[3] All variables have been deflated to constant 1984 dollars.

B Proofs

B.1 Derivation of the Value Functions

In this section we present the details of the derivation of the value functions in Section 3.

Starting from the no-arbitrage condition in equation (17):

$$\pi_D(a,Q)Mdt + E[d(M \cdot \mathcal{S}(a,Q))] + E[d(M \cdot V_D(a,Q^*))] = 0,$$

the terms in the expectations can be written as:

$$\begin{aligned} E[d(M \cdot \mathcal{S})] &= E[dM \cdot \mathcal{S} + M \cdot d\mathcal{S} + dM \cdot d\mathcal{S}] \\ &= M \cdot S \cdot E\left[\frac{dM}{M} + \frac{d\mathcal{S}}{\mathcal{S}} + \frac{dM}{M} \cdot \frac{d\mathcal{S}}{\mathcal{S}}\right] \\ &= M \cdot S\left[-rdt + E\left(\frac{d\mathcal{S}}{dS}\right) + E\left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{dS}\right)\right] \\ &= Mdt\left[-r\mathcal{S} + E\left(\frac{d\mathcal{S}}{dt}\right) + E\left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{dt}\right)\right] \end{aligned} \tag{A.1}$$

$$\begin{aligned} E[d(M \cdot V_D)] &= E[dM \cdot V_D + M \cdot dV_D + dM \cdot dV_D] \\ &= M \cdot V_D \cdot E\left[\frac{dM}{M} + \frac{dV_D}{V_D} + \frac{dM}{M} \cdot \frac{dV_D}{V_D}\right] \\ &= M \cdot V_D\left[-rdt + E\left(\frac{dV_D}{V_D}\right) + E\left(\frac{dM}{M} \cdot \frac{dV_D}{V_D}\right)\right] \\ &= Mdt\left[-rV_D + E\left(\frac{dV_D}{dt}\right) + E\left(\frac{dM}{M} \cdot \frac{dV_D}{dt}\right)\right] \end{aligned} \tag{A.2}$$

where the dependence of $\mathcal{S}$ on (a,Q) and the dependence of V_D on (a,Q^*) have been suppressed to ease the notation. Plugging (A.1) and (A.2) into the no-arbitrage condition:

$$\pi_D - r\mathcal{S} + E\left(\frac{d\mathcal{S}}{dt}\right) + E\left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{dt}\right) - rV_D + E\left(\frac{dV_D}{dt}\right) + E\left(\frac{dM}{M} \cdot \frac{dV_D}{dt}\right) = 0. \tag{A.3}$$

By applying Ito's Lemma and using the expressions for the Brownian motions ruling Q and Q^*, we can derive expressions for some of the terms in (A.3):

$$
\begin{aligned}
d\mathcal{S} &= \mathcal{S}'dQ + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt = \mathcal{S}'[\mu Q dt + \sigma Q dz] + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt \\
E[d\mathcal{S}] &= \mu Q \mathcal{S}'dt + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt \\
dV_D &= V_D' dQ^* + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt = V_D'[\mu^* Q^* dt + \sigma^* Q^* dz^*] + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt \\
E[dV_D] &= \mu^* Q^* V_D' dt + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt\ .
\end{aligned}
$$

Using these results and the expression for the evolution of M, we can rewrite (A.3) as:

$$
\begin{aligned}
&\pi_D - r\mathcal{S} + \mu Q \mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' + E\left[(-rdt + \sigma_M dz)\cdot\left(\mu Q \mathcal{S}' + \sigma Q \mathcal{S}'\frac{dz}{dt} + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''\right)\right] - ... \\
&...rV_D + \mu^* Q^* V_D' + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' + E\left[(-rdt + \sigma_M dz)\cdot\left(\mu^* Q^* V_D' + \sigma^* Q^* V_D'\frac{dz^*}{dt} + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D''\right)\right] = 0
\end{aligned}
$$

and multiplying every term for dt:

$$
\begin{aligned}
&\pi_D dt - r\mathcal{S}dt + \mu Q \mathcal{S}'dt + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt + E\left[(-rdt + \sigma_M dz)\cdot\left(\mu Q \mathcal{S}'dt + \sigma Q \mathcal{S}'dz + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt\right)\right] - ... \\
&...rV_D dt + \mu^* Q^* V_D' dt + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt + ... \\
&...E\left[(-rdt + \sigma_M dz)\cdot\left(\mu^* Q^* V_D' dt + \sigma^* Q^* V_D' dz^* + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt\right)\right] = 0. \qquad \text{(A.4)}
\end{aligned}
$$

For the properties of Brownian motions, the terms in expectation can be reduced to:

$$
\begin{aligned}
& E\left[(-rdt + \sigma_M dz)\cdot\left(\mu Q \mathcal{S}'dt + \sigma Q \mathcal{S}'dz + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt\right)\right] = ... \\
... &= E\left[-r\mu Q \mathcal{S}'dt^2 - r\sigma Q \mathcal{S}'dtdz - r\frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt^2 + \sigma_M \mu Q \mathcal{S}'dzdt + \sigma_M \sigma Q \mathcal{S}'dz^2 + \sigma_M \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dzdt\right] = ... \\
... &= \sigma_M \sigma Q \mathcal{S}' E(dz^2) = ... \\
... &= \sigma\sigma_M Q \mathcal{S}'dt.
\end{aligned}
$$

$$
\begin{aligned}
& E\left[(-rdt+\sigma_M dz)\cdot\left(\mu^* Q^* V_D' dt+\sigma^* Q^* V_D' dz^*+\frac{1}{2}(\sigma^*)^2(Q^*)^2 V_D'' dt\right)\right]=... \\
... \quad = \quad & E\Big[-r\mu^* Q^* V_D' dt^2 - r\sigma^* Q^* V_D' dtdz^* - r\frac{1}{2}(\sigma^*)^2(Q^*)^2 V_D'' dt^2 + \sigma_M \mu^* Q^* V_D' dzdt + ... \\
& ... + \sigma_M \sigma^* Q^* V_D' dzdz^* + \sigma_M \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dzdt\Big] = ... \\
... \quad = \quad & \sigma^* \sigma_M Q^* V_D' E(dzdz^*) = ... \\
... \quad = \quad & \rho\sigma^* \sigma_M Q^* V_D' dt.
\end{aligned}
$$

So we can rewrite (A.4) as:

$$
\begin{aligned}
\pi_D - r\mathcal{S} + \mu Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' + \sigma\sigma_M Q\mathcal{S}' - rV_D + \mu^* Q^* V_D' + \frac{1}{2}(\sigma^*)^2(Q^*)^2 V_D'' + \rho\sigma^*\sigma_M Q^* V_D' &= 0 \\
\pi_D - r\mathcal{S} + (\mu+\sigma\sigma_M) Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' - rV_D + (\mu^* + \rho\sigma^*\sigma_M) Q^* V_D' + \frac{1}{2}(\sigma^*)^2(Q^*)^2 V_D'' &= 0
\end{aligned}
$$

which is equation (18) in the paper. One possible solution of this equation is such that:

$$
\begin{aligned}
\pi_D(a,Q) - r\mathcal{S}(a,Q) + (\mu+\sigma\sigma_M)Q\mathcal{S}'(a,Q) + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''(a,Q) &= 0 \qquad \text{(A.5)} \\
-rV_D(a,Q^*) + (\mu^*+\rho\sigma^*\sigma_M)Q^* V_D'(a,Q^*) + \frac{1}{2}(\sigma^*)^2(Q^*)^2 V_D''(a,Q^*) &= 0. \qquad \text{(A.6)}
\end{aligned}
$$

We start by deriving the value function $\mathcal{S}(a,Q)$, solution of (A.5). Let $\hat{\mu} \equiv \mu + \sigma\sigma_M$ and recall that $\pi_D(a,Q) = B(aw)^{1-\eta}P^\eta Q \equiv \mathcal{P}_D(a)Q$:

$$
-r\mathcal{S} + \hat{\mu} Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' = -\mathcal{P}_D(a)Q.
$$

This differential equation admits a solution of the form $\mathcal{S}(a,Q) = Q^\chi + cQ$, where χ is the solution of the following quadratic equation:

$$
\begin{aligned}
-r + \left(\hat{\mu} - \frac{1}{2}\sigma^2\right)\chi + \frac{1}{2}\sigma^2\chi^2 &= 0 \\
-\bar{r}_s + (m_s - 1)\chi + \chi^2 &= 0
\end{aligned}
$$

where $m_s = \frac{2\hat{\mu}}{\sigma^2}$ and $\bar{r}_s = \frac{2r}{\sigma^2}$. Hence:

$$
\chi = \frac{(1-m_s) \pm \sqrt{(1-m_s)^2 + 4\bar{r}_s}}{2}.
$$

Similarly, c is the solution of:

$$
\begin{aligned}
-rc + c\hat{\mu} &= -\mathcal{P}_D \\
c &= \frac{\mathcal{P}_D}{r - \hat{\mu}}.
\end{aligned}
$$

Hence the value function describing the expected present discounted value of domestic profits takes the form:

$$\mathcal{S}(a, Q) = A_{\mathcal{S}}(a)Q^{\alpha_s} + B_{\mathcal{S}}(a)Q^{\beta_s} + \frac{\mathcal{P}_D Q}{r - \hat{\mu}}$$

where α_s and β_s are the negative and positive value of χ, respectively, and $A_{\mathcal{S}}(a)$ and $B_{\mathcal{S}}(a)$ are firm-specific parameters to be determined. Since there is no option value associated to domestic profits, we can impose: $A_{\mathcal{S}}(a) = B_{\mathcal{S}}(a) = 0$, so that the solution is simply given by the value of profits discounted with the risk-adjusted measure:

$$\mathcal{S}(a, Q) = \frac{\mathcal{P}_D Q}{r - \hat{\mu}} = \frac{B(aw)^{1-\eta} P^{\eta} Q}{r - (\mu + \sigma\sigma_M)} \ . \tag{A.7}$$

In the same way we can derive the value function $V_D(a, Q^*)$, solution of (A.6). Let $\tilde{\mu} \equiv \mu^* + \rho\sigma^*\sigma_M$:

$$-rV_D + \tilde{\mu}Q^*V_D' + \frac{1}{2}(\sigma^*)^2(Q^*)^2V_D'' = 0.$$

This differential equation admits a solution of the form $V_D(a, Q^*) = (Q^*)^{\xi}$, where ξ is the solution of the following quadratic equation:

$$
\begin{aligned}
-r + \left(\tilde{\mu} - \frac{1}{2}(\sigma^*)^2\right)\xi + \frac{1}{2}(\sigma^*)^2\xi^2 &= 0 \\
-\bar{r} + (m-1)\xi + \xi^2 &= 0
\end{aligned}
$$

where $m = \frac{2\tilde{\mu}}{(\sigma^*)^2}$ and $\bar{r} = \frac{2r}{(\sigma^*)^2}$. Hence:

$$\xi = \frac{(1-m) \pm \sqrt{(1-m)^2 + 4\bar{r}}}{2}.$$

The value function describing the expected present discounted value of foreign profits of a domestic firm takes the form:

$$V_D(a, Q^*) = A_D(a)(Q^*)^{\alpha} + B_D(a)(Q^*)^{\beta} \tag{A.8}$$

where α and β are the negative and positive value of ξ, respectively.

By following the same procedure, we can solve for the expected value of foreign profits of an exporter, $V_X(a, Q^*)$, and of a multinational, $V_I(a, Q^*)$:

$$V_X(a, Q^*) = A_X(a)(Q^*)^\alpha + B_X(a)(Q^*)^\beta + \frac{B(\tau a w)^{1-\eta}(P^*)^\eta Q^*}{r - (\mu^* + \rho\sigma^*\sigma_M)} - \frac{f_X}{r} \tag{A.9}$$

$$V_I(a, Q^*) = A_I(a)(Q^*)^\alpha + B_I(a)(Q^*)^\beta + \frac{B(a w^*)^{1-\eta}(P^*)^\eta Q^*}{r - (\mu^* + \rho\sigma^*\sigma_M)} - \frac{f_I}{r}. \tag{A.10}$$

B.2 Proof of Theorem 1: Ordering of the Quantity Thresholds

It is easy to prove that, if (11) holds, the following order of the deterministic thresholds holds:

$$W_{IX} < W_{ID} < W_{XD} < W_{DX} < W_{DI} < W_{XI}.$$

Dixit and Pindyck (1994) show that hysteresis, here defined as the difference $Q_{RS} - W_{RS}$, for $R, S, \in \{D, X, I\}$ is increasing in sunk costs. Hence: $Q_{DI} - W_{DI} > Q_{DX} - W_{DX}$. Since $W_{DI} > W_{DX}$, we have $Q_{DI} > Q_{DX}$. By applying the same reasoning, one can also show that $Q_{XI} > Q_{DI}$. Symmetrically, one can also show the ordering of the exit thresholds, so that:

$$Q_{IX} < Q_{ID} < Q_{XD} < Q_{DX} < Q_{DI} < Q_{XI}.$$

B.3 Proof of Theorem 2: Monotonicity of the Quantity Thresholds

Everything else constant, a higher productivity $1/a$ is equivalent to a lower variable operating cost. Dixit and Pindyck (1994), pp. 221-223, show that entry and exit thresholds are increasing in operating costs, hence $\frac{\partial Q_{RS}}{\partial(a)} > 0$.

B.4 Proof of Theorem 3: Hysteresis and Productivity

To be added.

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