When is the Government Spending Multiplier Large?

Lawrence Christiano, Martin Eichenbaum, and Sergio Rebelo

Northwestern University

September 2009

Abstract

When the nominal interest rate is constant.

1. Introduction

A classic question in macroeconomics is: what is the size of the government-spending multiplier? There is a large empirical literature that grapples with this question. Authors such as Barro (1981) argue that the multiplier is around 0.8 while authors such as Ramey (2008) estimate the multiplier to be closer to 1.2.[1] There is also a large literature that uses general-equilibrium models to study the size of the government-spending multiplier. In standard new-Keynesian models the government-spending multiplier can be somewhat above or below one depending on the exact specification of agent's preferences (see Gali, López-Salido, and Vallés (2007) and Monacelli and Perotti (2008)). In frictionless real-business-cycle models this multiplier is typically less than one (see e.g. Aiyagari, Christiano, and Eichenbaum (1992), Baxter and King (1993), Burnside, Eichenbaum and Fisher (2004), Ramey and Shapiro (1998), and Ramey (2008)). Viewed overall it is hard to argue, based on the literature, that the government-spending multiplier is substantially larger than one.

In this paper we argue that the government-spending multiplier can be much larger than one when the nominal interest rate does not respond to an increase in government spending. We develop this argument in a model where the multiplier is quite modest if the nominal interest rate is governed by a Taylor rule. When such a rule is operative the nominal interest rate rises in response to an expansionary fiscal policy shock that puts upward pressure on output and inflation.

There is a natural scenario in which the nominal interest rate does not respond to an increase in government spending: when the zero lower bound on the nominal interest rate binds. We find that the multiplier is very large in economies where the output cost of being in the zero bound state is also large. In such economies

[1] For recent contributions to the VAR-based empirical literature on the size of the government-spending multiplier see Fisher and Peters (2009) and Ilzetzki, Mendoza, and Vegh (2009).

it can be socially optimal to substantially raise government spending in response to shocks that make the zero lower bound on the nominal interest rate binding.

We begin by considering an economy with Calvo-style price frictions, no capital and a monetary authority that follows a standard Taylor rule. Building on insights in Eggertsson and Woodford (2003), we study the effect of a temporary, unanticipated rise in agents' discount factor. Other things equal, the shock to the discount factor increases desired saving. Since investment is zero in this economy, aggregate saving must be zero in equilibrium. When the shock is small enough, the real interest rate falls and there is a modest decline in output. However, when the shock is large enough, the zero bound becomes binding before the real interest rate falls by enough to make aggregate saving zero. In this model, the only force that can induce the fall in saving required to re-establish equilibrium is a large, transitory fall in output.

Why is the fall in output so large when the economy hits the zero bound? For a given fall in output, marginal cost falls and prices decline. With staggered pricing, the drop in prices leads agents to expect future deflation. With the nominal interest rate stuck at zero the real interest rate rises. This perverse rise in the real interest rate leads to an increase in desired saving which partially undoes the effect of a given fall in output. So, the total fall in output required to reduce desired saving to zero is very large.

This scenario resembles the paradox of thrift originally emphasized by Keynes (1936) and recently analyzed by Krugman (1998), Eggertsson and Woodford (2003), and Christiano (2004). In the textbook version of this paradox, prices are constant and an increase in desired saving lowers equilibrium output. But, in contrast to the textbook scenario, the zero-bound scenario studied in the modern literature involves a deflationary spiral which contributes to and accompanies the large fall in output.

Consider now the effect of an increase in government spending when the zero bound is strictly binding. This increase leads to a rise in output, marginal cost and expected inflation. With the nominal interest rate stuck at zero, the rise in expected inflation drives down the real interest rate which drives up private spending. This rise in spending leads to a further rise in output, marginal cost, and expected inflation and a further decline in the real interest rate. The net result is a large rise in inflation and output. In effect, the increase in government consumption unleashes an inflationary spiral that counteracts the deflationary spiral associated with the zero bound state.

The exact value of the government-spending multiplier depends on a variety of factors. However, we show that this multiplier is large in economies in which the output cost associated with the zero bound problem is more severe. We argue this point in two ways. First, we show that the value of the government-spending multiplier can depend sensitively on the model's parameter values. But, parameter values which are associated with large declines in output when the zero bound binds are also associated with large values of the government-spending multiplier. Second, we show that the value of the government-spending multiplier is positively related to how long the zero bound is expected to bind.

An important practical objection to using fiscal policy to counteract a contraction associated with the zero-bound state is that there are long lags in implementing increases in government spending. Motivated by this consideration, we study the size of the government-spending multiplier in the presence of implementation lags. We find that a key determinant of the size of the multiplier is the state of the world in which new government spending comes on line. If it comes on line in future periods when the nominal interest rate is zero then there is a large effect on current output. If it comes on line in future periods where the nominal interest rate is positive, then the current effect on government spending is smaller. So

our analysis supports the view that, for fiscal policy to be effective, government spending must come online in a timely manner.

In the second step of our analysis we incorporate capital accumulation into the model. In addition to the discount factor shock we also allow for a neutral technology shock and an investment-specific shock. For computational reasons we consider temporary shocks that make the zero bound binding for a deterministic number of periods. Again, we find that the government-spending multiplier is larger when the zero bound binds. Allowing for capital accumulation has two effects. First, for a given size shock it reduces the likelihood that the zero bound becomes binding. Second, when the zero bound binds, the presence of capital accumulation tends to increase the size of the government-spending multiplier. The intuition for this result is that, in our model, investment is a decreasing function of the real interest rate. When the zero bound binds, the real interest rate generally rises. So, other things equal, saving and investment diverge as the real interest rate rises, thus exacerbating the meltdown associated with the zero bound. As a result, the fall in output necessary to bring saving and investment into alignment is larger than in the model without capital.

The simple models discussed above suggest that the multiplier can be large in the zero bound state. The obvious next step would be to use reduced form methods, such as identified VARs, to estimate the government-spending multiplier when the zero bound binds. Unfortunately, this task is fraught with difficulties. First, we cannot mix evidence from states where the zero bound binds with evidence from other states because the multipliers are very different in the two states. Second, we have to identify exogenous movements in government spending when the zero bound binds.[2] This task seems daunting at best. Almost surely gov-

[2]To see how critical this step is, suppose that the government chooses spending to keep output exactly constant in the face of shocks that make the zero bound bind. A naive econometrician who simply regressed output on government spending would falsely conclude that the

ernment spending would rise in response to large output losses in the zero bound state. To know the government spending multiplier we need to know what output would have been had government spending not risen. For example, the simple observation that output did not grow quickly in Japan in the zero bound state, even though there were large increases in government spending, tells us nothing about the question of interest.

Given these difficulties, we investigate the size of the multiplier in the zero bound state using the empirically plausible DSGE model proposed by Altig, Christiano, Eichenbaum and Lindé (2004) (henceforth ACEL). This model incorporates price and wage setting frictions, habit formation in consumption, variable capital utilization and investment adjustment costs of the sort proposed by Christiano, Eichenbaum and Evans (2005) (henceforth CEE). ACEL estimate the parameters of their model to match the impulse response function of ten macro variables to a monetary shock, a neutral technology shock, and a capital-embodied technology shock.

Our key findings based on the ACEL model can be summarized as follows. First, when the central bank follows a Taylor rule the value of the government-spending multiplier is generally less than one. Second, the multiplier is much larger if the nominal interest rate does not respond to the rise in government spending. For example, suppose that government spending goes up for eight quarters and the nominal interest rate remains constant. In this case the impact multiplier is roughly two. Third, the value of the multiplier depends critically on how much government spending occurs in the period during which the nominal interest rate is constant. The larger is the fraction of government spending that occurs while the nominal interest rate is constant, the smaller is the value of the

government spending multiplier is zero. This example is, of course, just an application of Tobin's (1970) post hoc ergo propter hoc argument.

multiplier. Consistent with the theoretical analysis above, this result implies that for government spending to be a powerful weapon in combating output losses associated with the zero bound state, it is critical that the bulk of the spending come on line when the lower bound is actually binding.

As emphasized by Eggertsson and Woodford (2003), an alternative way to escape the negative consequences of a shock that makes the zero bound binding is for the central bank to commit to future inflation. We abstract from this possibility in this paper. We do so for a number of reasons. First, this theoretical possibility is well understood. Second, we do not think that it is easy in practice for the central bank to credibly commit to future high inflation. Third, the optimal trade-off between higher government purchases and anticipated inflation depends sensitively on how agents value government purchases and the costs of anticipated inflation. Studying this issue is an important topic for future research.

Our analysis is related to several recent papers on the zero bound. Eggertson (2009) focuses on the effects of transitory tax cuts when the zero bound on nominal interest rates binds. Bodenstein, Erceg, and Guerrieri (2009) analyze the effects of shocks to open economies when the zero bound binds. Braun and Waki (2006) use a model in which the zero bound binds to account for Japan's experience in the 1990s. Their results for fiscal policy are broadly consistent with our results. Braun and Waki (2006) and Coenen and Wieland (2003) investigate whether alternative monetary policy rules could have avoided the zero bound state in Japan.

Our paper is organized as follows. In section 2 we analyze the size of the government-spending multiplier when the interest follows a Taylor rule in a standard new-Keynesian model without capital. In section 3 we modify the analysis to assume that the nominal interest rate does not respond to an increase in government spending, say because the lower bound on the nominal interest rate binds. In section 4 we extend the model to incorporate capital. In section 5 we discuss

the properties of the government-spending multiplier in the medium size DSGE model proposed by ACEL. Section 6 concludes.

2. The standard multiplier in a model without capital

In this section we present a simple new-Keynesian model and analyze its implications for the size of the "standard multiplier," by which we mean the size of the government-spending multiplier when the nominal interest rate is governed by a Taylor rule.

Households The economy is populated by a representative household, whose life-time utility, U, is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left[C_t^{\gamma} (1 - N_t)^{1-\gamma} \right]^{1-\sigma} - 1}{1 - \sigma} + v\left(G_t\right) \right]. \tag{2.1}$$

Here E_0 is the conditional expectation operator, and C_t, G_t, and N_t denote time-t consumption, government consumption, and hours worked, respectively. We assume that $\sigma > 0$, $\gamma \in (0, 1)$, and that $v(.)$ is a concave function.

The household budget constraint is given by:

$$P_t C_t + B_{t+1} = B_t \left(1 + R_t\right) + W_t N_t + T_t, \tag{2.2}$$

where T_t denotes firms' profits net of lump-sum taxes paid to the government. The variable B_{t+1} denotes the quantity of one-period bonds purchased by the household at time t. Also, P_t denotes the price level and W_t denotes the nominal wage rate. Finally, R_t denotes the one-period nominal rate of interest that pays off in period t. The household's problem is to maximize utility given by equation (2.1) subject to the budget constraint given by equation (2.2) and the condition $E_0 \lim_{t \to \infty} B_{t+1} / [(1 + R_0)(1 + R_1)...(1 + R_t)] \geq 0$.

Firms The final good is produced by competitive firms using the technology,

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1, \tag{2.3}$$

where $Y_t(i)$, $i \in [0,1]$ denotes intermediate good i.

Profit maximization implies the following first-order condition for $Y_t(i)$:

$$P_t(i) = P_t \left(\frac{Y_t}{Y_t(i)} \right)^{\frac{1}{\varepsilon}}, \tag{2.4}$$

where $P_t(i)$ denotes the price of intermediate good i and P_t is the price of the homogeneous final good.

The intermediate good, $Y_t(i)$, is produced by a monopolist using the following technology:

$$Y_t(i) = N_t(i),$$

where $N_t(i)$ denotes employment by the i^{th} monopolist. We assume there is no entry or exit into the production of the i^{th} intermediate good. The monopolist is subject to Calvo-style price-setting frictions and can optimize its price, $P_t(i)$, with probability $1-\theta$. With probability θ the firm sets:

$$P_t(i) = P_{t-1}(i).$$

The discounted profits of the i^{th} intermediate good firm are:

$$E_t \sum_{j=0}^{\infty} \beta^{t+j} \upsilon_{t+j} \left[P_{t+j}(i) Y_{t+j}(i) - (1-\nu) W_{t+j} N_{t+j}(i) \right], \tag{2.5}$$

where $\nu = 1/\varepsilon$ denotes an employment subsidy which corrects, in steady state, the inefficiency created by the presence of monopoly power. The variable υ_{t+j} is the multiplier on the household budget constraint in the Lagrangian representation of the household problem. The variable W_{t+j} denotes the nominal wage rate.

Firm i maximizes its discounted profits, given by equation (2.5), subject to the Calvo price-setting friction, the production function, and the demand function for $Y_t(i)$, given by equation (2.4).

Monetary policy We assume that monetary policy follows the rule:

$$R_{t+1} = \max(Z_{t+1}, 0), \tag{2.6}$$

where

$$Z_{t+1} = (1/\beta)(1+\pi_t)^{\phi_1(1-\rho_R)}(Y_t/Y)^{\phi_2(1-\rho_R)}\left[\beta\left(1+R_t\right)\right]^{\rho_R} - 1.$$

Throughout the paper a variable without a time subscript denotes its steady state value, e.g. the variable Y denotes the steady-state level of output. The variable π_t denotes the time-t rate of inflation. We assume that $\phi_1 > 1$ and $\phi_2 \in (0,1)$.

According to equation (2.6) the monetary authority follows a Taylor rule as long as the implied nominal interest rate is non-negative. Whenever the Taylor rule implies a negative nominal interest rate, the monetary authority simply sets the nominal interest rate to zero. For convenience we assume that steady-state inflation is zero. This assumption implies that the steady-state net nominal interest rate is $1/\beta - 1$.

Fiscal policy As long as the zero bound on the nominal interest rate is not binding, government spending evolves according to:

$$G_{t+1} = G_t^{\rho}\exp(\eta_{t+1}). \tag{2.7}$$

Here G is the level of government spending in the non-stochastic steady state and η_{t+1} is an i.i.d. shock with zero mean. To simplify our analysis, we assume that government spending and the employment subsidy are financed with lump-sum taxes. The exact timing of these taxes is irrelevant because Ricardian equivalence holds under our assumptions. We discuss the details of fiscal policy when the zero bound is binding in Section 3.

Equilibrium The economy's resource constraint is:

$$C_t + G_t = Y_t. \tag{2.8}$$

A 'monetary equilibrium' is a collection of stochastic processes,

$$\{C_t, N_t, W_t, P_t, Y_t, R_t, P_t(i), Y_t(i), N_t(i), \upsilon_t, B_{t+1}, \pi_t\},$$

such that for given $\{G_t\}$ the household and firm problems are satisfied, the monetary and fiscal policy rules are satisfied, markets clear, and the aggregate resource constraint is satisfied.

To solve for the equilibrium we use a linear approximation around the non-stochastic steady state of the economy. Throughout, $\hat{Z}_t$ denotes the percentage deviation of Z_t from its non-stochastic steady state value, Z. The equilibrium is characterized by the following set of equations.

The Phillips curve for this economy is given by:

$$\pi_t = E_t\left(\beta\pi_{t+1} + \kappa\widehat{MC}_t\right), \tag{2.9}$$

where $\kappa = (1-\theta)(1-\beta\theta)/\theta$. In addition, MC_t denotes real marginal cost which, under our assumptions, is equal to the real wage rate. Absent labor market frictions, the percent deviation of real marginal cost from its steady state value is given by:

$$\widehat{MC}_t = \hat{C}_t + \frac{N}{1-N}\hat{N}_t. \tag{2.10}$$

The linearized intertemporal Euler equation for consumption is:

$$\begin{aligned} & [\gamma(1-\sigma)-1]\hat{C}_t - (1-\gamma)(1-\sigma)\frac{N}{1-N}\hat{N}_t \qquad (2.11)\\ = \; & E_t\left\{\beta(R_{t+1}-R) - \pi_{t+1} + [\gamma(1-\sigma)-1]\hat{C}_{t+1} - (1-\gamma)(1-\sigma)\frac{N}{1-N}\hat{N}_{t+1}\right\} \end{aligned}$$

The linearized aggregate resource constraint is:

$$\hat{Y}_t = (1-g)\,\hat{C}_t + g\hat{G}_t, \tag{2.12}$$

where $g = G/Y$.

Combining equations (2.9) and (2.10) and using the fact that $\hat{N}_t = \hat{Y}_t$ we obtain:

$$\pi_t = \beta E_t\left(\pi_{t+1}\right) + \kappa\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}_t - \frac{g}{1-g}\hat{G}_t\right]. \tag{2.13}$$

Similarly, combining equations (2.11) and (2.12) and using the fact that $\hat{N}_t = \hat{Y}_t$ we obtain:

$$\begin{aligned}&\hat{Y}_t - g\left[\gamma\left(\sigma-1\right)+1\right]\hat{G}_t = \\ &E_t\left\{-\left(1-g\right)\left[\beta\left(R_{t+1}-R\right)-\pi_{t+1}\right] + \hat{Y}_{t+1} - g\left[\gamma\left(\sigma-1\right)+1\right]\hat{G}_{t+1}\right\}.\end{aligned} \tag{2.14}$$

As long as the zero bound on the nominal interest rate does not bind, the linearized monetary policy rule is given by:

$$R_{t+1} - R = \rho_R\left(R_t - R\right) + \frac{1-\rho_R}{\beta}\left(\phi_1\pi_t + \phi_2\hat{Y}_t\right).$$

Whenever the zero bound binds, $R_{t+1} = 0$.

We solve for the equilibrium using the method of undetermined coefficients. For simplicity, we begin by considering the case in which $\rho_R = 0$. Under the assumption that $\phi_1 > 1$, there is a unique linear equilibrium in which π_t and $\hat{Y}_t$ are given by:

$$\pi_t = A_\pi\hat{G}_t, \tag{2.15}$$

$$\hat{Y}_t = A_Y\hat{G}_t. \tag{2.16}$$

The coefficients A_π and A_Y are given by:

$$A_\pi = \frac{\kappa}{1-\beta\rho}\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)A_Y - \frac{g}{1-g}\right], \tag{2.17}$$

$$A_Y = g\frac{(\rho-\phi_1)\,\kappa-[\gamma\,(\sigma-1)+1]\,(1-\rho)\,(1-\beta\rho)}{(1-\beta\rho)\,[\rho-1-(1-g)\,\phi_2]+(1-g)\,(\rho-\phi_1)\,\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)}. \quad (2.18)$$

The effect of an increase in government spending Using equation (2.12) we can write the government-spending multiplier as:

$$\frac{dY_t}{dG_t} = \frac{1}{g}\frac{\hat{Y}_t}{\hat{G}_t} = 1+\frac{1-g}{g}\frac{\hat{C}_t}{\hat{G}_t}. \quad (2.19)$$

This equation implies that the multiplier is less than one whenever consumption falls in response to an increase in government spending. Equation (2.16) implies that the government-spending multiplier is given by:

$$\frac{dY_t}{dG_t} = \frac{A_Y}{g}. \quad (2.20)$$

From this equation we see that the multiplier is constant over time. To analyze the magnitude of the multiplier outside of the zero bound we consider the following baseline parameter values:

$$\theta = 0.85,\ \beta = 0.99,\ \phi_1 = 1.5,\ \phi_2 = 0,\ \gamma = 0.29,\ g = 0.2,\ \sigma = 2,\ \rho_R = 0,\ \rho = 0.8. \quad (2.21)$$

These parameter values imply that $\kappa = 0.03$ and $N = 1/3$. Our baseline parameter values imply that the government-spending multiplier is 1.05. Figure 1 displays the impulse response of output, inflation, and the nominal interest rate to a government spending shock.

In our model Ricardian equivalence holds. From the perspective of the representative household the increase in the present value of taxes equals the increase in the present value of government purchases. In a typical version of the standard neoclassical model we would expect some rise in output driven by the negative wealth effect on leisure of the tax increase. But in that model the multiplier is generally less than one because the wealth effect reduces private consumption.

From this perspective it is perhaps surprising that the multiplier in our baseline model is greater than one. This perspective neglects two key features of our model, the frictions in price setting and the complementarity between consumption and leisure in preferences. When government purchases increase, total demand, $C_t + G_t$, increases. Since prices are sticky, price over marginal cost falls after a rise in demand. As emphasized in the literature on the role of monopoly power in business cycles, the fall in the markup induces an outward shift in the labor demand curve. This shift amplifies the rise in employment following the rise in demand. Given our specification of preferences, $\sigma > 1$ implies that the marginal utility of consumption rises with the increase in employment. As long as this increase in marginal utility is large enough, it is possible for private consumption to actually rise in response to an increase in government purchases. Indeed, consumption does rise in our benchmark scenario which is why the multiplier is larger than one.

To assess the importance of our preference specification we redid our calculations using the basic specification for the momentary utility function commonly used in the new-Keynesian DSGE literature:

$$u = \left(C_t^{1-\varsigma} - 1\right) / \left(1 - \varsigma\right) - \eta N_t^{1+\vartheta} / \left(1 + \vartheta\right), \tag{2.22}$$

where, ς, η, and ϑ are positive. The key feature of this specification is that the marginal utility of consumption is independent of hours worked. Consistent with the intuition discussed above, we found that, across a wide set of parameter values, dY/dG is always less than one with this preference specification.[3]

To provide additional intuition for the determinants of the multiplier, Figure 2 displays dY/dG for various parameter configurations. In each case we perturb

[3]See Monacelli and Perotti (2008) for a discussion of the impact of preferences on the size of the government spending multiplier in models with Calvo-style frictions when the zero bound is not binding.

one parameter at a time relative to the benchmark parameter values. The $(1,1)$ element of Figure 2 shows that a rise in σ is associated with an increase in the multiplier. This result is consistent with the intuition above which builds on the observation that the marginal utility of consumption is increasing in hours worked. This dependence is stronger the higher is σ. Note that multiplier can be bigger than unity even for σ slightly less than unity. This result presumably reflects the positive wealth effects associated with the increased competitiveness of the economy associated with the reduction in the markup.

The $(1,2)$ element of Figure 2 shows that the multiplier is a decreasing function of κ. In other words, the multiplier is larger the higher is the degree of price stickiness. The result reflects the fall in the markup when aggregate demand and marginal cost rise. This effect is stronger the stickier are prices. The multiplier exceeds one for all $\kappa < 0.13$. In the limiting case when prices are perfectly sticky ($\kappa = 0$) the multiplier is given by:

$$\frac{dY_t}{dG_t} = \frac{\left[\gamma\left(\sigma - 1\right) + 1\right]\left(1 - \rho\right)}{1 - \rho + \left(1 - g\right)\phi_2} > 0.$$

Note that when $\phi_2 = 0$ the multiplier is greater than one as long as σ is greater than one.

When prices are perfectly flexible ($\kappa = \infty$) the markup is constant. In this case the multiplier is less than one:

$$\frac{dY_t}{dG_t} = \frac{1}{1 + \left(1 - g\right)\frac{N}{1-N}} < 1.$$

This result reflects the fact that with flexible prices an increase in government spending has no impact on the markup. As a result, the demand for labor does not rise as much as in the case in which prices are sticky.

The $(1,3)$ element of Figure 2 shows that as ϕ_1 increases, the multiplier falls. The intuition for this effect is that the expansion in output increases marginal

cost which in turn induces a rise in inflation. According to equation (2.6) the monetary authority increases the interest rate in response to a rise in inflation. The rise in the interest rate is an increasing function of ϕ_1. Higher values of ϕ_1 lead to higher values of the real interest rate which are associated with lower levels of consumption. So, higher values of ϕ_1 lead to lower values of the multiplier.

The $(2,1)$ element of Figure 2 shows that as ϕ_2 increases, the multiplier falls. The intuition underlying this effect is similar to that associated with ϕ_1. When ϕ_2 is large there is a substantial increase in the real interest rate in response to a rise in output. The contractionary effects of the rise in the real interest rate on consumption reduce the size of the multiplier.

The $(2,2)$ element of Figure 2 shows that as ρ_R increases the multiplier rises. The intuition for this result is as follows. The higher is ρ_R the less rapidly the monetary authority increases the interest rate in response to the rise in marginal cost and inflation that occur in the wake of an increase in government purchases. This result is consistent with the traditional view that the government-spending multiplier is greater in the presence of accommodative monetary policy. By accommodative we mean that the monetary authority raises interest rates slowly in the presence of a fiscal expansion.

The $(2,3)$ element of Figure 2 shows that the multiplier is a decreasing function of the parameter governing the persistence of government purchases, ρ. The intuition for this result is that the present value of taxes associated with a given innovation in government purchases is an increasing function of ρ. So the negative wealth effect on consumption is an increasing function of ρ.

We conclude this subsection by noting that we redid Figure 2 using a forward-looking Taylor rule in which the interest rate responds to the one-period-ahead expected inflation and output gap. The results that we obtained were very similar to the ones discussed above.

Viewed overall, our results indicate that, from the perspective a simple new-Keynesian model, it is quite plausible that the multiplier is above one. However, it is difficult to obtain multipliers above 1.2 for plausible parameter values.

3. The constant-interest-rate multiplier in a model without capital

In this section we analyze the government-spending multiplier in our simple new-Keynesian model when the nominal interest rate is constant. We focus on the case in which the nominal interest rate is constant because the zero bound binds. We assume, as in Eggertsson and Woodford (2003) and Christiano (2004), that the shock that makes the zero bound binding is an increase in the discount factor. We think of this shock as representing a temporary rise in agent's propensity to save.

A discount factor shock We modify agent's preferences, given by (2.1), to allow for a stochastic discount factor,

$$U = E_0 \sum_{t=0}^{\infty} d_t \left[\frac{\left[C_t^{\gamma} (1 - N_t)^{1-\gamma} \right]^{1-\sigma} - 1}{1 - \sigma} + v\left(G_t\right) \right]. \tag{3.1}$$

The cumulative discount factor, d_t, is given by:

$$d_t = \begin{cases} \frac{1}{1+r_1} \frac{1}{1+r_2} \cdots \frac{1}{1+r_t}, & t \geq 1, \\ 1 & t = 0. \end{cases} \tag{3.2}$$

The time-t discount factor, r_t, can take on two values: r and r^l, where $r^l < 0$. The stochastic process for r_t is given by:

$$\Pr\left[r_{t+1} = r^l | r_t = r^l\right] = p, \ \Pr\left[r_{t+1} = r | r_t = r^l\right] = 1 - p, \ \Pr\left[r_{t+1} = r^l | r_t = r\right] = 0. \tag{3.3}$$

The value of r_{t+1} is realized at time t. We define $\beta = 1/(1+r)$, where r is the steady state value of r_{t+1}.

We consider the following experiment. The economy is initially in the steady state, so $r_t = r$. At time zero r_1 takes on the value r^l. Thereafter r_t follows the process described by equation (3.3). The discount factor remains high with probability p and returns permanently to its normal value, r, with probability $1-p$. In what follows we assume that r^l is sufficiently high that the zero-bound constraint on nominal interest rates binds. We assume that $\hat{G}_t = \hat{G}^l \geq 0$ in the lower bound and $\hat{G}_t = 0$ otherwise.

To solve the model we suppose (and then verify) that the equilibrium is characterized by two values for each variable: one value for when the zero bound binds and one value for when it is not. We denote the values of inflation and output in the zero bound by π^l and $\hat{Y}^l$, respectively. For simplicity we assume that $\rho_R = 0$, so there is no interest rate smoothing in the Taylor rule, (2.6). Since there are no state variables and $\hat{G}_t = 0$ outside of the zero bound state, as soon as the zero bound is not binding the economy jumps to the steady state.

We can solve for $\hat{Y}^l$ using equation (2.13) and the following version of equation (2.14), which takes into account the discount factor shock:

$$\begin{aligned} & \hat{Y}_t - g\left[\gamma(\sigma-1)+1\right]\hat{G}_t \qquad (3.4) \\ = \;& E_t\left\{\hat{Y}_{t+1} - g\left[\gamma(\sigma-1)+1\right]\hat{G}_{t+1} - \beta(1-g)\left(R_{t+1}-r_{t+1}\right) + (1-g)\pi_{t+1}\right\} \end{aligned}$$

We focus on the case in which the zero bound binds at time t, so $R_{t+1} = 0$. Equations (2.13) and (3.4) can be re-written as:

$$\hat{Y}^l = g\left[\gamma\left(\sigma-1\right)+1\right]\hat{G}^l + \frac{1-g}{1-p}\left(\beta r^l + p\pi^l\right), \qquad (3.5)$$

$$\pi^l = \beta p \pi^l + \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}^l - \frac{g}{1-g}\kappa\hat{G}^l. \qquad (3.6)$$

Equations (3.5) and (3.6) imply that π^l and $\hat{Y}^l$ are given by:

$$\pi^l = \frac{(1-g)\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\beta r^l}{\Delta} + g\kappa(1-p)\frac{\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\gamma(\sigma-1)+\frac{N}{1-N}}{\Delta}\hat{G}^l. \quad (3.7)$$

$$\hat{Y}^l = \frac{(1-\beta p)(1-g)\beta r^l}{\Delta} + \frac{(1-\beta p)(1-p)\left[\gamma(\sigma-1)+1\right]-p\kappa}{\Delta} g\hat{G}^l, \quad (3.8)$$

where:

$$\Delta = (1-\beta p)(1-p) - p\kappa\left[1+\frac{N}{1-N}(1-g)\right].$$

Since r^l is negative, a necessary condition for the zero bound to bind is that $\Delta > 0$. If this condition did not hold inflation would be positive and output would be above its steady state value. Consequently, the Taylor rule would call for an increase in the nominal interest rate so that the zero bound would not bind.

Equation (3.8) implies that the drop in output induced by a change in the discount rate, which we denote by Θ, is given by:

$$\Theta = \frac{(1-\beta p)(1-g)\beta r^l}{\Delta}. \quad (3.9)$$

By assumption $\Delta > 0$, so $\Theta < 0$. The value of Θ can be a large negative number for plausible parameter values. The intuition for this result is as follows. The basic shock to the economy is an increase in agent's desire to save. We develop the intuition for this result in two steps. First, we provide intuition for why the zero bound binds. We then provide the intuition for why the drop in output can to be very large when the zero bound binds.

To understand why the zero bound binds recall that in this economy saving must be zero in equilibrium. With completely flexible prices the real interest rate would simply fall to discourage agents from saving. There are two ways in which such a fall can occur: a large fall in the nominal interest rate and/or a substantial

rise in the expected inflation rate. The extent to which the nominal interest rate can fall is limited by the zero bound. In our sticky-price economy a rise in the rate of inflation is associated with a rise in output and marginal cost. But a transitory increase in output is associated with a further increase in the desire to save, so that the real interest rate must rise by even more. Given the size of the shock to the discount factor there may be no equilibrium in which the nominal interest rate is zero and inflation is positive. So the real interest rate cannot fall by enough to reduce desired saving to zero. In this scenario the zero bound binds.

Figure 3 illustrates this point using a stylized version of our model. Saving (S) is an increasing function of the real interest rate. Since there is no investment in this economy saving must be zero in equilibrium. The initial equilibrium is represented by point A. But the increase in the discount factor can be thought of as inducing a rightward shift in the saving curve from S to S'. When this shift is large, the real interest rate cannot fall enough to re-establish equilibrium because the lower bound on the nominal interest rate becomes binding prior to reaching that point. This situation is represented by point B.

To understand why the fall in output can be very large when the zero bound binds, recall that equation (3.7) shows how the rate of inflation, π^l, depends on the discount rate and on government spending in the zero bound state. In this state Δ is positive. Since r^l is negative, it follows that π^l is negative and so too is expected inflation, $p\pi^l$. Since the nominal interest rate is zero and expected inflation is negative, the real interest rate (nominal interest rate minus expected inflation rate) is positive. Both the increase in the discount factor and the rise in the real interest rate increase agent's desire to save. There is only one force remaining to generate zero saving in equilibrium: a large, transitory fall in income. Other things equal this fall in income reduces desired saving as agents attempt to smooth the marginal utility of consumption over states of the world. Because

the zero bound is a transitory state of the world this force leads to a decrease in agents desire to save. This effect has to exactly counterbalance the other two forces which are leading agents to save more. This reasoning suggest that there is a very large decline in income when the zero bound binds. In terms of Figure 3 we can think of the temporary fall in output as inducing a shift in the saving curve to the left.

We now turn to a numerical analysis of the government-spending multiplier, which is given by:

$$\frac{dY^l}{dG^l} = \frac{(1-\beta p)(1-p)[\gamma(\sigma-1)+1]-p\kappa}{\Delta}. \tag{3.10}$$

In what follows we assume that the discount factor shock is sufficiently large to make the zero bound binding. Conditional on this bound being binding, the size of the multiplier does not depend on the size of the shock. In our discussion of the standard multiplier we assume that the first-order serial correlation of government spending shocks is 0.8. To make the experiment in this section comparable we choose $p = 0.8$. This choice implies that the first-order serial correlation of government spending in the zero bound is also 0.8. All other parameter values are given by the baseline specification in (2.21).

For our benchmark specification the government-spending multiplier is 3.7, which is roughly three times larger than the standard multiplier. The intuition for why the multiplier can be large when the nominal interest rate is constant, say because the zero bound binds, is as follows. A rise in government spending leads to a rise in output, marginal cost and expected inflation. With the nominal interest rate equal to zero, the rise in expected inflation drives down the real interest rate, leading to a rise is private spending. This rise in spending generates a further rise in output, marginal cost, and expected inflation and a further decline in the real interest rate. The net result is a large rise in inflation and output.

We now consider the sensitivity of the multiplier to parameter values. The first row of Figure 4 displays the government-spending multiplier and the response of output to the discount rate shock in the absence of a change in government spending as a function of the parameter κ. The '$*$' indicates results for our benchmark value of κ. This row is generated assuming a discount factor shock such that r^l is equal to -2 percent on an annualized basis. We graph only values of κ for which the zero bound binds, so we display results for $0.02 \leq \kappa \leq 0.036$. Three key features of this figure are worth noting. First, the multiplier can be very large. Second, absent a change in government spending, the decline in output is increasing in the degree of price flexibility, i.e. it is increasing in κ, as long as the zero bound binds. This result reflects that, conditional on the zero bound binding, the more flexible are prices, the higher is expected deflation and the higher is the real interest rate. So, other things equal, higher values of κ require a large transitory fall in output to equate saving and investment when the zero bound binds. Third, the government-spending multiplier is also an increasing function of κ.

The second row of Figure 4 displays the government-spending multiplier and the response of output to the discount rate shock in the absence of a change in government spending as a function of the parameter p. The '$*$' indicates results for our benchmark value of p. We graph only values of p for which the zero bound binds, so we display results for $0.75 \leq p \leq 0.82$. Two key results are worth noting. First, absent a change in government spending the decline in output is increasing in p. So the longer is the expected duration of the shock the worst are the output consequences of the zero bound being binding. Second the value of the government-spending multiplier is an increasing function of p.

Figure 4 shows the precise value of the multiplier is sensitive to the choice of parameter values. But looking across parameter values we see that the government-

spending multiplier is large in economies where the drop in output associated with the zero bound is also large. Put differently, fiscal policy is particularly powerful in economies where the zero bound state entails large output losses. One more way to see this result is to analyze the impact of changes in N, which governs the elasticity of labor supply, on dY^l/dG^l and Θ. Equations (3.10) and (3.9) imply that:

$$\frac{dY^l}{dG^l} = \frac{(1-\beta p)(1-p)\left[\gamma(\sigma-1)+1\right]-p\kappa}{(1-\beta p)(1-g)\beta r^l}\Theta. \tag{3.11}$$

From equation (3.9) we see that changes in N that make Δ converge to zero imply that Θ, the impact of discount factor shock on output, converges to minus infinity. It follows directly from equation (3.11) that the same changes in N cause dY^l/dG^l to go to infinity. So, again we conclude that the government-spending multiplier is particularly large in economies where the output costs of being in the zero bound state are very large.[4]

Sensitivity to the timing of government spending In practice there is likely be a lag between the time at which the zero bound becomes binding and the time at which additional government purchases begin. A natural question is: how does the economy respond at time t to the knowledge that the government will increase spending in the future? Consider the following scenario. At time t the zero bound binds. Government spending does not change at time t, but it takes on the value $G^l > G$ from time $t+1$ on, as long as the economy is in the zero bound. Under these assumptions equations (2.13) and (3.4) can be written as:

$$\pi_t = \beta p \pi^l + \kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\hat{Y}_t, \tag{3.12}$$

$$\hat{Y}_t = (1-g)\beta r^l + p\hat{Y}^l - g\left[\gamma(\sigma-1)+1\right]p\hat{G}^l + (1-g)p\pi^l. \tag{3.13}$$

[4] An exception pertains to the parameter σ. The value of dY^l/dG^l is monotonically increasing in σ, but $d\hat{Y}^l/dr^l$ is independent of σ.

Here we use the fact that $\hat{G}_t = 0$, $E_t(\pi_{t+1}) = p\pi^l$, $E_t(\hat{G}_{t+1}) = p\hat{G}^l$, and $E_t(\hat{Y}_{t+1}) = p\hat{Y}^l$. The values of π^l and $\hat{Y}^l$ are given by equations (3.7) and (3.8), respectively. Using equation (3.8) to replace $\hat{Y}^l$ in equation (3.13) we obtain:

$$\frac{dY_{t,1}}{dG^l} = \frac{1-g}{g}\frac{p}{1-p}\frac{d\pi^l}{d\hat{G}^l}. \tag{3.14}$$

Here the subscript 1 denotes the presence of a one period delay in implementing an increase in government spending. So, $dY_{t,1}/dG^l$ represents the impact on output at time t of an increase in government spending at time $t+1$. One can show that the multiplier is increasing in the probability, p, that the economy remains in the zero bound. The multiplier operates through the effect of a future increase in government spending on expected inflation. If the economy is in the zero bound in the future, an increase in government purchases increases future output and therefore future inflation. From the perspective of time t this effect leads to higher expected inflation and a lower real interest rate. This lower real interest rate reduces desired saving and increases consumption and output at time t.

Evaluating equation (3.14) at the benchmark values we obtain a multiplier equal to 1.5. While this multiplier is much lower than the benchmark multiplier of 3.7, it is still large. Moreover, this multiplier pertains to an increase in today's output in response to an increase in future government spending that only occurs if the economy is in the zero bound state in the future.

Suppose that it takes two periods for government purchases to increase in the event that the zero bound binds. It is straightforward to show that the impact on current output of a potential increase in government spending that takes two periods to implement is given by:

$$\frac{dY_{t,2}}{dG^l} = p\frac{1-g}{g}\left[\frac{d\pi_{t,1}}{d\hat{G}^l} + \frac{1}{1-p}\frac{d\pi^l}{d\hat{G}^l}\right].$$

Here the subscript 2 denotes the presence of a two period delay. Using our benchmark parameters the value of this multiplier is 1.44, so the rate at which the

multiplier declines as we increase the implementation lag is relatively low.

Consider now the case in which the increase in government spending occurs only after the zero bound ends. Suppose, for example, that at time t the government promises to implement a persistent increase in government spending at time $t+1$, if the economy emerges from the zero bound at time $t+1$. This increase in government purchases is governed by: $\hat{G}_{t+j} = 0.8^{j-1}\hat{G}_{t+1}$, for $j \geq 2$. In this case the value of the multiplier, dY_t/dG_{t+1}, is only 0.46 for our benchmark values.

The usual objection to using fiscal policy as a tool for fighting recessions is that there are long lags in gearing up increases in spending. Our analysis indicates that the key question is: in which state of the world does additional government spending come on line? If it comes on line in future periods when the zero bound binds there is a large effect on current output. If it comes on line in future periods where the zero bound is not binding the current effect on government spending is smaller.

Optimal government spending The fact that the government-spending multiplier is so large in the zero bound raises the following question: taking as given the monetary policy rule described by equation (2.6) what is the optimal level of government spending when the representative agent's discount rates is higher than its steady state level? In what follows we use the superscript L to denote the value of variables in states of the world where the discount rate is r^l. In these states of the world the zero bound may or may not be binding depending on the level of government spending. From equation (3.7) we anticipate that the higher is government spending, the higher is expected inflation, and the less likely the zero bound is to bind.

We choose G^L to maximize the expected utility of the consumer in states of the world in which the discount factor is high and the zero bound binds. For

now we assume that in other states of the world $\hat{G}$ is zero. So, we choose G^L to maximize:

$$\begin{aligned} U^L &= \sum_{t=0}^{\infty} \left(\frac{p}{1+r^l}\right)^t \left[\frac{\left[\left(C^L\right)^{\gamma}\left(1-N^L\right)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma} + v\left(G^L\right)\right], \quad (3.15) \\ &= \frac{1+r^r}{1+r^l-p}\left[\frac{\left[\left(C^L\right)^{\gamma}\left(1-N^L\right)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma} + v\left(G^L\right)\right]. \end{aligned}$$

To ensure that U^L is finite we assume that $p < (1+r^l)$.

Note that:

$$\begin{aligned} Y^L &= N^L = Y\left(\hat{Y}^L+1\right), \\ C^L &= Y\left(\hat{Y}^L+1\right) - G\left(\hat{G}^L+1\right). \end{aligned}$$

Substituting these expressions into equation (3.15) we obtain:

$$\begin{aligned} U^L &= \frac{1+r^r}{1+r^l-p}\left[\frac{\left(\left[N\left(\hat{Y}^L+1\right) - Ng\left(\hat{G}^L+1\right)\right]^{\gamma}\left(1-N\left(\hat{Y}^L+1\right)\right)^{1-\gamma}\right)^{1-\sigma}-1}{1-\sigma}\right] \\ &\quad + \frac{1+r^r}{1+r^l-p} v\left[Ng\left(\hat{G}^L+1\right)\right]. \end{aligned}$$

We choose the value of $\hat{G}^L$ that maximizes U^L subject to the intertemporal Euler equation (equation (2.14)), the Phillips curve (equation (2.13)), and $\hat{Y}_t = \hat{Y}^L$, $\hat{G}_t = G^L$, $E_t(\hat{G}_{t+1}) = pG^L$, $\pi_{t+1} = \pi^L$, $E_t(\pi_{t+1}) = p\pi^L$, and $R_{t+1} = R^L$, where

$$R^L = \max\left(Z^L, 0\right),$$

and

$$Z^L = \frac{1}{\beta} - 1 + \frac{1}{\beta}\left(\phi_1\pi^L + \phi_2\hat{Y}^L\right).$$

The last constraint takes into account that the zero bound on interest rates may not be binding even though the discount rate is high.

Finally, for simplicity we assume that $v(G)$ is given by:

$$v\left(G\right)=\psi_{g}\frac{G^{1-\sigma}}{1-\sigma}.$$

We choose ψ_{g} so that $g=G/Y$ is equal to 0.2.

Since government purchases are financed with lump sum taxes, the optimal level of G has the property that the marginal utility of G is equal to the marginal utility of consumption:

$$\psi_{g}G^{-\sigma}=\gamma C^{\gamma(1-\sigma)-1}N^{(1-\gamma)(1-\sigma)}.$$

This relation implies:

$$\psi_{g}=\gamma\left(\left[N\left(1-g\right)\right]\right)^{\gamma(1-\sigma)-1}N^{(1-\gamma)(1-\sigma)}\left(Ng\right)^{\sigma}.$$

Using our benchmark parameter values we obtain a value of ψ_{g} equal to 0.015.

Figure 5 displays the values of U^{L}, $\hat{Y}^{L}$, Z^{L}, $\hat{C}^{L}$, R^{L}, and π^{L} as a function of $\hat{G}^{L}$. The '*' indicates the level of a variable corresponding to the optimal value of $\hat{G}^{L}$. The 'o' indicates the level of a variable corresponding to the highest value of $\hat{G}^{L}$ that satisfies $Z^{l}\leq 0$. A number of features of Figure 5 are worth noting. First, the optimal value of $\hat{G}^{L}$ is very large: roughly 30 percent (recall that in steady state government purchases are 20 percent of output). Second, for this particular parameterization the increase in government spending more than undoes the effect of the shock which made the zero bound constraint bind. Here, government purchases rise to the point where the zero bound is marginally non binding and output is actually above its steady state level. These last two results depend on the parameter values that we chose and on our assumed functional form for $v(G_{t})$. What is robust across different assumptions is that it is optimal to substantially increase government purchases and that the government-spending multiplier is large when the zero-bound constraint binds.[5]

[5] We derive the optimal fiscal policy taking monetary policy as given. Nakata (2009) argues

The zero bound and interest rate targeting Up to now we have emphasized the economy being in the zero bound state as the reason why the nominal interest rate might not change after an increase in government spending. Here we discuss an alternative interpretation of the constant interest rate assumption. Suppose that there are no shocks to the economy but that, starting from the non-stochastic steady state, government spending increases by a constant amount and the monetary authority deviates from the Taylor rule, keeping the nominal interest rate equal to its steady-state value. This policy shock persists with probability p. It is easy to show that the government-spending multiplier is given by equation (3.10). So the multiplier is exactly the same as in the case in which the nominal interest rate is constant because the zero bound binds.

4. A model with capital and multiple shocks

In the previous section we use a simple model without capital to argue that the government-spending multiplier is large whenever the output costs of being in the zero bound state are also large. Here we show that this basic result extends to a generalized version of the previous model in which we allow for capital accumulation. In addition, we consider three types of shocks: a discount-factor shock, a neutral technology shock, and a capital-embodied technology shock. These shocks have different effects on the behavior of the model economy. But, in all cases, the government-spending multiplier is large whenever the output costs of being in the zero bound state are large.

---

that it is also optimal to raise government purchases when monetary policy is chosen optimally. He does so using a second-order Taylor approximation to the utility function in a model with separable preferences where the natural rate of interest follows an exogenous stochastic process.

The model The preferences of the representative household are given by equations (3.1) and (3.2). The household's budget constraint is given by:

$$P_t\left(C_t + I_t e^{-\psi_t}\right) + B_{t+1} = B_t\left(1 + R_t\right) + W_t N_t + P_t r_t^k K_t + T_t, \tag{4.1}$$

where I_t denotes investment, K_t is the stock of capital, and r_t^k is the real rental rate of capital. The capital accumulation equation is given by:

$$K_{t+1} = I_t + \left(1 - \delta\right) K_t - \frac{\sigma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{4.2}$$

The variable ψ_t represents a capital-embodied technology shock. The price of investment goods in units of consumption is equal to $\exp(-\psi_t)$. A positive shock to ψ_t is associated with a decline in the price of investment goods. The parameter $\sigma_I > 0$ governs the magnitude of adjustment costs to capital accumulation. As $\sigma_I \rightarrow \infty$, investment and the stock of capital become constant. The resulting model behaves in a manner very similar to the one described in the previous section.

The household's problem is to maximize life-time expected utility, given by equations (3.1) and (3.2), subject to the resource constraints given by equations (4.1) and (4.2) and the condition $E_0 \lim_{t\rightarrow\infty} B_{t+1}/[(1+R_0)(1+R_1)...(1+R_t)] \geq 0$.

It is useful to derive an expression for Tobin's q, i.e. the value in units of consumption of an additional unit of capital. We denote this value by q_t. Equation (4.1) implies that the real cost of increasing investment by one unit is $e^{-\psi_t}$. Equation (4.2) implies that increasing investment by one unit raises K_{t+1} by $1 - \sigma_I\left(\frac{I_t}{K_t} - \delta\right)$ units. It follows that the optimal level of investment satisfies the following equation:

$$e^{-\psi_t} = q_t\left[1 - \sigma_I\left(\frac{I_t}{K_t} - \delta\right)\right]. \tag{4.3}$$

Firms The problem of the final good producers is the same as in the previous section. The discounted profits of the i^{th} intermediate good firm are given by:

$$E_t \sum_{j=0}^{\infty} \beta^{t+j} \upsilon_{t+j} \left\{ P_{t+j}(i) Y_{t+j}(i) - (1-\nu) \left[W_{t+j} N_{t+j}(i) + P_{t+j} r^k_{t+j} K_{t+j}(i) \right] \right\}. \tag{4.4}$$

Output of good i is given by:

$$Y_t(i) = e^{a_t} \left[K_t(i) \right]^{\alpha} \left[N_t(i) \right]^{1-\alpha},$$

where $N_t(i)$ and $K_t(i)$ denote the labor and capital employed by the i^{th} monopolist. The variable a_t represents a neutral technology shock that is common to all intermediate goods producers.

The monopolist is subject to the same Calvo-style price-setting frictions described in Section 2. Recall that $\nu = 1/\varepsilon$ denotes a subsidy that is proportional to the costs of production. This subsidy corrects the steady-state inefficiency created by the presence of monopoly power. The variable υ_{t+j} is the multiplier on the household budget constraint in the Lagrangian representation of the household problem. Firm i maximizes its discounted profits, given by equation (4.4), subject to the Calvo price-setting friction, the production function, and the demand function for $Y_t(i)$, given by equation (2.4).

The monetary policy rule is given by equation (2.6).

Equilibrium The economy's resource constraint is:

$$C_t + I_t e^{-\psi_t} + G_t = Y_t. \tag{4.5}$$

A 'monetary equilibrium' is a collection of stochastic processes,

$$\{C_t, I_t, N_t, K_t, W_t, P_t, Y_t, R_t, P_t(i), r^k_t, Y_t(i), N_t(i), \upsilon_t, B_{t+1}, \pi_t\},$$

such that for given $\{d_t, G_t, a_t, \psi_t\}$, the household and firm problems are satisfied, the monetary policy rule given by equation (2.6) is satisfied, markets clear, and the aggregate resource constraint holds.

Experiments At time zero the economy is in its non-stochastic steady state. At time one agents learn that one of the three shocks (r^L, a_t, or ψ_t) differs from its steady state value for T periods and then returns to its steady state value. We consider shocks that are sufficiently large so that the zero bound on the nominal interest rate binds between two time periods that we denote by t_1 and t_2, where $1 \leq t_1 \leq t_2 \leq T$. The values of t_1 and t_2 are different for different shocks.[6] We solve the model using a shooting algorithm. In practice the key determinants of the multiplier are t_1 and t_2. To maintain comparability with the previous section we keep the size of the discount factor shock the same and choose $T = 10$. In this case t_1 equals one and t_2 equals six. Consequently, the length for which the zero bound binds after a discount rate shock is roughly the same as in the model without capital.

With the exception of σ_I and δ all parameters are the same as in the economy without capital. We set δ equal to 0.02. We choose the value of σ_I so that the elasticity of I/K with respect to q is equal to the value implied by the estimates in Eberly, Rebelo, and Vincent (2008).[7] The resulting value of σ_I is equal to 17.

We compute the government spending multiplier under the assumption that G_t increases by $\hat{G}$ percent for as long as the zero bound binds. In general, the increase in G_t affects the time period over which the zero bound binds. Consequently we proceed as follows. Guess a value for t_1 and t_2. Increase G_t for the period $t \in [t_1,$

[6] The precise timing of when the zero bound constraint is binding may not be unique.

[7] Eberly, Rebelo and Vincent (2008) obtain a point estimate of b equal to 0.06 in the regression $I/K = a + b\ln(q)$. This estimate implies a steady state elasticity of I_t/K_t with respect to Tobin's q of $0.06/\delta$. Our theoretical model implies that this elasticity is equal to $(\sigma_I \delta)^{-1}$. Equating these two elasticities yields a value of σ_I of 17.

$t_2]$. Check that the zero bound binds for $t \in [t_1, t_2]$. If not revise the guess for t_1 and t_2.

Denote by $\hat{Y}_t$ the percentage deviation of output from steady state that results from a shock that puts the economy into the zero bound state holding G_t constant. Let $\hat{Y}_t^*$ denote the percentage deviation of output from steady state that results from the both the original shock and the increase in government purchases described above. We compute the government spending multiplier as follows:

$$\frac{dY_t}{dG_t} = \frac{1}{g}\frac{\hat{Y}_t^* - \hat{Y}_t}{\hat{G}}.$$

As a reference point we note that when the zero bound is not binding the government-spending multiplier is roughly 0.9. This value is lower than the value of the multiplier in the model without capital. This lower value reflects the fact that an increase in government spending tends to increase real interest rates and crowd out private investment. This effect is not present in the model without capital.

A discount factor shock We now consider the effect of an increase in the discount factor from its steady state value of four percent (APR) to -1 percent (APR). Figure 6 displays the dynamic response of the economy to this shock. The zero bound binds in periods one through six. The higher discount rate leads to substantial declines in investment, hours worked, output, and consumption. The large fall in output is associated with a fall in marginal cost and substantial deflation. Since the nominal interest rate is zero, the real interest rate rises sharply. We now discuss the intuition for how the presence of investment affects the response of the economy to a discount rate shock. We begin by analyzing why a rise in the real interest rate is associated with a sharp decline in investment. Ignoring covariance terms, the household's first-order condition for investment can

be written as:

$$E_t\left(\frac{1+R_{t+1}}{P_{t+1}/P_t}\right) = \frac{1}{q_t}E_t e^{a_t}\alpha K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha}s_{t+1} + \tag{4.6}$$
$$\frac{1}{q_t}E_t\left\{q_{t+1}\left[(1-\delta) - \frac{\sigma_I}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)^2 + \sigma_I\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\frac{I_{t+1}}{K_{t+1}}\right]\right\},$$

where s_t is the inverse of the markup rate. Equation (4.6) implies that in equilibrium the household equates the returns to two different ways of investing one unit of consumption. The first strategy is to invest in a bond that yields the real interest rate defined by the left-hand side of equation (4.6). The second strategy involves converting the consumption good into $1/q_t$ units of installed capital. The return to this capital has three components. The first component is the marginal product of capital (the first term on the right-hand side of equation (4.6)). The second component is the value of the undepreciated capital in consumption units ($q_{t+1}(1-\delta)$). The third component is the value in consumption units of the reduction in adjustment costs associated with an increase in installed capital.

To provide intuition it is useful to consider two extreme cases, infinite adjustment costs ($\sigma_I = \infty$) and zero adjustment costs ($\sigma_I = 0$). Suppose first that adjustment costs are infinite. Figure 7 displays a stylized version of this economy. Investment is fixed and saving is an increasing function of the real interest rate. The increase in the discount factor can be thought of as inducing a rightward shift in the saving curve. When this shift is very large, the real interest rate cannot fall by enough to re-establish equilibrium. The intuition for this result and the role played by the zero bound on nominal interest rates is the same as in the model without capital. That model also provides intuition for why the equilibrium is characterized by a large, temporary fall in output, deflation, and a rise in the real interest rate.

Suppose now that there are no adjustment costs ($\sigma_I = 0$). In this case Tobin's

q is equal to $e^{-\psi}$ and equation (4.6) simplifies to:

$$E_t \frac{1+R_{t+1}}{P_{t+1}/P_t} = E_t \left[e^{a_t} \alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} s_{t+1} + (1-\delta) \right].$$

According to this equation an increase in the real interest rate must be matched by an increase in the marginal product of capital. In general the latter is accomplished, at least in part, by a fall in K_{t+1} caused by a large drop in investment. In Figure 7 the downward sloping curve labeled 'elastic investment' depicts the negative relation between the real interest rate and investment in the absence of any adjustment costs. As drawn the shift in the saving curve moves the equilibrium to point C and does not cause the zero bound to bind. So, the result of an increase in the discount rate is a fall in the real interest rate and a rise in saving and investment.

Now consider a value of σ_I that is between zero and infinity. In this case both investment and q respond to the shift in the discount factor. For our parameter values the higher the adjustment costs the more likely it is that the zero bound binds. In terms of Figure 7 a higher value of σ_I can be thought of as generating a steeper slope in the investment curve, thus increasing the likelihood that the zero bound binds.

Suppose that the zero bound binds. Other things equal, a higher real interest rate increases desired saving and decreases desired investment. So the fall in output required to equate the two must be larger than in an economy without investment. This larger fall in output is undone by an increase in government purchases. Consistent with this intuition Figure 6 shows that the government-spending multiplier is very large when the zero bound binds (on impact dY/dG is roughly equal to four). This multiplier is actually larger than in the model without capital.[8]

[8] This multiplier is computed setting $\hat{G}$ to one percent.

A natural question is what happens to the size of the multiplier as we increase the size of the shock. Recall that in the model without capital, as long as the zero bound binds, the size of the shock does not affect the size of the multiplier. The analogue result here, established using numerical methods, is that the size of the shock does not affect the multiplier as long as it does not affect t_1 and t_2. For a given t_1 the size of the multiplier is decreasing in t_2. For example, suppose that shock is such that t_2 is equal to four instead of the benchmark value of six. In this case the value of the multiplier falls from 3.9 to 2.3. The latter value is still much larger than 0.9, the value of the multiplier when the zero bound does not bind.

A neutral technology shock We now consider the effect of a temporary, three-percent increase in the neutral technology shock, a_t. Figure 8 displays the dynamic response of the economy to this shock. The zero bound binds in periods one through eight. Strikingly, the positive technology shock leads to a decline in output, investment, consumption, and hours work. The shock also leads to a sharp rise in the real interest rate and to substantial deflation. To understand these effects it is useful to begin by considering the effects of a technology shock when we abstract from the zero bound. A transitory technology shock triggers a relatively small rise in consumption and a relatively large rise in investment. Other things equal, the expansion in output leads to a rise in marginal cost and the rate of inflation. However, the direct impact of the technology shock on marginal cost dominates and generates strong deflationary pressures. A Taylor rule with a large coefficient on inflation relative to output dictates that the central bank lower real rates to reduce the rate of deflation.[9] If the technology shock is large enough, the

[9] Our formulation of the Taylor rule assumes that the natural rate of output is constant and equal to the level of output in the steady state. In reality, the monetary authority could well revise its estimate of the natural rate in response to a persistent technology shock. For simplicity

zero bound becomes binding. At this point the real interest rate may simply be too high to equate desired saving and investment. The intuition for what happens when the zero bound binds is exactly the same as for the discount factor shock. The key point is that the only way to reduce desired saving is to have a temporary large fall in output. As with the discount rate shock once the zero bound binds, the government-spending multiplier rises dramatically (see Figure 8).[10]

An investment-specific shock We now consider the effect of a temporary eight percent increase in the price of investment goods (i.e. an eight percent fall in ψ_t). Figure 9 displays the dynamic response of the economy to this shock. Even though the shock that we consider is very large, the zero bound binds only in periods one through three. In addition, the effects of the shock on the economy are small relative to the effects of the other shocks that we discussed. So, while the multiplier is certainly large when the zero bound binds, it is much smaller than in the cases that we have already analyzed.[11]

The shock leads to a decline in output, investment, consumption, and hours worked. It is also associated with deflation and a rise in the real interest rate. To understand how the zero bound can become binding in response to this shock, consider the impact on the economy when the zero bound does not bind. In this case output falls. This fall occurs because the investments become more expensive, reducing the incentive to work. Consumption also falls because of the negative wealth effect of the shock. Other things equal, the fall in output is associated with strong deflationary pressures. Suppose that these deflationary pressures predominate. The fall in output and deflation lead the central bank

we abstract from this possibility.

[10] In computing the government spending multiplier we increased G_t to 0.5 percent in periods one through eight.

[11] In computing the government spending multiplier we set $\hat{G}$ to one percent in periods one through three.

to lower nominal interest rates. For a sufficiently large shock, the zero bound becomes binding. The intuition for what happens when the zero bound binds is exactly the same as for the discount factor shock and the neutral technology shock.

5. The multiplier in a medium-size DSGE model

In the previous sections we built intuition about the size of the government-spending multiplier using a series of simple new-Keynesian models. In this section we investigate the determinants of the multiplier in a medium-size DSGE model taken from ACEL. These authors estimate the parameters of their model to match the estimated impulse response function of ten aggregate U.S. time series to three identified shocks: a neutral technology shock, a capital-embodied technology shock, and a monetary policy shock. The ten aggregate time series include measures of the change in the relative price of investment, the change in average productivity, the change in the GDP deflator, capacity utilization, per capita hours worked, the real wage, the shares of consumption and investment in GDP, the Federal Funds Rate and the velocity of money.

The model includes a variety of frictions that are useful in matching the estimated impulse response functions. These frictions include: sticky wages, sticky prices, variable capital utilization, and the CEE investment adjustment-cost specification. The representative agent's momentary utility is given by equation (2.22), modified to include internal habit formation in consumption. With one exception we use the parameter values estimated in ACEL. The one exception is the parameter governing investment adjustment costs. The law of motion for capital used in ACEL is given by:

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]. \tag{5.1}$$

The point estimate of $S''(1)$ in ACEL is 3.28 with a standard error of 1.69. When we use that point estimate with our specification of the Taylor rule we find that the model has multiple equilibria. In what follows we assume that $S''(1) = 5$ which is close to the value estimated by Smets and Wouters (2007) and yields a unique equilibrium.

The second panel of Figure 10 shows the value of the government-spending multiplier when monetary policy is governed by a Taylor rule. We consider the case where government spending increases by a constant amount for four and eight periods, respectively. The key result here is that during the first four periods in which the experiments are comparable the multiplier is higher in the first case than in the second case. This result is consistent with the analysis in Section 2 which argues that, when the Taylor rule is operative, the magnitude of the multiplier is decreasing in the persistence of the shock to government spending.

The first panel reports the value of the government spending multiplier when an increase in government spending coincides with a nominal interest rate that is constant, say because the zero bound binds.[12] Interestingly, when government spending rises for only four periods, the government-spending multiplier is not very sensitive to whether the interest rate is constant or the Taylor rule is operative. However, when government spending rises for eight periods, there is a very large difference between the Taylor rule case and the zero bound case. In the latter case the impact multiplier is roughly two. The multiplier rises in a hump-shaped manner, attaining a peak value of roughly three after four periods. Output remains above its steady state value for three years, even though the fiscal stimulus lasts for only two years. Both the hump-shaped response of the multiplier and the persistent effects of the increase in government spending reflects the endogenous

[12]Recall that the value of the multiplier does not depend on why the nominal interest rate is constant. Given this property, we study the size of the multiplier in ACEL without specifying either the type or the magnitude of the shock that makes the zero bound binding.

sources of persistence present in ACEL, e.g. habit formation in consumption and investment adjustment costs.

In the third panel we address the following question: how sensitive is the multiplier to the proportion of government spending that occurs while the nominal interest rate is zero? To this end we calculate the government-spending multipliers when government spending goes up for eight, 16, and 24 periods, respectively. In all cases the nominal interest rate is zero for eight periods and follows a Taylor rule thereafter. So in the three cases the proportion of government spending that comes online while the nominal interest rate is zero is 100, 50, and 33.3 percent, respectively.

Our basic result is that the multipliers are higher the larger is the percentage of the spending that comes online when the nominal interest rate is zero. This result holds even in the first eight periods when the increase in government spending is the same in all three cases. For example, the impact multiplier falls from roughly two to 0.5 as we go from the first to the third case. This decline is consistent with our discussion of the sensitivity of the multiplier to the timing of government spending in Section 3. A key lesson from this analysis is that if fiscal policy is to be used to combat a shock that sends the economy into the zero bound, it is critical that the spending come on line when the economy is actually in the zero bound. Spending that occurs after that yields very little bank for the buck and actually dulls the impact of the spending that comes on line when the zero bound binds.

Using a model similar to ACEL, Cogan, Cwik, Taylor, and Wieland (2009) study the impact of increases in government spending when the nominal interest rate is set to zero for one or two years. A common feature of their experiments is that the bulk of the increase in government spending comes on line when the nominal interest rate is no longer constant. Consistent with our results, Cogan et

al. (2009) find modest values for the government-spending multiplier.

6. Conclusion

In this paper we argue that the government-spending multiplier can be very large when the nominal interest rates is constant. We focus on a natural case in which the interest rate is constant, which is when the zero lower bound on nominal interest rates binds. In these economies the government-spending multiplier is quite modest when monetary policy is governed by a Taylor rule. For the economies that we consider it is optimal to increase government spending in response to shocks that make the zero bound binding.

Our analysis abstracts from a host of political economy considerations which might make an increase in government spending less attractive than our analysis indicates. We are keenly aware that it is much easier to start new government programs than to end them. It remains very much an open question whether, in the presence of political economy considerations, tax policy of the sort emphasized by Eggertson (2009) is a better way of responding a zero bound episode than an increase in government purchases. What our analysis does indicate is that measures designed to increase aggregate demand are particularly powerful during such episodes.

References

[1] Adão, Bernardino, Isabel Correia, and Pedro Teles "Unique monetary equilibria with interest rate rules," mimeo, Bank of Portugal, 2009.

[2] Aiyagari, S. Rao, Lawrence Christiano, and Martin Eichenbaum "The output, employment, and interest rate effects of government consumption," *Journal of Monetary Economics*, vol. 30(1), pages 73-86, 1992.

[3] Altig, David, Lawrence Christiano, Martin Eichenbaum, and Jesper Lindé "Firm-specific capital, nominal rigidities and the business cycle," National Bureau of Economic Research Working Paper No. 11034, January 2005.

[4] Barro, Robert "Output effects of government purchases," *Journal of Political Economy*, 89(6): 1115, 1981.

[5] Baxter, Marianne and Robert G. King "Fiscal policy in general equilibrium," *The American Economic Review*, 83, 3: 315-334, Jun., 1993.

[6] Bodenstein, Martin, Christopher J. Erceg, and Luca Guerrieri "The effects of foreign shocks when U.S. interest rates are at zero," mimeo, Board of Governors of the Federal Reserve System, 2009.

[7] Braun, R. Anton and Yuichiro Waki "Monetary policy during Japan's lost decade," *The Japanese Economic Review*, 57:2, 324-344, June 2006.

[8] Burnside, Craig, Martin Eichenbaum, and Jonas Fisher, "Fiscal shocks and their consequences," *Journal of Economic Theory* 115: 89-117, 2004.

[9] Christiano, Lawrence "The zero-bound, zero-inflation targeting, and output collapse," mimeo, Northwestern University, 2004.

[10] Christiano, Lawrence, Martin Eichenbaum, and Charles Evans "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of Political Economy*, 113: 1–45, 2005.

[11] Coenen, Guenter and Volker Wieland "The zero-interest rate bound and the role of the exchange rate for monetary policy in Japan," *Journal of Monetary Economics* , Vol. 50 (5), July 2003, pp. 1071-1101.

[12] Cogan, John, Tobias Cwik, John Taylor, and Volker Wieland "New Keynesian versus old Keynesian government spending multipliers," mimeo, Stanford University, 2009.

[13] Eberly, Janice, Sergio Rebelo and Nicolas Vincent "Investment and value: A neoclassical benchmark," mimeo, Northwestern University, 2008.

[14] Eggertsson, Gauti and Michael Woodford "The zero interest-rate bound and optimal monetary policy." *Brookings Panel on Economic Activity*, 2003.

[15] Eggertsson, Gauti "What fiscal policy is effective at zero interest rates?," mimeo, Federal Reserve Bank of New York, 2009.

[16] Fisher, Jonas D.M. and Ryan Peters "Using Stock Returns to Identify Government Spending Shocks," mimeo, Federal Reserve Bank of Chicago, 2009.

[17] Gali, Jordi, 'Technology, employment, and the business cycle: do technology shocks explain aggregate fluctuations?' *American Economic Review*, 89(1), 249-271, 1999.

[18] Gali, Jordi, J.David López-Salido and Javier Vallés "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economics Association*, 5:1, 227-270, 2007.

[19] Ilzetzki, Ethan, Enrique Mendoza and Carlos A. Vegh "How big (small?) are fiscal multipliers?," mimeo, University of Maryland, 2009.

[20] Keynes, John Maynard *The General Theory of Employment, Interest, and Money*, London, Macmillan, 1936.

[21] Krugman, Paul "It's baaack: Japan's slump and the return of the liquidity trap," *Brookings Papers on Economic Activity*, 2: 137-187, 1998.

[22] Monacelli, Tommaso and Roberto Perotti "Fiscal policy, wealth effects and markups," National Bureau of Economic Research Working Paper 14584, 2008.

[23] Nakata, Taisuke "Optimal Monetary Policy and Government Expenditure under a Liquidity Trap," mimeo, New York University, 2009.

[24] Ramey, Valerie A. "Identifying government spending shocks: it's all in the timing," mimeo, University of California, San Diego, 2008.

[25] Ramey, Valerie A. and Matthew D. Shapiro "Costly capital reallocation and the effects of government spending," *Carnegie-Rochester Conference Series on Public Policy*, 48: 145-194, June 1998.

[26] Schmitt-Grohé Stephanie and Martín Uribe "Liquidity traps with global Taylor Rules," *International Journal of Economic Theory* 5:85–106, 2009.

[27] Smets, Frank and Raf Wouters "An estimated dynamic stochastic general equilibrium model of the Euro area," *Journal of the European Economic Association*, September 2003, Vol. 1, No. 5, Pages 1123-1175.

[28] Smets, Frank and Rafael Wouters "Shocks and frictions in U.S. business cycles: A Bayesian DSGE approach," *American Economic Review*, 97:3, 586-606, June 2007.

[29] Tobin, James "Money and income: post hoc ergo propter hoc?," *Quarterly Journal of Economics*, 84: 301-329 ,1970

Figure 1: Effect of an increase in government spending when the zero bound is not binding (model with no capital).

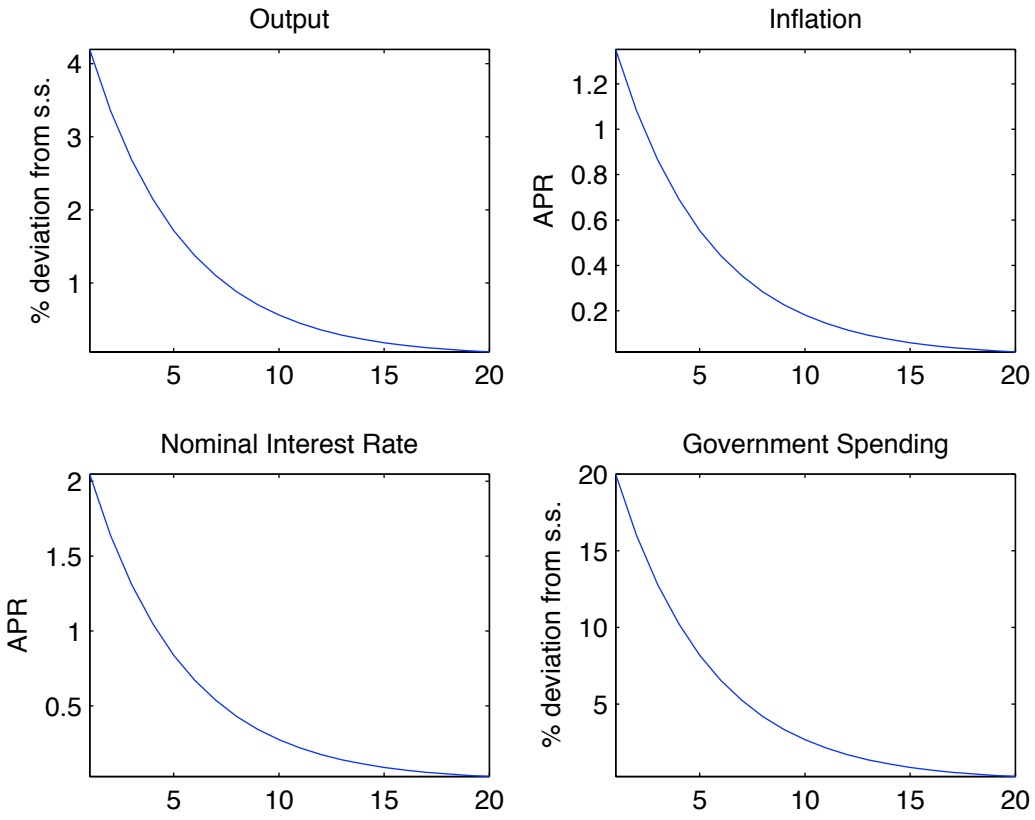

Figure 2: Government spending multiplier
when the zero bound is not binding
(model with no capital).

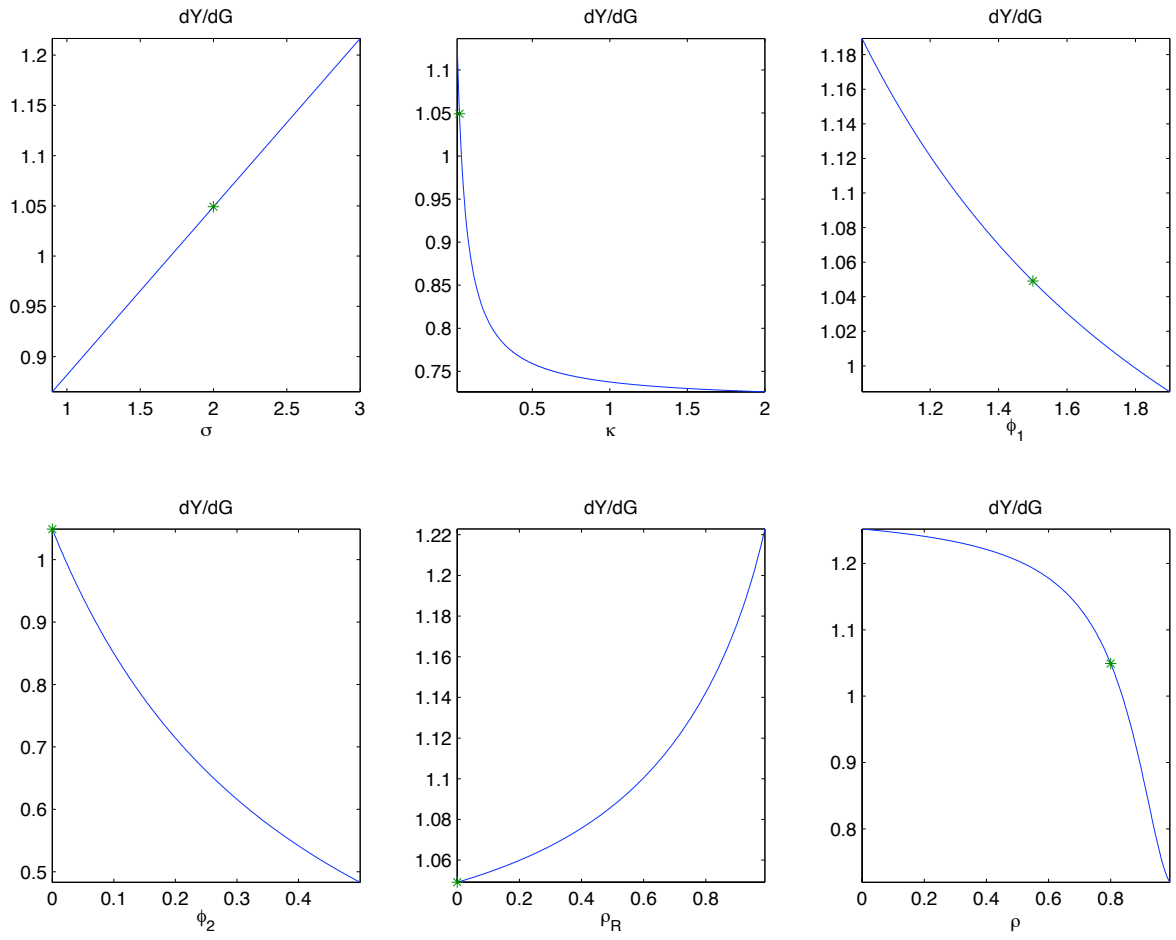

Figure 3: Simple diagram for model with no capital.

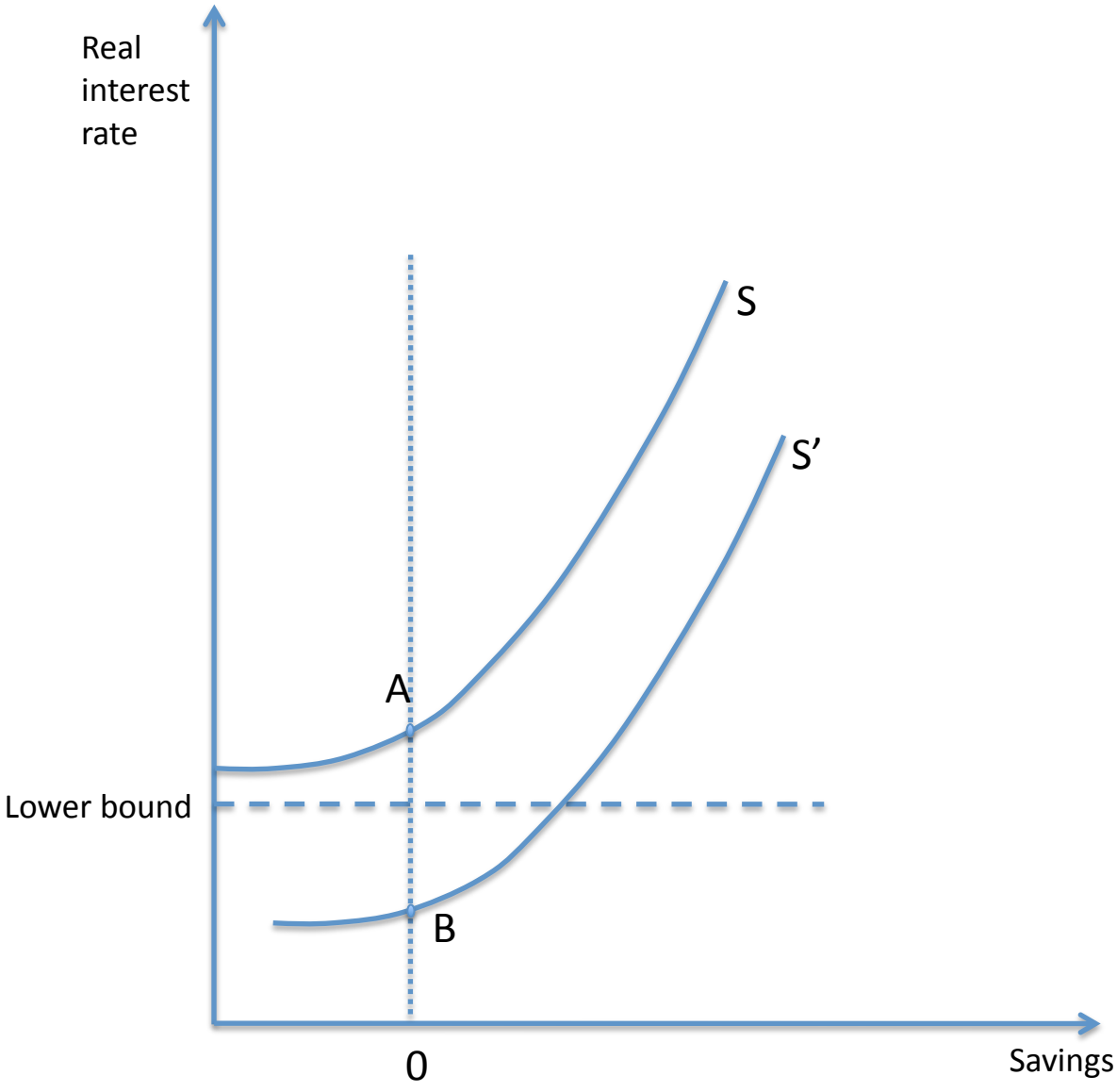

Figure 4: Government spending multiplier
when the zero bound is binding
(model with no capital).

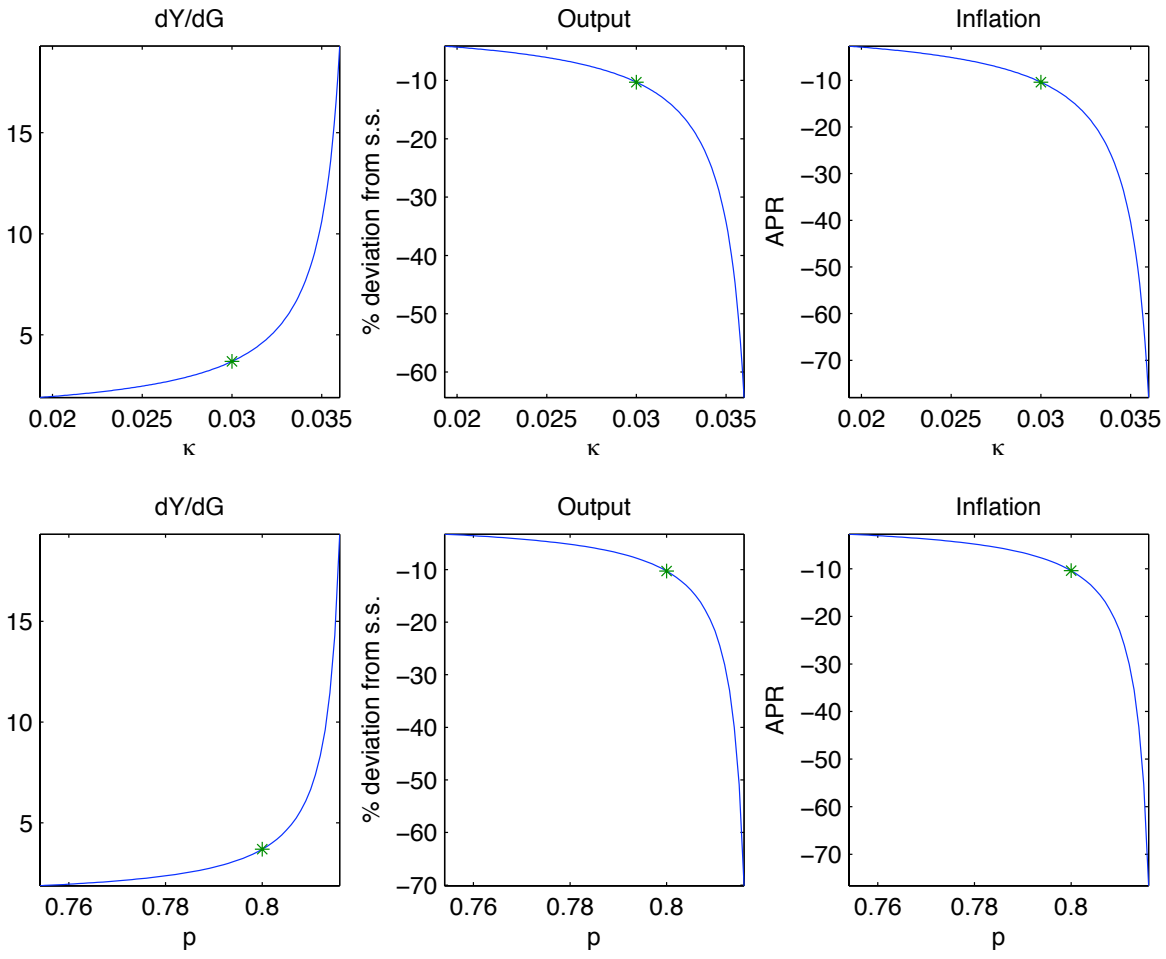

Figure 5: Optimal level of government spending in the zero bound.

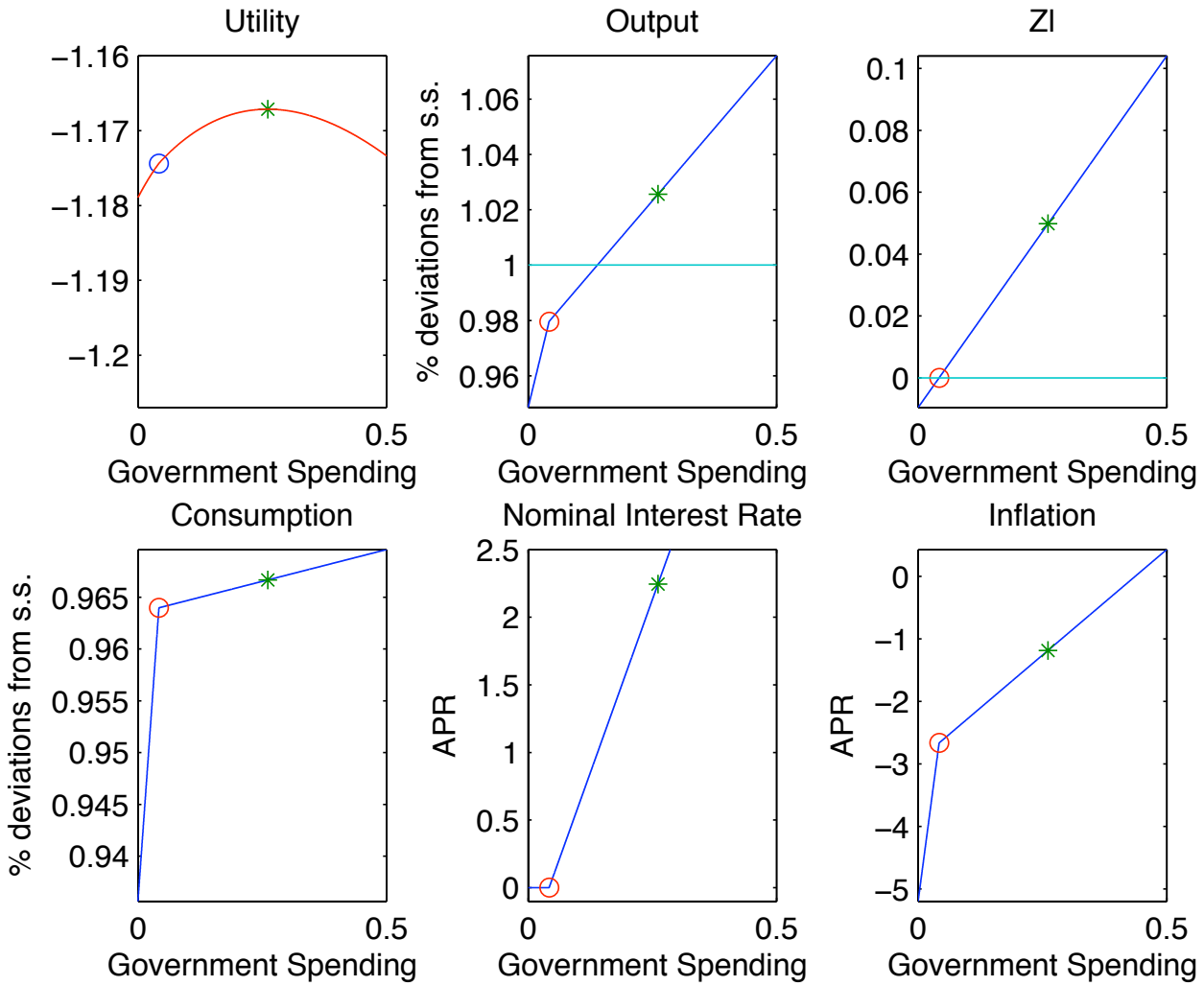

Figure 6: Effect of discount rate shock
when the zero bound is binding
(model with capital, 'standard' adjustment costs).

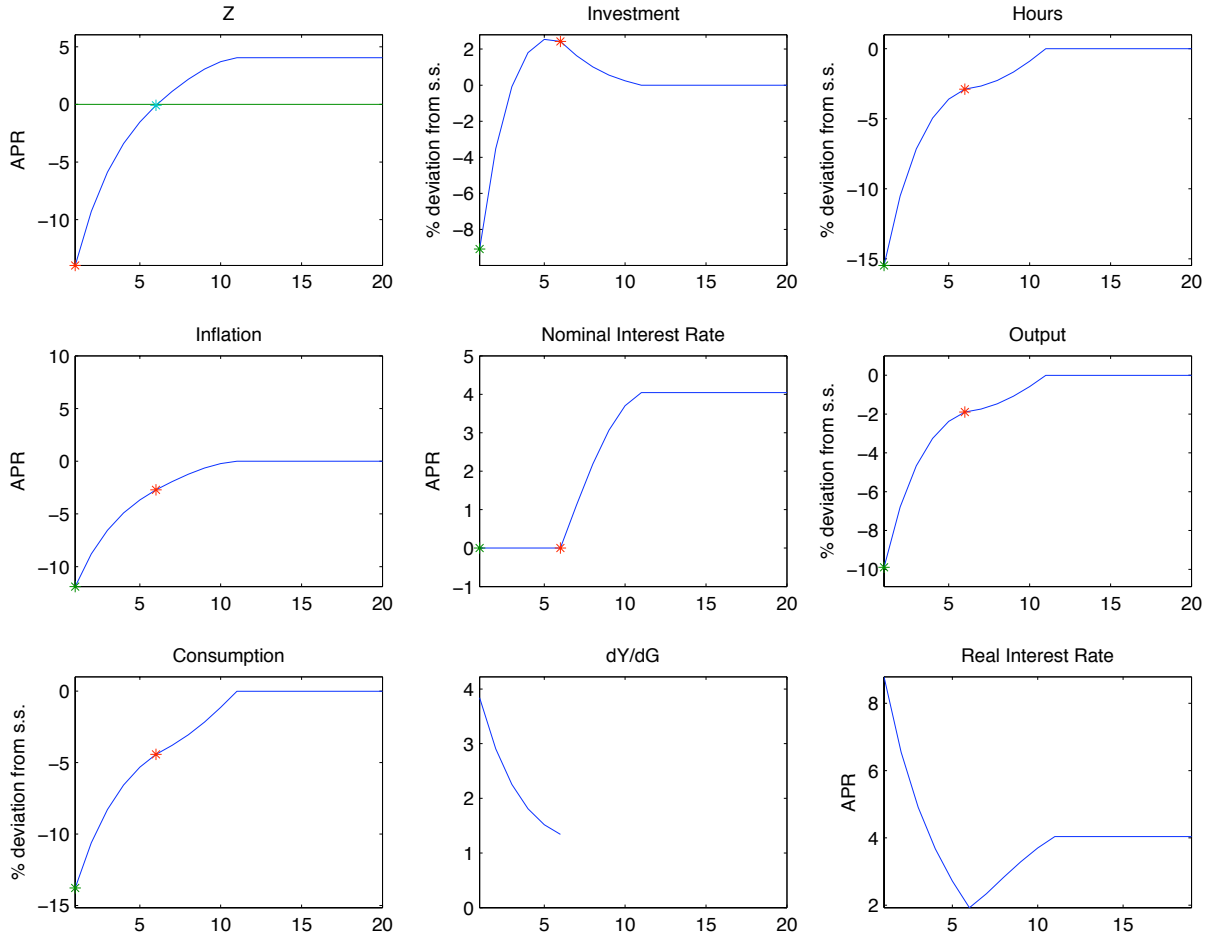

Figure 7: Simple diagram for model with capital.

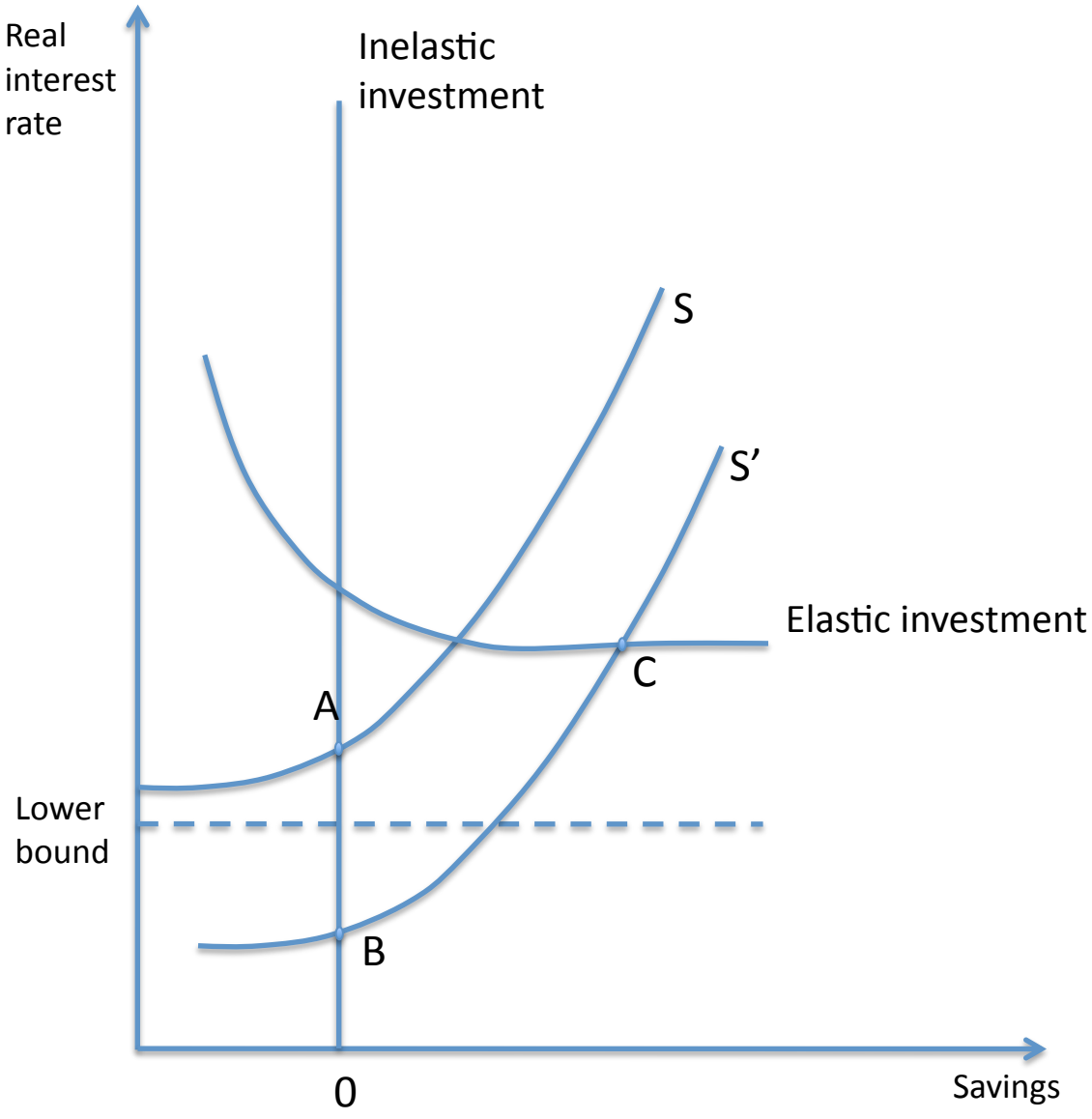

Figure 8: Effect of neutral technology shock
when the zero bound is binding
(model with capital, 'standard' adjustment costs).

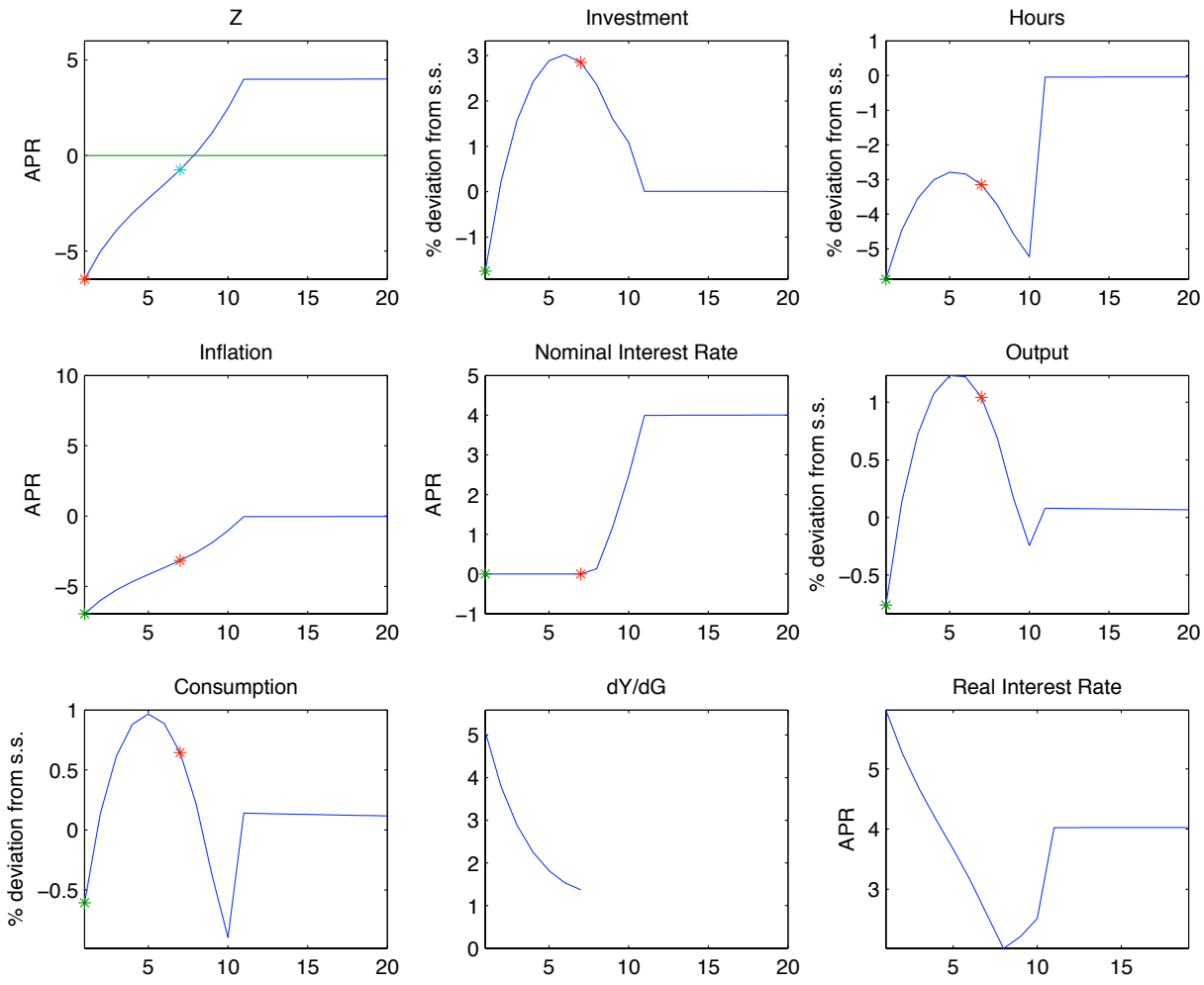

Figure 9: Effect of investment-specific shock
when the zero bound is binding
(model with capital, 'standard' adjustment costs).

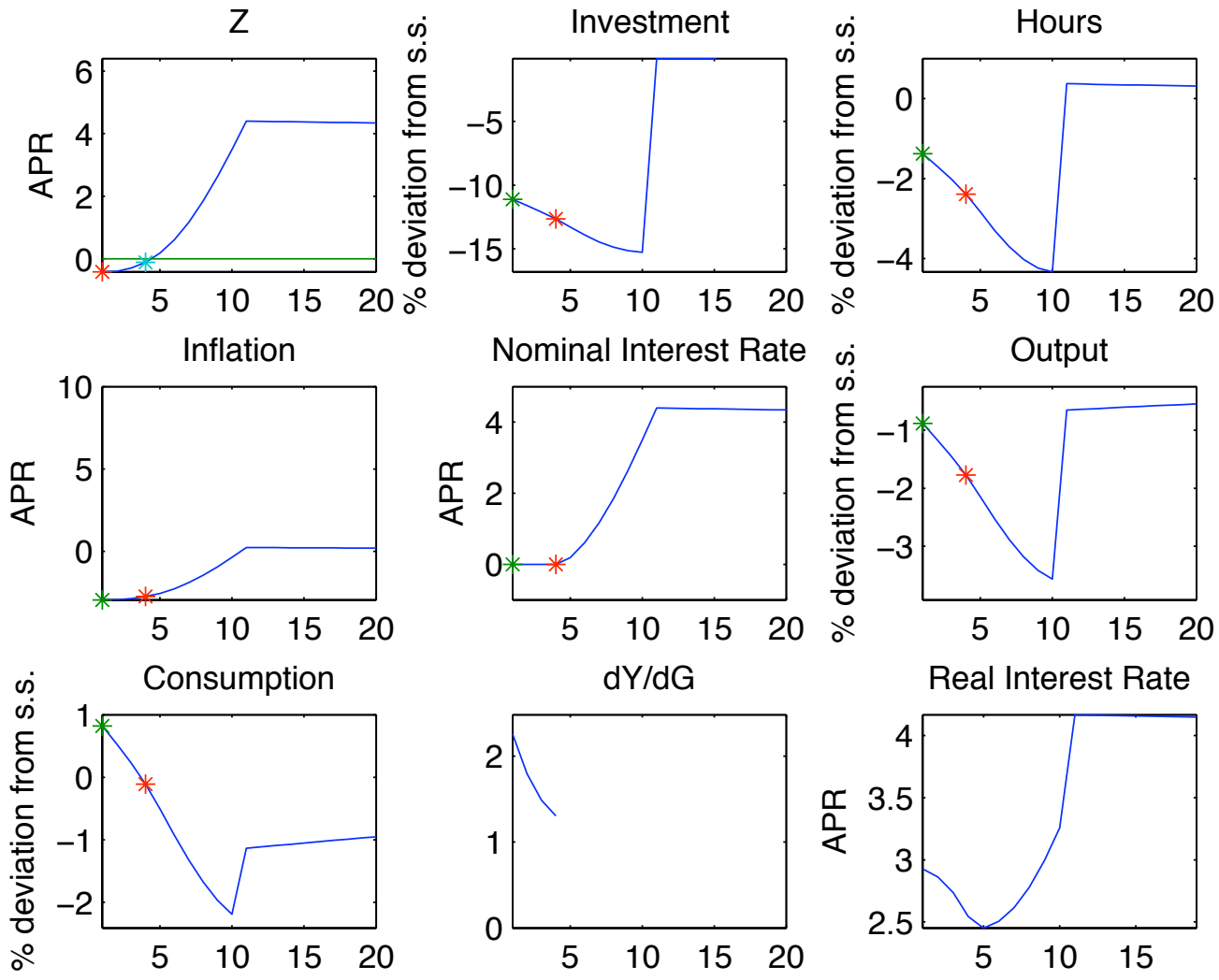

Figure 10: Government spending multiplier in the ACEL model.