The Extensive Margin of Exporting Goods: A Firm-level Analysis^{*}

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Abstract

We examine three-dimensional panel data for Brazilian and Chilean manufacturing exporters, their products and destinations. The data show that (i) the distribution of the exporters' number of goods (the exporter scope) is robust within destinations and approximately Pareto with most firms selling only one or two goods, and (ii) that the exporter scope is positively associated with average sales per good within destinations but not across destinations. We present a heterogeneous-firm model with product choice that implies these regularities and retains key predictions of previous trade models. The model explains the regularities with dis-economies of scope in product-entry costs on the distribution side.

Keywords: International trade; heterogeneous firms; multi-product firms; firm and product panel data; Brazil; Chile

JEL Classification: F12, L11, F14

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1 Introduction

Most exports are shipments by multiproduct firms.¹ We examine the extensive margin of introducing goods at export destinations to learn about determinants of trade flows and the nature of market-entry costs. We present a heterogeneous-firm model with product choice, and consistent empirical evidence, in which distribution-side costs determine exporter behavior at the extensive margin of adding goods. These distributionside product-entry costs are repeated at every destination and offer the key explanation for empirically observed regularities in exporter behavior.

We use three-dimensional panel data for Brazilian exporters, their destination markets and their products, and confirm the regularities with Chilean data.² We decompose total exports into the common extensive margin for the market-entry of firms and sales per firm. Our data allow us to further decompose sales per firm into the extensive margin of product-entry with goods and the remaining intensive margin of sales per good. We focus our investigation on the novel extensive margin of product-entry with additional goods. Two important regularities emerge from our data. First, the probability distribution of the average exporter's number of goods per destination (the *exporter*) scope at the destination) is remarkably robust across destination markets and resembles a Pareto distribution when firms are ranked by total sales. Second, the firms' average sales per good at the intensive margin (their *average product scale*) strictly increases with exporter scope worldwide, and destination by destination. The average product scale distribution is approximately Pareto in the upper tail. We explain these regularities with a heterogenous-firm model, where firms draw their productivity from a Pareto distribution and make choices at the three margins: entry by destination, exporter scope by destination, and product scale by good and destination. On the distribution side, firms face repeated product-entry costs by destination.

Specifically, exporters first incur a market-entry cost upon shipment of the first unit of the first good. Then, upon introduction of an additional good at a destination, the exporter incurs an additional fixed product-entry cost that strictly increases in exporter scope at the destination. Both cost components are related to distribution, not

¹Bernard, Jensen and Schott (2005) document for U.S. trade data in the year 2000, for instance, that firms that export more than five products at the HS 10-digit level make up 30 percent of exporting firms but account for 97 percent of all exports. In our Brazilian exporter data for 2000, 25 percent of all manufacturing exporters ship more than ten products at the HS 6-digit level and account for 75 percent of total exports.

²Evidence on Brazil is arguably informative for our understanding of a typical country's world export behavior. While Brazil ranks among the top 30 exporting countries in the world, its exports per capita are close to the world median. World trade flow (WTF) data for the year 2000, the final WTF year, show Brazil's total exports at the 88th percentile worldwide (top 27th out of 205). In terms of exports per capita, Brazil ranks at the 48th percentile (top 100th out of 192). Exporter behavior in Brazil is nevertheless strikingly similar to that in leading export countries such as France (Eaton, Kortum and Kramarz 2004) and the United States (Bernard, Redding and Schott 2007). Chilean data from (Álvarez, Faruq and López 2007) confirm the patterns for Brazil.

production, and important to match the data regularities. Consumers in every country have nested Dixit-Stiglitz preferences over firms' product mixes and the firms' individual products (similar to Allanson and Montagna 2005, Agur 2007). The model implies that exporter scope and total exports strictly increase with the firm's productivity and that they are both Pareto distributed. We show in the model that fixed product-entry costs must be strictly convex in exporter scope at every destination to match the empirical finding that average product scale is positively associated with exporter scope at every destination. Under strictly convex product-entry costs, average product scale strictly increases with the firm's productivity and is Pareto distributed. The intuition is that a firm equates the marginal profit from introducing an additional good with the marginal cost of product entry. Under a constant elasticity of substitution, the marginal profit of a good is a constant fraction of sales per good. So, unless the marginal product-entry cost strictly increases with the firm's scope (which is the case for strictly convex product-entry costs), the constant elasticity of substitution across the firm's products would imply that average sales per product do not increase with productivity—contrary to the empirical evidence. A constant elasticity of demand with respect to additional goods under nested Dixit-Stiglitz utility, and a constant elasticity of product-entry costs with respect to exporter scope, together imply that exporter scope and total exports are Pareto distributed under Pareto distributed productivity.

The model is a tractable extension of the Melitz (2003) model to the multi-product setting. In the limit of our model, as product-entry costs become arbitrarily convex in scope, our Pareto shape parameter in the total exports distribution converges to the Eaton, Kortum and Kramarz (2005) shape parameter and our decomposition of the total-exports response to trade costs along three margins converges to the two margins in the Chaney (2007) version of Melitz (2003). Our model also offers a straightforward extension to the case of product heterogeneity as in Bernard, Redding and Schott (2006). Our findings are consistent with heterogeneity in the productivity of goods (expertise) as in the seminal work by Bernard et al. (2006). When the distribution of goods expertise is Pareto, however, average sales increase only if product entry costs are strictly convex as in our model.

We carefully address the empirical concern whether worldwide firm-level explanations (such as product-adoption in production as in Bernard et al. 2006), and not repeated distribution-side product-entry costs, drive the observed positive association between average product scale and exporter scope. We perform covariance decompositions on our three-dimensional panel data for exporters, their products and their destinations, and uncover a negative association between firm-level worldwide scope and worldwide average product scale. This evidence is not consistent with a positive scope-scale association at the firm level. Within each destination market, however, there is a positive association between the firm's scope and its average product scale, and this repeated positive association dominates the aggregate data. In our model, distribution-side dis-economies of scope, under product-entry costs that are convex in scope within every destination, explain the observed exporter behavior.

The paper has four more sections. Section 2 discusses the three-dimensional panel data on Brazilian and Chilean exporters, their products and destinations, and documents main facts. (Details on complementary data sources are relegated to the Appendix.) Section 3 presents our theoretical model. Section 4 turns to empirical analysis that helps evaluate important implications of the model. Section 5 concludes.

2 Data

Our main data source is a three-dimensional panel of Brazilian exporters, their respective destination countries, and their export products at the Harmonized-System 6-digit level. We combine the exporter data with worldwide bilateral trade information from outside sources (for outside sources see the Appendix). The Brazilian exporter data derive from the universe of customs declarations for merchandize exports in the year 2000, by any firm. The pristine Brazilian NCM product codes are 8-digit numbers, of which the first six digits coincide with the first six digits in the Harmonized System. We aggregate these exporter data to the firm, year and Harmonized-System 6-digit level.

At the firm level, the data exhibit market-access patterns broadly similar to the French firm-destination data in Eaton et al. (2004) and the U.S. firm-destinationproduct data in Bernard et al. (2007). Similar to Eaton et al. (2004), for instance, firm entry into destinations explains around 70 percent of Brazil's exports covariation with gravity-equation variables across destinations. In contrast to the French and U.S. data, however, the Brazilian data do not cover domestic sales. This restricts our analysis to export-market access. To assess the empirical regularities for an additional country, we use a three-dimensional panel of Chilean exporters from the universe of customs declarations by Chilean manufacturing firms in 2000 (Álvarez et al. 2007). For Chile, product codes are reported at the Harmonized System 8-digit level and we aggregate the information to the Harmonized-System 6-digit level for comparability.

To relate our data to product-market information for destination countries and their sectors, we map the Harmonized System 6-digit codes to *ISIC revision 2* at the two-digit level and link our data to World Trade Flow (*WTF*) data for the year 2000 (Feenstra, Lipsey, Deng, Ma and Mo 2005).³ The link between our data and *WTF* also provides us with an estimate of the coverage of Brazil's self-reported exports declarations. In 2000, our *SECEX* data for manufactured merchandize sold by Brazilian firms from any sector, including commercial intermediaries, covers 95.9 percent of Brazilian exports in *WTF*. The small discrepancy might be related to underreported Brazilian exports, which *WTF* may uncover as imports elsewhere, or to valuation differences because of differently reported exchange rate fluctuations and transportation costs.

³Our novel concordance will become available at URL econ.ucsd.edu/muendler/brazil shortly.

From source s			Brazil			Chile
to destination d	USA	Argentina	Oecd	non-Oecd	World	World
	(1)	(2)	(3)	(4)	(5)	(6)
# of Firms (M)	3,083	$4,\!590$	5,041	$8,\!590$	10,215	4,099
# of Destinations (N)	1	1	23	141	170	140
# of HS-6 goods (G)	2,144	2,814	2,772	$3,\!535$	3,717	$3,\!199$
# of Observations (MNH)	10,775	$21,\!623$	$36,\!359$	$124,\!811$	$162,\!570$	$37,\!183$
Destination share in Total exp.	.257	.144	.559	.441	1	1
Firm shares in Total exports						
Single-prod. firms	.123	.086	.142	.069	.090	.041
Multi-prod. firms' top product	.662	.555	.625	.572	.597	.715
Multi-prod. firms' other prod.	.215	.359	.233	.359	.313	.243
Median Total exports $(T_d(m))$.120	.068	.137	.064	.089	.038
Median Exporter scope $(G_d(m))$	1	2	2	2	2	2
Median Avg. prod. scale $(z_d(m))$.068	.031	.070	.027	.037	.014
Mean Total exports (\bar{t}_d)	3.170	1.192	4.217	1.928	3.720	2.779
Mean Exporter scope (\bar{G}_d)	3.495	4.711	3.933	5.190	5.278	5.454
Mean Avg. prod. scale (\bar{z}_d)	.907	.253	1.072	.372	.705	.510

Table 1: Sample Characteristics by Destination

Sources: SECEX 2000 for Brazil, manufacturing firms and their manufactured products; Chilean customs data 2000 (Álvarez et al. 2007) for manufacturing firms.

Note: Aggregate regions (world, OECD, non-OECD) treated as single destinations, collapsing product shipments to different countries into single product shipment. Products at the Harmonized-System 6-digit level. Exports in US\$ million fob. Non-OECD includes all non-members in 1990; OECD includes all OECD members in 1990. The U.S. is Brazil's top export destination in 2000, Argentina second to top. Firms' mean product scale (z_d in US\$ million fob) is the scope-weighted arithmetic mean of exporters' average product scales.

For our analysis, we remove commercial intermediaries from the Brazilian data, and only keep manufacturing firms who report their direct export shipments. This sample restriction makes our findings most closely comparable to Eaton et al. (2004) and Bernard et al. (2007) but we lose many observations, mainly because of the importance of commercial intermediaries for export processing, partly because of missing sector information, and partly because of manufacturing firms' resales of non-manufactured goods. After restricting the sample to manufactured merchandize exported directly by Brazilian manufacturers, our sample covers 81.7 percent of the WTF manufactures exports.

As Table 1 shows in column 5 (column 6), our Brazilian (Chilean) data include 10,215 (4,099) manufacturing firms with shipments of 3,717 (3,199) manufacturing goods at the 6-digit Harmonized System level to 170 (140) foreign destinations, and a total of 162,570 (37,183) exporter-destination-product observations.⁴ Multi-product

⁴We remove export records with zero value from the Brazilian data, which include shipments of

exporters dominate. They ship more than ninety percent of all exports both from Brazil and Chile, and their single top-selling products account for almost sixty percent of all Brazilian exports and more than seventy percent of all Chilean exports.

To analyze export behavior, we decompose a firm ϕ 's total exports $t_{d\phi}$ from Brazil or Chile to destination market d into the firm's number of goods sold at d (the exporter scope) $G_{d\phi}$ and the firm's average sales per export good in d (the average product scale) $z_{d\phi} \equiv t_{d\phi}/G_{d\phi}$:

$$t_{d\phi} = \sum_{g=1}^{G_{d\phi}} p_{dg\phi} x_{dg\phi} = G_{d\phi} z_{d\phi}$$

where $p_{dg\phi}$ is the price of product g and $x_{dg\phi}$ its export quantity. To calculate summary medians and means of these variables for regional aggregates and the world as a whole in Table 1 (columns 3 to 6), we treat the aggregate as if it were a single destination and collapse all product shipments to different countries into a single product shipment. In all subsequent data treatments, in contrast, we will analyze these variables country by country, consistent with our main hypothesis that distribution-side determinants of trade matter repeatedly destination by destination.

The median exporter is a relatively small exporter, with sales to the rest of the world totalling around US\$ 90,000 in Brazil (column 5) and US\$ 40,000 in Chile (column 6). The mean Brazilian exporter, in contrast, sells around US\$ 3.7 million abroad, more than 40 times as much as the firm at the median. Similarly, the mean Chilean exporter sells around US\$ 2.8 million abroad, or 70 times as much as the firm at the median. Exporter scope and average product scale exhibit similar differences between mean and median. The Brazilian firm at the median sells two products worldwide, but the mean scope per firm is 5.3 products. The Brazilian firm at the median has a product scale of around US\$ 40,000 per product, but the mean product scale per exporter is US\$ 700,000, or around 20 times as high as that for the median firm.⁵

The importance of the top-selling product at multi-product exporters and the meanmedian difference patterns repeat across destinations. To investigate the robustness across countries, we select Brazil's top two export destinations (United States and Argentina), as well as two destination aggregates, OECD and non-OECD countries. Our theory emphasizes the importance of exporting behavior within destinations, and single countries are our main object of investigation. Within single countries, the mean firm's

commercial samples but also potential reporting errors, and lose 408 of initially 162,978 exporterdestination-product observations. Our results on exporter scope do not materially change when including or excluding zero-shipment products from the product count. There are no reported shipments with zero value in the Chilean data.

⁵The means in Table 1 are calculated as follows. A source country's total exports T_d are decomposed into $T_d = M_d \bar{G}_d \bar{z}_d$, where M_d is the number of exporters to destination d, $\bar{G}_d \equiv \sum_{\phi=1}^{M_d} G_d(\phi)/M_d$ is the exporters' mean exporter scope, and $\bar{z}_d \equiv \bar{t}_d/\bar{G}_d$ is their goods' mean product scale. Equivalently, \bar{z}_d is the weighted arithmetic mean of $z_d(\phi)$ over all ϕ , with weights $G_d(\phi)$: $\bar{z}_d = \sum_{\phi=1}^{M_d} G_d(\phi) z_d(\phi) / [\sum_{\phi=1}^{M_d} G_d(\phi)] = \bar{t}_d/\bar{G}_d$. Scope weighting is necessary for the mean scope and the mean product scale to yield total exports when multiplied.

exports exceed the median firm's exports by similarly large factors as in the aggregate, between 14 (in Argentina) and 26 (in the United States). While total exports to OECD countries are more than twice as large as exports to all non-OECD countries, the exporter distributions look alike in OECD and non-OECD countries. Exports of the mean firm exceed the exports of the median firm by a factor of about 30 for both the non-OECD and the OECD aggregates. The regularities are consistent with the idea that systematic features of distribution costs or exporter characteristics, or both, determine export behavior.

Brazilian exporters exhibit market access patterns similar to those of French exporters (Eaton et al. 2004). Following Eaton et al. (2004), total exports T_{sd} from source country s (Brazil, France) to destination market d can be decomposed into: $M_{sd}\bar{t}_{isd}$, where M_{sd} is the number of source country s's exporters with shipments to destination country d, and \bar{t}_{isd} are these exporters' average sales in destination country d. The same total exports T_{sd} can also be decomposed into: $\lambda_{sd}T_d$, where λ_{sd} is the market share of source country s's exports in destination d, and T_d is the market size of destination country d (manufacturing absorption). By definition, $M_{sd}\bar{t}_{isd} = \lambda_{sd}T_d$. We regress the log number of firms on the log of $\lambda_{sd}T_d$ to inspect how these market characteristics are associated with the market presence of additional firms M_{sd} (as opposed to additional sales per firm \bar{t}_{isd}):⁶</sup>

$$\ln M_{sd} = -5.710 + .719 \ln \lambda_{sd} + .626 \ln T_d.$$

The R^2 is 0.833.⁷ Firm presence explains most of the variation in Brazilian exports, but is somewhat less important than in France. Given market size and industry bias, a higher Brazilian (French) market share λ_{sd} in a destination typically reflects 72 (88) percent more firms selling there and 28 (12) percent more sales per firm. Given market share, larger market size T_d is associated with 63 (62) percent more firms and 37 (38) percent more sales per firm.

Brazilian exporters' average sales \bar{t}_{isd} to a destination industry can be decomposed further into $\bar{t}_{isd} = \bar{G}_d \bar{z}_d$, where \bar{G}_d is the exporters' mean exporter scope, and $\bar{z}_d \equiv \bar{t}_d/\bar{G}_d$ is their goods' mean product scale. We regress the log mean exporter scope on the log of $\lambda_{sd}T_d$ to examine how market characteristics are related to exporter scope (as opposed to $M_{sd}\bar{z}_d$, the market presence of additional firms and additional product

⁶We aggregate the *SECEX* exporter data to 16 SIC industries as in Eaton et al. (2004) for this purpose and link the data to destination information from WTF (Feenstra et al. 2005) and *Unido* Industrial Statistics (UNIDO 2005). The regression sample contains 67 destinations in the Brazilian data (excluding the domestic Brazilian market), whereas there are 113 destinations in the French data (including the domestic French market). Gomes and Ellery Jr. (2007) present similar regressions for a sub-sample of *SECEX* exporters linked to firm survey data (*Pesquisa Industrial Anual*) in 1999.

⁷Standard errors in parentheses. Because of the identity connecting the variables, a regression of $\ln \bar{t}_{isd}$ on the log of $\lambda_{sd}T_d$ yields coefficients of 1 minus the ones reported above.



Source: SECEX 2000, manufacturing firms and their manufactured products. *Note:* Products at the Harmonized-System 6-digit level.



scale per good and firm):

$$\ln \bar{G}_d = 2.324 + .087 \ln \lambda_{sd} - .058 \ln T_d.$$

The R^2 is 0.281. Neither market share nor market size are statistically significant predictors of exporter scope at conventional levels.⁸ Our model will allow for small variation in exporter scope across destinations. Though exporter scope has little explanatory power for exports in descriptive regressions, its association with scale by destination provides important insight into market access costs.

Figure 1 plots the firms' total exports against their total-exports percentile and exporter scope against the exporter-scope percentile for Brazil's top two export markets, the United States and Argentina. Except for the small firms, total exports in the

⁸Standard errors in parentheses. The R^2 drops to .212 when including $\ln M_{sd}$ and industry-fixed effects but coefficients become statistically significant at conventional levels except for market size, while magnitudes change little.

upper panel exhibit an approximate Pareto distribution. In this paper, we strive to explain the approximate Pareto shape of the distribution (for an explanation of deviant small-firm behavior in the lower tail see Arkolakis 2008). Exporter scope in the lower panel is a discrete variable but the overall shape of the distribution is similar to that of a Pareto variable.

Figure 2 illustrates the common relationship between the firm distribution of total exports and mean scope and scale across the same six export markets as in Table 1. All axes have a logarithmic scale. We group firms by their total exports percentile along the horizontal axis. At the origin of the horizontal axis, we plot the mean scope and mean product scale per firm for all firms in the sample, that is we plot the means that we also report in Table 1. Then we step one percentile to the right along the horizontal axis and restrict the sample to the firms in the top 99 percentiles of total exports. The figure depicts the mean scope and the mean product scale for this higher-up exporter group.⁹ We continue to move up in the total-exports ranking of firms, percentile by percentile, depicting the mean scope and the mean product scale for higher and higher-percentile groups of firms, until we reach the group of firms in the top (100th) percentile of total exports.

Both log mean scope and log mean scale increase roughly linearly with the percentile in the two individual export markets United States and Argentina and, on average, in the aggregate regions (OECD, non-OECD, world) for ten-country groups. The simultaneous increase in both scope and scale with the firm's percentile in total exports implies that scope and scale are positively related. The regularity across markets is consistent with the idea that firms' choices of scope and scale are positively associated within every destination market. We now turn to a model of exporting that is consistent with the evidence, and then revisit the data to evaluate the implied relationships.

3 A Model of Exporter Scope and Product Scale

We present a model of heterogeneous firms that sell one or multiple products. Firms incur destination-specific market entry costs that depend on the number of products they offer at a destination. Consumers in every country have nested preferences over firms' product mixes and the firms' individual products.

⁹Formally, we calculate total exports $t_d(\phi_1)$ at the first percentile of firms. We consider only the top 99 percent of firms ϕ' whose exports exceed the first-percentile threshold $t_d(\phi') > t_d(\phi_1)$. For these M'_d firms in the top 99 percentiles, we calculate the mean scope $\sum_{\phi'=1}^{M'_d} G_d(\phi')/M'_d$ and the scope-weighted mean product scale $\sum_{\phi'=1}^{M'_d} G_d(\phi') z_d(\phi') / [\sum_{\phi'=1}^{M'_d} G_d(\phi')]$. Scope weighting is necessary for the mean scope and the mean product scale to yield total exports when multiplied (see footnote 5). We repeat these calculations for total exports $t_d(\phi_p)$ at every percentile p.



Source: SECEX 2000 for Brazil, manufacturing firms and their manufactured products; Chilean customs data 2000 (Álvarez et al. 2007) for manufacturing firms.

Note: Left-most observations are all exporters; at the next percentile are exporter observations with shipments in the top 99 percentiles; up to the right-most observations with exporters whose shipments are in the top percentile. Aggregate regions include only destinations with more than 100 firms; destinations ranked by total exports and lumped into groups of ten destinations for which unweighted means over distributions are shown (20 OECD countries for Brazil, 49 non-OECD for Brazil, 70 worldwide for Brazil, 28 for Chile). Products at the Harmonized-System 6-digit level. Non-OECD includes all non-members in 1990; OECD includes all OECD members in 1990. Firms' mean product scale (\bar{z}_d in US\$ thousand fob) is the scope-weighted arithmetic mean of exporters' average product scales.

Figure 2: Scope, Average Scale and the Total Exports Distribution

3.1 Consumers

There are N countries. When we consider an export destination, we label the country with d. The source country of an export shipment receives the label s. There is a measure of L_d consumers at destination d. Consumers have symmetric preferences with a constant elasticity of substitution σ over a continuum of varieties. In our multiproduct setting, a conventional "variety" becomes the composite product

$$X_{isd}(\omega) \equiv \left[\int_0^{G_{isd}(\omega)} x_{isdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \,\mathrm{d}g$$

that a seller ω from industry *i* in source country *s* offers for sale at destination *d*. In marketing terminology, the composite product is a firm's *product mix*. The elasticity of substitution across goods in the product mix ε is constant and G_{isd} is the measure of goods. We assume that every product mix is uniquely made by a single firm, but a firm may ship different product mixes to different destinations.

The set of product mixes shipped from industry i in source country s to destination d is Ω_{isd} . So the consumer's utility at destination d is

$$\left(\sum_{s=1}^{N}\sum_{i=1}^{I(s)}\int_{\omega\in\mathbf{\Omega}_{isd}}\left[\int_{0}^{G_{isd}(\omega)}x_{isdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}g\right]^{\frac{\varepsilon}{\varepsilon-1}\frac{\sigma-1}{\sigma}}\,\mathrm{d}\omega\right)^{\frac{\sigma}{\sigma-1}}\,\mathrm{where}\,\,\varepsilon>1,\,\sigma>1,\varepsilon\neq\sigma.$$
(1)

A similar nested utility function is also used by Allanson and Montagna (2005) to study implications of the product life-cycle for market structure, and by Agur (2007) to analyze trade patterns in a two-country heterogeneous-firm model with a continuum of products. The specification generalizes monopolistic-competition models of trade (such as Krugman 1980).¹⁰ For preferences to be well defined, we require that $\varepsilon > 1$ and $\sigma > 1$. Subsequent derivations do not materially differ if we assume that products

$$\left(\sum_{s=1}^{N}\sum_{i=1}^{I(s)}\int_{\omega\in\mathbf{\Omega}_{isd}} \left[\sum_{g=1}^{G_{isd}(\omega)} x_{isdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}\frac{\sigma-1}{\sigma}} \mathrm{d}\omega\right)^{\frac{\sigma}{\sigma-1}} \quad \text{where} \quad \varepsilon>1, \, \sigma>1, \varepsilon\neq\sigma, \quad (2)$$

and Ω_{isd} is the set of product mixes shipped from s to d. Atkeson and Burstein (2007) adopt a similar nested CES form in a heterogenous-firms model of trade but do not consider multi-product firms. A discrete number of products facilitates empirical work with product-level data, and we have re-derived our main results for function (2). To compare our model's implications directly to related heterogeneous-firm models, however, we present the continuous version of the product space. We could make the elasticities of substitution country-specific (ε_d and σ_d), and all our results would continue to apply. We keep the elasticities the same across destinations to simplify notation and to emphasize that our results do not depend on preference assumptions.

¹⁰There is a counterpart to (1) in discrete product space, where consumers at d have preferences

within a product mix are more substitutable among each other than with outside goods $(\varepsilon > \sigma > 1)$, or less substitutable $(\sigma > \varepsilon > 1)$.

Every consumer receives per-capita income $y_d = w_d + \Pi_d$, where w_d is the wage for labor, inelastically supplied to producers in country d, and $\Pi_d \equiv \sum_{i=1}^{I} \sum_{\varsigma=1}^{N} \pi_{id\varsigma}/L_d$ is the consumer's share in total profits earned by the country's producers. Country d's total income is $y_d L_d$. The consumer's first-order conditions of utility maximization imply a product demand

$$x_{isdg}(\omega) = \left(\frac{p_{sdg}(\omega)}{P_{isd}(\omega; G_{isd})}\right)^{-\varepsilon} X_{isd}(\omega; G_{isd}), \tag{3}$$

given the firm's product mix $X_{isd}(\omega; G_{isd})$ and the product-mix price $P_{isd}(\omega; G_{isd})$:

$$X_{isd}(\omega;G_{isd}) \equiv \left[\int_{0}^{G_{isd}(\omega)} x_{isdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}g\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } P_{isd}(\omega;G_{isd}) \equiv \left(\int_{0}^{G_{isd}(\omega)} p_{sdg}(\omega)^{1-\varepsilon} \mathrm{d}g\right)^{\frac{1}{1-\varepsilon}}.$$

The first-order conditions also imply a product-mix demand

$$X_{isd}(\omega; G_{isd}) = \left(\frac{P_{isd}(\omega; G_{isd})}{P_d}\right)^{-\sigma} X_d,\tag{4}$$

where

$$X_d \equiv \left(\sum_{s=1}^N \sum_{i=1}^{I(s)} \int_{\omega \in \mathbf{\Omega}_{isd}} X_{isd}(\omega; G_{isd})^{\frac{\sigma-1}{\sigma}} \,\mathrm{d}\omega\right)^{\frac{\sigma}{\sigma-1}}$$

and

$$P_d \equiv \left(\sum_{s=1}^N \sum_{i=1}^{I(s)} \int_{\omega \in \mathbf{\Omega}_{isd}} P_{isd}(\omega; G_{isd})^{1-\sigma} \,\mathrm{d}\omega\right)^{\overline{1-\sigma}}$$

So, the demand for firm ω 's product g, produced in country s and industry i and sold to country d, is

$$x_{isdg}(\omega) = \left(\frac{p_{sdg}(\omega)}{P_{isd}(\omega; G_{isd})}\right)^{-\varepsilon} \left(\frac{P_{isd}(\omega; G_{isd})}{P_d}\right)^{-\sigma} \frac{y_d L_d}{P_d}$$
(5)

with $P_d X_d = y_d L_d$. This is an important relationship. Since $P_{isd}(\omega; G_{isd})$ strictly decreases in exporter scope for $\varepsilon > 1$, higher exporter scope diminishes infra-marginal sales and reduces $x_{isdg}(\omega)$ for $\varepsilon > \sigma$. In other words, if products within a product mix are more substitutable among each other than with outside goods, raising exporter scope is costly to the exporter. For the converse case with $\sigma > \varepsilon$, however, higher exporter scope boosts infra-marginal sales and raises $x_{isdg}(\omega)$. As long as the productentry cost is a sufficiently convex function of the firm's offered scope, our model will generate results consistent with the stylized facts in section 2, irrespective of whether there is a decline in infra-marginal sales or not. The only difference between the case of a decline in infra-marginal sales and the converse case is that product-entry cost need to be a more convex function of scope if there is no decline in infra-marginal sales in order for optimal scope to be well defined.

3.2 Firms

There is a continuum of firms that differ ex ante only in their worldwide unique productivity scalar ϕ . Each firm is located in a single source country s and belongs to an industry i. Firm ϕ manufactures every one of its products with the same constantreturns-to-scale technology $q_{sdg}(\phi) = \phi \ell$, independent of s, d and g, where ℓ is employment contracted at the source country's wage w_s . (We will allow for product-specific productivity, or expertise, in an extension below.) When exported, a product incurs a standard iceberg trade cost so that $\tau_{sd} > 1$ units must be shipped from s for one unit to arrive at destination d. We assume that $\tau_{ss} = 1$ for domestic sales.

We call an exporter's measure of goods G_{isd} shipped to destination d the exporter scope at destination d. We call the sales $p_{sdg} x_{isdg}$ of a firm's individual product g the product scale at destination d, where p_{sdg} is the product's price. A firm maximizes its profits by choosing its scope G_{isd} for every destination d and the scale $p_{sdg} x_{isdg}$ for every product g at destination d. Firms with a given productivity ϕ from a given country s face an identical optimization problem.

In order to sell in a market, a firm has to incur a fixed market-entry cost $\gamma_i \kappa_d > 0$ in terms of destination-country labor units.¹¹ The market-entry cost has an industryspecific component γ_i and a destination-specific component κ_d . In addition, a firm must pay a scope-dependent product-entry cost at every sales destination. This productentry cost increases in an exporter's offered scope

$$F_d(G_{isd}) = \gamma_i \gamma_d \left(G_{isd}\right)^{\delta} / \delta \qquad \text{where} \quad \gamma_i \gamma_d > 0, \ \delta > 0, \tag{6}$$

at destination d, also in terms of destination-country labor units. If $\delta > 1$, the productentry cost is a convex function of the firm's offered scope. The product-entry cost has the same industry-specific component γ_i as in market-entry cost and a potentially different destination-specific component γ_d .

¹¹In continuous product space with nested CES utility, every firm would otherwise export to all destinations worldwide. No fixed market-entry cost is needed in the discrete product-space version of our model where the firm's first good causes a nontrivial fixed product-entry cost.

The firm's profit in a given destination market d is

$$\pi_{isd}(\phi) = \max_{G_{isd}, P_{isd}, \{p_{sdg}\}_{g=1}^{G_{isd}}} \int_0^{G_{isd}} \left(p_{sdg} - \tau_{sd} \frac{w_s}{\phi} \right) (p_{sdg})^{-\varepsilon} \frac{\left(P_{isd}(\phi; G_{isd}) \right)^{\varepsilon - \sigma}}{\left(P_d \right)^{1 - \sigma}} y_d L_d \, \mathrm{d}g \, (7)$$
$$- w_d \gamma_i \kappa_d - w_d \gamma_i \gamma_d \frac{\left(G_{isd} \right)^{\delta}}{\delta}$$

by demand (5), where $\pi_{isd}(\phi)$ denotes maximized profits. In general, the product-mix price $P_{isd}(\phi; G_{isd})$ is a function of G_{isd} so that a firm takes into account how every single product competes with the neighboring products in the firm's product mix at a destination. For constant elasticities of substitution, however, the optimal markup does not depend on exporter scope.¹²

The firm's profit maximization problem can be formulated as a choice of the price index P_{isd} and the individual product prices p_{sdg} to maximize (7) subject to the constraint that $P_{isd}(\phi; G_{isd})^{1-\varepsilon} = \int_0^{G_{isd}(\phi)} p_{sdg}(\phi)^{1-\varepsilon} dg$ by (3).¹³ The first-order conditions with respect to the product-mix price P_{isd} and individual prices p_{sdg} imply identical product prices $p_{sdg}(\phi) = \eta \tau_{sd} w_s/\phi$ with a constant markup over marginal cost $\eta \equiv \sigma/(\sigma-1) > 1$ for $\sigma > 1$ in optimum (see Appendix A). So, firm ϕ optimally chooses to sell every product in its product mix at destination d for the same price, and therefore, within an industry i, also with the same quantity. Note that symmetric prices and supplies are an optimum outcome for multiproduct firms, not an assumption.

Given G_{isd} , optimal product scale is therefore the same for every product g,

$$p_{sdg}(\phi) x_{isdg}(\phi) = (G_{isd})^{-\frac{\varepsilon-\sigma}{\varepsilon-1}} y_d L_d \left(\frac{\phi P_d}{\eta \tau_{sd} w_s}\right)^{\sigma-1},$$
(8)

because $P_{isd}(\phi; G_{isd}) = (G_{isd})^{1/(1-\varepsilon)} p_{sdg}(\phi)$. Using optimal product scale in the profit function yields

$$\pi_{isd}(\phi) = \max_{G_{isd}} \frac{(G_{isd})^{\bar{\sigma}} y_d L_d}{\sigma} \left(\frac{\phi P_d}{\eta \tau_{sd} w_s}\right)^{\sigma-1} - w_d \gamma_i \kappa_d - w_d \gamma_i \gamma_d \frac{(G_{isd})^{\delta}}{\delta} \quad \text{where } \bar{\sigma} \equiv \frac{\sigma-1}{\varepsilon-1}.$$

The composite elasticity term $\bar{\sigma} \equiv (\sigma - 1)/(\varepsilon - 1)$ satisfies $\bar{\sigma} < 1$ iff $\varepsilon > \sigma > 1$ (that is iff products within the product mix are closer substitutes among each other than with outside goods).

Taking the first derivative of the scale-optimized profit function with respect to G_{isd} and setting it to zero, we find the optimal exporter scope for a firm from country s and

¹²Even in discrete product space, the optimal markup does not vary with exporter scope for constant elasticities of substitution. We choose a continuous product space because discrete product space results in some non-analytic solutions.

¹³We thank Elhanan Helpman for pointing out this solution strategy.

industry i selling to d,

$$G_{isd}(\phi) = \left[\frac{\bar{\sigma}}{\sigma} \frac{y_d L_d}{w_d \gamma_i \gamma_d} \left(\frac{\phi P_d}{\eta \tau_{sd} w_s}\right)^{\sigma-1}\right]^{\frac{1}{\delta-\bar{\sigma}}} \quad \text{for} \quad G_{isd}(\phi) \ge G_d^* \quad \text{and} \quad \delta > \bar{\sigma}.$$
(9)

For a well-defined profit-maximum to exist, profits must be concave in G_{isd} at the optimal $G_{isd}(\phi)$. Equivalently, product-entry costs must not be too concave and satisfy $\delta > \bar{\sigma}$. Otherwise, firms would choose an infinite exporter scope.¹⁴ So, more productive firms sell more products in a market. Using (9) in the zero-profit condition $\pi_{isd}(\phi) = 0$, the minimum optimal scope G_d^* of any firm in country d is

$$G_d^* = \left(\frac{\delta \bar{\sigma}}{\delta - \bar{\sigma}} \frac{\kappa_d}{\gamma_d}\right)^{\frac{1}{\delta}}.$$
 (10)

This minimum optimal scope solely depends on destination-specific elasticities and cost parameters at d. The minimum optimal scope is independent of industry and independent of the country's size L_d and per-capita income y_d .

Similarly, there is a productivity threshold for exporting from s to d. Using (10) in (9), the productivity threshold ϕ_{isd}^* for a firm in industry i from source country s to sell at d is

$$\phi_{isd}^* = \left(\frac{\sigma}{\bar{\sigma}} \frac{w_d \,\gamma_i \gamma_d}{y_d L_d} \left(G_d^*\right)^{\delta - \bar{\sigma}}\right)^{\frac{1}{\sigma - 1}} \frac{\eta \,\tau_{sd} \,w_s}{P_d}.\tag{11}$$

The productivity threshold for exporting to d decreases in destination-market expenditure $y_d L_d$, it increases in the scope-cost parameter $w_d \gamma_d$ and in the fixed market-entry cost $w_d \kappa_d$ (because $\delta > \bar{\sigma}$), and it increases in the firm's marginal cost $\tau_{sd} w_s$. (A firm with cutoff productivity ϕ_{isd}^* is considered to be selling to d.)

Using (10) and (11), the optimal exporter scope (9) for shipments from source country s to country d can also be expressed as

$$G_{isd}(\phi) = G_d^* \left(\frac{\phi}{\phi_{isd}^*}\right)^{\frac{\sigma-1}{\delta-\bar{\sigma}}}$$
(12)

in terms of the minimum optimal scope and the productivity threshold, and conditional on the firm's exporting from s to d ($\phi \ge \phi_{isd}^*$). Note that optimal exporter scope at destination d is a function of firm ϕ 's ranking relative to the least productive competitor from the same source country s and industry i, ϕ/ϕ_{isd}^* . So data for a single source country suffice for empirical tests.

¹⁴Derivation shows that the second-order condition is equivalent to $\delta > \bar{\sigma}$. The reason is that the ratio between the marginal revenue and the marginal cost of adding a product is proportional to $(G_{isd})^{\bar{\sigma}-\delta}$. Suppose $\delta < \bar{\sigma}$. Then the marginal-revenue-to-marginal-cost ratio tends to positive infinity as the measure of products goes to positive infinity.

Similarly, multiplying the optimal product scale (8) by the optimal measure of products (9), and using productivity threshold (11), yields a firm's optimal total exports

$$t_{isd}(\phi) \equiv G_{isd}(\phi) \, p_{sdg}(\phi) \, x_{isdg}(\phi) = \frac{\sigma \, w_d \, \gamma_i \gamma_d}{\bar{\sigma}} \left(G_{isd}(\phi) \right)^{\delta}$$
(13)
$$= \frac{\delta \, \sigma \, w_d \, \gamma_i \kappa_d}{\delta - \bar{\sigma}} \left(\frac{\phi}{\phi_{isd}^*} \right)^{\delta \frac{\sigma - 1}{\delta - \bar{\sigma}}},$$

conditional on the firm's exporting from s to d ($\phi \ge \phi_{isd}^*$). More productive firms have larger total sales to a market because $\delta > \bar{\sigma}$.

Finally, firm ϕ 's optimal average product scale is

$$z_{isd}(\phi) \equiv \frac{t_{isd}(\phi)}{G_{isd}(\phi)} = \frac{\sigma w_d \gamma_i \gamma_d}{\bar{\sigma}} \left(G_{isd}(\phi) \right)^{\delta - 1}$$

$$= \frac{\sigma w_d \gamma_i \gamma_d}{\bar{\sigma}} \left(G_d^* \right)^{\delta - 1} \left(\frac{\phi}{\phi_{isd}^*} \right)^{(\delta - 1)\frac{\sigma - 1}{\delta - \bar{\sigma}}},$$
(14)

conditional on the firm's exporting from s to $d \ (\phi \ge \phi_{isd}^*)$.

We summarize these findings in

Proposition 1 Suppose product-entry costs are not too concave $(\delta > \overline{\sigma} \equiv (\sigma - 1)/(\varepsilon - 1))$ and the elasticity of substitution within product mixes differs from the elasticity between product mixes ($\varepsilon \neq \sigma$). Then for all $s, d \in \{1, ..., N\}$

- there exists a productivity threshold $\phi_{isd}^* > 0$ such that exporter scope $G_{isd}(\phi) > 0$ and shipments $x_{isdg}(\phi) > 0$ for all products $g \in [0, G_{isd}(\phi)]$ iff $\phi \ge \phi_{isd}^*$;
- exporter scope $G_{isd}(\phi)$ strictly increases in ϕ for $\phi \ge \phi^*_{isd}$;
- average product scale $z_{isd}(\phi)$ strictly increases in $G_{isd}(\phi)$ and in ϕ for $\phi \ge \phi^*_{isd}$ iff $\delta > 1$ (convex product-entry costs in scope).

Whereas the optimal scope increases with the firm's productivity, optimal scale per product increases in productivity only if product-entry costs are convex ($\delta > 1$).¹⁵ The intuition is that a firm equates the marginal profit from introducing an additional good with the marginal cost of product entry in optimum. Under a constant elasticity of substitution, the marginal profit of a good is a constant fraction of sales per good. So, all firms would have identical average sales per product if marginal product entry costs were constant. When marginal product entry costs increase in exporter scope (which is the case for strictly convex product-entry costs), more productive firms that choose wider exporter scope will also exhibit larger average sales per product.

¹⁵If $\sigma > \varepsilon > 1$ (products within the product mix are closer substitutes with outside goods than among each other), then $\delta > \overline{\sigma} > 1$ must be satisfied for an optimum in exporter scope to exist. If $\varepsilon = \sigma$, then optimal scale per product also strictly increases in ϕ because $\sigma > 1$.

3.3 Scope, average-scale and total-exports distributions

We revisit our three main exporter statistics—exporter scope G_{isd} , total exports t_{isd} , and average product scale $z_{isd} = t_{isd}/G_{isd}$ —and derive their relationship to the firm's percentile in the productivity distribution. Motivated by the approximate Pareto shape of observed total exports and exporter scope, we consider the firms' productivity to be Pareto.¹⁶

A firm's productivity in source country s is drawn from a Pareto distribution with a source-country dependent location parameter b_s and a worldwide shape parameter θ over the support $[b_s, +\infty)$ for $s = 1, \ldots, N$. The cumulative distribution function of productivity is $F_s(\phi) = 1 - (b_s/\phi)^{\theta}$, and the density $f_s(\phi) = \theta(b_s)^{\theta}/(\phi)^{\theta+1}$. More advanced countries have a higher b_s threshold (b_s is also the mode of the Pareto distribution). The mean, median, variance and skewness of the Pareto distribution strictly decrease in the shape parameter θ .

By the properties of the Pareto distribution, the conditional density of firms from source country s with at least one shipment to destination d is

$$\mu_{isd}(\phi) = \begin{cases} \theta \left(\phi_{isd}^*\right)^{\theta} / \left(\phi\right)^{\theta+1} & \text{if } \phi \ge \phi_{isd}^*, \\ 0 & \text{otherwise.} \end{cases}$$
(15)

So the productivity distribution of active firms with at least one shipment from s to d is Pareto with the same shape parameter θ but with an industry, source and destination specific location parameter ϕ_{isd}^* . The Pareto distribution carries over to exporter scope G_{isd} , total exports t_{isd} , and average product scale $z_{isd} = t_{isd}/G_{isd}$.

Proposition 2 Suppose that the distribution of firm productivity is Pareto with shape parameter θ , that product-entry costs are not too concave ($\delta > \bar{\sigma} \equiv (\sigma - 1)/(\varepsilon - 1)$) and that the elasticity of substitution within product mixes differs from the elasticity between product mixes ($\varepsilon \neq \sigma$). Then

- the distribution of exporter scope is Pareto with shape parameter $\theta(\delta \bar{\sigma})/(\sigma 1)$ and location parameter G_d^* given by eq. (10);
- the distribution of total exports is Pareto with shape parameter $\theta(\delta \bar{\sigma})/[\delta(\sigma 1)]$ and location parameter $\delta \sigma w_d \gamma_i \kappa_d / (\delta - \bar{\sigma});$
- the distribution of average product scale is Pareto with shape parameter $\theta(\delta \bar{\sigma})/[(\delta 1)(\sigma 1)]$ and location parameter $(G_d^*)^{\delta 1} \sigma w_d \gamma_i \kappa_d / (\delta \bar{\sigma})$, where G_d^* is given by eq. (10), if product-entry cost are strictly convex in exporter scope $(\delta > 1)$.

¹⁶Theoretical results that give rise to Pareto distributed productivity across firms include Kortum (1997), Luttmer (2007), Arkolakis (2007).

Proof. The statements follow directly from the following property of the Pareto distribution. For a Pareto distributed random variable ϕ with shape parameter θ and location parameter ϕ_{isd}^* , the transformed random variable $x = A(\phi)^B$ is Pareto distributed with shape θ/B and location $A(\phi_{isd}^*)^B$. To see this, apply the change of variables theorem to $\phi(x) = (x/A)^{1/B}$ and $\mu(\phi)$ from (15) to find that $\int_a^b \mu(\phi) \, d\phi = \int_{x(a)}^{x(b)} \mu(\phi(x))\phi'(x) \, dx = \int_{x(a)}^{x(b)} (\theta/B) \left[A(\phi_{isd}^*)^B\right]^{\theta/B}/(x)^{\theta/B+1} \, dx.$

The statements in proposition 2 are closely related to Figure 2. By the properties of the Pareto distribution, the probability that an active firm is at the (1 - Pr)-th percentile in the productivity distribution or above, attaining a productivity of at least ϕ_0 , is

$$1 - \Pr = 1 - F_{sd}(\phi_0) = \left(\frac{\phi_0}{\phi_{isd}^*}\right)^{-\theta}$$

This implies that the mean exporter scope $G_{sd}(1-\Pr)$ of firms at the $(1-\Pr)$ -th percentile or above is

$$\bar{G}_{sd}(1-\Pr) = G_d^* \cdot \left(1-\Pr\right)^{-\frac{\sigma-1}{(\delta-\bar{\sigma})\theta}}$$
(16)

by (12). Figure 2 plots average exporter scope $\bar{G}_{sd}(1-\Pr) = A_G (1-\Pr)^{-1/\theta_G}$ in logs, after reverting the horizontal axis, where the power is the negative of the inverse of the Pareto shape parameter $\theta_G = \theta(\delta - \bar{\sigma})/(\sigma - 1)$. Note that, if the market-entry cost for the first shipment $\gamma_i \kappa_d$ and the product-entry cost parameter $\gamma_i \gamma_d$ vary proportionally across destinations, the exporter-scope distribution is identical across countries by (10).

Mean total exports $\bar{t}_{sd}(1-Pr)$ of firms at the (1-Pr)-th percentile or above are

$$\bar{t}_{sd}(1-\Pr) = \frac{\delta \sigma w_d \,\overline{\gamma} \kappa_d}{\delta - \bar{\sigma}} \cdot \left(1-\Pr\right)^{-\delta \frac{\sigma-1}{(\delta - \bar{\sigma})\theta}}$$

by (13), where $\overline{\gamma}$ is the firm-average cost parameter across all industries. The power (the negative of the inverse of the Pareto shape parameter) generalizes the power $-(\sigma-1)/\theta$ in Eaton et al. (2005). As introducing additional good becomes infinitely costly to the firm in the limit ($\delta \to \infty$), our shape parameter converges to the Eaton et al. (2005) shape parameter.

Finally, the mean of average product scale for firms at the (1-Pr)-th percentile or above is

$$\bar{z}_{sd}(1-\Pr) = \frac{\sigma \, w_d \, \overline{\gamma} \gamma_d}{\bar{\sigma}} \, \left(G_d^*\right)^{\delta-1} \cdot \left(1-\Pr\right)^{-(\delta-1)\frac{\sigma-1}{(\delta-\bar{\sigma})\theta}} \tag{17}$$

by (14), where $\overline{\gamma}$ is the firm-average cost parameter across all industries. Figure 2 plots average product scale $\overline{z}_{sd}(1-\Pr) = A_z (1-\Pr)^{-1/\theta_z}$ in logs, after reverting the horizontal axis, where $\theta_z = \theta(\delta - \overline{\sigma})/[(\delta - 1)(\sigma - 1)]$. Note that variation in $\gamma_i \kappa_d$ and $\gamma_i \gamma_d$ across destinations affects the average product scale distribution even if $\gamma_i \kappa_d$ and $\gamma_i \gamma_d$ change proportionally (in contrast to the exporter-scope distribution). Higher entry costs for larger markets, as in Eaton et al. (2004) and Arkolakis (2008), are consistent with such variation.

3.4 Margin Decompositions

The measure of firms in industry *i* from country *s* who choose to ship to *d* is $M_{isd} \leq J_{is}$, where J_{is} is the measure of potential entrants who have received a productivity draw at no cost.

Total exports by industry. Summing the firms' total exports (13) over all firms in industry i with shipments from s to d, we find

$$T_{isd} = J_{is} \left(\frac{b_s}{\phi_{isd}^*}\right)^{\theta} \int_{\phi_{isd}^*}^{+\infty} \frac{\delta \sigma w_d \gamma_i \kappa_d}{\delta - \bar{\sigma}} \left(\frac{\phi}{\phi_{isd}^*}\right)^{\frac{\delta(\sigma-1)}{\delta - \bar{\sigma}}} \mu_{isd}(\phi) \, \mathrm{d}\phi$$

$$= \underbrace{J_{is} \left(\frac{b_s}{\phi_{isd}^*}\right)^{\theta}}_{\# \text{ of exporters}} \cdot \underbrace{\frac{\delta \theta \sigma w_d \gamma_i \kappa_d}{\delta[\theta - (\sigma - 1)] - \theta \bar{\sigma}}}_{\text{from s to } d \ (M_{isd})} \tag{18}$$

and require that

$$\delta > \frac{\theta \,\bar{\sigma}}{\theta - (\sigma - 1)}$$

for average exports per firm \bar{t}_{isd} to be well defined.

The market share of country s's exports in country d's total consumption of industry i goods is

$$\lambda_{isd} \equiv \frac{T_{isd}}{\sum_{\varsigma=1}^{N} T_{\varsigma d}} = \frac{J_{is} \left(b_s / \phi_{i\varsigma d}^* \right)^{\theta}}{\sum_{\varsigma} J_{i\varsigma} \left(b_{\varsigma} / \phi_{i\varsigma d}^* \right)^{\theta}} = \frac{J_{is} \left(b_s \right)^{\theta} \left(\tau_{sd} w_s \right)^{-\theta}}{\sum_{\varsigma} J_{i\varsigma} \left(b_{\varsigma} \right)^{\theta} \left(\tau_{\varsigma d} w_{\varsigma} \right)^{-\theta}}$$
(19)

by average total exports (18) and the productivity threshold (11). It is remarkable that the elasticity of trade with respect to trade costs is $-\theta$, exactly as in Eaton and Kortum (2002). Moreover, the model is reminiscent of the finding in Eaton et al. (2004) that France's exports to destinations are predominantly explained by the measure of active exporters $J_{is} (b_s/\phi_{isd}^*)^{\theta}$, and only to a small extent by the exports per firm. We find a similar pattern for Brazilian and Chilean exporters.

The Extensive Margin of Exporting Goods and Trade Costs. To relate the extensive margin of goods to earlier heterogeneous-firm models of trade, we analyze the common comparative-statics experiment of a change in trade costs. We decompose the change in total exports from a source country to a destination into two extensive margins—the entry of firms and the introduction of goods—and the remaining intensive margin of sales per good. This extends the analysis in Chaney (2007), who studies a model of monopolistic competition with only one extensive margin—the entry of firms—and the composite intensive margin of sales per firm. We decompose the

intensive margin of sales per firm further into a firm's number of goods and the sales per good.

Total exports from country s's industry i to d are

$$T_{isd} = J_{is} \int_{\phi_{isd}^*}^{\infty} G_{isd}(\phi) x_{isdg}(\phi) \mu_{isd}(\phi) \mathrm{d}\phi.$$

Consider the response of total exports to a change in trade costs. Applying Leibniz integral rule yields 17

$$\begin{aligned} \frac{\partial \ln T_{isd}}{\partial \ln \tau_{sd}} &= \frac{\tau_{sd}}{T_{isd}} \Bigg(\int_{\phi_{isd}^*}^{\infty} \frac{\partial G_{isd}(\phi)}{\partial \tau_{sd}} x_{isdg}(\phi) \,\mu_{isd}(\phi) \mathrm{d}\phi \\ &+ \int_{\phi_{isd}^*}^{\infty} G_{isd}(\phi) \frac{\partial x_{isdg}(\phi)}{\partial \tau_{sd}} \mu_{isd}(\phi) \mathrm{d}\phi + \frac{\partial \phi_{isd}^*}{\partial \tau_{sd}} G_{isd}\left(\phi_{isd}^*\right) x_{isdg}\left(\phi_{isd}^*\right) \mu_{isd}(\phi_{isd}^*) \Bigg), \end{aligned}$$

and, after substitution of the optimality conditions,

$$\frac{\partial \ln T_{isd}}{\partial \ln \tau_{sd}} = \underbrace{-\left[\theta - \delta\left(\frac{\sigma - 1}{\delta - \bar{\sigma}}\right)\right]}_{\text{extensive margin}} \underbrace{-\frac{\sigma - 1}{\delta - \bar{\sigma}}}_{\text{extensive margin of introducing goods}} \underbrace{-(\delta - 1)\left(\frac{\sigma - 1}{\delta - \bar{\sigma}}\right)}_{\text{intensive margin of sales per good}}$$

As introducing more than one good becomes infinitely costly in the limit $(\delta \to \infty)$, firms turn into single-product exporters and the elasticities converge to those in the Chaney (2007) model. At the other extreme, when δ approaches $\bar{\sigma}$, responses at the extensive margin of introducing goods become very important. The margin of sales per good is now different by a multiplicative factor $(\delta - 1)/(\delta - \bar{\sigma})$. The extensive margin of introducing goods is augmented while the extensive margin of firm entry is diminished accordingly. But the overall elasticity of trade remains $-\theta$.

3.5 **Product Heterogeneity**

We generalize our model by allowing for heterogeneous productivity of goods (expertise) in the spirit of Bernard et al. (2006). In order to fit the Bernard et al. (2006) model possibly well to the data, we allow for demand-side factors behind export-market access. For this purpose, we extend the Bernard et al. (2006) model and give the firm not only a worldwide choice of products to manufacture as in Bernard et al. (2006). In addition, we allow the firm to decide locally which of its manufactured products to sell at any given

¹⁷We assume that marginal changes in τ_{sd} do not affect the general equilibrium of the model (marginal new trade flows are considered small compared to the size of the source and destination countries).

destination d. To make our extension closely comparable, we equalize the elasticities of substitution within and across firms' product mixes to $\varepsilon = \sigma$ as in Bernard et al. (2006). Even for equal elasticities, product heterogeneity and increasing product-entry cost suffice to characterize the optimal exporter scope because the additional profits per product from additional goods drop as the firm adopts lower-expertise products.

A firm's productivity in making a specific good consists of a common firm-level component ϕ as before and a component λ that is specific to the individual good, called expertise. Firm ϕ can adopt products of varying expertise levels λ so that production under the constant-returns-to-scale technology $q_{sd}(\phi\lambda) = \phi\lambda\ell$ becomes product specific, but remains independent of s and d, where ℓ is employment. Similar to (5) above, demand for firm ϕ 's product λ , manufactured in country s and sold to country d, becomes

$$x_{sd}(\phi\lambda) = \left(\frac{p_{sd}(\phi\lambda)}{P_d}\right)^{-\sigma} \frac{y_d L_d}{P_d}$$

for $\varepsilon = \sigma$ and with $P_d X_d = y_d L_d$.

The density function of expertise is $h(\lambda)$. There is a countable number of products in the data, so we choose not to impose an upper bound on the support of λ .¹⁸ After firm-level productivity ϕ and the expertise distribution $h(\lambda)$ are revealed, the firm decides which products to manufacture and which of its manufactured products to sell at destination d. As before, firms with productivity ϕ from a given industry i in country s face an identical optimization problem at destination d

$$\pi_{isd}(\phi) = \max_{\lambda_{isd}^*, p_{sd\lambda}} G(\lambda_{isd}^*) \int_{\lambda_{isd}^*}^{\infty} \left(p_{sd\lambda} - \tau_{sd} \frac{w_s}{\phi \lambda} \right) (p_{sd\lambda})^{-\sigma} \frac{y_d L_d}{(P_d)^{1-\sigma}} \frac{h(\lambda)}{G(\lambda_{isd}^*)} \, \mathrm{d}\lambda \\ - w_d \gamma_i \kappa_d - w_d \gamma_i \gamma_d \frac{[G(\lambda_{isd}^*)]^{\delta}}{\delta},$$

where $\pi_{isd}(\phi)$ denotes maximized profits and we define exporter scope as

$$G(\lambda_{isd}^*) \equiv \int_{\lambda_{isd}^*}^{\infty} h(\lambda) \,\mathrm{d}\lambda \tag{20}$$

given the scope-relevant lower cutoff λ_{isd}^* for a firm's product mix. Note that the division of $h(\lambda)$ by $G_{isd}(\lambda_{isd}^*)$ standardizes the distribution $h(\lambda)$ to a probability distribution.

The first-order condition with respect to price $p_{sd\lambda}$ yields optimal product price $p_{sd}(\phi\lambda) = \eta \tau_{sd} w_s/(\phi\lambda)$ as before, with a constant markup over marginal cost $\eta \equiv \sigma/(\sigma-1) > 1$ for $\sigma > 1$. Note that price for product λ strictly drops in λ , whereas profits $p_{sd}(\phi\lambda)^{1-\sigma}/\sigma$ for every individual product λ strictly increase in λ for $\sigma > 1$. So, if a firm chooses to adopt a certain product λ at destination d, then it must optimally

¹⁸We do not normalize $h(\lambda)$ to a probability density so that $G(\lambda_{isd}^*) = \int_{\lambda_{isd}^*}^{\infty} h(\lambda) \, d\lambda > 1$ is possible. Instead, we assume $\lim_{\lambda \to \infty} h(\lambda) = 0$.

adopt all products λ' with more expertise $\lambda' > \lambda$. This implies that there is a lower cutoff for a firm's product mix at destination d.

The first-order condition with respect to the cutoff λ_{isd}^* is

$$\frac{1}{h(\lambda_{isd}^*(\phi))}\frac{\partial \pi_{isd}(\phi)}{\partial \lambda_{isd}^*} = -\frac{p_{sd}(\phi\lambda_{isd}^*(\phi))^{1-\sigma}}{\sigma}\frac{y_d L_d}{P_d^{1-\sigma}} + w_d \gamma_i \gamma_d G(\lambda_{isd}^*(\phi))^{\delta-1} = 0,$$

and implicitly yields the unique optimal cutoff $\lambda_{isd}^*(\phi)$ if $\delta > 1$.¹⁹ Uniqueness follows because profits $p_{sd}(\phi\lambda_{isd}^*)^{1-\sigma}$ for the cutoff product strictly monotonically increase in λ_{isd}^* , whereas $G(\lambda_{isd}^*)$ strictly monotonically drops in λ_{isd}^* by (20). The optimal cutoff $\lambda_{isd}^*(\phi)$ monotonically drops in ϕ because profits $p_{sd}(\phi\lambda_{isd}^*)^{1-\sigma}$ per product strictly monotonically increase in productivity ϕ . So, more productive firms adopt a wider exporter scope $G(\lambda_{isd}^*)$ that includes more lower-expertise products than at less productive firms.

An equivalent formulation of this scope-relevant optimality condition is

$$p_{sd}(\phi\lambda)^{1-\sigma} \frac{y_d L_d}{P_d^{1-\sigma}} = \sigma \, w_d \, \gamma_i \gamma_d \, G(\lambda_{isd}^*(\phi))^{\delta-1} \left(\frac{\lambda_{isd}^*(\phi)}{\lambda}\right)^{1-\sigma} \tag{21}$$

for all products with an expertise above the optimal cutoff $\lambda > \lambda_{isd}^*(\phi)$ and therefore sold at destination d.

We are interested in firm ϕ 's optimal average product scale similar to (14) for an empirical evaluation of the model predictions. By definition, average product scale is

$$z_{isd}(\phi) \equiv \frac{t_{isd}(\phi)}{G_{isd}(\lambda_{isd}^*(\phi))} = \int_{\lambda_{isd}^*(\phi)}^{\infty} p_{sd}(\phi\lambda)^{1-\sigma} \frac{y_d L_d}{P_d^{1-\sigma}} \frac{h(\lambda)}{G_{isd}(\lambda_{isd}^*(\phi))} d\lambda$$
$$= \sigma w_d \gamma_i \gamma_d \left(G_{isd}(\lambda_{isd}^*(\phi)) \right)^{\delta-1} \int_{\lambda_{isd}^*(\phi)}^{\infty} \left(\frac{\lambda}{\lambda_{isd}^*(\phi)} \right)^{\sigma-1} \frac{h(\lambda)}{G_{isd}(\lambda_{isd}^*(\phi))} d\lambda, \quad (22)$$

where the second equality follows using (21) for optimum under product heterogeneity, conditional on the firm's exporting from s to d ($\phi \ge \phi_{isd}^*$).

This relationship offers three noteworthy insights. First, compared to average product scale (14) in the benchmark model (for $\sigma = \varepsilon$ so that $\bar{\sigma} = 1$), equation (22) contains one and only one additional factor (the integral), which is also a function of exporter scope because λ_{isd}^* and exporter scope are inversely related. Second, if the probability distribution of expertise λ is Pareto, then the additional factor (the integral) is a constant so that product heterogeneity is empirically indistinguishable from the benchmark model and average product scale increases in productivity if and only if $\delta > 1$ similar to proposition 1 before. Third, if δ approaches unity as in Bernard et al. (2006), then the probability distribution of expertise λ must be such that the additional factor

¹⁹Derivation shows that the second-order condition can only be satisfied if $\delta > 1$.

(the integral) strictly increases in productivity ϕ , that is it strictly decreases in λ_{isd}^* and equivalently strictly increases in exporter scope. This requirement is violated for the Pareto distribution of expertise and in the limit as σ approaches unity for any distribution function $h(\lambda)$.²⁰ In these circumstances, $\delta > 1$ is required for average product scale to increase with productivity and exporter scope, as in the benchmark model.

4 An Empirical Evaluation of Model Implications

We evaluate three main aspects of the model. In our first evaluation, we query the model prediction that an exporter's local scope repeats across its export destinations and that it is fully predicted by the exporter's global characteristics relative to its source-country competitors. We then investigate how average product scale and exporter scope are related to uncover the implied scope elasticity of product-entry costs in two ways. Our second evaluation compares the exporters' average-product-scale distribution to the exporter-scope distribution after ranking the exporters by their total exports within destinations. This provides a test of the model prediction that log average product scale and log exporter scope linearly increase in firms' log total-exports percentiles and yields an estimate for the scope elasticity of product-entry costs. In our third evaluation, we impose no ranking on the exporters and revisit the relationship between average product scale and exporter scope in the pristine data to check consistency. This third evaluation also affords us with a test of the model prediction that exporter scope and average product scale are associated within every destination market because of repeated local product-entry costs. We document the sources of the scope-scale association through a decomposition of the scale-scope covariation into cross-firm, cross-destination and within firm-and-destination covariation.

Prediction: Firm-level determinants of exporter scope are identical across destinations. We take the natural log of optimal exporter scope (9) and augment the relationship by a disturbance $\epsilon_{d\phi}$,

$$\ln G_{d\phi} = \ln G_d^* + \frac{\sigma - 1}{\delta - \bar{\sigma}} \frac{1}{\theta} \ln (1 - \Pr_d) + \epsilon_{d\phi}$$

 20 Define

$$H(\lambda_{isd}^*) \equiv \int_{\lambda_{isd}^*}^{\infty} \left(\frac{\lambda}{\lambda_{isd}^*}\right)^{\sigma-1} \frac{h(\lambda)}{G_{isd}(\lambda_{isd}^*)} \, \mathrm{d}\lambda$$

and take the first derivative of $H(\lambda_{isd}^*)$ to find

$$H'(\lambda_{isd}^*) = -\frac{\sigma - 1}{\lambda_{isd}^*} H(\lambda_{isd}^*) + \frac{h(\lambda_{isd}^*)}{G_{isd}(\lambda_{isd}^*)} \left[H(\lambda_{isd}^*) - 1 \right].$$

and $\lim_{\sigma \to 1} H'(\lambda_{isd}^*) = 0.$

Log # Products	Brazil				Chile		
estimator controls	OLS OLS Dest eff.		Firm FE Dest. eff.	OLS	OLS Dest eff.	Firm FE Dest. eff.	
	(1)	(2)	(3)	(4)	(5)	(6)	
Log Local total-exp. percentile	$.393 \\ (.006)$	$.394 \\ (.006)$	5.11e-25 (9.00e-15)	.247 $(.011)$.244 $(.011)$	-2.32e-26 (3.36e-15)	
Constant	2.572 (.008)	$\underset{(.021)}{2.201}$	2.236 (2.68e-14)	$\begin{array}{c} 1.763 \\ (.014) \end{array}$	$1.430 \\ (.034)$	1.523 (8.82e-15)	
Observations Firm panels R^2 (within)	68,055 .054	68,055 .118	$68,055 \\ 10,209 \\ 0$	12,423 .04	12,423 .085	$12,423 \\ 4,091 \\ 0$	

Table 2: Exporter Scope and Local Total-Exports Percentile Correlations

Sources: Brazilian SECEX 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products.

Note: Aggregation to exports by firm and destination. Products at the Harmonized-System 6-digit level. Standard errors in parentheses: * significance at ten, ** five, *** one percent.

We estimate the relationship with linear regression for a single source country (and consequently drop s from the notation). By (13) and the properties of the Pareto distribution, $\ln(\phi/\phi_d^*)$ is proportional to a firm's log percentile in local exports among its source-country competitors at d. So we use a Brazilian (Chilean) exporter's local log percentile $1 - \Pr_d$ in total exports compared to its Brazilian (Chilean) competitors at destination d to measure $\ln(\phi/\phi_d^*)$. We use destination indicators to estimate $\ln G_d^*$ by destination. The regression allows us to assess whether the firms' scope-rank relationship is systematically related across destinations.

Table 2 shows in columns 1 and 4 the coefficient estimates from a regression of log exporter scope on the firm's local log rank and a constant, omitting destination indicators for now in lieu of a single constant (under the assumption that there is no variation in $\ln G_d^*$ across destinations). The coefficient on the firm's local log rank is positive as the model predicts and statistically significant. Including destinationindicators in the regression does not alter this finding, as a comparison to columns 2 and 5 shows. Destination-fixed effects themselves improve the goodness fit, but much idiosyncratic variation that is not related to destination attributes or the firm's export ranking remains. Suppose it is a firm's destination-specific appeal to consumers, ϕ_d , that determines its rank $\ln(\phi_d/\phi_{isd}^*)$ and that the firm's global characteristic ϕ cannot explain local variation. Then the positive association between log scope and destination-specific log rank would remain statistically significant also in a regression that conditions on firm-fixed effects. This, however, is not the case empirically. Table 2 shows in columns 3 and 6 that firm-fixed worldwide effects completely absorb the covariation between a firm's local log rank. So, the firm-related determinant of exporter scope is the same across destinations.

			_			
From source s	Brazil			Chile		
to destination d	USA	Argentina	World	USA	Argentina	World
	(1)	(2)	(3)	(4)	(5)	(6)
Log # Products						
Log Percentile $(1 - Pr)$	479 $(.002)$	540 (.003)	417 (.006)	175 (.008)	273 (.008)	145 (.010)
R^2	.998	.996	.402	.828	.917	.076
Log exports/product						
Log Percentile $(1 - Pr)$	422 (.006)	357 (.002)	469 (.013)	733 (.009)	594 (.004)	752 (.027)
R^2	.979	.996	.165	.985	.996	.223
Implied scope elasticity						
of product-entry cost (δ)	1.882	1.661	2.123	5.179	3.180	6.187

Table 3: Linear Fits of Scope and Average Scale Distributions

Sources: Brazilian SECEX 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products.

Note: Ordinary-least-squares regressions of firms' mean scope at given percentile or above and firms' mean product scale (the scope-weighted arithmetic mean of exporters' average product scales \bar{z}_d in US\$ thousand fob) at given percentile or above on log percentile $\ln(1-Pr)$ and a constant, using one hundred percentile observations per destination (Figure 2). World includes only destinations with more than 100 source-country firms (70 countries for Brazil, 28 for Chile); destination observations weighted by total exports. Products at the Harmonized-System 6-digit level.

Prediction: Log average product scale and log exporter scope linearly increase in firms' log total-exports percentiles. We take the natural logs of (16) and (17) and fit the relationships

$$\ln \bar{G}_{d,(1-\Pr)} = \ln G_d^* - \frac{\sigma - 1}{(\delta - \bar{\sigma})\theta} \ln(1 - \Pr) + \epsilon_{d,(1-\Pr)}^G,$$

$$\ln \bar{z}_{d,(1-\Pr)} = \left[\ln \left(\frac{\sigma w_d \bar{\gamma} \gamma_d}{\bar{\sigma}} \right) + (\delta - 1) \ln G_d^* \right] - \frac{(\delta - 1)(\sigma - 1)}{(\delta - \bar{\sigma})\theta} \ln(1 - \Pr) + \epsilon_{d,(1-\Pr)}^z,$$

where $\epsilon_{d,1-\text{Pr}}^G$ and $\epsilon_{d,1-\text{Pr}}^z$ are potentially percentile-specific disturbances, using ordinary least squares destination by destination on one hundred percentile observations each. A firm's percentile is its local rank in total exports at the destination across all industries. These regressions fit curves for individual countries as in Figure 2, and an average relationship for the country aggregates. Note that the ratio between the log-percentile coefficient in the first and second regression is $1/(\delta - 1)$ and provides an estimate of the scope elasticity of product-entry costs.

Table 3 reports the coefficient estimates. The goodness of fit is close to one in the regressions for the two individual destinations U.S. and Argentina and consistent with log-linear relationships (column 1, 2, and 4, 5). For Brazil, the coefficient in the exporter-scope regression is around -.5 and implies that the Pareto shape parameter of exporter scope is around 2. Similarly, the coefficient in the average-product-scale

Log Exp./prd.	\mathbf{Firms}^a	Firm-destination data ^{b}			Firm-des	Firm-destination-product data c		
estimator controls	OLS	OLS	OLS Dest.	Ind. FE Dest.	Firm FE Dest.	Firm FE Dest.	OLS Dest., prd.	Firm FE Dest., prd.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			\mathbf{Br}	azil				
Log # Prod.	.341 (.022)	160 (.011)	068 (.011)	.072 (.011)	.260 (.013)	$1.180 \\ (.014)$.336 $(.014)$.977 $(.014)$
Obs. Panels	10,215	46,208	46,208	$46,\!208$ 259	$46,208 \\ 10,215$	$76,964 \\ 10,,215$	76,964	$76,964 \\ 10,215$
R^2 (within)	.023	.004	.091	.074	.131	.133	.237	.229
Corr. coeff. Firm	n FE, Log	# Prod.			234	309		279
			Cł	nile				
Log # Prod.	.135 (.035)	303 $(.024)$	111 $(.025)$.226 (.027)	.840 (.028)	$.338 \\ (.030)$.792 (.028)
Obs. Panels	4,099	12,777	12,777		$12,777 \\ 4,099$	$21,\!142 \\ 4,\!099$	21,142	$21,\!142 \\ 4,\!099$
R^2 (within)	.004	.012	.102		.124	.082	.294	.176
Corr. coeff. Firm	n FE, Log	# Prod.			235	218		182

 Table 4: Decomposition of Product Scale and Exporter Scope Correlations

^aAggregation: Firm $(\ln z_{\cdot\phi} = \ln(\varepsilon w_{\cdot} \overline{\gamma}_i \gamma_{\cdot} / \overline{\sigma}) + (\delta - 1) \ln G_{\cdot\phi} + \eta_{\cdot\phi}).$

^bAggregation: Firm and destination $(\ln z_{d\phi} = \ln(\varepsilon w_d \gamma_i \gamma_d / \bar{\sigma}) + (\delta - 1) \ln G_{d\phi} + \eta_{d\phi}).$

^cAggregation: Firm, dest. and product group $(\ln z_{dh\phi} = \ln(\varepsilon w_d \gamma_i \gamma_d / \bar{\sigma}) + (\delta - 1) \ln G_{dh\phi} + \eta_{dh\phi}).$

Sources: Brazilian SECEX 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products.

Note: Products at the Harmonized-System 6-digit level; product-group fixed effects at the Harmonized-System 2-digit level. Constant, destination fixed and product fixed effects not reported. Industry fixed effects at the CNAE two-digit level for Brazil. R^2 is within fit for firm FE regressions. Correlation coefficient between firm fixed effects and log number of products.

regression of about -.4 implies that the Pareto shape parameter of average product scale is around 2.5 for Brazil. The ratio of the two regression coefficients implies a scopeelasticity of product-entry costs δ of around 1.7. So, product-entry costs are convex in scope. The degree of convexity is remarkably similar across the two destinations worldwide for Brazil. The relationships are less robust for Chile. In all cases, however, the estimate for δ clearly exceeds unity.

Prediction: The average-product-scale and exporter-scope association is explained by within-destination variation. Figure 2 illustrates, and results in Table 3, document that average-product-scale and exporter-scope are positively associated. The stable scope-elasticity of product-entry costs across destinations raises the empirical concern, however, that the regularity may not be driven by repeated destination-market dis-economies of scope, as in the model, but in fact by firm-wide determinants such production choices (e.g. Bernard et al. 2006). To query this issue, we decompose the covariation between average product scale and exporter scope into cross-firm, cross-destination, and within destination-and-firm covariation.

We take the natural log of optimal average product scale (14) as a function of optimal exporter scope and augment the relationship by a disturbance $\epsilon_{d\phi}$

$$\ln z_{d\phi} = \ln \frac{\sigma w_d \gamma_i \gamma_d}{\bar{\sigma}} + (\delta - 1) \ln G_{d\phi} + \epsilon_{d\phi}.$$
(23)

We estimate the relationship with linear regression, and let industry and destination fixed effects capture $\ln \gamma_i$ and $\ln w_d \gamma_d$. Bernard et al. (2006) take firm-level means across destinations to estimate a comparable relationship

$$\ln z_{\cdot\phi} = \ln \frac{\sigma w_{\cdot} \overline{\gamma} \gamma_{\cdot}}{\overline{\sigma}} + (\delta - 1) \ln G_{\cdot\phi} + \epsilon_{\cdot\phi},$$

where dots indicate the means over destinations (the averaged-out indexes) and the bar indicates the mean over industries.²¹ Table 4 shows the regression result on the means in column 1. Similar to Bernard et al. (2006), the regression coefficient for log scale on scope is significantly positive.

To query the source of this positive covariation between log scale and scope, we turn to firm and destination data as in (23). In the absence of any decomposition, the overall correlation between average product scale and exporter scope is negative (column 2). Removing the contribution of the covariation between the destinationmeans of product scale $\ln z_{.\phi}$ and exporter scope raises the coefficient estimate (and the fit) slightly (column 3). So destinations with lower product scale are typically also destinations with higher exporter scope $\ln G_{.\phi}$, consistent with lower product-entry costs $\gamma_i \gamma_d$ at destinations where market-entry cost $\gamma_i \kappa_d$ are higher.

Removing the covariation between the industry's mean product scale across its destinations $\ln z_d$ and the industry's mean scope across destinations $\ln G_d$ from the regression raises the coefficient estimate finally into the positive range (column 4). So industries with high exporter scope worldwide are typically also industries with low product scale worldwide (as also reflected in the negative correlation coefficient between industry-fixed effects and the log number of products in the final row of Table 4). This decomposition of the covariation between log scale and scope is consistent with the interpretation that a main source of the positive scale-scope covariation worldwide (in column 1) is the repeated positive scale-scope relationship by destination and by industry (column 4). The destination effects in the scale-scope relationship are in turn

$$\ln z_{\cdot g\phi} = \ln \frac{\sigma \, w_{\cdot} \, \overline{\gamma} \gamma_{\cdot}}{\bar{\sigma}} + (\delta - 1) \ln G_{\cdot \phi} + \epsilon_{\cdot g\phi}$$

 $^{^{21}\}mathrm{Bernard}$ et al. (2006) report a product-level version of the regression equation, where product scale

is not averaged by firm. The regressor $\ln \bar{G}_{.\phi}$ does not vary by product, however, so that the coefficient on log exporter scope is implicitly re-weighted towards large-scope firms. Our main interest is a decomposition of the covariation between average scale and scope as in (14) by source.

correlated with common gravity-equation predictors in an expected way (see Table 5 in the appendix).

Finally, we remove from the regression the covariation between the firm's worldwide mean product scale across its destinations $\ln z_d$ and the firm's worldwide mean scope across destinations $\ln G_d$. This raises the coefficient estimate even further into the positive range (column 5). Our model allows for an industry-specific entry cost component but does not explicitly permit a firm-specific entry cost component. So, a full explanation for the systematic negative scale-scope association at the worldwide firm level remains beyond our model. We conjecture, however, that a random draw of a firm-specific white-noise entry cost component, realized after the firm's optimal scale and scope choices, would give rise to a firm fixed effect while integrating to zero and vanishing in aggregate exports data over the continuum of all exporters. Controlling for the firm fixed effect documents, however, that worldwide production-side explanations of the positive scale-scope association (such as in Bernard et al. 2006), or worldwide marketing explanations, cannot explain the overall covaration in the data (column 1 and 5^{22} The reason is that there is a negative scale-scope association at the worldwide firm level, in contrast with the overall positive scale-scope association, which we explain with repeated positive scale-scope associations within every destination.

The single source of positive covariation within destinations and firms in the data is consistent with a version of our model where product-entry costs convexly increase in exporter scope. Note that firms need to incur these product-entry costs repeatedly market by market. The coefficient in column 5 of around .3 implies dis-economies of scope with a scope-elasticity of product-entry costs δ of around 1.3. So, product-entry costs are convex in scope also by this measure but the degree of convexity is somewhat smaller than in the regressions of Table 3 for Brazil, where the ranking of firms by total exports imposed additional structure.

A final empirical concern is that firms may more frequently adopt product types that offer high product scale so that the positive scale-scope association would be driven by product selection and not by dis-economies of scope. We therefore decompose the covariation between log scale and scope further into sources of covaration within product groups (at the Harmonized System 2-digit level) and across product groups. Columns 5 through 7 in Table 4 show that the positive association between log scale and scope becomes even more pronounced. Dis-economies of scope for product-entry by destination are significantly higher within product groups (column 7) than across (column 4 where across and within product-group variation is not decomposed). The productgroup effects in the scale-scope relationship are in turn correlated with measures for the products' degree of differentiation in an expected way (see Table 6 in the appendix). So

²²The expected difference between the ordinary least squares estimate (column 2) and the firm fixed effects estimator (column 5) is proportional to the correlation between the firm fixed effects and the mean explanatory variables: $\mathbb{E}[\hat{\beta}^{\text{OLS}} - \hat{\beta}^{\text{FE}}] = \mathbb{E}[(X'X)^{-1}X'A\alpha]$, where X is the data matrix, A is a matrix of firm indicators, and α is the coefficient vector of firm-fixed effects.

it is not the firms' adoption of high-scale products across product groups but a strong positive scale-scope association within product groups that accounts for the positive scale-scope relationship. The positive covariation within destinations, industries and their product groups in the data is fully consistent with our model where dis-economies of scope stem from repeated product-entry costs product-market by product-market and firm by firm.

5 Conclusion

The extensive margin of introducing additional export goods offers new insight into exporter behavior. Data on the universe of exporters in Brazil, their sales destinations and individual products, document that an exporter's number of products (the exporter scope) and the exporter's average sales per product (the product scale) are jointly Pareto distributed in the upper tails and positively associated destination by destination. To be consistent with the data, a model needs to account for the repeated positive association between scale and scope within every destination. We introduce a heterogeneous-firm model where exporters face a repeated product-entry cost that convexly increases in the exporter's scope in every destination market. The model predicts main features of the data. The model also preserves main predictions of prior heterogeneous-firm models of trade such as a single elasticity of exporting and the extensive margin of firm entry. The empirical evidence and the model show that, beyond production-side explanations, distribution-side features such as dis-economies of scope in product-market access are salient determinants of exporter behavior.

Appendix

A Optimal markup choice

Maximizing the firm's constrained objective function

$$\max_{P_{isd}, \{p_{sdg}\}_{g=1}^{G_{isd}}} \pi_{isd}(\phi) + \lambda \left(P_{isd}(\phi; G_{isd}) - \left[\int_{0}^{G_{isd}} p_{sdg}(\phi)^{1-\varepsilon} \mathrm{d}g \right]^{\frac{1}{1-\varepsilon}} \right),$$

where profit $\pi_{isd}(\phi)$ is given by (7), yields the first order conditions

$$p_{sdg}(\phi) = \frac{\varepsilon \, y_d L_d}{\left(\varepsilon - 1\right) \, y_d L_d + \lambda \left(P_d\right)^{1 - \sigma} P_{isd}(\phi; G_{isd})^{\sigma}} \frac{\tau_{sd} \, w_s}{\phi} \tag{24}$$

for individual product prices p_{sdg} and the first-order condition

$$\lambda(\phi) = -(\varepsilon - \sigma) G_{isd} \left(p_{sdg}(\phi)^{1-\varepsilon} - \tau_{sd} \frac{w_s}{\phi} p_{sdg}(\phi)^{-\varepsilon} \right) \frac{P_{isd}(\phi; G_{isd})^{\varepsilon - \sigma - 1}}{\left(P_d\right)^{1-\sigma}} y_d L_d \quad (25)$$

for the product-mix price P_{isd} , where λ is the Lagrange multiplier on the constraint $(\lambda < 0 \text{ iff } \varepsilon > \sigma)$ and G_{isd} is given. Note that prices $p_{sdg}(\phi)$ are identical in optimum for all products of a firm by (24). To solve out for optimal individual prices, guess that $p_{sdg}(\phi) = \eta(\cdot) \tau_{sd} w_s / \phi$ for some markup $\eta(\cdot)$, which could be a function of any variable or parameter of the model. Using (25), $\eta(\cdot)$ and the fact that $P_{isd}(\phi; G_{isd}) = (G_{isd})^{1/(1-\varepsilon)} p_{sdg}(\phi)$ in (24) implies that

$$\eta(\cdot) = \frac{\sigma}{\sigma - 1}$$
 so that $p_{sdg}(\phi) = \frac{\sigma}{\sigma - 1} \frac{\tau_{sd} w_s}{\phi}$.

B Exports data for Brazil

The Brazilian customs office SECEX (Secretaria de Comércio Exterior) collects and compiles export reports by product code at the plant, month and NCM (Nomenclatura Comum do Mercosul) level. NCM coincides with the Harmonized System at the 6-digit level. We use the year 2000 only and aggregate the data to the firm, destination and Harmonized-System 6-digit level. This facilitates comparisons to other Brazilian and international data sources.²³

We map destination information from Brazilian country codes into the international ISO system. Product codes at the 6-digit level in the Brazilian data include codes in

 $^{^{23}}$ The aggregation is comparable to export-country studies at the six-digit Harmonized System level such as Feenstra (1994) or Hummels and Klenow (2005), and to firm-level studies such as Eaton et al. (2004).

the 999000s, for which there exist no corresponding Harmonized System entries. These codes are not closely related to traded merchandize and relate to entries such as onboard aircraft consumption of combustibles or merchandize for non-financial rental. We remove the codes from the data. To compare our data to sector-level productmarket information by destination country, we map the Harmonized System 6-digit codes to *ISIC revision* 2 at the two-digit level.²⁴

C Exports data for Chile

The Chilean data for the year 2000 are courtesy of Álvarez et al. (2007) and derive from the universe of Chilean customs declarations for merchandize exports, similar to the Brazilian *SECEX* data. The Chilean customs authorities collect the reports by firm and Harmonized System eight-digit code. We aggregate the pristine eightdigit Harmonized System information to information by exporting firm at the sixdigit Harmonized System level. This ensures comparability to the Brazilian data (and international sources, as mentioned above).

We map destination country names into the international ISO system. The sector affiliation of Chilean exporters is reported at *ISIC revision* 2 three-digit level. We use the *ISIC revision* 2 for the export firm from the original data. Robustness checks using product-level information for sector affiliates from the Harmonized System six-digit level and using the *ISIC revision* 2 product code of the top selling product for the firm do not yield substantively different results.

D Auxiliary data for Brazil and Chile

Trade flow data by industry and destination. We link the firm-level product and destination information for Brazil and Chile to WTF (World Trade Flow) data for the year 2000 (Feenstra et al. 2005). We extract sector-level trade flow statistics in current US\$ for Brazil's and Chile's export destination markets. For Brazil, we map the SITC Rev. 2 four-digit sector information to the SITC Rev. 2 two-digit level, and then to the two-digit ISIC revision 2 level for combination with SECEX. For Chile, we map the ISIC revision 2 information at the three-digit level to the two-digit ISIC revision 2 level for combination exports data.

Output data by industry and destination. We obtain manufacturing output by destination country and manufacturing industry for 2000 from the *Unido* Industrial Statistics Database at the two-digit *ISIC revision 2* level in current US\$ (UNIDO

²⁴Our novel concordance will become available at URL econ.ucsd.edu/muendler/brazil shortly.

2005). We map the Harmonized System six-digit codes to ISIC revision 2 at the two-digit level for this purpose.

Country and geographic data by destination. National accounts information for host-country regressors comes from the World Bank's World Development Indicators and the IMF's International Financial Statistics (population, GDP, consumption expenditure and household consumption expenditure in current US\$). We use CEPII bilateral geographic data;²⁵ the data include the mean distance between Brasília or Santiago de Chile on the one hand and foreign capital cities (km) on the other hand, common borders with Brazil or Chile, and a common language with Brazil (Portuguese-speaking Angola, China Macão SAR, Guinea Bissau, Mozambique and Portugal) or Chile (Spanish speaking countries).

Goods data. We calculate Balassa (1965) comparative-advantage measures for Brazilian and Chilean goods from UN Comtrade trade data for the year 2000 at the *ISIC* rev. 2 four-digit level. Good h's Balassa advantage is

$$BADV_h \equiv \frac{X_h^{\text{Brazil}} / \sum_k X_{k,t}^{\text{Brazil}}}{X_h^{\text{World}} / \sum_k X_{k,t}^{\text{World}}},$$

where X_h are exports. Note that this index measures revealed comparative advantage from international comparisons of exports data, and is blind to possible sources of advantage. Any explanation of comparative advantage is consistent with this measure. We first map the *ISIC rev.* 2 information to the Harmonized System six-digit level and then aggregate to the Harmonized System two-digit level by taking the unweighed average across six-digit goods in the Brazilian data.

We use the Rauch (1999) classification of goods by degree of differentiation under Rauch's conservative definition.²⁶ We first map Rauch's *SITC Rev.* 2 four-digit sector information to the Harmonized System six-digit level and then aggregate to the Harmonized System two-digit level by taking the unweighed average across six-digit goods in the Brazilian data.

We reuse the WTF data for the year 2000 (Feenstra et al. 2005) to obtain goodslevel measures of typical import destinations. For this purpose, we drop Brazilian or Chilean exports and imports from the WTF data and calculate for the rest of the world the number of destinations to which goods at the SITC Rev. 2 four-digit level (Brazil) or the ISIC rev. 2 three-digit level (Chile) ship, and what import values they exhibit worldwide, in the OECD and Mercosur (Argentina, Paraguay, Uruguay). For Brazil, we map the SITC Rev. 2 four-digit sector information to the Harmonized System six-digit level and then aggregate to the Harmonized System two-digit level by taking

 $^{^{25}}$ From www.cepii.fr/anglaisgraph/bdd/distances.htm.

²⁶We use Rauch's revision 2 from 2007 (available at www.econ.ucsd.edu/~jrauch/intltrad)

	Destination Eff	. on Prod. Scale	Destination Eff. on Exp. Scope		
	from Log Exp./	prod. regressions	from Log $\#$ Products regression		
	(1)	(2)	(3)	(4)	
	Brazil	Chile	Brazil	Chile	
	Firm & dest.	Firm & dest.	Firm & dest.	Firm & dest.	
	FE, & scope	FE, & scope	FE, & scale	FE, & scale	
Mean Log Market size	042	002	.043	003	
	(.031)	(.010)	(.058)	(.013)	
Log Population	.348 (.048)	.032 (.014)	.287 (.085)	.033 $(.018)$	
Log GDP per cap.	.287	.028	.291	.005	
	(.044)	(.014)	(.086)	(.019)	
Log Distance	331	236	462	155	
	(.138)	(.044)	(.304)	(.057)	
Common borders	171	.227	282	.255	
	(.276)	(.081)	(.630)	(.138)	
Common language	078	.048	.007	.099	
	(.300)	(.092)	(.380)	(.085)	
Const.	-8.354	1.907	-8.124	1.278	
	(1.302)	(.434)	(2.980)	(.562)	
Obs. R^2	$\begin{array}{c} 106 \\ .56 \end{array}$	$ 102 \\ .574 $	94 .401	94 .396	

Table 5: Correlates of Destination Effects on Product Scale and Exporter Scope

Sources: Brazilian *SECEX* 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products, linked to *Cepii* distance data (Mayer and Zignago 2006) and *Unido* Industrial Statistics (UNIDO 2005).

Note: Aggregation to exports by firm and destination. Regressions of destination fixed effects on destination-level predictors, where destination fixed effects on product scale are from a destination fixed effects regression controlling for scope and firm fixed effects (see column 3 in Table 4). Destination fixed effects on exporter scope are from a destination fixed effects regression controlling for scale and firm fixed effects. Mean log market size is average sectoral absorption over *ISIC rev. 2* industries at destination level. Standard errors in parentheses: * significance at ten, ** five, *** one percent.

the unweighed average across six-digit goods. For Chile, we just aggregate from the Harmonized System six-digit level to the Harmonized System two-digit level by taking the unweighed average across six-digit goods.

	Destination Eff.	on Prod. Scale	Destination Eff. on Exp. Scope			
	from Log Exp./p	orod. regressions	from Log $\#$ Pro	from Log $\#$ Products regressions		
	(1)	(2)	(3)	(4)		
	Brazil	Chile	Brazil	Chile		
	Firm, dest. & prd. FE, & scope	Firm, dest. & prd. FE, & scope	Firm, dest. & prd. FE, & scale	Firm, dest. & prd. FE, & scale		
Comparative adv.	.186 (.119)	.010 (.020)	.012 (.037)	.004 (.006)		
Reference priced	-2.964 (.881)	.062 (.151)	-1.594 (.833)	.072 (.142)		
Differentiated	-2.031 (.813)	.125 (.139)	-1.858 (.761)	007 (.130)		
Log ww. $\#$ Dest.	-1.765 (.856)	.253 (.147)	883 (.899)	.212 (.153)		
No OECD imp.	$21.525 \\ (47.204)$	3.034 (8.093)	-6.657 (11.326)	991 (1.929)		
Log OECD Imp. ^{a}	.544 (.254)	.012 (.044)	.234 (.228)	.108 (.039)		
No Mercosur imp.	-1.661 (2.090)	109 (.358)	$1.180 \\ (2.204)$.132 (.376)		
Log Mercos. Imp. ^{a}	.083 (.213)	.013 (.037)	042 (.216)	009 (.037)		
Const.	$5.304 \\ (4.546)$	644 (.779)	-1.826 (4.824)	822 (.822)		
Obs.	91	91	94	94		
$\frac{R^2}{}$.202	.25	.167	.37		

Table 6: Correlates of Product Effects on Product Scale and Exporter Scope

^{*a*}Log of nonzero imports \times indicator.

Sources: Brazilian SECEX 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products, linked to WTF (Feenstra et al. 2005) and Unido Industrial Statistics (UNIDO 2005).

Note: Aggregation to exports by firm, destination, product group (Harmonized System 2-digit level). Regressions of product fixed effects at the Harmonized-System 2-digit level on product-level predictors, where product fixed effects on product scale are from a product fixed effects regression controlling for scope as well as destination and firm fixed effects (see column 6 in Table 4). Product fixed effects on exporter scope are from a product fixed effects regression controlling for scale as well as destination and firm fixed effects. Balassa (1965) comparative-advantage for Brazil from UN Comtrade trade data for 2000 at the *ISIC Rev.* 2 level: product h's comparative advantage is $BADV_h \equiv [T_h^{\text{Brazil}} / \sum_k T_k^{\text{Brazil}}] / [T_h^{\text{World}} / \sum_k T_k^{\text{World}}]$, where T_h are worldwide exports. Goods classification by degree of differentiation from Rauch (1999), conservative definition, revision 2 (2007): share of Harmonized-System 6-digit goods at the Harmonized-System 2-digit level; omitted benchmark category is homogeneous goods (traded on an organized exchange). Worldwide product-group imports exclude Brazil as importer and exporter. Standard errors in parentheses: * significance at ten, ** five, *** one percent.

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