# Why Nonlinear/Non-Gaussian DSGE Models?

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# Motivation

- These lectures review recent advances in nonlinear and non-gaussian macro model-building.
- First, we will justify why we are interested in this class of models.
- Then, we will study both the solution and estimation of those models.
- We will work with discrete time models.
- We will focus on DSGE models.

# Nonlinearities

- Most DSGE models are nonlinear.
- Common practice (you saw it yesterday): solve and estimate a linearized version with Gaussian shocks.
- Why? Stochastic neoclassical growth model is nearly linear for the benchmark calibration (Aruoba, Fernández-Villaverde, Rubio-Ramírez, 2005).
- However, we want to depart from this basic framework.
- I will present three examples.

# Example I: Epstein-Zin Preferences

- Recursive preferences (Kreps-Porteus-Epstein-Zin-Weil) have become a popular way to account for asset pricing observations.

- Natural separation between IES and risk aversion.

- Example of a more general set of preferences in macroeconomics.

- Consequences for business cycles, welfare, and optimal policy design. Link with robust control.

- I study a version of the RBC with inflation and adjustment costs in The Term Structure of Interest Rates in a DSGE Model with Recursive Preferences.

# Household

- Preferences:

$$U_t = \left[ \left( c_t^{\upsilon} (1 - l_t)^{1-\upsilon} \right)^{\frac{1-\gamma}{\theta}} + \beta \underbrace{\left( \mathbb{E}_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}}}_{\text{Risk-adjustment operator}} \right]^{\frac{\theta}{1-\gamma}}$$

where:

$$\theta = \frac{1-\gamma}{1 - \frac{1}{\psi}}.$$

- Budget constraint:

$$c_t + i_t + \frac{b_{t+1}}{p_t} \frac{1}{R_t} = r_t k_t + w_t l_t + \frac{b_t}{p_t}$$

- Asset markets.

# Technology

- Production function:

$$y_t = k_t^{\zeta} \left(z_t l_t\right)^{1-\zeta}$$

- Law of motion:

$$\log z_{t+1} = \lambda \log z_t + \chi \sigma_{\varepsilon} \varepsilon_{zt+1} \text{ where } \varepsilon_{zt} \sim \mathcal{N}(0,1)$$

- Aggregate constraints:

$$c_t + i_t = k_t^{\zeta} \left(z_t l_t\right)^{1-\zeta}$$
$$k_{t+1} = (1-\delta) k_t + i_t$$

# Approximating the Solution of the Model

- Define $s_t = \left(\widehat{k}_t, \log z_t; 1\right)$ where $\widehat{k}_t = k_t - k_{ss}$.
- Under differentiability conditions, third-order Taylor approximation of the value function around the steady state:

$$V\left(\widehat{k}_t, \log z_t; 1\right) \simeq V_{ss} + V_{i,ss} s_t^i + \frac{1}{2} V_{ij,ss} s_t^i s_t^j + \frac{1}{6} V_{ijl,ss} s_t^i s_t^j s_t^l,$$

- Approximations to the policy functions:

$$var\left(\widehat{k}_t, \log z_t; 1\right) \simeq var_{ss} + var_{i,ss} s_t^i + \frac{1}{2} var_{ij,ss} s_t^i s_t^j + \frac{1}{6} var_{ijl,ss} s_t^i s_t^j s_t^l$$

- and yields:

$$\begin{aligned} R_m\left(\widehat{k}_t, \log z_t, \log \pi_t, \omega_t; 1\right) \simeq & \; R_{m,ss} + R_{m,i,ss} sa_t \\ & + \frac{1}{2} R_{m,ij,ss} sa_t^i sa_t^j + \frac{1}{6} R_{m,ijl,ss} sa_t^i sa_t^j sa_t^l \end{aligned}$$

# Structure of Approximation

1. The constant terms $V_{ss}$, $var_{ss}$, or $R_{m,ss}$ do **not** depend on $\gamma$, the parameter that controls risk aversion.

2. **None** of the terms in the first-order approximation, $V_{.,ss}$, $var_{.,ss}$, or $R_{m,.,ss}$ (for all $m$) depend on $\gamma$.

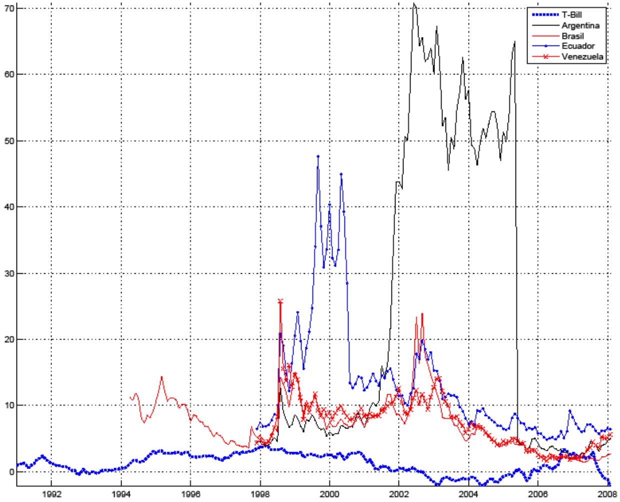
3. **None** of the terms in the second-order approximation, $V_{..,ss}$, $var_{..,ss}$, or $R_{m,..,ss}$ depend on $\gamma$, except $V_{33,ss}$, $var_{33,ss}$, and $R_{m,33,ss}$ (for all $m$). This last term is a constant that captures precautionary behavior.

4. In the third-order approximation **only** the terms of the form $V_{33.,ss}$, $V_{3.3,ss}$, $V_{.33,ss}$ and $var_{33.,ss}$, $var_{3.3,ss}$, $var_{.33,ss}$ and $R_{m,33.,ss}$, $R_{m,3.3,ss}$, $R_{m,.33,ss}$ (for all $m$) that is, terms on functions of $\chi^2$, depend on $\gamma$.

# Example II: Volatility Shocks

- Data from four emerging economies: Argentina, Brazil, Ecuador, and Venezuela. Why?
- Monthly data. Why?
- Interest rate $r_t$: international risk free real rate+country spread.
- International risk free real rate: Monthly T-Bill rate. Transformed into real rate using past year U.S. CPI inflation.
- Country spreads: Emerging Markets Bond Index+ (EMBI+) reported by J.P. Morgan.
  EMBI data coverage: Argentina 1997.12 - 2008.02; Ecuador 1997.12 - 2008.02; Brazil 1994.04 - 2008.02; and Venezuela 1997.12 - 2008.02.

# Data


T-Bill
Argentina
Brasil
Ecuador
Venezuela
70
60
50
40
30
20
10
0
1992
1994
1996
1998
2000
2002
2004
2006
2008

# The Law of Motion for Interest Rates

- Decomposition of interest rates:

$$r_t = \underbrace{r}_{\text{mean}} + \underbrace{\varepsilon_{tb,t}}_{\text{T-Bill shocks}} + \underbrace{\varepsilon_{r,t}}_{\text{Spread shocks}}$$

- $\varepsilon_{tb,t}$ and $\varepsilon_{r,t}$ follow:

$$\varepsilon_{tb,t} = \rho_{tb}\varepsilon_{tb,t-1} + e^{\sigma_{tb,t}} u_{tb,t},\ u_{tb,t} \sim \mathcal{N}(0,1)$$

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t},\ u_{r,t} \sim \mathcal{N}(0,1)$$

- $\sigma_{tb,t}$ and $\sigma_{r,t}$ follow:

$$\sigma_{tb,t} = \left(1-\rho_{\sigma_{tb}}\right)\sigma_{tb} + \rho_{\sigma_{tb}}\sigma_{tb,t-1} + \eta_{tb} u_{\sigma_{tb},t},\ u_{\sigma_{tb},t} \sim \mathcal{N}(0,1)$$

$$\sigma_{r,t} = \left(1-\rho_{\sigma_r}\right)\sigma_r + \rho_{\sigma_r}\sigma_{r,t-1} + \eta_r u_{\sigma_r,t},\ u_{\sigma_r,t} \sim \mathcal{N}(0,1)$$

- I could also allow for correlations of shocks.

# A Small Open Economy Model I

- Risk Matters: The Real Effects of Volatility Shocks.

- Prototypical small open economy model: Mendoza (1991), Correia *et al.* (1995), Neumeyer and Perri (2005), Uribe and Yue (2006).

- Representative household with preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t - \omega^{-1} H_t^{\omega}\right]^{1-v} - 1}{1-v}.$$

- Why Greenwood-Hercowitz-Huffman (GHH) preferences? Absence of wealth effects.

# A Small Open Economy Model II

- Interest rates:

$$r_t = r + \varepsilon_{tb,t} + \varepsilon_{r,t}$$
$$\varepsilon_{tb,t} = \rho_{tb}\varepsilon_{tb,t-1} + e^{\sigma_{tb,t}} u_{tb,t},\ u_{tb,t} \sim \mathcal{N}(0,1)$$
$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t},\ u_{r,t} \sim \mathcal{N}(0,1)$$
$$\sigma_{tb,t} = \left(1 - \rho_{\sigma_{tb}}\right)\sigma_{tb} + \rho_{\sigma_{tb}}\sigma_{tb,t-1} + \eta_{tb} u_{\sigma_{tb},t},\ u_{\sigma_{tb},t} \sim \mathcal{N}(0,1)$$
$$\sigma_{r,t} = \left(1 - \rho_{\sigma_r}\right)\sigma_r + \rho_{\sigma_r}\sigma_{r,t-1} + \eta_r u_{\sigma_r,t},\ u_{\sigma_r,t} \sim \mathcal{N}(0,1)$$

- Household's budget constraint:

$$\frac{D_{t+1}}{1+r_t} = D_t - W_t H_t - R_t K_t + C_t + I_t + \frac{\Phi_d}{2}\left(D_{t+1} - D\right)^2$$

- Role of $\Phi_d > 0$ (Schmitt-Grohé and Uribe, 2003).

# A Small Open Economy Model III

- The stock of capital evolves according to the following law of motion:

$$K_{t+1} = (1-\delta) K_t + \left(1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t$$

- Typical no-Ponzi condition.

- Production function:

$$Y_t = K_t^{\alpha}\left(e^{X_t} H_t\right)^{1-\alpha}$$

where:

$$X_t = \rho_x X_{t-1} + e^{\sigma_x} u_{x,t},\ \ u_{x,t} \sim \mathcal{N}(0,1).$$

- Competitive equilibrium defined in a standard way.

# Solving the Model

- Perturbation methods.
- We are interested on the effects of a volatility increase, i.e., a positive shock to either $u_{\sigma_r,t}$ or $u_{\sigma_{tb},t}$, while $u_{r,t} = 0$ and $u_{tb,t} = 0$.
- We need to obtain a *third* approximation of the policy functions:
  1. A first order approximation satisfies a certainty equivalence principle. Only level shocks $u_{tb,t}$, $u_{r,t}$, and $u_{X,t}$ appear.
  2. A second order approximation only captures volatility indirectly via cross products $u_{r,t}u_{\sigma_r,t}$ and $u_{tb,t}u_{\sigma_{tb},t}$,. Thus, volatility only has an effect if the real interest rate changes.
  3. In the third order, volatility shocks, $u_{\sigma,t}$ and $u_{\sigma_{tb},t}$, enter as independent arguments.
- Moreover:
  1. Cubic terms are quantitatively important.
  2. The mean of the ergodic distributions of the endogenous variables and the deterministic steady state values are quite different. Key for calibration.

# Example III: Fortune or Virtue

- Strong evidence of time-varying volatility of U.S. aggregate variables.
- Most famous example: the Great Moderation between 1984 and 2007.
- Two explanations:
  1. Stochastic volatility: fortune.
  2. Parameter drifting: virtue.
- How can we measure the impact of each of these two mechanisms?
- We build and estimate a medium-scale DSGE model with:
  1. Stochastic volatility in the shocks that drive the economy.
  2. Parameter drifting in the monetary policy rule.

# The Discussion

- Starting point in empirical work by Kim and Nelson (1999) and McConnell and Pérez-Quirós (2000).
- Virtue: Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004).
- Sims and Zha (2006): once time-varying volatility is allowed in a SVAR model, data prefer fortune.
- Follow-up papers: Canova and Gambetti (2004), Cogley and Sargent (2005), Primiceri (2005).
- Fortune papers are SVARs models: Benati and Surico (2009).
- A DSGE model with both features is a natural measurement tool.

# The Goals

1. How do we write a medium-scale DSGE with stochastic volatility and parameter drifting?
2. How do we evaluate the likelihood of the model and how to characterize the decision rules of the equilibrium?
3. How do we estimate the model using U.S. data and assess model fit?
4. How do we build counterfactual histories?

# Model I: Preferences

- Household maximizes:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t \left\{ \log\left(c_{jt} - hc_{jt-1}\right) + \upsilon \log\left(\frac{m_{jt}}{p_t}\right) - \varphi_t \psi \frac{l_{jt}^{1+\vartheta}}{1+\vartheta} \right\}$$

- Shocks:

$$\log d_t = \rho_d \log d_{t-1} + \sigma_{d,t} \varepsilon_{d,t}$$

$$\log \varphi_t = \rho_\varphi \log \varphi_{t-1} + \sigma_{\varphi,t} \varepsilon_{\varphi,t}$$

- Stochastic Volatility:

$$\log \sigma_{d,t} = \left(1 - \rho_{\sigma_d}\right) \log \sigma_d + \rho_{\sigma_d} \log \sigma_{d,t-1} + \eta_d u_{d,t}$$

$$\log \sigma_{\varphi,t} = \left(1 - \rho_{\sigma_\varphi}\right) \log \sigma_\varphi + \rho_{\sigma_\varphi} \log \sigma_{\varphi,t-1} + \eta_\varphi u_{\varphi,t}$$

# Model II: Constraints

- Budget constraint:

$$c_{jt} + x_{jt} + \frac{m_{jt}}{p_t} + \frac{b_{jt+1}}{p_t} + \int q_{jt+1,t} a_{jt+1} d\omega_{j,t+1,t} =$$

$$w_{jt} l_{jt} + \left( r_t u_{jt} - \mu_t^{-1} \Phi \left[ u_{jt} \right] \right) k_{jt-1} + \frac{m_{jt-1}}{p_t} + R_{t-1} \frac{b_{jt}}{p_t} + a_{jt} + T_t + \digamma_t$$

- The capital evolves:

$$k_{jt} = (1 - \delta) k_{jt-1} + \mu_t \left( 1 - V \left[ \frac{x_{jt}}{x_{jt-1}} \right] \right) x_{jt}$$

- Investment-specific productivity $\mu_t$ follows a random walk in logs:

$$\log \mu_t = \Lambda_\mu + \log \mu_{t-1} + \sigma_{\mu,t} \varepsilon_{\mu,t}$$

- Stochastic Volatility:

$$\log \sigma_{\mu,t} = \left( 1 - \rho_{\sigma_\mu} \right) \log \sigma_\mu + \rho_{\sigma_\mu} \log \sigma_{\mu,t-1} + \eta_\mu u_{\mu,t}$$

# Model III: Nominal Rigidities

- Monopolistic competition on labor markets with sticky wages (Calvo pricing with indexation).

- Monopolistic intermediate good producer with sticky prices (Calvo pricing with indexation):

$$y_{it} = A_t k_{it-1}^{\alpha} \left( l_{it}^{d} \right)^{1-\alpha} - \phi z_t$$

$$\log A_t = \Lambda_A + \log A_{t-1} + \sigma_{A,t} \varepsilon_{A,t}$$

- Stochastic Volatility:

$$\log \sigma_{A,t} = \left(1 - \rho_{\sigma_A}\right) \log \sigma_A + \rho_{\sigma_A} \log \sigma_{A,t-1} + \eta_A u_{A,t}$$

# Model IV: Monetary Authority

- Modified Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left( \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_{\Pi,t}} \left( \frac{\frac{y_t^d}{y_{t-1}^d}}{\exp\left(\Lambda_{y^d}\right)} \right)^{\gamma_{y,t}} \right)^{1-\gamma_R} \exp\left(\sigma_{m,t}\varepsilon_{mt}\right)$$

- Stochastic Volatility:

$$\log \sigma_{m,t} = \left(1 - \rho_{\sigma_m}\right) \log \sigma_m + \rho_{\sigma_m} \log \sigma_{m,t-1} + \eta_m u_{m,t}$$

- Parameter drifting:

$$\log \gamma_{\Pi,t} = \left(1 - \rho_{\gamma_\Pi}\right) \log \gamma_\Pi + \rho_{\gamma_\Pi} \log \gamma_{\Pi,t-1} + \eta_\Pi \varepsilon_{\pi,t}$$

$$\log \gamma_{y,t} = \left(1 - \rho_{\gamma_y}\right) \log \gamma_y + \rho_{\gamma_y} \log \gamma_{y,t-1} + \eta_y \varepsilon_{y,t}$$

# More About Nonlinearities I

- The previous examples are not exhaustive.
- Unfortunately, linearization eliminates phenomena of interest:
  1. Asymmetries.
  2. Threshold effects.
  3. Precautionary behavior.
  4. Big shocks.
  5. Convergence away from the steady state.
  6. And many others....

# More About Nonlinearities II

Linearization limits our study of dynamics:

1. Zero bound on the nominal interest rate.
2. Finite escape time.
3. Multiple steady states.
4. Limit cycles.
5. Subharmonic, harmonic, or almost-periodic oscillations.
6. Chaos.

# More About Nonlinearities III

- Moreover, linearization induces an approximation error.
- This is worse than you may think.

1. Theoretical arguments:
    1. Second-order errors in the approximated policy function imply first-order errors in the loglikelihood function.
    2. As the sample size grows, the error in the likelihood function also grows and we may have inconsistent point estimates.
    3. Linearization complicates the identification of parameters.
2. Computational evidence.

# Arguments Against Nonlinearities

1. Theoretical reasons: we know way less about nonlinear and non-gaussian systems.

2. Computational limitations.

3. Bias.

**Mark Twain**

To a man with a hammer, everything looks like a nail.

**Teller's Law**

A state-of-the-art computation requires 100 hours of CPU time on the state-of-the art computer, independent of the decade.

# Solving DSGE Models

- We want to have a general formalism to think about solving DSGE models.
- We need to move beyond value function iteration.
- Theory of functional equations.
- We can cast numerous problems in macroeconomics involve functional equations.
- Examples: Value Function, Euler Equations.

# Functional Equation

- Let $J^1$ and $J^2$ be two functional spaces, $\Omega \subseteq \Re^l$ and let $\mathcal{H} : J^1 \to J^2$ be an operator between these two spaces.

- A *functional equation problem* is to find a function $d : \Omega \to \Re^m$ such that

$$\mathcal{H}(d) = \mathbf{0}$$

Regular equations are particular examples of functional equations.

- Note that $\mathbf{0}$ is the space zero, different in general that the zero in the reals.

# Example: Euler Equation I

- Take the basic RBC:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

$$c_t + k_{t+1} = e^{z_t} k_t^{\alpha} + \left(1-\delta\right) k_t, \ \forall \ t > 0$$

$$z_t = \rho z_{t-1} + \sigma \varepsilon_t, \ \varepsilon_t \sim \mathcal{N}(0,1)$$

- The first order condition:

$$u'\left(c_t\right) = \beta \mathbb{E}_t \left\{ u'\left(c_{t+1}\right)\left(1 + \alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} - \delta\right)\right\}$$

- There is a policy function $g : \Re_+ \times \Re \rightarrow \Re_+^2$ that gives the optimal choice of consumption and capital tomorrow given capital and productivity today.

# Example: Euler Equation II

- Then:

$$u'\left(g^{1}\left(k_t, z_t\right)\right) = \beta \mathbb{E}_t \left\{ u'\left(g^{1}\left(k_{t+1}, z_{t+1}\right)\right)\left(1 + f\left(g^{2}\left(k_t, z_t\right), z_{t+1}\right) - \delta\right)\right\}$$

or, alternatively:

$$u'\left(g^{1}\left(k_t, z_t\right)\right) - \beta \mathbb{E}_t \left\{ u'\left(g^{1}\left(g^{2}\left(k_t, z_t\right), z_{t+1}\right)\right)\left(1 + f\left(g^{2}\left(k_t, z_t\right), z_{t+1}\right) - \delta\right)\right\} = 0$$

- We have functional equation where the unknown object is the policy function $g(\cdot)$.
- More precisely, an integral equation (expectation operator). This can lead to some measure theoretic issues that we will ignore.

# Example: Euler Equation III

- Mapping into an operator is straightforward:

$$\mathcal{H} = u'(\cdot) - \beta \mathbb{E}_t \left\{ u'(\cdot)\left(1 + f(\cdot, z_{t+1}) - \delta\right)\right\}$$
$$d = g$$

- If we find $g$, and a transversality condition is satisfied, we are done!

# Example: Euler Equation IV

- Slightly different definitions of $\mathcal{H}$ and $d$ can be used.

- For instance if we take again the Euler equation:

$$u'(c_t) - \beta \mathbb{E}_t \left\{ u'(c_{t+1}) \left(1 + \alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} - \delta\right) \right\} = 0$$

  we may be interested in finding the unknown conditional expectation $\mathbb{E}_t \left\{ u'(c_{t+1}) \left(1 + \alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} - \delta\right) \right\}$.

- Since $\mathbb{E}_t$ is itself another function, we write

$$\mathcal{H}(d) = u'(\cdot) - \beta d = \mathbf{0}$$

  where $d = E_t \{\vartheta\}$ and $\vartheta = u'(\cdot)(1 + f(\cdot, z_{t+1}) - \delta)$.

# How Do We Solve Functional Equations?

Two Main Approaches

1. Perturbation Methods:

$$d^n(x,\theta) = \sum_{i=0}^{n} \theta_i (x - x_0)^i$$

We use implicit-function theorems to find coefficients $\theta_i$.

2. Projection Methods:

$$d^n(x,\theta) = \sum_{i=0}^{n} \theta_i \Psi_i(x)$$

We pick a basis $\{\Psi_i(x)\}_{i=0}^{\infty}$ and "project" $\mathcal{H}(\cdot)$ against that basis.

# Relation with Value Function Iteration

- There is a third main approach: the dynamic programing algorithm.
- Advantages:
  1. Strong theoretical properties.
  2. Intuitive interpretation.
- Problems:
  1. Difficult to use with non-pareto efficient economies.
  2. Curse of dimensionality.

# Evaluating the Likelihood Function

- How do we take the model to the data?
- Usually we cannot write the likelihood of a DSGE model.
- Once the model is nonlinear and/or non-gaussian we cannot use the Kalman filter to *evaluate* the likelihood function of the model.
- How do we evaluate then such likelihood? Using Sequential Monte Carlo.

# Basic Estimation Algorithm 1: Evaluating Likelihood

Input: observables $Y^T$, DSGE model $M$ with parameters $\gamma \in \Upsilon$.

Output: likelihood $p\left(y^T;\gamma\right)$.

1. Given $\gamma$, solve for policy functions of $M$.
2. With the policy functions, write the state-space form:

$$S_t = f\left(S_{t-1}, W_t; \gamma_i\right)$$
$$Y_t = g\left(S_t, V_t; \gamma_i\right)$$

3. With state space form, evaluate likelihood:

$$p\left(y^T;\gamma_i\right) = \prod_{t=1}^{T} p\left(y_t|y^{t-1};\gamma_i\right)$$

# Basic Estimation Algorithm 2: MLE

Input: observables $Y^T$, DSGE model $M$ parameterized by $\gamma \in \Upsilon$.

Estimates: $\widehat{\gamma}$

1. Set $i = 0$. Fix initial parameter values $\gamma_i$.
2. Compute $p\left(y^T; \gamma_i\right)$ using algorithm 1.
3. Is $\gamma_i = \arg\max p\left(y^T; \gamma\right)$?
    1. Yes: Make $\widehat{\gamma} = \gamma_i$. Stop.
    2. No: Make $\gamma_i \rightsquigarrow \gamma_{i+1}$. Go to step 2.

# Basic Estimation Algorithm 3: Bayesian

Input: observables $Y^T$, DSGE model $M$ parameterized by $\gamma \in \Upsilon$ with priors $\pi(\gamma)$.

Posterior distribution: $\pi\left(\gamma | Y^T\right)$

1. Fix $I$. Set $i = 0$ and chose initial parameter values $\gamma_i$.
2. Compute $p\left(y^T; \gamma_i\right)$ using algorithm 1.
3. Propose $\gamma^* = \gamma_i + \varepsilon$ where $\varepsilon \sim N(0, \Sigma)$.
4. Compute $\alpha = \min\left\{\frac{p\left(y^T; \gamma^*\right)\pi(\gamma^*)}{p(y^T; \gamma_i)\pi(\gamma_i)}, 1\right\}$.
5. With probability $\alpha$, make $\gamma_{i+1} = \gamma^*$. Otherwise $\gamma_{i+1} = \gamma_i$.
6. If $i < M$, $i \rightsquigarrow \bar{\imath} + 1$. Go to step 3. Otherwise Stop.