The Demand for Annuities with Stochastic Mortality Probabilities

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Abstract

The conventional wisdom dating back to Yaari (1965) is that households without a bequest motive should fully annuitize their assets. Various market frictions do not break this sharp result. This paper demonstrates that incomplete annuitization can be optimal in the presence of stochastic mortality probabilities, even without any liquidity constraints. Moreover, stochastic mortality probabilities are a mechanism for various other market frictions to further reduce annuity demand.

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is presented using a calibrated lifecycle dynamic programming model. Implications for Social Security are discussed. Keywords: Annuities, stochastic mortality, annuity puzzle, Social Security.
1 Introduction

The classic paper of Yaari (1965) demonstrated that the demand for life annuities should be so strong that lifecycle consumers without a bequest motive invest all of their savings inside of an annuity. Annuities statewise dominate all non-annuity investments since annuities produce a mortality credit – derived from the pooled participants who die and forfeit their assets – in addition to the return from the underlying asset. If an investor wanted to invest in bonds then a fixed return annuity invested in bonds would produce the bond yield plus the mortality credit. If an investor wanted to invest in stocks then a variable return annuity invested in stocks would produce the same realized yield plus the mortality credit. Annuities could, therefore, significantly increase welfare, especially if risk averse households are not able to manage mortality risk in other ways (Brown 2002).

Yaari’s seminal paper has received considerable attention in the subsequent literature, especially since true life annuities are uncommon. As is well known, Yaari’s model assumed costless and complete markets. In practice, however, annuities are not fairly priced: premiums incorporate sales charges as well as adjustments for adverse selection (Brugiavini 1993; Mitchell, Poterba, Warshawsky, and Brown 1999; Walliser 2000; Finkelstein and Poterba 2004). In addition, other sources of longevity pooling exist that might crowd out some of the demand for annuities, including Social Security and defined-benefit pensions (Townley and Boadway 1988) and even marriage (Kotlikoff and Spivak 1981). Moreover, people might face liquidity constraints after annuitization (Sinclair and Smetters 2004; Turra and Mitchell 2008; Davidoff, Brown and Diamond 2005). Still, the careful analysis of Davidoff, Brown and Diamond (2005) demonstrates that many of Yaari’s original assumptions are not necessary for his original result to hold. As, Brown, Kling, Mullainathan, and Wrobel (2008, P. 304) conclude: “As a whole, however, the literature has failed to find a sufficiently general explanation of consumer aversion to annuities.” Indeed, the apparent under-annuitization is commonly referred to as “the annuity puzzle” (Ameriks, Caplin, Laufer and van Nieuwerburgh 2011, and many others).

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1 Most products in the marketplace that are called annuities are tax vehicles that offer minimal insurance protection against longevity risk. Premiums for individual immediate annuities in the United States totaled just $X billion in 2011 (LIMRA XX).
The current paper adopts the Yaari framework but allows for the mortality survival probabilities themselves to be stochastic, which is quite natural and consistent with an investor's health status evolving over time. Rather than adding restrictions to the Yaari model, we simply relax an existing implicit constraint by allowing for non-fixed probabilities over the lifecycle. We otherwise leave the Yaari model unfettered. Insurers and households have the same information. There are no loadings. Households do not face any binding “liquidity constraints,” an often cited (albeit unmotivated) reason for incomplete annuitization.

Stochastic mortality introduces valuation (or principal) risk, much like a long-term bond. If households are sufficiently impatient, a long-term traditional annuity no longer dominates, even with no other sources of uncertainty or market frictions. The optimal level of annuitization falls below 100%. If, however, households are sufficiently patient then annuities again dominate if there are no additional frictions. In that case, stochastic mortality probabilities provides a mechanism by which these additional frictions can smoothly reduce annuity demand, something does not exist with deterministic probabilities, as we show. Even the role of bequests because more meaningful with stochastic survival probabilities, since this uncertainty interacts more with the annuitization choice of wealthy households – where we think these bequest motives are actually operative – instead of indiscriminately reducing the annuity demand across the board, as happens with deterministic survival probabilities and homothetical utility.

To be clear, we don’t intend our model to explain market behavior or a broad range of stylized facts. Many puzzles remain – especially regarding annuity contract design – which could maybe be better explained with some behavioral models (Gottlieb 2012). Rather, we interpret our analysis as purely normative in nature and simply argue that the optimal baseline annuitization rate in a fully unconstrained Yaari model can fall below 100%. However, we maybe contribute indirectly to the annuity puzzle by “lowering the bar” a bit about what really needs to be explained.

Still, we note that our mechanism is at least consistent with both industry research and academic experimental evidence which indicates that households view annuities as increasing their risk rather than risk reducing. Brown et al (2008) interpret this evidence as consistent with narrow framing inherit in prospect theory. In our model, even with rational expec-
tations, annuities can deliver both a larger expected consumption stream and more risk relative to bonds. As a result, a greater level of risk aversion can produce less annuitization. In contrast, with deterministic survival probabilities, the mortality pooling provided by an annuity provides more consumer surplus at higher levels of risk aversion.

The rest of the paper is organized as follows. Section 2 develops a three-period model with deterministic survival probabilities and argues that Yaari’s 100% annuitization result is even stronger than likely previously understood. This discussion helps to demonstrate the value that stochastic survival probabilities play in reducing the optimal annuity demand. Section 3 then analyzes the impact of stochastic survival probabilities. Section 4 presents a multiple-period lifecycle model while Section 5 presents simulation evidence that includes various frictions. We find that, even without any liquidity constraints, the median American household should not annuitize any assets at the point of retirement (especially with Social Security). However, the rich households should annuitize, if they have no bequest motives. Section 6 concludes.

2 Three-period model

Consider an individual age $j$ in state $h$ who can live at most up to three additional periods: $j$, $j+1$, and $j+2$. The chance of surviving from age $j$ to reach $j+1$ is denoted as $s_j(h)$, which is condition on state $h$ at time $j$. State $h$ is drawn from a countable set $H$ with a cardinality exceeding 1 (formally, $h, h' \in H$ with $|H| > 1$). We can interpret these elements as “health states,” although they generally represent anything that impacts survival probabilities. The cardinality assumption ensures that there is more than one such state, and so we model the Markov transitional probability between states as $P(h'|h)$ where $h$ is the current state and $h'$ is the state next period.

An annuity contract with a single premium $\pi_j$ at age $j$ is available that pays 1 unit in each period, $j+1$ and $j+2$, conditional on survival. In a competitive environment without any additional frictions (i.e., fair pricing), the premium paid today must equal the actuarial present value of the payment of 1 received from the annuity in each of the future two periods. The premium can, therefore, be written recursively as follows:
\[ \pi_j(h) = \frac{s_j(h)}{(1 + r)} \cdot 1 + \frac{s_j(h) \cdot \sum_{h'} P(h'|h) s_{j+1}(h') \cdot 1}{(1 + r)^2} \]

\[ = \frac{s_j(h)}{(1 + r)} \cdot \left( 1 + \frac{\sum_{h'} P(h'|h) s_{j+1}(h') \cdot 1}{(1 + r)} \right) \]

\[ = \frac{s_j(h)}{(1 + r)} \cdot \left( 1 + \sum_{h'} P(h'|h) \pi_{j+1}(h') \right) \quad (1) \]

where \( h' \in H \) is the health state realized in period \( j + 1 \) and \( \pi_{j+2}(h) = 0 \). Hence:

\[ \pi_{j+1}(h) = \frac{s_{j+1}(h)}{(1 + r)} \quad (2) \]

The realized (ex-post) gross annuity rate of return, denoted as \( 1 + \rho_j(h) \), is derived similar to any investment: the dividend yield (1 in this case) plus new price \( (\pi_{j+1}(h')) \), all divided by the original price \( (\pi_j(h)) \). Hence, the net return for a survivor to age \( j + 1 \) is:

\[ \rho_j(h'|h) = \frac{1 + \pi_{j+1}(h')}{\pi_j(h)} - 1. \quad (3) \]

2.1 Deterministic Survival Probabilities (The Yaari Model)

In the deterministic (Yaari) model, survival is uncertain but the probabilities themselves are deterministic. The deterministic model, therefore, can be viewed as a restriction on the stochastic survival probability process:

\[ P(h'|h) = \begin{cases} 
1, & h = h' \\
0, & h \neq h' 
\end{cases} \]

In other words, for the corresponding Markov transition matrix \([P(h'|h)]\), the off-diagonal elements must be zero. But, survival probabilities are not restricted to be constant across age. For a person with health status \( h \) we can allow for standard lifecycle “aging” effects:

\[ s_{j+1}(h) < s_j(h) < 1 \]

In particular, survival is allowed to decrease with age, consistent with diminishing health that is predictable by age alone. (The second inequality simply recognizes that some people
However, the probabilities themselves are not stochastic, which we can interpret as there being no changes in survivor probabilities that can’t already be predicted by the initial health status $h$ and age alone.

By equation (1), the premium for a person of health status $h$ at age $j$ is given by:

$$\pi_j(h) = \frac{s_j(h)}{(1 + r)} \cdot (1 + \pi_{j+1}(h))$$

which implies:

$$\frac{(1 + r)}{s_j(h)} = \frac{1 + \pi_{j+1}(h)}{\pi_j(h)}$$

The realized net rate of return to an annuity, therefore, is equal to

$$\rho_j(h) = \frac{1 + \pi_{j+1}(h)}{\pi_j(h)} - 1$$

$$= \frac{(1 + r)}{s_j(h)} - 1.$$

Notice that the realized annuity return (5) is identical to that of a single-period annuity, that is, independent of the survival probability at age $j + 1$. Equation (4) includes the actuarial present value of the fair-premium, $\pi_{j+1}(h)$, at age $j + 1$. Hence, the survival probability at age $j + 1$ is already priced into the annuity premium, $\pi_j(h)$, paid at age $j$. Since $\pi_{j+1}(h)$ is deterministic and known at age $j$, there is then no additional risk over the life of the annuity contract. It follows that a multiple-year annuity can be created with a sequence of single-period annuities, a well-known result in the literature.

We say that annuities statewise dominate bonds if $\rho_j(h) > r$ for all values of $h$. We have the following result.

**Proposition 1.** With deterministic survival probabilities, annuities statewise dominate bonds for any initial health state at age $j$.

**Proof.** By equation (5), $\rho_j(h) > r$ for all values of $h$ provided that $s_j(h) < 1$ (i.e., people can die).
Proposition (1) implies that annuities should be held by all people for all wealth in the Yaari economy. Statewise dominance is the strongest notion of stochastic ordering. Any person with preferences exhibiting positive marginal utility – even very non-standard preferences that place weight on \textit{ex-post} realizations – will prefer a statewise dominant security. Statewise dominance implies that annuities are also \textit{first-order} dominant (and, so will be chosen by all \textit{expected utility} maximizers) and \textit{second-order} dominant (and, so will be chosen by all \textit{risk averse expected utility} maximizers).

\subsection{2.2 Robustness}

It is well known that Yaari’s full annuitization result is pretty strong. But it is even stronger than is often appreciated and robust to many of the market frictions often thrown at the model. Understanding the strength of the Yaari result doesn’t just create a good “straw man.” It allows to understand the role that stochastic survival probabilities play in providing a gateway for many of these frictions to be material.

Figure 1 gives some graphical insight into the statewise dominance that is produced with deterministic survival probabilities. Consider an investor age $j$ who is deciding between investing a fixed amount in bonds and or buying an annuity with a competitive return that is conditional on her health $h$ at age $j$.\footnote{Of course, we are being simple. In the presence of inter-temporal substitution and standard utility the amount of saving varies by the return of each investment. But this secondary effect is inmaterial for our purposes.} Her “budget constraint” between bonds and annuities is simply a straight line with slope of -1: she can either invest $1 into bonds or $1 into annuities. Her “indifference curve” between these investments is also a straight line because the investments only differ in their deterministic rate of return, i.e., they are statewise stochastically identical. But notice that the slope of the indifference curve is steeper than -1 and equal to $-\frac{1}{s_j(h)}$. Intuitively, she would be willing to give up $1 in annuity investment if she could trade it for $\frac{1}{s_j(h)} > $1 worth of bonds: both investments would be worth the same amount at age $j + 1$. Of course, the market would not allow for this trade and so the maximum indifference curve that she can achieve intersects the budget constraint at the corner point of full annuitization, as shown in Figure 1.
As it turns out, this “corner optimality” is terribly hard to break with deterministic survival probabilities – and, when it changes, it “breaks, not bends.” To see why, suppose that we reduced the size of the mortality credit by increasing the value of $s_j(h)$. Notice that we would simply rotate the indifference curve toward the budget set around the kink point at full annuitization. So, we are still at full annuitization. In fact, advancing away from the corner requires a rotation greater than the value of the mortality credit itself. This situation is shown in Figure 2. The budget constraint still has a slope of -1: as before, $1$ can be invested in bonds or annuities. But now the indifference curve is less steep than -1 because the agent would be willing to give up more than $1$ in annuities for less than $1$ in bonds. But, now, the optimal solution “jumps corners” from full annuitization to zero annuitization. Once again, there is no interior solution.

There a couple market frictions that can rotate the indifference curve. The most obvious one is transaction costs. While fixed transaction costs could wipe out the small mortality credits earned by younger households, they would have to be substantial at older ages to have any effect on annuitization. Hence, we would expect to see 0% annuitization for very young consumers and 100% annuitization for older ones. Moral hazard could also reduce the effective size of the mortality credit if agents invest in living longer after annuitization. However, moral hazard can’t exist without annuitization, and the optimal asset mix must still be located at the 100% annuitization corner.

In fact, most of commonly cited market frictions do no rotate the indifference curve at all, thereby having no effect. Hidden information leading to adverse selection would seem to reduce the size of the mortality credit. But annuities must still statewise dominate, and so everyone would annuitize in equilibrium, implying no reduction in the mortality credit. While social security crowds some personal saving, the asset-annuity slope tradeoff for the remaining saving is unchanged. Insurance within marriage can reduce the level of precautionary saving, but it does not eliminate the statewise dominance of annuities or

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3Specifically, for a given set of preferences, agents allocate their consumption in order to equalize their marginal utilities across ages weighted by their rate of time preference and survival probabilities. With standard Inada conditions (where marginal utility shoots to infinity as consumption approaches zero), agents already invest in preserving consumption flows in high utility states before annuitization. Annuitization, therefore, causes moral hazard through the income effect from receiving the additional mortality credit.
change the slope. *Uncertainty income* and *uncertain expenses* – whether correlated or not with the deterministic lifecycle changes in the survival probabilities – has no impact as well.

At first glance, the presence of “liquidity constraints,” a commonly cited friction, would seem to undermine the case for full annuitization in the Yaari model that is augmented with shocks to income or expenses. In particular, it would appear that households should not want to fully annuitize their wealth in case they need access to the wealth after a negative shock that increases their marginal utility; short-term bonds should be more accessible for liquidity purposes. But it is important to note this argument is actually quite different than the standard borrowing constraint assumption found in the literature. Standard borrowing constraints prevent people from borrowing against future income. Recent work has provided the microeconomics foundations for standard borrowing constraints based on inability of the private sector to fully enforce two-sided contracts in the presence of hidden information.⁴ A constraint against borrowing from future income, however, does not undermine the case for full annuitization. Any existing savings, should always be invested in a statewise dominate asset.

Rather, the “liquidity argument” is actually imposing a constraint on asset rebalancing. For incomplete annuitization to occur, households must be unable to rebalance their existing assets from annuities into bonds. This constraint, however, is a much stronger assumption than a standard borrowing constraint. The microeconomics foundation prohibiting rebalancing is unclear with deterministic survival probabilities. Indeed, since there is no mortality “reclassification risk” with deterministic survival then even “surrender fees” intended to reduce rebalancing would inefficiently distort marginal utility after a shock and, therefore, could not survive competition.⁵ Indeed, annuity-bond rebalancing would be competitively provided. It could take the form of a secondary market, much like the growing “life settlements” market for life insurance contracts. Or, households could simply rebalance on their own by purchasing a term life insurance with the 1 unit of annuity payment received at

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⁴See, for example, Zhang (1997) and Clementi and Hopenhayn (2006).

⁵Surrender fees have been shown to be optimal in the presence of reclassification risk with life insurance contracts (XXX). In the context of life insurance contracts, Gottlieb and Smetters (2012) show that surrender fees could survive competition, even without reclassification risk, if agents “narrowly frame” their purchase decisions.
age \( j + 1 \) and borrowing \( \pi_{j+1} \). Subsequent hidden information would not undermine either type of market action: if annuity providers can observe the initial survival probabilities (e.g., health state \( h \)) necessary for underwriting the competitively-priced annuity for a person at age \( j \) then they must also know the health state at age \( j + 1 \).\(^6\)

### 3 Stochastic Survival Probabilities

Now suppose that we allow \( p(h'|h) > 0 \) when \( h' \neq h \), which implies stochastic survival probabilities. In other words, for the corresponding Markov transition matrix \([P(h'|h)]\), the off-diagonal elements are allowed to be positive.

#### 3.1 Stochastic Rankings

Allowing for stochastic survival probabilities breaks the statewise dominance of annuities over bonds.

**Proposition 2.** With stochastic survival probabilities, annuities do not generically statewise dominate bonds.

**Proof.** Inserting equation (1) into equation (3) and rearranging:

\[
\rho_j (h'|h) = \frac{1 + \pi_{j+1} (h')}{\frac{s_j(h)}{(1+r)} \cdot (1 + \sum_{h'} P(h'|h) \pi_{j+1} (h'))} - 1
= \frac{1 + \pi_{j+1} (h')}{\frac{s_j(h)}{(1+r)} \cdot (1 + E_H (\pi_{j+1} (h')))} - 1
\]

Since \(|H| > 1\) then \( \pi_{j+1} (\inf (H)) < E_H (\pi_{j+1} (h)) \). It is easy, therefore, to construct examples where \( \rho_j (h'|h) < r \), thereby violating FOSD. Consider, for example, a set \( H \) with the elements \( h \) and \( h' \), where \( s_j (h) \to 1 \) and \( s_{j+1} (h') \to 0 \) (and, hence, \( \pi_{j+1} (h') \to 0 \)). Then, we can further refine \( H \) so that \( E_H (\pi_{j+1} (h')) \) is sufficiently large, producing that \( \rho_j (h'|h) < r \) since \( E_H (\pi_{j+1} (h')) \to \infty \) implies \( \rho_j (h'|h) \to -(1 + r) \). \( \square \)

\(^6\)In the case of a secondary market, firms could perfectly observe health status at age \( j + 1 \) from the contract signed at age \( j \).
Intuitively, the competitive annuity premium at age $j$ is set equal to the present value of the \textit{expected} annuity payments received at ages $j + 1$ and $j + 2$, conditional on the health state $h$ at age $j$. But a sufficiently negative health \textit{realization} $h'$ at age $j + 1$ can reduce the expected payout at age $j + 2$ such that the capital depreciation at age $j + 1$ on the annuity contract is larger than the mortality credit received from the previous period. In effect, when survival probabilities are stochastic, the annuity contract has a “valuation” (or “principal”) risk similar to a long-dated bond\footnote{If participants can sell their contracts in a secondary market, which should be competitively provided in world without hidden information, then we can also interpret valuation risk as “resale” risk.} A key difference, however, is that standard valuation risk with bonds can be eliminated if they are held to maturity. Holding to maturity is obviously impossible for an annuity.

The fact that annuities do not statewise dominate bonds in the presence of stochastic survival probabilities only means that annuities will not necessarily be optimal across a wide range of preferences with a positive marginal utility. It is still possible, however, that annuities could dominate for more specific types of preferences, including expected utility maximizers. Indeed, annuities will be strictly preferred by risk-neutral consumers.

\textbf{Proposition 3.} The expected return to annuities exceeds bonds if the chance of mortality is positive.

\textit{Proof.} The expected annuity return for a survivor to age $j + 1$ is equal to

$$E [\rho_j (h'|h)] = \frac{1 + \sum_{h'} P (h'|h) \pi_{j+1} (h')}{\pi_j (h)} - 1$$

$$= \frac{(1+r) \pi_j (h')}{s_j (h)} - 1$$

$$= \frac{(1 + r)}{s_j (h)} - 1$$

$$> r$$

if $s_j (h) < 1$. \hfill \Box

Intuitively, risk neutral agents care only about the greater expected return derived from the mortality credit and ignore valuation risk.
However, annuities do not necessarily (that is, generically) dominate bonds for the more restricted class of risk averse expected utility maximizers.

**Proposition 4.** With stochastic survival probabilities, annuities do not generically second-order stochastically dominate (SOSD) bonds.

*Proof.* See next subsection.

In sum, the reliance on dominance rankings allows us to make fairly general statements across a wide range of utility functions and only narrow when necessary. In particular, Proposition (2) shows that annuities do not statewise dominate bonds in the presence of stochastic survival probabilities. Hence, we can no longer expect that, across a wide range of preferences, zero bonds will be held. Proposition (4) then shows that annuities are not necessarily dominate even for just risk averse agents. Annuitzation provides a hedge against longevity risk but also creates valuation risk. The standard full annuitization result, therefore, breaks down simply by allowing for non-zero diagonal terms on the Markov transition $[P(h'|h)]$.

### 3.2 Examples

We demonstrate Propositions (2) through (4) with a series of successive simple examples. Proposition (4) in particular, is especially surprising because it cuts against the conventional wisdom that ex-ante identical agents will prefer long-term contracts as a way of pooling future “reclassification risk” (i.e., changes in the survival probabilities).

#### 3.2.1 Failure of Statewise Dominance

Continuing with our three-period setting, consider an agent at age $j$ with current health state $h$. To really focus on the key mechanisms at work, lets make a range of unrealistic but simplifying assumptions. Specifically, set the probability of survival to $j + 1$ equal to $s_j(h) = 1$, which eliminates any positive mortality credit from annuitization during period

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8Specifically, gamble $A$ second-order stochastically dominates gamble $B$ iff $E_A u(x) \geq E_B u(x)$ for any nondecreasing, concave utility function, $u(x)$. 

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The bond net return $r$ (and, hence, discount rate) is also 0. Both of these assumptions allow us to focus on the value of pooling reclassification risk (changes in mortality) realized at age $j+1$. The main results are robust to small changes in these parameters.

At age $j+1$, one of two mutually exclusive health states can be realized with equal (that is, 50%) probability: $h_G$ ("Good") and $h_B$ ("Bad"). If the Good health state $h_G$ is realized then the probability of surviving to age $j+2$ is one: $s_{j+1}(h_G) = 1$. Similarly, if the Bad health state $h_B$ is realized then the probability of surviving to age $j+2$ is zero: $s_{j+1}(h_B) = 0$. In words, a person who realizes Good health will live periods $j+1$ and $j+2$ while a person realizing a Bad state will only live for the period $j+1$. Again, these extreme assumptions allow us to focus on the important theoretical drivers.

By equation (2), $\pi_{j+1}(h_G) = 1$ and $\pi_{j+1}(h_G) = 0$. Hence, by equation, (1), the competitive premium paid today at age $j$ for an annuity that pays $1$ at ages $j$ and $j+1$ (of course, conditional on surviving) is equal to $\pi_j(h) = 1.0 \cdot (1 + 0.5)$, or $1.5$. This amount is simply equal to the $1$ annuity payment that is received with certainty at age $j+1$ (since $s_j(h) = 1.0$) plus the expected value of the $1$ annuity payment made at age $j+2$, which is paid 50% of the time to people who realize Good health at age $j+1$.

Suppose that an agent, therefore, is thinking about whether to invest $1.5$ in the annuity or bond. Consider two cases:

- **Case 1 ($h_G$ is realized at age $j+1$):** Consider a household that realizes the Good health state $h_G$ at age $j+1$. Then, by equation (3), the realized net return $\rho_j(h_B)$ to the annuity is equal to $\frac{2}{1.5} - 1 > 0$, thereby beating bonds. In terms of dollars, the agent at age $j+1$ has $2$ in assets with the annuity, equal to the $1$ produced by the annuity at age $j+1$ plus the present value (at a zero discount rate) of the $1$ in annuity payment that will be paid (for sure) at age $j+2$. Had this household instead invested $1.5$ at age $j$ into bonds, it would have had only $1.5$ at age $j+1$.

- **Case 2 ($h_B$ is realized at age $j+1$):** If, however, the Bad health state $h_B$ is realized at age $j+1$ then the realized net return to annuitization is equal to $\frac{1}{1.5} - 1 < 0$, thereby under-performing bonds. In terms of dollars, this household has only $1$ in assets at age $j+1$, which was produced by the annuity that they previously purchased for $1.5$ at age $j$; the present value of the remaining annuity is zero (a value depreciation). Had this household
instead invested the value of the annuity premium into bonds then it would have had $1.5 at age $j + 1$, and so they are worse off with annuity.

Hence, annuities fail to \textit{statewise dominate} bonds in the presence of a devaluation of the annuity contract after a negative survival shock. Intuitively, the competitively priced annuity contract at age $j$ was calculated based on expected survival outcomes. Obviously, survival realizations below expectation can leave some buyers worse off \textit{ex-post}.

\subsection*{3.2.2 Failure of Second-Order Dominance}

The violation of statewise dominance, however, is only one chink in the annuity armor. It simply means that annuities will not necessarily be optimal across a wide range of preferences that, for example, place some weight on ex-post realizations. Annuities, however, can still be the dominate security for \textit{risk-averse expected utility} maximizing agents whose preferences full weigh risky gambles from an \textit{ex-ante} position, that is, at age $j$.

Hence, continuing with our example from Section 3.2.1, lets now backward induct to the decision that is being made age $j$. The household with health $h$ at age $j$ is choosing between the following two lotteries:

- Lottery A (Bond): Invest $1.5 into a bond that will be worth at age $j + 1$ an amount of $1.5$ with 100% certainty
- Lottery B (Annuity): Invest $1.5$ into an annuity that will be worth at age $j + 1$ an amount $1$ with 50% probability or $2$ with 50% probability.

Notice that, based purely on asset values, Lottery B (Annuity) represents a mean-preserving \textit{increase} in risk relative to Lottery A (Bond). However, we still can’t make any definitive statement yet about dominance because we ultimately care about consumption values. Still, the increasing asset value risk suggests a way of how we might construct a set of preferences where Lottery A could be preferred. Indeed, as we know demonstrate, such preferences can be consistent with expected utility. So, we need to now explicitly consider consumer preferences.

Continuing with our example, suppose that our agent at age $j$ with health $h$ is endowed with $1.5$ and only consumes in age $j + 1$ and $j + 2$. The agent has standard \textit{conditional} expected utility preferences over consumption equal to
\[ u(c_{j+1}|h_{j+1}) + \beta \cdot s_{j+1} (h_{j+1}) u(c_{j+2}|h_{j+1}), \] (6)

where the period felicity function \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \) takes the constant relative risk aversion form\(^9\) \( \sigma \) is the level of risk aversion, and \( \beta \) is the weight placed on future utility. Recall \( s_{j+1} (h_{j+1} = h_G) = 1 \) and \( s_{j+1} (h_{j+1} = h_B) = 0 \). Hence, the **unconditional** expected utility at age \( j \) is equal to

\[ EU = \frac{1}{2} \left[ u(c_{j+1}|h_G) + \beta \cdot u(c_{j+2}|h_G) \right] + \frac{1}{2} \cdot u(c_{j+1}|h_B), \] (7)

As we now show, the rate of time preference plays a key role in the lottery choice.

**High Patience** \((\beta = 1)\)  Suppose that there is no rate of time preference, and so agents are fully patient, weighting future utility equal to current utility: \( \beta = 1 \). Now consider the conditional consumption streams associated with the two competing investments, the Bond versus the Annuity, which is made at age \( j \). Independent of the level of risk aversion, we get:

- **Bond**: If Good health is realized at age \( j + 1 \) then \( c_{j+1} = 0.75 \) and \( c_{j+2} = 0.75 \); if Bad health is realized then \( c_{j+1} = 1.5 \).

- **Annuity**: If Good health is realized at age \( j + 1 \) then \( c_{j+1} = 1.0 \) and \( c_{j+2} = 1.0 \); if Bad health is realized then \( c_{j+1} = 1.0 \).

Notice that the choice of the Bond creates very non-smooth consumption choices across the two health states. In contrast, the Annuity effectively shifts 0.5 units of consumption from the Bad health state to the Good state, thereby creating perfectly smooth consumption across states and time. Annuities, therefore, will be preferred by anyone with a reasonable felicity function \( u \) exhibiting risk aversion. This result is consistent with the previous literature demonstrating that longer-term contracts provide superior risk pooling than spot contracts among ex-ante identical individuals who want to hedge reclassification risk.

**Low Patience** \((\beta \to 0)\)  Now consider the opposite extreme where agents are very impatient: \( \beta \to 0 \). The conditional consumption streams are now as follows:

\(^9\)As will be evident below, our analysis holds for any risk averse function. However, the CRRA assumption allows us to report a few numerical examples.
• Bond: If Good health is realized at age $j+1$ then $c_{j+1} \rightarrow 1.5$ and $c_{j+2} \rightarrow 0$; if Bad health is realized then $c_{j+1} = 1.5$.

• Annuity: If Good health is realized at age $j+1$ then $c_{j+1} \rightarrow 2.0$ and $c_{j+2} \rightarrow 0$; if Bad health is realized then $c_{j+1} = 1.0$.

Notice that the Annuity now actually increases consumption variation across the two health states. At the limit point ($\beta = 0$), no weight is placed on consumption at age $j+2$, and so bonds are unambiguously more effective at smoothing consumption across those allocations that are actually valued by the agent. With the Bond, consumption is equal to a smooth value of 1.5 in all health states and ages that are valued. With the Annuity, consumption is equal to 2.0 with Good health and 1.0 with Bad health.

Notice that agents in this example do not face any asset rebalancing constraint, thereby allowing them to consume the entire value of the Annuity at age $j+2$ when the Good state is reached. Hence, time-based “liquidity constraints” are not at play. Instead, the Annuity is inferior because its fail to properly shift consumption across health states in a way that is actually informed by the agent’s preferences. In particular, the fair (competitive) Annuity premium must be based on objective survival probabilities and the market discount rate, which can result in very different set of allocations across risky states than desired by the agent.

Of course, the choice of $\beta = 0$ is an extreme case and made for purposes of illustration to focus on the pooling potential of reclassification risk at age $j+1$. Indeed, recall that we effectively eliminated any potential value of the annuity other than pooling this reclassification risk. Indeed, agents do not care about mortality risk itself at all. In particular, for age $j+1$, they place a positive on their utility but also survive to that age with complete certainty (i.e., recall that we set $s_j(h) = 1$). For age $j+2$, we have just the opposite situation: they do not survive to that age with compete certainty but also they place no weight on its actual utility. Of course, we relax these assumptions a bit and easily introduce a mortality role for the Annuity if we allow $s_j(h) < 1$. In this case, annuitization at age $j$ introduces introduces a small mortality credit to those who survive to age $j+1$, reducing the value of the initial premium. Hence, the consumption associated with the constant-payout Annuity remains at the values shown above. But the consumption values shown for the Bond would decrease to a value slightly below 1.5 because it does not receive a mortality credit. However, for a
small change, the smooth value of consumption produced by Bonds could be more valuable to a risk averse agent than the variable but larger expected consumption produced by the Annuity.

Now suppose that the value of $\beta$ is small but still strictly positive. Hence, the marginal utility associated with consumption at age $j + 2$ must also be considered. In general, the highest expected utility is obtained when we equalize the *weighted marginal utilities* across different states, which is not equivalent to equalizing consumption when $\beta < 1$. Given the analysis above, not surprisingly, the Bond can still be more effective at smoothing the weighted marginal utilities as well. As a specific example, suppose that $\beta = 0.10$. Also, set the level of risk aversion $\sigma$ equal to 2. Then the $EU(bonds) = -0.91$ while the $EU(stocks) = -0.93$. Of course, if we were to increase the value of $\sigma$ then the curvature of the utility function becomes more important and could tilt the balance back toward the Annuity.

### 3.3 A Gateway Mechanism

Maybe most importantly, the presence of stochastic survival probabilities provides a mechanism for many of the “additional frictions” noted earlier to actually reduce the optimal level of annuitization, either individually or jointly, in a smooth manner. For a standard risk averse investor, the “indifference curve” between bonds and annuities now takes the more usual “bulge toward the origin” property, as shown in Figure 3.2.1. As before, the “budget constraint,” of course, remains a -1 slope since $1 can be invested in bonds or annuities. But annuities are no longer stochastically equivalent to bonds, thereby removing the sharp “corner optimality” of annuities that exists with deterministic survival probabilities where most of these additional frictions are simply infra-marginal.

The roll of additional shocks that are *correlated* with the survival become especially material. A negative health shock should increase both medical costs and potentially reduce future human capital returns (e.g., disability). With *deterministic* survival probabilities, a negative health shock is no different than any other shock to expenses or income that increases marginal utility; full annuitization should still occur in the presence of asset rebal-

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10 If we interpret each period in our simple model as 30 years, then this small value is equivalent to annual rate equal to about 0.92.
ancing. With stochastic survival probabilities, however, a negative health shock reduces the value of the annuity at the same time that expenses increase and income falls.

Now, a reduction in the size of the mortality credit, whether from hidden information or transaction costs, further reduces the marginal interior point demand for annuities. Graphically, the indifference curve shown in Figure 3.2.1 rotates (or “rolls”) along the budget constraint. The presence of social security can crowd out annuity demand for other personal saving since both funded or unfunded social security systems already provide a mortality credit. Even “background risks,” such as shocks to income that are otherwise uncorrelated with survival (e.g., unemployment), can become a bit more important through interaction effects: the higher marginal utility after the income loss could happen at the same time as a negative shock to the survival probabilities.

3.4 Robustness

While allowing for stochastic survival probabilities breaks the standard full annuitization result, it might seem that allowing for an even richer set of contracts could then restore the classic full annuitization result. We now consider a few such potential modifications.

3.4.1 Shorter Contracts

In the three-period model, the annuity contract purchased at age $j$ lasts until death or age $j + 2$, whichever occurred first. The annuity produced a mortality credit but at the cost of valuation risk. Suppose, however, that we replaced the two-period long annuity contract with a sequence of one-period contracts, the first one issued at age $j$ and the second issued at age $j + 1$. There is no valuation risk with a one-period contract (formally, $\pi_{j+1} = 0$ in equation (1)), and so the annuity return is simply equal to the bond yield plus any mortality credit, as in the original Yaari model. Annuities would again statewise dominate bonds. Indeed, as shown above, a shorter contract might be preferred in the presence of valuation

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11A funded actuarial-fair social security system is directly substitutable for personal savings into a fair annuity. An unfunded system provides the mortality credit in the form of a benefit that is larger than could otherwise be afforded by the payroll taxes that were otherwise saved into bonds after subtracting any tax needed to service the implicit debt in a dynamically efficient economy.
risk and high enough impatience.

However, in the presence of stochastic survival probabilities, a shorter contract length also magnifies the “reclassification risk” problem that has been explored recently in the life insurance literature.\footnote{See, for example, [REF’s]. In the case of annuities, the premiums are paid prior to the disbursement of benefits, the relationship can effectively be viewed as a two-sided commitment problem.} A competitive equilibrium will cause consumers to dynamically re-contact as they receive updates about their survivor probabilities so that most of the \textit{ex-ante} potential from pooling is essentially erased \textit{ex-post}.\footnote{Hence, in a subgame perfect game, the ex-ante potential is also erased.} We, however, leave a fuller welfare analysis of these tradeoffs to future work, especially in light of confounding market frictions. We are more focused on annuity demand.

It suffices to note for our purposes that even a shorter annuity contract might not dominate if agents also receive updates about their survival probabilities (and can die) at even a higher frequency. Indeed, one can interpret our three periods from before as representing an interval of length $\tau$ in total time, with each period representing time length $\frac{\tau}{3}$. Annuities might not dominate even as $\tau \to 0$, if information innovations also happen at higher frequency.\footnote{As an extreme example, consider a person named Fred who trades an annuity contract in “near continuous” time of length $\tau$. Now suppose that Fred is having an acute myocardial infarction (“heart attack”) that will lead to death. Fred’s health can naturally also be thought of as going through a near continuum of health states as well, from better to worse.}

### 3.4.2 A Richer Space of Mortality-Linked Contracts

Suppose now that households could also purchase additional mortality-linked contracts that make positive or negative payments based on changes in their individual health. Naturally, we won’t consider an entire set of Arrow-Debreu securities; more rigid contracts like annuities exist precisely because a full set of Arrow-Debreu securities are not available. (In other words, a security that has any resemblance to a traditional-looking annuity would be spanned by existing securities in a full Arrow-Debreu economy.) Instead, we ask, what is the minimum type of mortality-linked contract that, when combined with an annuity, would restore annuities to their statewise (or, even second-order) position of dominance?
Importantly, insurance for medical costs, such as long-term care, and related expenses is *not* fully sufficient to restore annuity dominance. Such payments would certainly help improve the choice of annuities by reducing some of the correlated costs. But, as we showed earlier, we can still get imperfect annuitization without additional medical costs.

Instead, to produce full annuitization, we would need an even more complete “health state” Arrow-like security that pays positive or negative amounts based on the health care state $h'$ at age $j + 1$ that covers a wider range of risks, including valuation risk and its potential interaction with income. A rich enough set of “health state” securities could essentially undo the long-dated nature of the annuity contract similar in spirit to a time sequence of interest rate swaps can undo the duration risk associated with a long-term bond. One crucial difference, however, is that the payoff to the Arrow “health state” securities must be based on idiosyncratic private information rather than observable transactions such as market prices (interest rates) or even observable individual health care costs. Moreover, these “health state” securities in a competitive setting would need to be quite rich in design since they must be functions of both initial health (to capture initial valuation) and age (to capture duration).

### 3.4.3 “Hybrid Annuities”

Thus far, we have defined a “life annuity” in the traditional sense as a contract that *pays a constant amount in each state contingent on survival*, as in the original Yaari model. Most of the literature about the “annuity puzzle” is relative to such a contract. It is straightforward, however, to construct a “hybrid annuity” with bond-like features – specifically, that includes some *non-contingent* payments – that will at least weakly dominate a simple bond. By subsuming both annuity and bond types of contracts, this hybrid annuity can never do worse than either a bond or standard annuity, by construction.

Consider, for example, the case “Low Patience” ($\beta = 1$) considered earlier. A “hybrid annuity” that paid 0.75 at ages $j + 1$ and $j + 2$, *not contingent* of actual survival, would allow the agent to consume 1.5 in both Good and Bad health states at age $j + 1$. The non-contingency of the payments allows even an agent in the Bad state to borrow at the

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15 Since we have no inflation in our model, we could also interpret our annuity payments as being indexed.
zero risk-free rate against the payment that will be made at age $j + 2$, even though he or she does not survive until then. (If payments were contingent on survival then the agent could never borrow in the Bad health state since the mortality-adjusted interest rate would be infinite.) The “hybrid annuity,” therefore, would perfectly smooth consumption like a bond by providing a non-contingent stream of payments. More generally, a “hybrid annuity” could reproduce any combination of bonds and traditional annuities when $0 < \beta < 1$.

Of course, our purpose is to shed insight into the optimal allocation into traditional annuities, which the literature (and our analysis) suggests should be in strong demand with deterministic survival probabilities. If our terminology were too flexible then almost everything could be an “annuity.” Still, a valid interpretation of our results herein is that the traditional annuity itself is simply not the optimal contract in the presence of survival shocks and $\beta < 1$.

4 Multi-Period Model

We now present simulation evidence from a multi-period lifecycle dynamic programming model, which also incorporate the impact of various observable market frictions, including uninsured medical costs, Social Security, correlated and uncorrelated risky income, and management fees. We then consider the role of bequests. We also spent a considerable amount of time trying to the properly incorporate the effects of one-sided transaction costs, but we found it to be quite challenging to solve numerically. Importantly, since there is no

\[16^*\] Next version: In fact, for the Example considered earlier, we can derive the first-best allocation as follows ... Note: this trick does not work when the probabilities of being in each state are unequal since we can’t exactly match the number of choice variables with constraints; we would either need more linear securities (zero-coupon bonds with specific maturities) or Arrow securities.

\[17^*\] For example, empirically, there are many people whose total assets do not exceed the cost of a year’s stay in a nursing home. Annual cost of nursing home is roughly $50,000 (e. g., McGarry and Schoeni 2005), about the same as the mean wealth of the 3rd decile in the Health and Retirement Study (HRS), excluding Social Security wealth which cannot be liquidated (Moore and Mitchell 2000). The median HRS family wealth, excluding Social Security, equals approximately the cost of four years in a nursing home.

\[18^*\] One-sided transaction costs tie actual transactions that changes the level of annuitization between periods. However, that requires the addition of another state variable so that we can track both bonds and
hidden health information in our model\textsuperscript{19} we allow for full rebalancing between annuities and bonds. The rebalancing option, therefore, removes a “liquidity constraint” that would otherwise artificially decrease the demand for annuities.

### 4.1 Individuals

The economy is populated by overlapping generations of individuals who live to a maximum of $J$ periods (years), with one-period survival probabilities $s(j, h_j)$ at age $j$ dependent on health status $h_j$, which in turn is also uncertain. The state variable $h$ takes values and follows an $M$-state Markov process with an age-dependent transition matrix $P_{mn}(j)$; $m, n = 1, \ldots, M$. For our purposes, which focus on explaining annuity demand, $M = 3$ suffices: healthy ($h_1$), disabled ($h_2$), and very sick ($h_3$)\textsuperscript{20}. Most workers are healthy and able to work. A disabled worker is unable to work but receives some disability benefits until retirement. A very sick person is unable to work and receives disability plus some additional transfers.

Individuals can save into a life annuity and a non-contingent bond. In each period, each unit of annuity pays $1 contingent on survival. Hence, the net return $\rho$ is equal to $1$ plus the value of next-year annuity premium, as shown previously in equation (1), which allows for full rebalancing, although at a cost of valuation risk. Bonds pay a return equal to $r$.

Earning capacity varies according to predictable component $\varepsilon_j$ that is equal to the average productivity of a worker of age $j$. Earnings also changes a random term that captures the individual’s idiosyncratic productivity shocks, modeled as a Markov process with state variable $\eta$ and a transition matrix $Q_{kl}(j)$; $k, l = 1, \ldots, R$, where $R$ is the retirement age.

\textsuperscript{19}As a practical matter, it is not obvious that insurers who engage in medical underwriting have substantially less information about health than participants. The evidence of adverse selection in annuities is mixed [REF’s].

\textsuperscript{20}More health states might be required in different contexts, such as pricing long-term care insurance or evaluating its money’s worth, as in Brown and Finkelstein (2003).
A disabled ($h_2$) and very sick worker ($h_3$) can’t work. An individual’s wage, therefore, is a product of four factors: age-related productivity $\varepsilon_j$; individual productivity $\eta$; an indicator of the health status; and, the general-equilibrium market wage rate per unit of labor. A disabled worker, however, receives disability before retirement, and a very sick worker receives an even larger payment, discussed below. Very sick people also face a loss $L$ in the form of long-term care expenses.

We model Social Security income as a pay-as-you-go transfer from workers to retirees in each period, so that it is effectively a mandatory life annuity. The Social Security tax rate is determined endogenously under the balanced budget constraint from the equilibrium distribution of individuals and a targeted average income replacement rate.

Individuals have preferences for consumption and possibly for leaving bequests, which time-separable, with a constant relative risk aversion (CRRA) felicity. To avoid problems with tractability and uniqueness that arise in models with altruism, bequest motives are modeled as “joy of giving”:

$$U = \sum_{j=1}^{J} \beta^j u(c_j) = \sum_{j=1}^{J} \left[ \frac{c_j^{1-\sigma}}{1-\sigma} + \xi D_j A_{j+1} \right]$$

where $\beta$ is the rate of time preference, $c_j$ is consumption at age $j$, $\sigma$ is the risk aversion, $A_j$ is bequeath-able wealth at age $j$, $D_j$ is an indicator that equals 1 in the year of death and 0 otherwise, and $\xi$ is a parameter that determines the strength of the bequest motive.

Households also face insured medical cost shocks, which we model as long-term care expenses. Since they have limited liability protection, they can’t face negative consumption after a shock. To avoid infinite marginal utility associated with zero consumption under CRRA felicity, the government, therefore, also provides a small transfer to those individuals whose long-term care costs have exhausted their total available assets. As with Social Security, this transfer is financed through a balanced-budget tax.

An individual’s optimization problem, therefore, is fully described by four state variables: age $j$, health $h$, idiosyncratic productivity $\eta$, and wealth (assets) $A$. He or she solves the following problem taking the prices $w, r, \rho$ as given:
$$V_t(A, \eta, h, j) = \max_{a, b, c} \left\{ u(c) + \beta s(j, h) \int_{h'} \int_{\eta'} V_{t+1}[A'(a, b; h'), \eta', h', j + 1]Q(\eta, d\eta')P_j(h, dh') \right\}$$

subject to:

$$A' = a(1 + \rho) + b(1 + r) + \varepsilon_j \eta \mathbb{I}(h = h_1)w(1 - T) - c + B + Tr - L$$

$$b \geq 0$$

$$0 < c \leq A + \varepsilon_j \eta \mathbb{I}(h = h_1)w(1 - T) + B + Tr - L$$

where $B$ is the amount received from the distribution of unintended bequests, $L$ is the financial loss in the sick state, $Tr$ is the sum of all government transfers received (Social Security, disability, welfare), and $T$ is the total tax rate required to finance those transfers. The indicator function $\mathbb{I}(h = h_1) = 1$ if person is healthy and able to work; otherwise, $\mathbb{I}(h \neq h_1) = 0$, if person is disabled or very sick. Here, $A \in \mathbb{R}_+$, $\eta \in D = \{\eta_1, \eta_2, \ldots, \eta_n\}$, $h \in H = \{h_1, h_2, h_3\}$, $j \in J = \{1, 2, \ldots, J\}$, and the functions $\{V_t, a_t, b_t, c_t : S \rightarrow \mathbb{R}_+\}_{t=1}^{\infty}$ are measurable with respect to $F$, where $S = \mathbb{R}_+ \times D \times H \times J$, $F = B(\mathbb{R}_+) \times \mathbb{P}(D) \times \mathbb{P}(H) \times \mathbb{P}(J)$, and $\mathbb{P}(\cdot)$ denote power sets and $B(\mathbb{R}_+)$ is the Borel $\sigma$-algebra of $\mathbb{R}_+$.

The budget constraints have the following interpretations. Bonds $b$ must be non-negative, thereby recognizing that a competitive market would never allow an individual, who might die before the loan repayment, to borrow at the risk-free rate without also carrying life insurance in the amount of the loan. (However, annuities $a$ can be negative, which is equivalent to borrowing at the risk-free rate and purchasing life insurance to ensure repayment.) Moreover, an individual’s consumption $c$ must always remain non-negative. Without health shocks, this constraint would never bind under the standard Inada utility conditions. However, with medical expense shocks, we must explicitly enforce the constraint by calculating the welfare payment accordingly.

### 4.2 Production and General Equilibrium

The production side of our economy is less central in our focus. Nonetheless, we want to ensure that our factor prices are derived from a reasonable technology specification and the
government’s policy parameters add up. Total aggregate bequests made should also equal
bequests received. More importantly, since we run several different simulations, having a
firm production side of the economy allows us to recalibrate to the same observable economy
so that we can better isolate and interpret any effect from model changes. As an example,
we also consider a change to Social Security that will also impact factor prices in a closed
economy.

Accordingly, output $Y_t$ of the economy in period $t$ is determined by the constant returns-
to-scale technology with a Cobb-Douglas production function $Y = \theta_t K^\alpha L^{1-\alpha}$, where $K$
denotes capital and $L$ labor, in efficiency units. The economy, as a collection of individuals,
is then described by the measure $\Phi(j, \chi, \eta, A)$ of individuals by state, and by the values of
market wage $w$, interest rate $r$, capital stock $K$, and labor supply $L$.

A general equilibrium, therefore, is fairly standard and so a formal definition will be
skipped. In particular, (A) households optimize, taking as given the set of factor prices and
policy parameters; (B) the factor prices are derived from the production technology with the
level of household saving and labor supply; (C) the policy parameters are consistent with
balanced budget constraints; and, (D) bequests given (if any) equal bequests received. The
entire recursive household partial equilibrium program (9), therefore, is solved many times,
inside of a Gauss-Siedel like iteration, until general equilibrium is reached, defined as having
small Euler equation errors away from any boundaries, following Judd (XX).

4.3 Model Calibration

Mortality data for the population as a whole are based on the U.S. Social Security Adminis-
tration tables (Faber and Wade 1983; Bell et al 1992). Since the “sick state” corresponds
most closely with long-term disability during working years and with a need for long-term
care for retirees, the sources of mortality data for “sick” individuals are the RP-2000 dis-
babled life mortality tables (Society of Actuaries 2000) for ages 21 - 65 and nursing home
discharge and mortality data from the 1998 Green Book and the National Nursing Home
Survey (NNHS) for ages over 65. The latter data are in coarse age groups, so the rates by age

\footnote{Next version: update this source and a couple others.}
by year were obtained using the relative rates from population mortality and from mortality in continuing-care retirement communities (Barney 1998). Survival probabilities by age and health state are shown in Figure 4.

As noted earlier, three health states are generally enough to capture the information available from the existing data. The intermediate disability state is useful to represent impaired health with significantly higher mortality, but without actually being institutionalized and suffering the associated expense. For working years, we use the intermediate state to represent the first year of disability so that we can vary recovery rates with the duration of disability. Transition probabilities between health states are shown in Figure 5. Probabilities for the working-age population are based on long-term disability morbidity tables (Society of Actuaries 1982) and Social Security data regarding the number of persons receiving benefits. For older ages we used nursing home population and admission data from the Census Bureau, National Nursing Home Survey, Medicare, and the 1998 Green Book; we refined these transition probabilities based on the actuarial model of (Robinson 1996).

The expenditure connected with poor health for retired population is based on average nursing home costs as reported in the Green Book and NNHS. The information of long-term costs is necessarily incomplete: on one hand, some expenditures are maybe higher than necessary and in fact represent voluntary (and, hence, elastic) consumption; on the other hand, expenditures from loss of income of a family member caring for a non-institutionalized person requiring long-term care are not captured.

Individual productivity states and transition probabilities are taken from Nishiyama and Smetters (XXX), with the original eight states combined into two for most computations. Macroeconomic variables are also calibrated consistent with Nishiyama and Smetters (XXX): the capital share of output is $\alpha = 0.32$, the depreciation rate of physical capital is $\delta = 0.046$, and the capital-to-output ratio is $2.8^{22}$.

Individuals are followed from the beginning of their working lives and start out healthy. This obviously does not capture congenital or early-onset disabilities, but those complications would deprive the individual of earning power before he or she could make any decisions on saving anyway. Hence, such cases are thus irrelevant for the present model. In principle,

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22[Let’s update to include all 8 states in next version.]
disabled children would, however, have an effect on their parents, so it would perhaps be ideal to increase effective disability rates during working years. Nevertheless, as the most affected ages precede significantly the ages typical for major annuity purchases, there is little harm in ignoring such cases at this stage.

The rate of population growth is assumed to be a constant 1 percent. This roughly approximates population growth in the United States, but the particular value is not critical. The rate of productivity increase in the production function is also not critical and, for the sake of simplicity, productivity is assumed to be unchanging. The capital-output ratio is set at 2.8 by varying the subjective discount rate \( \beta \), producing a marginal product of capital of 6.8%.

Social Security benefits are modeled with the primary goal of matching the average level. A worker who earns the average wage (according to SSA) throughout his or her career and retires at the normal retirement age, will get a replacement ratio of approximately 45 percent.\(^{23}\) Our Social Security formula is redistributive and incorporates the “bend point” calculations contained in the law. Hence, the worker who remains in the high productivity state throughout his or her career will have higher benefits. Ideally, we would track each person’s average wages throughout their lifetime, but this would require an additional state variable and would be computationally very costly. Instead, consistent with some other papers, we use the conditional expectation of benefits by final productivity state: an individual who reaches the last working year is assigned the expected benefit amount conditional on this last working year state, which is adjusted to match the average replacement rate.\(^{24}\)

## 5 Simulation Results

We now report simulation evidence, starting with a “Basic Model” without any bequest motives and management fees. We then proceed to more elaborate settings as part of sensitivity analysis.

\(^{23}\)Author’s calculation based on benefit information published on the SSA web site, http://www.ssa.gov.

\(^{24}\)[Should we relax in next version?]
5.1 Basic Model

In our Basic Model, Social Security exists and uninsured long-term care costs are equal 1.5 times average annual earnings. Asset management fees and bequest motives are absent.

5.1.1 Annuitization at Retirement

Figure 7 shows the optimal amount of assets annuitized at the retirement age 65 by a healthy person \((h = 1)\) at different levels of wealth achieved by that age. Wealth is reported as a fraction of average wage in the economy. Notice that the level of annuitization varies significantly by level of risk aversion \(\sigma\) and assets. For households with \(\sigma = 2\), a relatively low level of risk aversion, annuities are not purchased at levels of wealth below 7 times average annual earnings. For households with a higher level of risk aversion of \(\sigma = 5\), annuities are not purchased at levels of wealth below 10 times the average annual earnings. In either case, a majority of 65-year olds in the U.S. population have assets in the range of one to five times annual earnings (Moore and Mitchell 2000). Hence, annuities are not optimal for most Americans reaching retirement, even in our Basic Model.

As noted earlier, these results are not driven by “liquidity constraints” that prevent rebalancing between annuities and bonds. Specifically, households facing a negative shock can always sell their annuities at fair prices and purchase bonds. However, the fair value of the annuity decreases after a negative shock to survival, which can occur exactly when they might need to rebalance for the purpose of paying for health expenses.

Figure 7 also shows that annuitization, however, becomes more optimal at larger values of wealth for a retiree. After a negative survival shock, a wealthy retiree can pay for any potentially correlated expenses from the annuity stream itself. Unlike workers, the retiree also does not have to worry about any reduction in earnings from becoming disabled or very sick. Hence, a wealth retiree can have a “hold to maturity” attitude for the annuity, in much

\[\text{Notice that some annuitization occurs at very low levels of wealth, which is an artifact of our welfare system that guarantees a minimum amount of consumption. Agents at this level of wealth do not have to worry as much about any additional loss from annuitization, and so they fairly attracted to the mortality credit. This kink, however, diminishes are higher levels of risk aversion because agents still value the opportunity to build safe wealth with a bond.}\]
the same way that a long-term bond holder is less concerned with the valuation risk along the duration.

### 5.1.2 Annuitization across the Lifecycle

Figure 8 then shows the fraction of *all wealth* that is annuitized across a range of ages. The total amount of wealth and annuities at any given age is calculated across the measure of households at each level of wealth that is alive at that age. Notice that the amount of annuitization is *smaller* at larger values of risk aversion $\sigma$. On one hand, more risk averse agents gain consumer surplus from the mortality protection provided by the annuity, as in the standard model with deterministic survival probabilities. On the other hand, annuitization now also contains valuation risk.

Also notice that we need to distinguish between the number of households who purchase an annuity and the amount of *annuitized wealth*. For example, the annuitized fraction wealth at age 65 varies from around 20% (for $\sigma = 5$) to almost 70% (for $\sigma = 2$). Whereas, as we previously showed, most households in our model should not purchase annuity at retirement, a potentially large fraction of wealth at age 65 should still be annuitized. The difference is due to skewness of wealth, both in the real world and contained our model. Wealthier households should annuitize and they hold most of the wealth. Of course, recall that our Basic Model contains no bequest motivations, which are often concentrated among the wealthy as well. Instead, any bequests made are purely accidental and distributed evenly across all households.\(^{26}\)

Figure 8, however, also shows that the optimal level of annuitization is actually U-shaped in age. Specifically, it is more optimal for both the very young and the very old to annuitize relative to the middle-age households. We can understand this interesting by referencing to Figure 6 which presents the calibrated *realized* annuity return $\rho$ as a function of age the health state transitions. The realized return includes two components: (I) for a given health care state, the standard mortality credit found with standard deterministic probability models, which is monotonically increasing in size over the lifecycle as the probability of death increases; and (II) the additional mortality credit due to the specific health state transitions.

\(^{26}\)In next version, let’s report the distribution of wealth and also consider a more regressive distribution of bequests.]
increases, and (II) the valuation risk associated with changes in the health care state. The “humpback whale” appearance of the annuity returns reflects these two competing effects. Notice that movements from healthy state $h_1$ to the disabled $h_2$ or very sick $h_3$ states leads to a large depreciation in the value of the annuity, whereas movement in the opposite direction can actually lead to value appreciation.

Consider a younger person under the age of 30. The potential realized returns to annuitization is small but positive. Young people have a small chance of mortality at the current health state and a small chance of transition to a different health state. The small chance of mortality implies a small mortality credit from annuitization while the small chance of transition to another health state also produces low valuation risk. The net effect is a small but positive realized return across all the health states when young. However, annuitization does not go to 100% because younger workers must also worry about the correlation of their investment returns with their human capital returns (wages) if they become disabled or very sick.

Now consider an older person above age 90. Annuitization is also relative high for them, but the reasons are exactly the opposite as for young people. Now, the potential for depreciation in the value of the annuity is quite high since there is a lot more movement between health states. However, the mortality credit is also large because the chance of death, from a given health state, is even larger in magnitude. The net effect also produces a potentially strong realized return from annuitization. Unlike younger people, however, older people do not have to worry about the correlation of investment returns with their human capital, which is zero.

5.2 Sensitivity Analysis

We now consider several changes to the Basic Model, including the importance of long-term care costs, management fees and bequests. For each case, we consider an incremental change to the Basic Model.
5.2.1 Long-Term Care Costs

Table 1 investigates the importance of reducing uninsured long-term care costs. It shows the fraction of total wealth that is annuitized as well as the fraction of retiree-only wealth annuitized. Recall, that in our Base Model, these costs were set equal to 1.5 times of average wage income.

Not surprisingly, the total amount of wealth that is annuitized increases as uninsured long-term care costs are reduced. However, notice that imperfect annuitization can still occur in the economy even with zero long-term care costs due to correlated human capital risk. Only retirees, who no longer have any more human capital, fully annuitize their wealth if long-term costs vanish.

But, once again, notice that a level of risk aversion reduces the level of annuitization, due to the presence of valuation risk. This result is exactly opposite as predicted by a model with deterministic survival probabilities. With deterministic survival, a greater level of risk aversion increases the consumer surplus and so very risk averse agents are more likely to buy the annuity in the presence of policy loading.

5.2.2 Management Fees

Management fees for a typical annuity range from 0.80% to 1.2% of assets, not including any additional “surrender fees.” Management fees effectively reduce the mortality credit received from annuitization. Recall that in a model with deterministic survival probabilities, a management fee has no impact on annuitization for those households that still receive a net positive mortality credit; for the other households, the level of annuitization drops to zero.

With stochastic survival probabilities, however, management fees have a smoother impact on the level of annuitization. Table 2 shows the impact of management fees on annuitization at a conservatively low level of risk aversion equal to \( \sigma = 2 \). Notice that total assets

---

27[Right now, uncorrelated human capital risks play very little role since our wage distribution is bounded above and they are patient. Allow for a greater wage range in the next draft. If it includes any zeros then full annuitization could not happen even with no correlated human capital shocks.]

28[References.]
annuitized drops to 26% at a management fee of 1.10%. A full 54% of retiree wealth is still annuitized, but only 8% of non-retiree wealth is annuitized.\textsuperscript{29} Intuitively, younger people have a smaller pre-expense mortality credit, and so a proportional loading will have a larger impact of their annuity demand.

5.2.3 Bequest Motives

Table 3 shows the impact of adding bequests to the model, again with a low level of risk aversion. Not surprisingly, the amount of wealth annuitized falls sharply with increases in the bequest motive.\textsuperscript{30} Unlike the pure lifecylce model with deterministic survival probabilities, however, the inclusion of a bequest motive more isolates the annuitization decisions of wealthy households.\textsuperscript{31}

5.3 Implications for Social Security

Our analysis provides three important implications for Social Security reform, which we now discuss.

With \textit{deterministic} survival probabilities, Social Security provides annuity protection by insuring against longevity risk. Unless households are driven to a corner in their level of savings (a true borrowing constraint), the annuity provided by Social Security at least weakly dominates bonds, of holding the total level of implicit and explicit debt constant.

If private annuities are available (on a basically fair basis to match Social Security’s low administrative costs) then $1 in Social Security wealth should crowd out an equal amount of private annuities. If private annuities are not generally available then Social Security wealth also provides some additional income effects and reduces some precautionary saving. In either case, personal savings accounts that augment Social Security should be highly

\textsuperscript{29}[Next version: to really demonstrate the smoother role of management fees in the presence of stochastic mortality, we need to report on both the intensive and extensive margins.]

\textsuperscript{30}[Next version: let’s seriously calibrate this experiment and compare the literature on the size of the capital stock that is intended for bequests. For example, the implied B/K ratio is equal to (B/Y)/(K/Y) = 0.037/2.8, which is too small.] [Next version: let’s also consider a “kitchen sink” sensitivity analysis with everything thrown in.

\textsuperscript{31}[Next version: finish this up and show the level of annuitization by wealth, as in Figure 7.]
annuitized. A reform to Social Security that, for example, includes “add-on” accounts would then optimally need to be considered about how that wealth would be optimally annuitized upon retirement.

With stochastic survival probabilities, however, the annuity provided by Social Security is no longer substitutable with even fairly priced private annuities. In essence, Social Security is a restricted annuity because it can’t be “rebalanced” since it is not legal to borrow against future benefits. While there is no valuation risk with Social Security, it comes at the cost of the loss of the rebalance option. (In other words, valuation risk is the cost of the option to rebalance.) Our simulation results demonstrate that, with the current Social Security program in place, most American households should not annuitize any additional assets upon retirement. Moreover, the greater the concern about risk, the lower the annuitization. While wealthy households are in a position to annuitize, they are also more likely to have a bequest motive. Our results, therefore, show very little evidence for policymakers to be concerned about making fair annuities available as part of reform that includes “add-on” accounts.

We also wanted to investigate a reform that replaced at least part of Social Security with a carved-out account. For the sake of argument (rather than necessarily political realism), we considered a full privatization of Social Security where Social Security is fully removed. The result are reported in Tables 4 and 5, which focuses only on steady states. General-equilibrium effects on the capital stock, factor prices and taxes are fully incorporated into the simulation. Table 4 shows that privatization significantly increases the capital output ratio from 2.8 to 3.6 or even larger, depending on the level of risk aversion. Table 5 then shows the share of that savings that is saved into annuities versus bonds. As before, it varies with the level of risk aversion. However, notice that very risk averse households still have a strong preference for bonds even with the full remove of Social Security, the exact opposite of traditional thinking on this issue. Moreover, most of the saving in annuities still comes from the rich, who are modeled as being without a bequest motive. Although not reported, the median household still has a very strong preference for bonds even at a risk aversion level of 3.32 These results suggest that providing annuity options, even for a large reform,

32[Next draft: show.]
is likely not a major concern if the objective is to help households achieve the optimal level of risk.

Finally, to help households achieve their optimal level of savings and investments, public policy programs that provide insurance against loss income or increased expenses after a health shock could have a material impact on the optimal design of either add-on or carve-out accounts. Our results earlier demonstrated that increases in medical expenses reimbursements, in particular, would likely lead to a need to an increased demand for annuitization. But, just the opposite is also true: a reduction in programs like Medicare and Medicaid over time could actually lead to an even lower demand for annuities to augment Social Security while an increased in the provision of health care would result in an increase.

6 Conclusion

Stochastic mortality introduces valuation risk, much like a long-term bond. If households are sufficiently impatient, a long-term traditional annuity no longer dominates, even with no other sources of uncertainty or market frictions. The optimal level of annuitization falls below 100%. If, however, households are sufficiently patient then annuities again dominate if there are no additional frictions. In that case, stochastic mortality probabilities provides a mechanism by which these additional frictions can smoothly reduce annuity demand, something does not exist with deterministic probabilities. We derived several implications for Social Security. Most notably, it is optimal for most households to not annuitize any wealth upon retirement, even if they face no liquidity constraints. Moreover, the more risk averse the household, the more optimal it is for them to invest in bonds. Concerns about providing annuity options as part of Social Security reform are likely overstated.
Appendix

A Discretization of State Space

Total wealth at age \( j \), \( A_j \), is represented as one of 101 points of the wealth grid, \( A_{jk} \), \( k = 0, 1, \ldots, 100 \). We fix point \( A_{j0} = 0 \), \( A_{j100} \) equals the assumed maximum wealth, and the value of \( A_{jk} \) increases with \( k \). Ideally, for best interpolation during optimization and evaluation, the spacing between adjacent grid points should be narrower where the value function’s curvature is the most pronounced. We chose equidistant points at the low end of the wealth distribution, and geometrically increasing values for intermediate to high wealth.

As most people’s wealth increases during the early part of life, the maximum wealth \( A_{j100} \) does not have to be the same for all ages; we also allow the grid to be expanded during the computation if the maximum wealth is actually reached by a positive measure of agents.

When the optimal policy (consumption, bond saving and annuity saving) is computed for an agent at the node \( (j, h, i, k) \), where the indices represent age, health, productivity, and wealth, respectively, the wealth \( A' \) next period (age \( j+1 \)) is allowed to take any positive value, rather than be limited to the values of the grid points. The value function \( V(j+1, h', i', A') \) corresponding to that wealth is determined by interpolation between the two grid points bracketing it, for the given final health and productivity state \((h', i')\) and age \( j+1 \).

The simplest way to interpolate \( V \) is by piecewise linear base functions: \( V \) is assumed to be linear in \( A \) between \( A_k \) and \( A_{k+1} \) such that \( A_k < A < A_{k+1} \). The disadvantage of this approach is that it alters the shape (curvature) of the value function. In optimization problems, concavity is often very important and it is desirable to keep concave functions concave not just globally but also locally, on any scale. We thus use interpolation by Schumaker’s shape-preserving quadratic splines (Judd 1999, pp. 231–234). They preserve local concavity/convexity of \( V \) as a function of wealth, as well as its smoothness, avoiding artificial kinks at grid points.

With respect to the curvature of the value function, it is important to keep in mind that this is not a standard, well-behaved concave problem. Limited liability introduces nonconcavity in wealth, and there are further complications due to the discrete nature of
health. Risk involved in annuity returns arises from the difference in the next-period present value of the annuity in healthy and sick states, as well as the transition probabilities for the respective states, so this is the main determinant of the optimal portfolio weights. The objective function can, and frequently does, have multiple local optima. For this reason, it is necessary to perform optimization at each node starting from several different initial guesses chosen by randomized search techniques.

In most cases we find insignificant difference between results of runs using linear and spline interpolation. Greater difference is normally a sign that the grid spacing is too wide, and it is then generally best to rerun the computation using a finer grid.

When the measure of agents is computed, it is also represented as having a discrete distribution, with probability masses at the nodes of the grid. For this purpose, a value from the continuum must be apportioned to the nearest two grid points. This process must preserve expected utility as well as the total measure, so the weights given to the two points will be inversely proportional to the distance to them (i.e., they will be based on a linear interpolation scheme).

The number of nodes in the full dynamic-programming tree is \( J \times m \times n \times (k^{\text{max}} + 1) \), where \( J \) is the maximum age (actually the age span between the minimum and the maximum), \( m \) is the number of health states, \( n \) the number of productivity states, and \( k^{\text{max}} \) the highest index of the wealth grid. We use ages from 21 to 120, so \( J = 100 \); as defined above, \( k^{\text{max}} = 100 \), and as discussed in the paper, \( m = 3 \) and \( n = 8 \) (or less in some versions). Therefore, \( J \times m \times n \times (k^{\text{max}} + 1) = 100 \times 3 \times 8 \times 101 = 242,400 \). (The smallest version, with \( n = 2 \), has 60,600 nodes.)


[33]Need to continue to update]


on the money’s worth of individual annuities. American Economic Review 89 (December), 1299–1318.


Tables and Figures

Figure 1: Optimal annuitization without health shocks and without transaction costs
Figure 2: Optimal annuitization without health shocks and with transaction costs
Figure 3: Optimal annuitization with health shocks
Figure 4: Survival Probabilities

Source: Authors’ calculations based on references cited in Section 3.
Figure 5: Health Transition Probabilities

Source: Authors’ calculations based on references cited in Section 3.
Note: The three panels show the annuity return by age for different health state transitions. The top panel shows the annuity returns for a healthy person, the middle panel shows the annuity returns for an impaired person, and the bottom panel shows the annuity returns for a nursing home resident.
Note: Optimal fraction of wealth held in life annuity form by a healthy person age 65 with different coefficients of risk aversion, plotted as a function of beginning of period wealth. The unit of wealth is the average annual earnings. Asset management fees and bequest motives are absent, but Social Security exists and long-term care costs equal 1.5 times average annual earnings. The capital-output ratio is set to 2.8 by varying the subjective discount rate.
Figure 8: Annuitized Fraction of Wealth by Age Group

Note: Optimal fraction of wealth held in life annuity form with different coefficients of risk aversion, plotted as a function of age. Asset management fees and bequest motives are absent, but Social Security exists and long-term care costs equal 1.5 times average annual earnings. The capital-output ratio is set to 2.8 by varying the subjective discount rate.
Table 1: Sensitivity Analysis: Reducing Long-Term Care costs

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annuitized fraction of total wealth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With health shocks, LTC costs = 0.0</td>
<td>100%</td>
<td>100%</td>
<td>89%</td>
<td>75%</td>
<td>64%</td>
</tr>
<tr>
<td>With health shocks, LTC costs = 0.5</td>
<td>98%</td>
<td>92%</td>
<td>82%</td>
<td>72%</td>
<td>47%</td>
</tr>
<tr>
<td>With health shocks, LTC costs = 1.5</td>
<td>73%</td>
<td>48%</td>
<td>28%</td>
<td>18%</td>
<td>13%</td>
</tr>
<tr>
<td>Annuitized fraction of retiree wealth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With health shocks, LTC costs = 0.0</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>With health shocks, LTC costs = 0.5</td>
<td>96%</td>
<td>92%</td>
<td>84%</td>
<td>81%</td>
<td>69%</td>
</tr>
<tr>
<td>With health shocks, LTC costs = 1.5</td>
<td>58%</td>
<td>36%</td>
<td>26%</td>
<td>22%</td>
<td>18%</td>
</tr>
</tbody>
</table>

Note: Entries show annuitized fraction of aggregate and retired wealth, for various levels of risk aversion and long-term care (LTC) costs, holding the capital-output ratio constant at 2.8 by varying the subjective discount rate. Annuity premiums are actuarially fair, individuals are completely selfish (i.e. have no bequest motives). The existence of health shocks and the associated costs are the only deviation from Yaari’s assumptions.
Table 2: Sensitivity Analysis: Including Asset Management Fees

<table>
<thead>
<tr>
<th>Management fees</th>
<th>Annuited fraction of aggregate wealth</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Retirees only</td>
<td>Non-retirees only</td>
</tr>
<tr>
<td>0.00%</td>
<td>82%</td>
<td>70%</td>
<td>89%</td>
</tr>
<tr>
<td>0.10%</td>
<td>80%</td>
<td>69%</td>
<td>88%</td>
</tr>
<tr>
<td>0.30%</td>
<td>75%</td>
<td>67%</td>
<td>80%</td>
</tr>
<tr>
<td>0.50%</td>
<td>57%</td>
<td>65%</td>
<td>51%</td>
</tr>
<tr>
<td>0.70%</td>
<td>40%</td>
<td>61%</td>
<td>25%</td>
</tr>
<tr>
<td>0.90%</td>
<td>31%</td>
<td>58%</td>
<td>13%</td>
</tr>
<tr>
<td>1.10%</td>
<td>26%</td>
<td>54%</td>
<td>8%</td>
</tr>
</tbody>
</table>

Note: Entries in the three left columns show annuitized fraction of aggregate wealth, for various levels of management fees, holding the capital-output ratio constant at 2.8 by varying the subjective discount rate. Annuity premiums are actuarially fair, individuals have access to social security, and long-term care costs are equal to 1.5 times average annual income. The coefficient of relative risk aversion is assumed to be 2. The management fees are calculated annually as a percent of total assets.
Table 3: Sensitivity Analysis: Including Bequest Motives

<table>
<thead>
<tr>
<th>Altruism parameter</th>
<th>Annuitized fraction of aggregate wealth</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>Retirees only</td>
<td>Non-retirees only</td>
</tr>
<tr>
<td>0.00</td>
<td></td>
<td>82%</td>
<td>70%</td>
<td>89%</td>
</tr>
<tr>
<td>0.05</td>
<td></td>
<td>80%</td>
<td>68%</td>
<td>88%</td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td>76%</td>
<td>63%</td>
<td>85%</td>
</tr>
<tr>
<td>0.15</td>
<td></td>
<td>69%</td>
<td>57%</td>
<td>77%</td>
</tr>
<tr>
<td>0.20</td>
<td></td>
<td>56%</td>
<td>47%</td>
<td>63%</td>
</tr>
<tr>
<td>0.25</td>
<td></td>
<td>46%</td>
<td>38%</td>
<td>52%</td>
</tr>
<tr>
<td>0.30</td>
<td></td>
<td>38%</td>
<td>29%</td>
<td>44%</td>
</tr>
</tbody>
</table>

Note: Entries in the middle three columns show annuitized fraction of aggregate wealth, for various levels of altruism, holding the capital-output ratio constant at 2.8 by varying the subjective discount rate. Annuity premiums are actuarially fair, individuals have access to social security, and long-term care costs are equal to 1.5 times average annual income. The coefficient of relative risk aversion is assumed to be 2. The Bequest/output ratio is given to illustrate the overall effect of the given value of the altruism parameter on bequests.
Table 4: Social Security: It’s Role in Annuitzation

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>With OASI</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total annuity holdings</td>
<td>2.3</td>
<td>1.8</td>
<td>1.3</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Annuity holdings of retirees</td>
<td>0.8</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Annuity holdings of non-retirees</td>
<td>1.5</td>
<td>1.3</td>
<td>1.0</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>Total bond holdings</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>1.9</td>
<td>2.3</td>
</tr>
<tr>
<td>Bond holdings of retirees</td>
<td>0.3</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Bond holdings of non-retirees</td>
<td>0.2</td>
<td>0.5</td>
<td>0.8</td>
<td>1.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Capital/output ratio</td>
<td>2.8</td>
<td>2.8</td>
<td>2.8</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td><strong>Without OASI</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total annuity holdings</td>
<td>2.7</td>
<td>2.2</td>
<td>1.7</td>
<td>1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Annuity holdings of retirees</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Annuity holdings of non-retirees</td>
<td>1.8</td>
<td>1.5</td>
<td>1.1</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>Total bond holdings</td>
<td>0.9</td>
<td>1.9</td>
<td>2.8</td>
<td>3.7</td>
<td>3.6</td>
</tr>
<tr>
<td>Bond holdings of retirees</td>
<td>0.6</td>
<td>0.9</td>
<td>1.0</td>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Bond holdings of non-retirees</td>
<td>0.3</td>
<td>1.0</td>
<td>1.8</td>
<td>2.6</td>
<td>2.5</td>
</tr>
<tr>
<td>Capital/output ratio</td>
<td>3.6</td>
<td>4.0</td>
<td>4.5</td>
<td>4.9</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Note: The panels show aggregate, retiree, and non-retiree annuity and bond holdings relative to output, for various levels of risk aversion, assuming long-term care (LTC) costs equal to 1.5 times average annual income. Annuity premiums are actuarially fair, individuals are completely selfish (i.e. have no bequest motives). The top panel assumes that social security is available and holds the capital-output ratio constant at 2.8 by varying the subjective discount rate. The bottom panel shows the general equilibrium response to terminating the social security program (in steady-state).
Table 5: Social Security: Optimal Annuitzation after Reform

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase in assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(in % of total increase)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total annuity holdings</td>
<td>64%</td>
<td>44%</td>
<td>30%</td>
<td>19%</td>
<td>18%</td>
</tr>
<tr>
<td>Annuity holdings of retirees</td>
<td>18%</td>
<td>13%</td>
<td>12%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Annuity holdings of non-retirees</td>
<td>46%</td>
<td>31%</td>
<td>18%</td>
<td>9%</td>
<td>8%</td>
</tr>
<tr>
<td>Total bond holdings</td>
<td>36%</td>
<td>56%</td>
<td>70%</td>
<td>81%</td>
<td>82%</td>
</tr>
<tr>
<td>Bond holdings of retirees</td>
<td>23%</td>
<td>22%</td>
<td>20%</td>
<td>19%</td>
<td>22%</td>
</tr>
<tr>
<td>Bond holdings of non-retirees</td>
<td>13%</td>
<td>34%</td>
<td>50%</td>
<td>62%</td>
<td>60%</td>
</tr>
</tbody>
</table>

Note: The top panel shows the steady-state general equilibrium increase in aggregate, retiree, and non-retiree annuity and bond holdings in response to terminating the social security program, for various levels of risk aversion, assuming long-term care (LTC) costs equal to 1.5 times average annual income. Annuity premiums are actuarially fair, individuals are completely selfish (i.e. have no bequest motives). The bottom panel shows the annuitized fraction of the additional wealth when moving from an economy with social security to one without social security.