Propagation and Risk Spreading in Alternative Social Security Systems

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Introduction

A main function of old-age pension systems is to provide resources to elderly retirees, of course. But these systems can satisfy many other important government functions as well. Indeed, in circumstances where access to capital markets is good and many individuals can, alone or in conjunction with private employers, save for retirement, broad-based public pension systems may not be needed simply to provide retirement income and their other functions may take on greater prominence. One such function is the allocation and spreading of economic and demographic shocks among different generations. Even with well-developed capital markets, there is no private market mechanism for trading between current and future generations, so government policy stands as the only option. While many government policies, including national debt management, infrastructure investment, and expenditures on public education, have important intergenerational consequences, the size and variety of public pension schemes makes them a natural place to focus when considering intergenerational policy.

Like private defined-contribution pension arrangements, funded defined-contribution public pension schemes result in one particular allocation of economic and demographic shocks among generations. For example, a demographic shock that leads to one age cohort being large relative to others will lead that cohort to experience relatively low lifetime wages (because of its high labor supply) and relatively low rates of return on its retirement saving (because of its high demand for retirement assets). But public schemes may deviate from the defined-contribution approach with respect to two criteria: asset accumulation and determination of contributions and benefits. With respect to the first criterion, systems may adhere to some form of strict pay-as-you-go (PAYG) approach, or to a more flexible approach that allows a fluctuation in the system’s financial assets or liabilities within some stable range. With respect to the second
criterion, systems may adjust either contributions or benefits to maintain financial stability, and when adjusting benefits may adjust them immediately or in the future.

In two earlier papers (Auerbach and Lee 2009, 2011), we studied a variety of existing and hypothetical unfunded arrangements, some adhering strictly to a PAYG approach and others allowing small fluctuations in trust fund balances. The first of these papers evaluated the stability of the existing Swedish system and several variants, while the second considered the performance of several stable unfunded systems, including the actual and hypothetical Swedish systems, the actual German system, and three stable variants of the existing U.S. social security system, according to a variety of welfare criteria, such as internal rates of return and an approximation of expected utility. Our findings, particularly in the second paper, suggested that the methods of spreading shocks across generations can have significant effects on welfare. But questions remain about the channels through which these effects operate.

Understanding the effects of an existing or proposed system on welfare is, ultimately, our objective in studying how different risks are spread among generations, but looking more closely at the pattern of risk-spreading can help us understand why certain systems seem to perform better in the welfare dimension and also how systems different than those we have considered would perform in response to different patterns of shocks. That is, our past welfare analysis was based on empirically estimated demographic and economic stochastic processes for the United States, but patterns in other countries, or in the United States in the future, may differ, and it would be useful to have a more general picture of how different systems perform with respect to the allocation of different types and patterns of risk. Therefore, in this paper, we develop a methodology for isolating the effects of different types of shocks on the welfare of different
generations, looking in particular at the extent to which such shocks are efficiently spread across cohorts.

**Previous Research**

The model developed in our earlier papers provides a framework for the current project, and is described in detail in these earlier papers. We give only a brief summary of its major features here.

Following Lee and Tuljapurkar (1994), we incorporate estimated stochastic processes for fertility and mortality in the United States along with an assumed deterministic immigration level to generate a stochastic population process. The real rate of return and the rate of labor productivity growth are also modeled as stochastic time series, with the long run mean values of these stochastic processes constrained to equal the central assumptions of recent Trustees’ projections (from 2004). We assume that wages track labor productivity. In order to make the study of long-term patterns easier, we adopt two strategies. First, we start our stochastic simulation from initial population and economic conditions but throw out the first hundred years to eliminate the significance of initial conditions. Second, we modify the stochastic processes to remove drift terms. That is, we wish to study a stochastic equilibrium in which the means or expected values of fertility, mortality, immigration, productivity growth, and interest rates have no trend, and the population age distribution is stochastically stable rather than reflecting peculiarities of the initial conditions. (The resulting population processes have some trend in variances if not mean, but our simulation experiments have shown these trends to be small within the simulation horizons we use, over the range one hundred to six hundred years). To determine the distributions of outcomes, we draw 1000 random trajectories, each for a period of 500 years.
after the initial 100-year period. We use these trajectories as a platform for studying different social security systems.

We limit our consideration to social security systems that are stable, in the sense of not being subject to excessive debt-payroll ratios on some trajectories. Systems that fail this stability test will require future policy changes, and so it is not meaningful to consider how they spread risk without incorporating these policy feedbacks. The systems considered in Auerbach and Lee (2011) include the following:

1. The existing Swedish system, which uses Notional Defined Contribution (NDC) accounts, a system that bases “notional pension wealth” accumulation and ultimate annuitization at retirement on the rate of productivity growth and includes an automatic balancing mechanism that can come into effect to ensure stability.

2. Three variants of the Swedish system developed in Auerbach and Lee (2009) that use different versions of the balancing mechanism (the “brake”) and/or incorporate labor force growth as well as productivity growth in the calculation of pensions.

3. The new German system, which bases pension growth on productivity growth and the growth rate of the old-age dependency ratio and uses payroll taxes as a residual to accomplish annual PAYG balance.

4. Three variants of the US system that achieve fiscal stability through the use of uniform tax and benefit adjustments that accomplish PAYG balance, as illustrations of what the US system might look like if it were financially stable.

Auerbach and Lee (2011) considered welfare properties of each of these systems, calibrated to US economic and demographic data.
New Analysis

Under the public pension systems just discussed, we wish to consider how shocks of particular types play out over time and generations. Our approach is to estimate impulse responses to shocks.

One initial thought might be to carry out such analysis using the stochastic modeling approach of our previous work by looking only at the particular shocks of interest, one at a time. However, such an approach is difficult, because each type of shock has complex economic effects and channels that cannot be determined without an explicit general equilibrium model. For example, a fertility shock would affect not only the relative sizes of different cohorts, and hence the finances of a public pension system, but also the returns to labor and capital over time. Thus, to determine how a particular pension system spreads the risks arising from a fertility shock, we need a full general equilibrium model to trace through all of these effects. Such a model was not needed and hence not specified in our earlier analysis.

The model we use is adapted from that laid out in Auerbach and Kotlikoff (1987, chapter 11) and used subsequently by Auerbach et al. (1989) to evaluate the economic effects of public pension systems in several countries. That original model was a perfect foresight, dynamic general equilibrium model with variations in fertility that permitted analysis of the interactions of demographic transitions and different public pension systems. However, several modifications are necessary to adapt the model to make it suitable for the current task. In particular, that model had a very simplistic approach to fertility, assuming that it was concentrated at one age, had no individual uncertainty as to life expectancy, and assumed a smooth rate of productivity growth. We will go through the model, indicating the adaptations developed for use in this paper and
how we use the newly modified model to measure the effects of shocks to productivity, fertility and mortality.

**The Model**

The model we use is one in which individuals live for up to $T$ years, the first 20 of which are spent as minor dependents of parents who make consumption decisions on their behalf. At any given time, the household consists of one parent and minor children. Household utility in each year is based on the parent’s consumption and leisure, following a CES function, and each child’s consumption. The household maximizes family utility subject to a lifetime budget constraint.

For simplicity, we assume that live children are born to parents between the ages of 21 and 40, and that mortality begins at age 60, after children have left the home.\(^1\) We assume that births follow a baseline age-specific fertility profile $z_a$ between ages 21 and 40, which may be shifted by an AR(1) shock $\zeta_t$ so that fertility at age $a$ and date $t$ – the number of children born to a household of age $a$ in year $t$ – is $f_{at} = z_a \zeta_t$. The baseline mortality hazard rate for age $a$ at date $t$ is $m_a$. We assume that this vector of age-specific mortality rates can be hit by a multiplicative shock that also follows an AR(1) process, $\mu_t$, so that the survival probability hazard for age $a$ and date $t$ is $s_{at} = (1 - m_a \mu_t)$.\(^2\) We assume, for the sake of simplicity, that there are no trends or aggregate uncertainty in either fertility or mortality.

As to intergenerational linkages after children become adults, we assume that there are no inter vivos gifts or intentional bequests. Given uncertain lifetimes, though, individuals dying

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\(^1\) In future work we intend to relax this assumption by assuming that minor children are reallocated among surviving adults of the same age as parents who die between ages 41 and 59.

\(^2\) A more general specification for the survival probability would be easy to include in the model. The same is true for the fertility specification.
before age $T$ could still leave accidental bequests. Rather than dealing with accidental bequests at different ages, which would make solution of the model extremely complicated, we assume perfect Yaari annuity markets, so that individuals fully annuitize their retirement savings and therefore leave no bequests regarding of when they die. This means that the gross return to saving should equal $(1 + r^n_t)/s_{at}$ for age-$a$ individuals at date $t$. Note that the combination of mortality and perfect annuity markets should leave the household optimization problem unaffected, as higher rates of return on annuities will just offset higher discount rates induced by mortality. That is, even though the household’s objective function now incorporates expected mortality, we can determine the household’s optimal planned consumption and labor supply paths (contingent on survival) ignoring mortality in both the objective function and the rate of return.

Figure 1 displays the baseline fertility and mortality profiles we use. These profiles are taken from Auerbach and Lee (2011), except that, to accommodate modeling assumptions, values for fertility are set to zero below age 21 and above age 40, and values of mortality are set to zero for ages below 60.³

We assume that the household maximizes a lifetime family utility function that is time-separable, separable across individuals, and having a nested CES structure for adults within periods (between consumption and leisure) and between periods. Taking fertility and mortality into account, the household’s objective function at age 21 may be written:

$$
\sum_{a=21}^{\infty} \frac{(1 + \delta)^{(a-21)}}{1 - 1/\gamma} \left( \prod_{i=60}^{21} s_{1,i+a-21} \right) \left\{ \left( C_{a,j+a-21}^{(1-1/\rho)} + \omega H(a)^{1-1/\gamma} \right)^{1-1/\rho} + \sum_{k=\max(1,a-D+1)}^{k=\min(a-20,60)} f_{a-k+1,1,a-21} \omega H(a)^{1-1/\gamma} \right\}
$$

³ We inflate the remaining fertility profile to offset the excluded years of fertility in order to produce the same number of births per adult as for the full fertility profile. We do not adjust the mortality profile, which gives us approximately the correct measure of life expectancy at age 60, although overstating life expectancy at earlier ages.
where $C_{a,t}$ is adult consumption at age $a$ in year $t$, $l_{a,t}$ is the corresponding leisure, $H(a)_{k,t}$ is the consumption of a child of age $k$ in year $t$ for a parent of age $a$, $T$ is maximum life (set to 100 in our simulations), $D$ is the maximum age of child-bearing (here, assumed to be age 40), and $G$ is the age after which children are adults and leaders of their own families (here, assumed to be 20). As in Auerbach and Kotlikoff (1987), the terms $\omega_j$ are weights of children in the utility function which are assumed to increase linearly from 0.25 at age 1 to 0.50 at age 20, i.e., $\omega_j = .25 + .25*(j-1)/19$. In expression (1), there are also three household preference parameters: $\delta$ is the pure rate of time preference, $\rho$ is the intratemporal elasticity of substitution between consumption and leisure, and $\gamma$ is the intertemporal elasticity of substitution over consumption (and, in the case of adults, leisure as well) at different dates. We set these three parameters equal to 0.006, 0.8, and 0.35, respectively.

The economy has one production sector, in which the representative firm is assumed to behave competitively in factor and output markets and produce output subject to a constant-returns-to-scale Cobb-Douglas function in capital and labor. Hence there are no pure profits, with returns to capital and labor exhausting the firm’s income. The economy is closed in the simulations we consider, so the production sector’s capital stock is determined by household plus government asset accumulation. Labor equals the sum of labor supplied by cohorts of different ages, where we assume that different ages of labor are perfect substitutes but differentially efficient as described by an empirically estimated age-based efficiency profile, $e_{at}$, also taken from Auerbach and Kotlikoff (1987).

We assume that individuals work starting at age 21, with the date of retirement being endogenously determined by preferences and factor prices. This date of retirement may vary
over time and is distinct from the initial age at which benefits are received, which we set at 67 for all social security systems considered, consistent with the normal retirement age under the current US system once it is fully phased in. There is a deterministic trend in productivity growth, at a rate of 1.5 percent per year, which is implemented in two pieces: an additive factor in the efficiency profile, $e_a$, which increases the individual’s efficiency at supplying labor, plus a trend in the labor endowment, which increases equally the individual’s efficiency at supplying labor and in the consumption of leisure. This method produces the right wage profile for each cohort but also avoids any trend in the work/leisure ratio, as discussed in Auerbach et al. (1989). We assume the presence of multiplicative productivity shocks around a steady state value, with these shocks again following an AR(1) process. These productivity shocks will affect the market returns to labor and capital at each date $t$, $w_t$ and $r_t$.4

Finally, we include the government sector in the model. The government sector consists of two components, general government and the public pension system. General government follows a parsimonious specification of government purchases as consisting of age-based and non-age-based components, with the age-based components (e.g., education spending, old-age medical care) held constant relative to their respective population groups and non-age-based components (e.g., defense spending) held constant relative to total population. We break spending down into age-specific and non-age-specific categories and hold spending each category, $i$, constant at $g_i$ per member of the relevant population, $N_i$, for $i = y, m,$ and $o$ (young, middle-aged, and old) or for the total population. That is, overall general government spending at any date $t$ equals:

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4 While productivity shocks hit both interest rates and wage rates, it would also be straightforward to evaluate separate shocks to the two processes by introducing another shock that affects only wages, via a shift in the age-wage productivity profile, $e_a$. 
We solve for the values of $g_i$ by entering relative values of spending ($gN$) in each of the four categories, taken from Auerbach and Kotlikoff (1987, chapter 11) – .306, .172, .141, and .381, respectively – and then scaling them so that government spending equals the exogenously determined level of revenue in the initial steady state. During the transition, we keep these government spending weights, $g_i$, constant except for trend productivity growth, so that government spending grows smoothly over time except as a result of changes in the age structure of the population.

General government is funded with a proportional income tax and a consumption tax, and we ignore the use of government debt for the general government budget. In the initial steady state, we set the proportional income tax to 20 percent and the consumption tax equal to 3 percent, which are similar to estimates for the current US tax system from Auerbach (2002). Because revenue requirements to meet the required spending, as specified in expression (2), fluctuate during the transition after a shock, we allow the consumption tax rate to vary to ensure annual budget balance for the general (non-pension) government.

The public pension system is modeled on one of the various systems described above. In the results that follow, we focus on three of these systems that by design have year-by-year budget balance: the US “benefit adjust” system, in which the payroll tax is fixed and all adjustments occur to benefits, the US “tax adjust” system, in which the replacement rate is fixed and all adjustments occur to payroll taxes, and the German system, which incorporates adjustments to both taxes and benefits in any given year.

For either US system, we calculate benefits roughly as under current law, taking the average of past labor earnings, inflated by wage growth between the date of earnings and the
date of benefits receipt to calculated average indexed monthly earnings.\textsuperscript{5} We then solve for the payroll tax such that budget balance is achieved, according to the expression,

\begin{equation}
\theta_t \sum_{j=21}^{T} w_{j,t} (1 - l_{j,t}) N_{j,t} = R_t \sum_{j=67}^{T} AIME_{t-j+1} N_{j,t}
\end{equation}

where $\theta_t$ is the payroll tax in year $t$, $w_{j,t}$ is the wage rate of an age-$j$ individual in year $t$, $N_{j,t}$ is the population of age $j$ in year $t$, $AIME_{t}$ is the average indexed monthly earnings for an individual born in year $t$, and $R_t$ is the replacement rate in year $t$. We set $\theta = .106$ in the initial steady state, consistent with the current OASI portion of for the United States, and solve for $R$ according the expression (3). During the transition, when shocks occur, we adjust either $R$ or $\theta$ annually to ensure that (3) continues to hold, according to whether we are considering the benefit-adjust or tax-adjust scenario.

For the German system, we follow the description in Auerbach and Lee (2011). Each beneficiary at date $t$ receives the same benefit, $B_t$, so that budget balance requires that

\begin{equation}
\theta_t \sum_{j=21}^{T} w_{j,t} (1 - l_{j,t}) N_{j,t} = B_t \sum_{j=67}^{T} N_{j,t}
\end{equation}

The benefit itself evolves from one year to the next according to the following formula:

\begin{equation}
B_t = B_{t-1} \frac{w_{r-1}(1-\theta_{r-1})}{w_{r-2}(1-\theta_{r-2})} \left[ 1 - .25 \left( \frac{OA_{t-1} - OA_{t-2}}{OA_{t-2}} \right) \right]
\end{equation}

\textsuperscript{5} For simplicity we include all years of work in this calculation, rather than the 35 years with highest earnings, as currently used for the US system.
where $w_t$ is the aggregate wage in year $t$, normalized for age-specific productivity differences, and $OA_t$ is the old-age dependency ratio in year $t$, which we define to be the ratio of the adult population in retirement (age > 66) to those not in retirement (ages 21-66). While expression (5) determines how benefits evolve over time, it does not fix the level of benefits. To facilitate comparison with the US systems, we fix $B$ in the initial steady state so that expression (4) is satisfied by the same payroll tax rate as is assumed for the US systems. Thereafter, during the transition, the benefit evolves according to expression (5) and the tax rate $\theta$ is determined so that expression (4) is satisfied. Thus, both $B$ and $\theta$ will change from year to year during the transition.

For all public pension systems, we assume that individuals perceive some linkage between social security benefits and contributions, that is, that a portion of payroll taxes are viewed as being offset by the incremental benefits they generate. The higher the perceived tax-benefit linkage, the lower the labor supply distortion caused by the payroll taxes. For the simulations reported below, we set the tax-benefit linkage at 0.25, meaning that one fourth of payroll taxes are ignored when individuals make labor supply decisions.

To consider the solution of the model, first assume that there are no shocks to mortality, fertility or productivity. In this case, the economy eventually follows a steady state path, so we start by solving for this steady state, using the Gauss-Seidel solution technique laid out in Auerbach and Kotlikoff (1987). Now, suppose that the economy is initially in this steady state, in year 1, and is then hit by one of the three types of shocks in year 2, with no further shocks thereafter (but the shock itself fading out only gradually in accordance with the AR(1) specifications for each type of shock). Since the shocks eventually die out completely, the economy will gradually return to the same steady state. To solve for the transition path, we
assume that the shocks occur by surprise in year 2, after which all agents in the economy are endowed with perfect foresight. Thus, the transition back to the steady state corresponds to a perfect foresight transition path, along which the actual paths of all variables are taken into account in household and firm optimization decisions at each date. We allow the transition to last for 150 years, to give the shocks and their consequences time to die out, and pin down values thereafter using those already solved for the steady state.

Once the transition path is determined, we can then calculate how the welfare of each cohort is affected by each particular type of shock, where the household’s welfare is based on its expected utility given in expression (1). From this, we calculate the “wealth equivalent” of the cohort’s utility change, $x$, as the scalar that, when multiplied by the household’s vector of consumption and leisure, equalizes steady state utility and actual utility along the economy’s transition path in the presence of the shock. By analyzing how the effects of the three different types of shocks (mortality, fertility, and productivity) are spread among different cohorts, we can gain insight into how and why the different public pension arrangements lead to different overall welfare when analyzed in the context of multiple shocks of all three types, as in our previous analysis (Auerbach and Lee 2011).

For shocks to productivity growth, the wealth equivalent as just described will give us a clear measure of changes in well-being due to the shocks. For shocks to fertility and mortality, however, the issue is more complicated, since changes in the vectors $s$ and $f$ in expression (1’) amount to changes in the household’s utility function. For example, higher fertility will increase the weight on children’s utility, and higher mortality will reduce the weight on consumption and leisure in later periods of adult life. These changes in the utility function are important to account for when determining the household’s optimal allocation of resources, but comparisons
of utility across different utility functions are not well defined. For example, simply having more children would lower the measured level of utility (since $\gamma < 1$), even if every element of the vectors of consumption and leisure were the same.

To deal with this issue, we measure wealth equivalents using the utility function that would apply in the absence of shocks. That is, we measure the utility of consumption and leisure profiles in transition relative to those in the absence of shocks using the utility function that would apply in the absence of shocks. With this approach, the wealth equivalent for a household will exceed 1 if and only if the household’s observed vector of consumption and leisure would be preferred to the bundle chosen in the absence of shocks, for the fertility and mortality profiles that would apply in the absence of shocks.\(^6\)

**Results**

For each of the shocks considered, to fertility, mortality, and productivity, we assume a one-time shock of 100%, that is an initial doubling of the fertility rate, the mortality rate, or the rate of productivity, followed by decay based on an AR(1) process, with the AR coefficient equal to 0.9. We consider shocks that are unrealistically large in magnitude to make it easier to see patterns in the results. Because the model is nonlinear, the results cannot simply be scaled down in proportion to determine the effects of smaller shocks, but the size of the shocks should not lead to differences in sign in the effects on different cohorts.

Figure 2 shows the evolution over time of the population age structure in response to a fertility shock, which increases proportionally the fertility rate at every child-bearing age.

Starting with a smooth population structure in the initial steady state, there is a jump up in the

\(^6\) Our approach to measuring utility makes sense if changes in fertility and mortality are due to exogenous shocks, but the issue would be more complicated if fertility and mortality shocks reflect endogenous behavior by the household. For example, fertility might change because of a reduction in the cost of raising children. To adequately evaluate the welfare effects of such a change, it would be necessary to incorporate household fertility decisions explicitly in our model.
young population by year 10. By year 50, there is a broader increase in the population showing both the initial fertility shock and a smoothed echo effect of this initial shock. In the final steady state, the population is larger as a consequence of the shock, but the original age structure is restored.

Figure 3 shows the comparable population evolution in response to a mortality shock. The shock, which increases mortality rates uniformly across the population at risk of death, reduces the elderly population by year 10, but this reduction disappears over time as mortality rates return to their original levels. Unlike in the case of a shock to fertility, where the population is permanently higher, under a mortality shock (given our assumption that changes in mortality do not interact with child bearing) the population has returned in the final steady state to its pre-shock pattern and level.

Figures 4 and 5 show the corresponding time paths of the aggregate population for each of these two scenarios, with the population converging to its higher long-run level after the fertility shock and recovering to its original level after the mortality shock.

Now, consider the effects of these shocks on the well-being of individuals in different cohorts. As a benchmark for each experiment, we consider the impact in the absence of any social security system.

**Productivity Shock**

Figures 6a-d show the effects of a productivity shock on the welfare of individual cohorts, the economy, and the social security system. Figure 6a shows the impact on well-being, based on the wealth-equivalent measure described above. As one would expect, a positive productivity shock increases well being, with the largest effects being experienced by the generations reaching adulthood around the time that the productivity shock hits. While the
effects differ slightly across the different social security systems, these differences are very minor compared to the common impact of the shock itself. Figures 6b and 6c shows the corresponding impact over time on the aggregate wage rate (the wage rate for labor supply of unit efficiency, normalized to 1 in the initial steady state) and the interest rate. Not surprisingly, both jump up when the productivity shock hits, since both labor and capital become more productive. Again, the differences across social security scenarios are minor, except for the interest rate being generally lower in the absence of social security (Figure 6c). This difference is due to the well-known negative effect of pay-as-you-go social security systems on capital accumulation; a lower capital stock leads to a lower capital-labor ratio and a higher interest rate.7

One interesting phenomenon in Figure 6c is that interest rates overshoot when adjusting back to their original level, falling below their initial value before recovering. The explanation for this is that, under perfect foresight, individuals who know that their wages are higher now than they will be in the future concentrate labor supply during the period of temporarily high productivity. In doing so, they accumulate a lot of capital to be used to finance higher levels of consumption later in life, which temporarily lifts the capital-labor ratio above its long run value after the productivity shock itself has dissipated.

As to the effects of the different social security systems, if one compares the patterns of gains to those in the absence of social security, all three systems appear to concentrate gains more among generations that reach adulthood around the time the productivity shock hits. This makes sense given that pay-as-you-go social security systems provide a rate of return based on the economic growth rate. The productivity shock makes social security a “less bad” deal, a source of gain that is absent when there is no social security system. Among the three social

7 Note that there is no comparable impact on the wage rate in Figure 6b because the wage rate shown here is normalized to 1 in the initial steady state.
security systems, the US tax adjust system appears to shift the gains slightly toward older
generations, while the US benefit adjust system shifts the gains slightly toward younger ones,
with the German system’s effects lying in between. Some insight into the reason for this
relationship comes from Figure 6d, which shows the time path of the social security tax rate
under the three systems. The tax rate is constant, by assumption, under the US benefit adjust
system. Under the US tax adjust system, the tax rate drops initially, since it is easier to finance
social security benefits with a more productive work force. Over time, however, the payroll tax
rate actually rises above its long-run level, because the productivity shock has dissipated but left
in its wake an impact on social security benefits, which are based on lifetime earnings. That is,
workers whose productivity has reverted to its original level must pay elevated benefits to those
who worked during a period of higher productivity. A similar, but muted effect, occurs under
the German system, which combines adjustments to both taxes and benefits.

Mortality Shock

Figures 7a-d show the effects of a mortality shock, beginning with the impact on
individual welfare in Figure 7a. To interpret this figure, it is important to remember that it
assesses the change in the bundle of consumption and leisure using pre-shock mortality profiles.
By this measure, we would expect a shortening of lifespan to increase well-being, ceteris
paribus, because it would make more resources available during the period in which an
individual is alive. This outcome is most evident in the figure for the no-social security case, in
which older generations – those who are primarily affected by the temporary increase in
mortality, experience an increase in welfare.

Why do adults reaching adulthood shortly after the transition begins experience a small
decline in welfare? The explanation appears in Figure 7b, which shows the wage-rate trajectory
over time. Wages dip temporarily, because older generations have less reason to save for old age and therefore accumulate less capital. Hence, those who reach adulthood shortly after the transition begins, who themselves will be largely unaffected by the mortality shock, experience lower wages and hence lower welfare. The same general equilibrium effects help older generations further, through a temporary rise in interest rates (Figure 7c).

This pattern of effects across generations is modified under the different social security systems, particularly those that involve at least some adjustment of social security taxes. Under the US tax adjust system, and to a lesser extent the German system, younger adults now gain as a result of the shock, while older generations see their gains reduced. The reason for this shift is the reduced payroll tax, shown in Figure 7d. A mortality shock temporarily reduces the old-age dependency ratio and hence allows a reduction in payroll taxes. Unlike the case of the productivity shock, there is no subsequent need for a payroll tax increase. Thus, a social security system that incorporates payroll tax adjustments does well at spreading the gains from a mortality shock, offsetting the negative general equilibrium effects on young workers’ wages and also distributing to them some of the surplus made possible by the lower consumption needs of the elderly.

**Fertility Shock**

Figures 8a-8d show the effects of a fertility shock. Again, recall that the welfare effects are measured for fixed fertility profiles. Ignoring general equilibrium effects, we would expect a reduction in well being for young adults alive at the time of the shock, because these adults must commit more resources to children as a result of larger family sizes. That is indeed what occurs in Figure 8a for the simulation in which there is no social security system. Under that scenario, there are essentially no “winners” from the boom in fertility. However, the largest losses are
experienced by those around 20 years into adulthood as of the shock, who themselves have just
moved out of their child-bearing years and thus are not directly affected by the fertility shock.
The explanation comes through general equilibrium effects. As shown in Figure 8b, the wage
rate drops starting around 20 years after the baby boom begins, just as the larger cohorts begin to
enter the labor force. Because labor is homogeneous, differing in productivity among workers of
different ages but otherwise perfectly substitutable, this drop in the wage rate affects all workers,
not just those in large cohorts.

Once social security systems are taken into account, the effects on welfare change. In
particular, a reliance on tax adjustments helps those who reach adulthood during the transition,
reducing the losses of those who lose the most and actually making winners out of those reaching
adulthood a few decades into the transition. The explanation comes from inspection of Figure
8d, which shows the evolution of the social security tax over time. Once larger cohorts hit the
labor force, the decline in the old-age dependency ratio permits a reduction in the tax rate, which
helps those in the labor force at the time. Later on, the social security tax rate rises above its
original level, as the large cohorts enter retirement. But this effect is muted by the echo effects
in population structure (see Figure 2), which limit the increase in the old age dependency ratio.

Conclusions

We have simulated the way in which different public pension structures spread the effects
of isolated deterministic macro shocks across the leading and trailing generations in a general
equilibrium setting. This enables us to make comparisons of outcomes across systems, across
kinds of shocks, and across generations. The impulse-response approach pinpoints the particular
consequences of different kinds of shocks, making it possible to observe and interpret these
outcomes in a way that is not possible with stochastic simulations which show us the results of a mixture of kinds of shocks initiated at many different times.

The general equilibrium setting provides new insights about the effects of shocks filtered through different pension structures, which affect the way that shocks alter the saving and labor supply behavior of generations. For example, following a mortality shock, older working age generations have less need to save for retirement (life expectancy is shorter and annuity rates of return are higher, reflecting higher old age mortality), so capital per worker is reduced and wages fall, while the rate of return earned by the elderly on their assets rises. But these general equilibrium effects are modified in different ways by the pension structures, as discussed earlier. Under the tax adjust system, for example, the mortality shock reduces the number of elder retirees leading to a lower tax rate for workers, offsetting the reduction in wages. Following a fertility shock, once the incremental births enter the labor force, wages fall and interest rates rise. The higher interest rates benefit the elderly retirees, while their benefit levels respond in opposite directions under tax and benefit Adjust programs.

It appears that the pension systems we consider are effective to some degree in spreading risk across generations, since the simulations with no social security system show greater intergenerational variation in wealth equivalents in response to mortality and fertility shocks. However, for a productivity shock, although the response was similar in all cases, the no-system simulations had very slightly less intergenerational variability.

Our conclusions are tentative pending some improvements in our model design. For example, we will include mortality below the age of 60, which will require modeling the assignment of orphans to surviving adults. We will add simulations that exclude the general equilibrium feedbacks so that we can make explicit assessments of their effects on the outcomes.
References


Figure 1. Baseline Fertility and Mortality Profiles
Figure 2. Population Age Structure, Fertility Shock
Figure 3. Population Age Structure, Mortality Shock
Figure 4. Aggregate Population (Normalized) Over Time, Fertility Shock
Figure 5. Aggregate Population (Normalized) Over Time, Mortality Shock
Figure 6a. Productivity Shock: Wealth Equivalents by Year of Adulthood

- ---- No SS
- Red US Tax
- Blue US REPL
- Green German
Figure 6b. Productivity Shock: Aggregate Wage Rate by Year
Figure 6c. Productivity Shock: Interest Rate by Year

- --- No SS
- Red US Tax
- Blue US Repl
- Green Germany
Figure 6d. Productivity Shock: Social Security Tax Rate by Year
Figure 7a. Mortality Shock: Wealth Equivalents by Year of Adulthood

- Dashed line: No SS
- Red line: US Tax
- Blue line: US Repl
- Green line: German

Year axis ranges from -78 to 151.
Figure 7b. Mortality Shock: Aggregate Wage Rate by Year

- ----- No SS
- red  US Tax
- blue US Repl
- green German
Figure 7c. Mortality Shock: Interest Rate by Year
Figure 7d. Mortality Shock: Social Security Tax Rate by Year
Figure 8a. Fertility Shock: Wealth Equivalents by Year of Adulthood

- No SS
- US Tax
- US Repl
- German
Figure 8b. Fertility Shock: Aggregate Wage Rate by Year
Figure 8c. Fertility Shock: Interest Rate by Year
Figure 8d. Fertility Shock: Social Security Tax Rate by Year