Optimal Capital Income Taxation with Housing*

Makoto Nakajima
University of Illinois at Urbana-Champaign†

This Draft: April 9, 2008
First draft: April 23, 2007

Abstract

I quantitatively study the optimal capital income taxation in the general equilibrium overlapping generations model with uninsurable idiosyncratic income shocks and with housing and financial assets. Following key characteristics of housing are explicitly modeled: *(i)* housing is held for the dual purpose of consumption and savings, *(ii)* housing can be either owned or rented, *(iii)* if owned, housing can be used as a collateral for mortgage loans, and *(iv)* there is a preferential tax treatment for owner-occupied housing through tax-exemption of imputed rents and mortgage interest payment deduction. Using calibrated models, I investigate whether and how the optimal capital income tax rate differs between the model with both housing and financial assets and the standard model without housing. I find that the optimal capital income tax changes significantly depending on the housing tax policy. This is mainly because capital income tax affects both portfolio choice between housing and financial assets, and tenure decision. In particular, I find that, when preferential tax treatment for owner-occupied housing like the current U.S. economy is maintained, there is a large welfare gain by lowering the capital income tax rate and thus narrowing the tax wedge between housing and financial assets, and owning and renting. The optimal capital income tax rate in the baseline model with housing is 13%, which is substantially lower than the optimal capital income tax rate in the model without housing, which is 37%. On the other hand, if the preferential tax treatment for owner-occupied housing is eliminated, it becomes optimal to tax both assets heavily, like in the one-asset life-cycle model. Indeed, the optimal tax rate for capital income and housing is 43%, which is higher than in the model without housing. Finally, welfare gain form implementing the optimal capital income tax rate is large in the model with housing; in both cases above, the welfare gain is larger than 2% of flow consumption.

JEL Classification: E21, E62, H21, H24, R2
Keywords: Capital Taxation, Housing Taxation, Optimal Taxation, Heterogeneous Agents, Incomplete Markets

*I thank In-Koo Cho for generously letting me use his parallel cluster. I also thank participants at Missouri Economic Conference, especially Michael Owyang, for their comments and suggestions.

†Department of Economics, University of Illinois at Urbana-Champaign. 1206 South 6th Street, Champaign, IL 61820. E-mail: makoto@uiuc.edu.

1 Introduction

In this paper I quantitatively investigate the optimal taxation on housing and financial assets in the general equilibrium overlapping generations model where agents face uninsurable labor income risk and mortality risk, and key characteristics of housing assets are explicitly modeled. The paper bridges the gap between the literature on macroeconomic public finance, which typically ignores the housing assets, and that on housing taxation, where the quantitative general equilibrium model is rarely used.

One of the most celebrated results in the macro-public finance literature is the zero optimal capital income tax rate. Using the Ramsey approach, Chamley (1986) and Judd (1985) show that the optimal capital income tax rate is zero in the long run in the model with the infinitely-lived representative agent. It is further shown that the result holds true in less restrictive environment.[1]

On the other hand, the optimal capital income tax rate is known to be different from zero in overlapping generation models where agents are finitely-lived. Erosa and Gervais (2002) and Garriga (2003) show theoretically that the optimal capital income tax rate is *not zero*. Moreover, Conesa et al. (2007) recently show quantitatively that the optimal capital income tax rate is not only non-zero but *very large*, using a calibrated overlapping generations model.

However, what has been neglected in the discussion above is that close to half of the total reproducible capital stock in the U.S. and the biggest single asset for the majority of households in the U.S. is housing capital, which is different from non-housing capital in a variety of ways. The main purpose of the paper is to ask whether and how the optimal capital income tax is affected by explicitly considering the unique nature of housing capital.

It is important to explicitly model housing in studying optimal taxation for the following reasons. First, as argued above, housing is big. Second, the distribution of housing and financial assets is known to be very different. Housing assets are more evenly distributed both within and across different age groups. Young households typically hold a larger amount of housing assets than the value of their total wealth, leveraged by mortgage loans. Third, housing is held mainly for enjoying services that the it generates. Besides, the value of services generated by owner-occupied housing is not taxed in the U.S. Together with the mortgage interest payment deduction, there is a substantial degree of preferential tax treatment for owner-occupied housings.

Considering these reasons, welfare and macroeconomic implications of changing capital income tax rate could be very different depending on how housing assets and the value of services generated from housing are treated. In other words, the optimal capital income tax rate computed by ignoring the differences between housing and non-housing capital could be very different from the one which is computed by explicitly modeling the housing capital. I will show in the paper that the magnitude of the mistake is sizable.

The model used here captures the following key properties of housing assets. First, housing

[1] Chari and Kehoe (1999) offers a good survey on the optimal taxation results within the Ramsey framework. Atkeson et al. (1999) show that the optimality of zero capital income tax rate holds even if some assumptions are relaxed.

assets are held for *dual purpose* of consumption and saving. Housing assets are held not only as a means of saving but also to enjoy living in. Housing assets are substitutes for financial assets as a means of saving, but not a perfect one because of the additional motive for holding. Second, housing assets are either owned or rented. Third, housing assets can be used as a collateral for mortgage loans. I will carefully investigate how these key characteristics of housing are linked to the optimal tax rates.

I use the Ramsey approach to the optimal taxation problem. In this approach, the size of government expenditure in every period is exogenously given, and lump-sum taxation is assumed to be unavailable. A set of distortionary tax instruments is assumed, and the optimal tax scheme within the set is explored. I fix the size of the government expenditure at the size implied by the tax regime that resembles the current U.S. tax regime. I use proportional taxes for labor and capital income, and imputed rents of owner-occupied housing assets. The baseline tax rates for capital and labor income are the effective tax rates in the current U.S. economy. Housing assets are tax-exempt in the baseline specification, which is also a feature of the current U.S. economy. I also introduce mortgage interest payment deduction, which is another key element in the current U.S.housing tax policy. The rate of deduction is set to match the effective rate of deduction in the current U.S. economy.

This paper is most closely related to Gervais (2002). Gervais (2002) uses a general equilibrium overlapping generations model and shows a large welfare gain from taxing housing and eliminating the tax deduction for mortgage interest payment. There are three key differences between Gervais' (2002) paper and the current paper. First, my focus is the capital income tax rate, while Gervais (2002) focuses on the welfare gain of eliminating preferential tax treatment of owner-occupied housing. Second, there is no intra-generational heterogeneity in Gervais' (2002) model. For considering the optimal taxation, it is important to capture the large inequality of income and assets, a large part of which is due to intra-generational heterogeneity. Third, there is no labor-leisure decision in Gervais' (2002) model. As convincingly argued in Conesa et al. (2007), labor supply distortion is a key element in shaping the optimal tax regime in a life-cycle model.

There are two main findings. First and most importantly, I find that the optimal capital income tax changes significantly depending on how housing is taxed. This is mainly because capital income tax affects both portfolio choice between housing and financial assets, and tenure decision. The optimal capital income tax rate when the preferential tax treatment for owner-occupied housing is maintained is 13%, which is substantially lower than the optimal capital income tax rate in the model without housing, which is 37%. On the other hand, if the preferential tax treatment is eliminated, it becomes optimal to tax both assets heavily, like in the one-asset life-cycle model. Indeed, the optimal tax rate for capital income and housing is 43%, which is higher than in the model without housing.

Second, the welfare gain associated with implementing the optimal capital income tax rate is large, in the model with housing. It is true no matter whether the preferential tax treatment for owner-occupied housing is maintained or nor. In the baseline model, the welfare gain by implementing the optimal capital income tax while keeping the preferential tax treatment for owner-occupied housing is 2.3% of flow consumption. If the preferential tax treatment is eliminated, the welfare gain is as large as 3.3%. There is a striking contrast from the welfare gain by implementing the optimal capital income tax rate in the model without housing, which is a mere 0.01%. I argue that the main source of welfare gain is nullifying the preferential tax treatment for owner-occupied housing and thus mitigating the efficiency loss from the tax benefit. Changing capital income tax rate indirectly affects the tax benefit of owner-occupied housing.

The remaining parts of the paper are organized as follows. Section 2 reviews related literature. Section 3 sets up the model. Section 4 describes how the model is calibrated. Section 5 comments on how the model is numerically solved. The properties of the baseline model economy are studied in Section 6. In Section 7, the methodology for counterfactual experiments is explained. In Section 8, I investigate welfare and macroeconomic effects of changing capital income tax rate. Variety of sensitivity analysis is offered in Section 9. Section 10 concludes.

2 Related Literature

The list of related literature starts from Chamley (1986) and Judd (1985), who show that the optimal capital income tax rate is zero in the long run in the standard growth model. The crucial assumption for this celebrated result is that the economy is inhabited by the infinitely-lived representative agent. There is no ex-ante heterogeneity within or across cohorts, and the complete markets wipe away any ex-post heterogeneity. If the economy is populated by ex-ante heterogeneous agents, or markets are incomplete, a zero capital income tax rate might no longer be optimal.

Aiyagari (1995) argues that, in the presence of market incompleteness, the optimal capital income tax is not zero in the long run. In the economy with uninsured idiosyncratic shocks to earnings, agents have a precautionary savings motive, which pushes the aggregate savings above the optimal level in the complete markets model. A positive capital income tax can fix the over-accumulation of assets by countering the incentive to hold precautionary savings.

Domeij and Heathcote (2004) build on the model used by Aiyagari (1995) and show that the optimal capital income tax rate is actually close to the effective capital income tax rate in the U.S, which is 40%, if the model captures the observed large degree of income and wealth inequality in the U.S. economy. Like Aiyagari (1995), they also use the model with uninsured idiosyncratic shocks to individual labor productivity. Moreover, they calibrate the model such that the income and wealth inequality implied by the model replicate those of the U.S. economy. Since the magnitude of the inequality of individual labor productivity and wealth inequality observed in the U.S. is so large, it is optimal to use extensively the redistribution or insurance function of the capital income tax. Capital income tax has a stronger insurance or redistribution effect than labor income tax because inequality with respect to capital income is substantially higher than that of labor income both in the model and in data.

On the other hand, in overlapping generations models populated with finitely-lived agents, Erosa and Gervais (2002) and Garriga (2003) theoretically show that the optimal capital income tax rate is *not zero* in general. The key intuition is that marginal utility with respect to both consumption and leisure changes over the life-cycle. Consequently, the optimal taxation must include age-dependent tax rates.

Moreover, Conesa et al. (2007) show that the optimal capital income tax rate is *very large* positive in the calibrated overlapping generations model. The result holds even if the markets are complete, or the progressivity of labor income tax provides a substantial degree of redistribution or insurance. In their baseline model with life-cycle, uninsured idiosyncratic productivity shocks, non-separable utility function between consumption goods and leisure, and progressive labor income taxation, they find that the optimal capital tax rate is as high as 36%. The life-cycle savings motive makes saving less elastic to changes in the after-tax return, which makes the efficiency loss associated with capital income taxation smaller and the efficiency loss from taxing labor income relatively larger.

Regarding housing taxation, a long list of studies argue the optimality of taxing imputed rents of owner-occupied housing and eliminating mortgage interest payment deduction. Rosen (1985) offers a good summary of the literature analyzing the effects of the government's policy toward housing. The current paper is related to the literature on housing taxation because the welfare gain form implementing the optimal capital income taxation is closely related to the welfare gain of eliminating inefficiency associated with the preferential tax treatment of owner-occupied housing. A value-added that the paper offers is that the welfare gain is quantitatively analyzed using a calibrated general equilibrium overlapping generations model.

The model used in the current paper is built on the literatures that construct general equilibrium models with uninsured idiosyncratic shocks. The pioneer papers are Aiyagari (1994) and Huggett (1996). The papers which introduce housing or durable assets into the standard general equilibrium framework with uninsured idiosyncratic uncertainty are Fernández-Villaverde and Krueger (2005), Díaz and Luengo-Prado (2006), and Nakajima (2005). Gervais (2002) first analyzed housing taxation in the calibrated general equilibrium overlapping generations model. Chambers et al. (2007) use the general equilibrium model with housing to investigate the recent rise in homeownership rate.

3 The Model

The model I use is based on the general equilibrium overlapping generations model with uninsured idiosyncratic shocks to labor productivity and mortality, in particular Conesa and Krueger (1999) and Conesa et al. (2007). The nobel feature of the model is that there are both housing and financial assets. The following four key characteristics of housing assets are explicitly captured in the model. First, housing assets play a dual role; housing generates services consumed by those who live in it, and at the same time is a means for saving. Second, housing can be owned or rented. Third, homeowners can use their housing as a collateral for mortgage loans. Using mortgage loans, agents can live in a house whose value is larger than the value of their total wealth. Fourth, there is a preferential tax treatment for owner-occupied housing through tax-exemption of imputed rents and mortgage interest payment deduction. Since the government can tax owner-occupied and rented housing and financial assets differently, the model can naturally be used to understand how the difference in taxes for housing, either owned or rented, and financial assets affect allocations, and welfare.

3.1 Demographics

Time is discrete. In each period, the economy is populated by I overlapping generations of agents. In time t, a measure $(1+\gamma)^t$ of agents are born. γ is the population growth rate. Each generation is populated by a mass of measure-zero agents. agents are born at age 1 and could live up to age I. There is a probability of early death. Specifically, s_i is the probability with which an age-i agent survives to age $i+1$. With probability $(1-s_i)$, an age-i agent does not survive to age $i+1$. I is the maximum possible age, which implies $s_I = 0$.

Because of the probability of early death, there are accidental bequests. These accidental bequests are taxed away by the government and redistributed to all the surviving agents as a lump-sum transfer tr.

Agents retire at age $1 < I_R < I$. Agents with age $i \leq I_R$ are called *workers*, and those with age $i > I_R$ are called *retirees*. I_R is a parameter, implying that retirement is mandatory.

3.2 Preference

An agent maximizes its expected lifetime utility. The utility function of an agent takes the standard time-separable form as follows:

$$\mathbb{E} \sum_{i=1}^{I} \beta^{i-1} u(c_i, d_i, m_i) \tag{1}$$

where c_i is the consumption of non-housing goods at age i, d_i is the consumption of housing services at age i, and m_i is the leisure enjoyed at age i. $\mathbb{E}$ is the expectation operator. β is the time discount factor. $u(.,.,.)$ is strictly increasing and strictly concave in all three arguments.

3.3 Endowment

Agents are endowed with one unit of time in each period and housing asset h_1 and financial asset a_1 at birth. I assume that $h_1 = 0$ and $a_1 = 0$. Agents can use their time either for work ℓ or for leisure m. Formally:

$$1 = \ell_i + m_i \tag{2}$$

for each age-i.

Agents are heterogeneous in terms of labor productivity. Labor productivity has two components, $\overline{e}_i$ and e. $\overline{e}_i$ is a component related to the age or working experience of agents. Since agents are forced to retire at age I_R, $\overline{e}_i = 0$ for $i > I_R$. e is the stochastic component and independent of the age of agents. Each newborn draws the initial $e \in E = \{e_1, e_2, ..., e_n\}$ from $\{p_e^0\}$, where each of p_e^0 represents the probability assigned to each possible realization of e. The stochastic process for e is identical for all agents and independent across agents. In particular, $\log(e)$ is assumed to follow a finite-state first order Markov process $(E, \{p_{ee'}\})$, where $p_{ee'}$ represents the Markov transition probability from e to e'. For an agent who supplies ℓ_i hours for work, the product $\ell_i \overline{e}_i e$ represents the individual labor supply of an age i agent, measured in efficiency units.

3.4 Technology

There is a representative firm which has access to the following constant returns to scale technology:

$$Y_t = Z_t F(K_t, L_t) \tag{3}$$

where Y_t is output, Z_t is the level of total factor productivity, K_t is aggregate non-housing capital input, and L_t is aggregate labor input measured in efficiency units in period t, respectively. Because of Euler's theorem, if the inputs are traded in competitive markets, the profit of the firm will be zero in equilibrium. Non-housing capital depreciates at a constant rate δ_K. Housing capital is denoted by H_t and depreciates at a constant rate δ_H. There is a linear technology that converts between one unit of housing capital and one unit of non-housing capital costlessly. In sum, the aggregate resource constraint of the economy is the following:

$$C_t + G_t + K_{t+1} + H_{t+1} + O_t = (1 - \delta_H)H_t + (1 - \delta_K)K_t + Y_t \tag{4}$$

where C_t is total private consumption, and G_t is public consumption. G_t is not valued by agents. O_t is the sum of costs associated with owned housing.

Housing capital H_t yield housing services D_t. The following linear production function is assumed:

$$H_t = D_t \tag{5}$$

Because of the structure of the transformation technology, I can use H_t and D_t interchangeably.

3.5 Real Estate Sector

Real estate sector works as the intermediary for agents who rent housing.[2] In each period, a real estate firm borrows financial assets from agents and use the assets to buy housing assets. The

[2]Chambers et al. (2008) construct a model where homeowners become landlords and supply rental properties to renters.

housing assets are rent to renters, and the real estate firm receives the rent q_t, and use the rent to pay back the cost of debt and other costs. The following equation specifies problem of a real estate firm:

$$\max_{h_t} \{(1-\delta_h)h_t + q_t h_t - (1+r_t)h_t\} \tag{6}$$

where $(1-\delta_h)h_t$ is the value of the house after depreciation, $q_t h_t$ is the rent income of the real estate firm, and $(1+r_t)h_t$ is the financial cost associated with the housing assets. Assuming free entry to the real estate sector, the equilibrium rent is determined by zero profit condition and takes the following form:

$$q_t = r_t + \delta_h \tag{7}$$

Basically, renters pay for the financial cost of the value of housing that they rent plus the maintenance cost (depreciation) for the rented housing, through real estate sector which is acting as the intermediary.

3.6 Market Structure

First of all, without loss of generality, I assume that agents own *financial assets* instead of *non-housing capital stock*. One unit of financial assets is a claim to one unit of non-housing capital. In addition, financial assets capture *mortgage loans* as well. In particular, a positive amount of financial assets is a claim to the same amount of non-housing capital stock, while a negative amount of financial assets denote mortgage debt of the absolute value of the financial asset position. The use of financial assets help easing the notation by combining the non-housing capital and mortgage loans. I also use housing asset and housing capital interchangeably. Housing asset can be either owned or rented from real estate sector.

Labor and financial assets are traded in competitive markets. By assumption, agents cannot trade state contingent assets to insure away the shocks with respect to labor productivity or mortality. However, agents can save in the form of housing and financial assets and self-insure.

As for the housing assets, agents can either own or rent housing assets but the choice is exclusive. When renting, the unit cost of housing is the rental cost q_t paid to a real estate firm. When owning, an agent has to pay for the depreciation to keep the value of housing. The interpretation of the depreciation is the maintenance cost. In addition, there is a fixed cost χ for owning housing assets. It is a parsimonious way to capture various costs associated with owning; large moving costs, costs for closing mortgage loans, costs for insurance, and so on. It is important to have χ; otherwise, the equilibrium homeownership rate will be substantially higher than the rate in the U.S. economy, given the current preferential tax treatment for owner-occupied housing. Naturally, χ is calibrated to match the homeownership rate in the U.S. economy. An important implication of the fixed cost of ownership is that homeownership rate increases with income, and thus age, like in the U.S. data.[3]

[3] Alternative way to introduce the difference between owning and renting is to assume that there is a minimum

When owning, an agent can use the value of housing assets as collateral. In particular, an agent can borrow up to $(1-\lambda)$ of the value of housing assets that the agent owns. Collateralized borrowing is called mortgage loans. Mortgages loans in the model captures both primary mortgage loans and other types of loans that are secured by the value of housing. There is no unsecured loan. If interpreted as the standard primary mortgage loans, λh is the down payment to own housing of value h. If interpreted as secondary mortgage loans or home equity loans, $(1-\lambda)h$ is the maximum value of mortgages an agent can take out from the housing asset of value h.

Housing services cannot be traded. It implies that, regardless of the tenure status, an agent consumes all the housing services generated by the housing asset that it owns or rents.

3.7 Government Policy

The government is engaged in the following three activities: *(i)* collecting various forms of taxes to finance the public expenditure G_t, *(ii)* collecting estate taxes and distributing them to all surviving agents in lump-sum, and *(iii)* running the pay-as-you-go (PAYGO) social security program.

The government must spend G_t in period t. $\{G_t\}_{t=0}^{\infty}$ is exogenously given. It is the standard set-up in the Ramsey problem. For simplicity, I assume that the government must balance budget each period. In other words, the government must collect taxes whose total amount is G_t in period t. The government can tax capital and labor income. In addition, the government can tax imputed rents of owner-occupied housing assets. I assume that only proportional taxes are available. The tax rates for capital and labor income taxes are denoted as $\tau_{K,t}$, $\tau_{L,t}$, respectively. The tax rate for imputed rents of owner-occupied housing assets is denoted as $\tau_{H,t}$. Notice that capital income taxes are levied only when an agent holds a positive balance of the financial asset. Moreover, the government can allow mortgage interest payment deduction. I use $\tau_{M,t}$ to denote the rate of mortgage interest payment deduction.

Since time of death is stochastic, and there is no private annuity market by assumption, there are accidental bequests. The government imposes a 100% estate tax rate for accidental bequests and distributes all the proceeds equally to all the surviving agents using a lump-sum transfer, in each period. tr_t denotes the lump-sum transfer for each agent in period t.

Finally, the government runs a simple PAYGO Social Security program. The government collects payroll taxes from labor income of working agents at the flat-rate $\tau_{S,t}$. All the proceeds are equally distributed to all the retired agents in each period. The social security benefit is denoted by $b_{i,t}$, where $b_{i,t} = 0$ for $i \leq I_R$, and $b_{i,t} = \bar{b}_t$ for all $i > I_R$. Notice that, since the amount of benefit is the same for all agents regardless of the amount contributed, this particular Social Security program has a strong redistribution effect.

---

size of properties and it is different between owned and rental properties. If the minimum size of owned properties is larger than the minimum size of rental properties, the model also implies that owned properties are on average larger than rented properties. This is the assumption used by Gervais (2002). For the results of the current paper, the results are similar even if the difference in minimum size is used instead of fixed cost of ownership.

3.8 Agents' Problem

The problem faced by agents is formulated recursively. I use a prime to denote a variable in the next period. An agent is characterized by the set of individual state variables (i, e, x), where i is the age, e is the stochastic component of individual productivity, and x is the total wealth. The use of the total wealth x instead of a pair of housing and financial assets (h, a) as a state variable greatly simplifies the problem. But the transformation becomes invalid if there is a fixed cost of changing housing or financial asset holding, and thus it is necessary to keep track of the asset allocation determined in the previous period. The recursive problem for an agent with individual state (i, e, x) and in time t is below:

$$V_t(i, e, x) = \max\left\{V_t^o(i, e, x), V_t^r(i, e, x)\right\} \tag{8}$$

$$V_t^o(i, e, x) = \max_{c\geq 0, h^o\geq 0, a\geq -(1-\lambda)h^o, \ell\in[0,1]} \left\{ u(c, h^o, 1-\ell) + \beta s_i \sum_{e'} p_{ee'} V_{t+1}(i+1, e', x') \right\} \tag{9}$$

subject to

$$x = h^o + a \tag{10}$$

$$(1+\widetilde{r}_t)a + (1-\delta_H - r_t\tau_{H,t})h^o + w_t e\overline{e}_i\ell\left(1-\tau_{S,t}-\tau_{L,t}\left(1-\frac{\tau_{S,t}}{2}\right)\right) + b_{i,t} + tr_t = c + \chi + x' \tag{11}$$

$$\widetilde{r}_t = \begin{cases} r_t(1-\tau_{K,t}) & \text{if } a \geq 0 \\ r_t(1-\tau_{M,t}) & \text{if } a < 0 \end{cases} \tag{12}$$

$$V_t^r(i, e, x) = \max_{c\geq 0, h^r\geq 0, \ell\in[0,1]} \left\{ u(c, h^r, 1-\ell) + \beta s_i \sum_{e'} p_{ee'} V_{t+1}(i+1, e', x') \right\} \tag{13}$$

subject to

$$(1+\widetilde{r}_t)x + w_t e\overline{e}_i\ell\left(1-\tau_{S,t}-\tau_{L,t}\left(1-\frac{\tau_{S,t}}{2}\right)\right) + b_{i,t} + tr_t = c + x' + q_t h^r \tag{14}$$

$$\widetilde{r}_t = r_t(1-\tau_{K,t}) \tag{15}$$

Equation (8) represents the tenure decision. $V_t^o(i, e, x)$ and $V_t^r(i, e, x)$ are the values conditional on owning and renting, respectively. The following two Bellman equations define the values conditional on the tenure choice.

The Bellman equation (9) is the problem for a homeowner. A homeowner chooses consumption c, financial assets a, owned housing assets h^o, wealth carried over to the next period x', and hours worked ℓ to maximize the sum of the current utility and the expected discounted value in the next period, subject to the constrains listed above and explained below.

The first constraint (10) is the asset allocation constraint. The total wealth x is allocated into housing assets h^o and financial assets a. Notice that the agent can borrow up to $-(1-\lambda)h^o$

using mortgage loans collateralized by the value of owned housing assets h^o. In case an agent is using mortgage loans, The size of housing h will be larger than the total wealth x. Any positive amount of housing assets can be owned.

The second constraint (11) is the budget constraint. The first term on the left hand side is the principal and after-tax interest income of financial assets. More explanation on the after-tax interest income is found below. The second terms represents the value of owned housing assets after paying for the housing tax and the depreciation. The interpretation of the depreciation is the maintenance cost to keep the quality of housing. The housing tax is represented as $r_t \tau_{H,t}$, which makes it easier later to compare the cost of renting and owning. The third term is the after-tax labor income. $w_t e \bar{e}_i \ell$ is the before-tax labor income. Since half of the social security contribution is made from the employer and is not subject to labor income tax, $\left(1 - \frac{\tau_{S,t}}{2}\right)$ is multiplied to the labor income tax rate. Since $\bar{e}_i = 0$ for $i > I_R$, labor income is always zero for retired agents. The last two terms on the left hand side are the social security benefit $b_{i,t}$ and the lump-sum transfer tr_t. As $b_{i,t} = 0$ for $i \leq I_R$, the social security benefit is zero for working agents. The lump-sum transfer is originated from the accidental bequests. The right hand side consists of non-housing consumption c, fixed cost associated with owning housing assets χ and total wealth carried over to the next period x'.

Equation (12) defines the after-tax interest income. When the agent is saving ($a \geq 0$), the savings yield the before-tax return of r_t but is subject to the capital income tax at the rate of $\tau_{K,t}$. When the agent is borrowing ($a < 0$), the agent pays the interest rate for the amount of the mortgage loans, but there is a tax deduction of proportion $\tau_{M,t}$ of mortgage interest payments.

Finally, when an agent is owning housing asset, the agent cannot rent, i.e., $h^r = 0$ in this case.

The Bellman equation (13) is the problem for a renter. A renter chooses h^r instead of h^o, and h^r is bounded from below by 0. A renter does not make an asset allocation decision because all the wealth is invested into financial assets by definition of a renter. (14) is the budget constraint for a renter. There is no term for the owner-occupied housing asset and there is a cost of rental properties $q_t h^r$ on the right hand side. x in the first term on the left hand side is replaced by a because $x = a$ for a renter. Finally, for a renter, $h^o = 0$, because of the exclusivity assumption. Also notice that there is no fixed cost χ in the budget constraint, in case housing assets are rented.

The solution to the dynamic programming problem above yields optimal decision rules $c = g_{c,t}(i, e, x)$, $h^o = g_{o,t}(i, e, x)$, $h^r = g_{r,t}(i, e, x)$, $a = g_{a,t}(i, e, x)$, $\ell = g_{\ell,t}(i, e, x)$, and $x' = g_{x,t}(i, e, x)$. The tenure decision is included in $h^o = g_{o,t}(i, e, x)$ and $h^r = g_{r,t}(i, e, x)$. In particular, if an agent is an owner then $h^r = g_{r,t}(i, e, x) = 0$ and $h^o = g_{o,t}(i, e, x) > 0$, and opposite if an agent is a renter.

3.9 Definition of Recursive Competitive Equilibrium

I define the recursive competitive equilibrium and the stationary recursive competitive equilibrium of the economy. In the latter, prices are constant over time. The population size is growing at the constant rate γ, but the age composition of the population is time invariant.

Let $\mathbf{M} = \{\mathbf{1}, \mathbf{2}, ..., \mathbf{I}\} \times \mathbf{E} \times \mathbf{X}$, where $x \in \mathbf{X} \subset \mathbb{R}^{+}$. $\mathbf{X}$ is assumed to be compact. The upper bounds are set such that these bounds are not binding and thus the solution to the problem with the bounds is the same as the one without. The lower bound of $\mathbf{X}$ is zero. $\mathbf{M}$ is the space of individual state variables. Let $m \in \mathbf{M}$ be an element of $\mathbf{M}$. Let $\mathcal{M}$ be the Borel $\sigma - algebra$ generated by $\mathbf{M}$, and let $B \in \mathcal{M}$ be an element of $\mathcal{M}$. Let μ the probability measure defined over $\mathcal{M}$. I will use a probability space $(\mathbf{M}, \mathcal{M}, \mu)$ to represent a type distribution of agents.

Definition 1 (Recursive competitive equilibrium)
Given sequences of government expenditures $\{G_t\}_{t=0}^{\infty}$, social security tax rates $\{\tau_{S,t}\}_{t=0}^{\infty}$, total factor productivity $\{Z_t\}_{t=0}^{\infty}$, and initial conditions K_0, H_0, μ_0, a recursive competitive equilibrium is a sequence of value functions $\{V_t(i,e,x)\}_{t=0}^{\infty}$, optimal decision rules, $\{g_{c,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{o,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{r,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{a,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{\ell,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{x,t}(i,e,x)\}_{t=0}^{\infty}$, aggregate stock of housing and non-housing capital and aggregate labor supply $\{K_t\}_{t=0}^{\infty}$, $\{H_t\}_{t=0}^{\infty}$, $\{L_t\}_{t=0}^{\infty}$, prices $\{r_t\}_{t=0}^{\infty}$, $\{w_t\}_{t=0}^{\infty}$, $\{q_t\}_{t=0}^{\infty}$, transfers $\{tr_t\}_{t=0}^{\infty}$, tax rates $\{\tau_{K,t}, \tau_{L,t}, \tau_{H,t}, \tau_{M,t}\}_{t=0}^{\infty}$, social security benefits $\{b_{i,t}\}_{t=0}^{\infty}$, measures $\{\mu_t\}_{t=0}^{\infty}$, such that:

1. *$\{V_t(i,e,x)\}_{t=0}^{\infty}$ is a solution to the agent's problem defined above. $\{g_{c,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{o,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{r,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{a,t}(i,e,x)\}_{t=0}^{\infty}$, $\{g_{\ell,t}(i,e,x)\}_{t=0}^{\infty}$, and $\{g_{x,t}(i,e,x)\}_{t=0}^{\infty}$, are the associated optimal decision rules.*

2. *The representative firm maximizes its profit. Equivalently, r_t and w_t satisfy the following marginal conditions for all t:*

$$r_t = Z_t F_K(K_t, L_t) - \delta_K$$

$$w_t = Z_t F_L(K_t, L_t)$$

3. *Real estate sector is competitive. Consequently, the rent is determined as follows:*

$$q_t = r_t + \delta_H$$

4. *The following market clearing conditions are satisfied for all t:*

$$K_t = \int_{\mathbf{M}} g_{a,t}(i,e,x) - g_{r,t}(i,e,x)\ d\mu$$

$$H_t = \int_{\mathbf{M}} g_{o,t}(i,e,x) + g_{r,t}(i,e,x)\ d\mu$$

$$L_t = \int_{\mathbf{M}} \overline{e}_i e g_{\ell,t}(i,e,x)\ d\mu$$

5. *Construct the transition function $Q_t(m,B)$ that is consistent with the optimal decision rules and the laws of motion for i and e. Then $\{\mu_t\}_{t=0}^{\infty}$ satisfies the following law of motion:*

$$\mu_{t+1}(B) = \int_{\mathbf{M}} Q(m,B)\ d\mu_t$$

6. *Budget balance regarding the social security program. In particular, the following budget balance condition is satisfied:*

$$\int_{\mathbf{M}} \overline{e}_i e g_{\ell,t}(i,e,x) w_t \tau_{S,t} \ d\mu_t = \int_{\mathbf{M}} b_{i,t} \ d\mu_t$$

7. *The total amount of accidental bequests is equal to the total amount of lump-sum transfers. In particular, the following budget balance condition is satisfied:*

$$\int_{\mathbf{M}} tr_{t+1} \ d\mu_{t+1} = \int_{\mathbf{M}} (1-s_i)\left(g_{h,t}(i,e,x)(1-\delta_H - r_t\tau_{H,t}) + g_{a,t}(i,e,x)(1+\widetilde{r}_t)\right) \ d\mu_t$$

8. *Government budget balance. The following budget balance condition is satisfied:*

$$\begin{aligned} G_t = \int_{\mathbf{M}} & \overline{e}_i e w_t g_{\ell,t}(i,e,x)\left(1-\frac{\tau_{S,t}}{2}\right)\tau_{L,t} \\ & + \max(g_{a,t}(i,e,x),0) r_t \tau_{K,t} + \min(g_{a,t}(i,e,x),0)\tau_{M,t} + g_{o,t}(i,e,x) r_t \tau_{H,t} \ d\mu_t \end{aligned}$$

Definition 2 (Stationary recursive competitive equilibrium)
A stationary recursive competitive equilibrium is a recursive competitive equilibrium where tax rates, total factor productivity, value functions, optimal decision rules, prices, transfers, social security benefits are time invariant. Government expenditures, and aggregate variables are growing at the constant rate γ and thus time invariant at per-capita.

Notice that the market clearing condition for non-housing capital stock includes $-g_{r,t}(i,e,x)$. This is because real estate firms borrow exactly the same amount as housing assets that they rent. The market clearing condition for housing capital stock includes owner-occupied housing assets and the amount of housing assets rented. The four terms in the integrand in the government budget constraint denote labor income taxes, capital income taxes, mortgage interest payment deduction, and housing taxes, respectively.

Since I focus on the stationary equilibrium, I drop the time scripts altogether hereinafter.

4 Calibration

I will describe how the baseline model economy with both housing and financial assets is calibrated. In the last subsection, I will describe how the version of the model economy only with financial assets is calibrated and compare the two economies.

4.1 Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 22. I is set at 79, meaning that the maximum actual age is 100. I_R is set at 43, implying

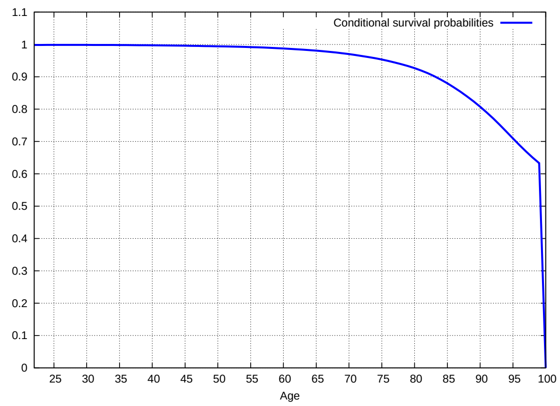


Figure 1: Conditional survival probabilities

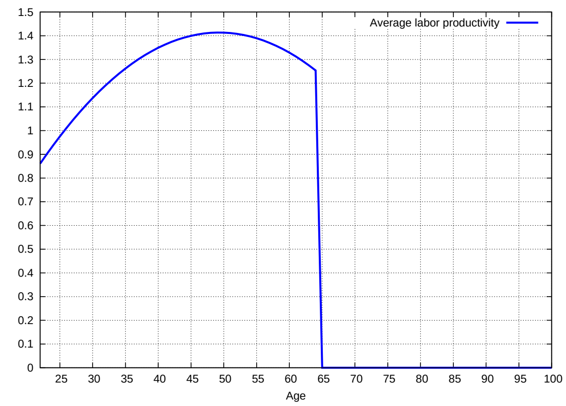


Figure 2: Average life-cycle profile of labor productivity

that the agents become retired at the actual age of 65. The annual population growth rate, γ, is set at 1.2%. This growth rate corresponds to the average annual population growth rate of the U.S. over the last 50 years. The survival probabilities $\{\pi_i\}_{i=1}^{I}$ are taken from the life table (Table 4.C6) in Social Security Administration (2005). Figure 1 shows the conditional survival probabilities used.

4.2 Preference

For the baseline calibration, the following functional form is used:

$$u(c,d,m) = \frac{(c^{\psi}d^{1-\psi})^{1-\sigma}}{1-\sigma} + \eta\frac{m^{1-\rho}}{1-\rho} \tag{16}$$

Leisure m is separable from consumption of aggregated goods, and consumption of non-housing goods c and housing services d are non-separable. The assumption of unit elasticity between housing and non-housing goods is also used by Fernández-Villaverde and Krueger (2005). They refer to some empirical studies estimating the elasticity and claims that the unit elasticity is in the middle of various estimates.

σ is set at 2.0, which is the commonly used value in the literature. ψ is pinned down later to match the relative size of the housing and non-housing capital stock in equilibrium. η is pinned down such that the average hours worked is 0.33 of the disposable time for workers, in equilibrium. ρ is closely associated with the labor supply elasticity. ρ is set at 3, which implies the Frisch elasticity associated with the average hours worked of 0.68.[4] The Frisch elasticity of 0.68 is in the middle of various estimates.

[4] Frisch elasticity can be computed by $\frac{1}{\rho}\frac{1-\overline{\ell}}{\overline{\ell}}$, where $\overline{\ell}$ is the average fraction of time for work.

For the robustness check, I also use the following non-separable functional form:

$$u(c, d, m) = \frac{((c^{\psi} d^{1-\psi})^{\eta} m^{1-\eta})^{1-\sigma}}{1-\sigma} \tag{17}$$

ψ and η are pinned down in the same way as in the baseline specification. σ is pinned down such that the coefficient of relative risk aversion associated with the composite goods of housing services and non-housing consumption goods is 2.[5]

The other parameter for preference, β, will be calibrated such that aggregate amount of wealth in the model matches the U.S. counterpart.

4.3 Endowment

The average life-cycle profile of the earnings $\{\overline{e}_i\}_{i=1}^{I}$ is taken from Hansen (1993). Since Hansen (1993) estimates labor productivity for 5-age groups (age 20-24, 25-29,...), Hansen's (1993) estimates are smoothed out using a quadratic function (second degree polynomial). Figure 2 shows the life-cycle profile of the average labor productivity used in the model. Since mandatory retirement at the model age of I_R, $\overline{e}_i = 0$ for $i > I_R$.

As for the shock component of agents' earnings, I use the data on the cross-sectional variances of log of the hourly wage of the heads of households of Panel Study on Income Dynamics (PSID). According to PSID, the cross-sectional variance of log of the hourly wage of heads of household of age 22 is 0.197, and the same statistics for heads of household of age 64 is 0.674, and the cross sectional variance is almost linearly increasing. Appendix A.1 includes details about the empirical exercise. I basically follow the empirical exercise by Storesletten et al. (2004) but derive the cross-sectional variances of hourly wages of the heads of households over the life-cycle, instead of those of the total earnings of households.

In the model, I assume that the initial distribution of $\log e$ is the normal distribution $N(0, \sigma_e^2)$ and $\log e$ follows the following AR(1) process:

$$\log e' = \rho_e \log e + \epsilon \tag{18}$$

with $\epsilon \sim N(0, \sigma_\epsilon^2)$. There are three parameters, ρ_e, σ_e and σ_ϵ, and these three parameters are pinned down to capture the properties of the PSID data described above. First of all, σ_e^2 is set at 0.197 so that the cross-sectional variances of $\log e$ for age-1 (corresponding to the actual age of 22) agents in the model is equal to the cross-sectional variance of log of the hourly wage of age-22 households. Second, in the data, cross-sectional variance almost linearly increases. It means that the persistence parameter ρ_e must be close to unity for the stochastic process of the model to replicate the property. Therefore, ρ_e is set at 0.99. Finally, σ_ϵ is chosen such that the stochastic process used in the model implies that the cross-sectional variance of $\log e$ for age-44 agents (corresponding to the actual age of 65) is 0.674. This procedure leads to $\sigma_\epsilon^2 = 0.02058$.

[5] Specifically, σ satisfies $1 - CRRA = \eta(1-\sigma)$, where $CRRA$ is the coefficient of relative risk aversion and is 2.0 in the current case.

Finally, the AR(1) process is approximated using a finite state fist order Markov process. I use $n = 9$ as the number of states. The AR(1) process obtained above is converted into the Markov process using the method proposed by Tauchen (1986). The initial distribution of $\log e$ is approximated by assigning the probabilities to each of the grids obtained by applying the Tauchen (1986) method, in the similar way as the Tauchen (1986) method.

4.4 Technology

The production function is the standard Cobb-Douglas type:

$$Y = ZK^{\theta}L^{1-\theta} \tag{19}$$

with $\theta = 0.247$ computed using the National Income and Product Accounts (NIPA). The value of θ is lower than the value usually used in the literature. This is because, in the current model, a part of the widely defined capital income associated with housing capital is removed from the definition of capital income for this economy with two kinds of capital.[6] For comparison, I also calibrate the model with only non-housing capital and financial assets. I recalibrate θ such that there is no distinction between housing and non-housing capital and obtain $\theta = 0.326$, which is close to commonly used value for one-asset models. The depreciation rate for non-housing capital is $\delta_K = 0.109$. The depreciation rate for housing capital is $\delta_H = 0.017$. Both are computed using NIPA. Z is a scaling parameter. I normalize at $Z = 1$.

4.5 Housing Market

There are two parameters related to the housing market, the down payment requirement ratio λ, and the fixed cost associated with owning χ. I set $\lambda = 0.20$. This is consistent with the typical down payment ratio of primary mortgage loans (20%) or loan-to-value (LTV) ratio of 80%. I calibrate χ such that the model generates the homeownership rate in the recent U.S. economy. Except for the very recent years, the homeownership rate stayed around 64% in the U.S. This number is going to be the calibration target. Notice that, without the fixed cost χ (or, equivalently, if $\chi = 0$ is assumed), the homeownership rate in the model will be substantially higher than the number in the U.S. economy.

4.6 Government Policy

Following Domeij and Heathcote (2004), who also use proportional taxes for capital and labor income, I use $\tau_K = 40\%$ and $\tau_L = 27\%$ for the baseline tax rates.[7] As for housing taxes, I assume that owner-occupied housing assets are not taxed and thus set $\tau_H = 0\%$ for the baseline rate.

[6] Díaz and Luengo-Prado (2006) follow the same calibration strategy and come up with similarly low θ.

[7] The tax rates are the averages between 1990 to 1996 of the effective tax rates computed by Mendoza et al. (1994). McGrattan (1994) and Joines (1981) come up with the similar effective tax rates for the U.S.

The baseline rate for the mortgage interest payment deduction is set at 23%. This number is the average marginal subsidy associated with mortgage interest payments, computed by Feenberg and Poterba (2004).

In the U.S., there is no federal tax for owner-occupied housing, but different local governments impose residential property taxes with different rates. For example, according to the Government of District of Columbia, if the tax rates applied in the largest city in each state are compared, the median effective tax rate in 2004 is 1.54%. National Association of Home Builders (NAHB) report that, according to self-reported property tax rates in Census 2000, the national average property tax rate in 2000 is 1.127%. Assuming that housing assets yield 6.0% of their market value annually, the median residential property tax rate roughly corresponds to a tax rate of about 20% for imputed income of housing assets.[8] However, the property taxes levied by local governments are generally considered benefit taxes whose proceeds are used by local government to provide goods and services necessary for homeowners. Following the argument, I consider a zero housing tax rate to be a reasonable assumption. Moreover, as it turns out, the key mechanism in the model is the tax wedge between owned and rented housing. Since the property tax does not change the tax wedge, as long as the property tax is passed on to renters by real estate sector, I expect that the introduction of property tax does not change the main results of the paper.

The size of the government expenditure is obtained ex-post in the stationary equilibrium of the model economy with the baseline specification. In the baseline model with the tax rates described above, the amount of government expenditures relative to output turns out to be 23.6%, which is close to the average size of expenditures of the U.S. federal government.

The Social Security tax rate is set at 7.4%. According to Social Security Administration (2005), the average labor income in 2003 is USD 32,808, while the average annual benefit of retired workers is USD 11,065.[9] The replacement ratio, defined as the ratio between the two, is 33.7%. The 7.4% social security tax rate in the model is determined such that, when the government is balancing budget in each period, the model replicates the replacement ratio.[10]

In the current U.S. economy, the social security tax rate associated with the retirement benefit (Old-Age and Survivors Insurance, OASI) is 10.6%. According to Social Security Administration (2005), the taxable labor income is 86% of the labor income, implying the effective tax rate is about 9%. The social security tax rate used in the baseline specification is slightly lower than 9%. The difference is due to a combination of reasons: *(i)* the average retirement age is getting lower and actually lower than 64 in the recent years, *(ii)* the relative size of working population is large right now due to the baby boomer generation. *(iii)* whereas the budget balance is imposed in the model for simplicity, the Social Security Trust Fund is accumulating in the U.S. economy.

[8] 6% is the equilibrium interest rate in the baseline model economy and close to the historical average of the net (after-depreciation) return of capital of the U.S. economy.

[9] This number is computed by multiplying the monthly benefit of retired workers of USD 922.1 by 12.

[10] Government budget balance implies $\tau_S m_W \overline{e} = \overline{b} m_R$ where m_W and m_R are measures of workers and retirees, respectively, and $\overline{e}$ and $\overline{b}$ represent the average labor income and benefits, respectively. Plugging in $\frac{\overline{b}}{\overline{e}} = 0.337$ and $\frac{m_R}{m_W} = 0.221$ yields $\tau_S = 0.074$.

In other words, the social security contribution exceeds the total amount of benefits in the U.S. economy. *(iv)* there is no dependent nor survivor in the model.

4.7 Endogenously Calibrated Parameters

As I mentioned above, three parameters regarding the preference, the time discount factor β, the parameter that determines the relative value of the utility from housing services, ψ, the parameter which determines the relative value of leisure, η, and the fixed cost associated with ownership, χ, are pinned down jointly such that, in the stationary equilibrium with the baseline specifications, the total value of housing capital stock and that of non-housing capital stock, the average hours spent for working, and the homeownership rate are close to their counterparts in the U.S. economy. In particular, according to the NIPA, the average value for the period 2002-2006 of private housing capital relative to output ($\frac{H}{Y}$) is 1.29, while the same statistic for the non-housing capital ($\frac{K}{Y}$) for the same period is 1.47. In total, the average value of total private capital stock over output is 2.76 in the U.S. As for the time spent for work, on average, workers spent one-third of their disposable time for work. Therefore, I use $\bar{\ell} = 0.33$ as the target. The target homeownership rate is 64%.

To pin down the four parameters, I need to compute the equilibrium of the model repeatedly with different parameter values, until the statistics generated by the model are close to the corresponding targets. Even though there is no guarantee that all the targets can be satisfied, because of the non-linear nature of the problem, the calibration process turned out to be successful, and I found that $\beta = 0.9734$ $\psi = 0.8793$, $\eta = 4.6507$, and $\chi = 0.0060$ jointly satisfy the four targets: $\frac{H}{T} = 1.29$, $\frac{K}{Y} = 1.47$, $\bar{\ell} = 0.33$, and the homeownership rate of 0.64.

For models other than the baseline, the parameters are re-calibrated such that the same set of statistics are simultaneously satisfied.

4.8 Model Economy with One Asset

Since one of the goals of the paper is to compare the optimal capital taxation in the economy with two types of assets and in the economy with one type of asset, I construct the version of the model economy where only financial assets are available. The one-asset model is constructed by treating housing assets as part of financial assets. Table 1 compares the two model economies.

In the economy with one asset, the parameter controlling the capital share of income, θ is higher because the capital income includes what is generated by housing capital. According to NIPA, θ for the one-asset model turns out to be 0.326, which is close to the value which is usually used in the models with one asset.

The depreciation rate for capital δ_K is adjusted, taking into account that capital in the one-asset model also includes housing capital which depreciates more slowly than non-housing capital. Naturally, the depreciation rate is lower. According to NIPA, the annual depreciation rate

Table 1: Comparison of the Model Economies

Economy	Baseline (two-asset) model	One-asset model
Aggregate statistics		
(H+K)/Y	2.7600	2.7600
H/Y	1.2900	–
K/Y	1.4700	2.7600
Parameters		
β	0.9734	0.9806
ψ	0.8793	–
η	4.6507	3.4255
χ	0.0060	–
θ	0.2470	0.3260
δ_H	0.0170	–
δ_K	0.1090	0.0660

associated with the one-asset model is 6.6%.

Notice that, the parameters β and η are re-calibrated for the one-asset model such that the model satisfies the capital output ratio of 2.76 and the average fraction of time spent on working at 0.33. ψ and χ are not used in a meaningful way in the one-asset model.

5 Computation

Since the model cannot be solved analytically, numerical methods are used to compute the stationary equilibrium of the model. The solution method is a standard one for the overlapping generation models.[11] In solving the problem of an individual agent, the optimal decision rules are approximated using piecewise-linear functions, and the optimal decision rules are obtained backwards, starting from the last period of life.

A challenge for the current model is that there are two types of assets. If the set of individual state variables includes two endogenous continuous state variables, then the model is very difficult to solve with a decent level of accuracy. This is especially so if there is a tenure choice as well as labor-leisure decision. However, it is feasible to solve the current model, because there is only one continuous state variable, which is the total wealth x. The set of individual state variables of agents does not include h and a separately but does include only $x = h + a$, because the distinction is not important for agents' optimal decision.[12]

[11] For more details on the computational methods employed here, see Ríos-Rull (1999).

[12] A negative a means taking mortgage loans, which enables an agent to own housing assets whose value (h) is

In obtaining the aggregate statistics, I implement a simulation with $1,000,000$ agents in each generation. Appendix A.2 includes details of the computation.

6 Properties of the Baseline Model Economy

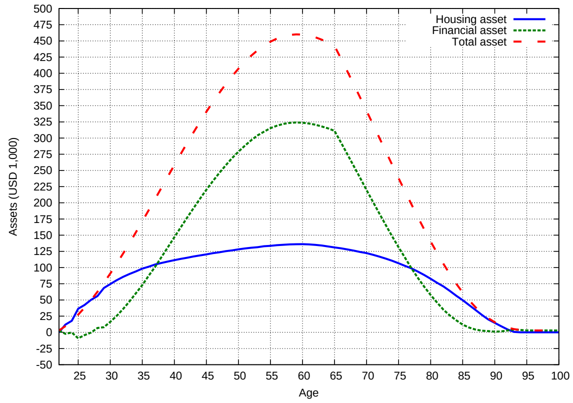
Figure 3 summarizes the average life-cycle profiles in the baseline model economy. Figure 3(a) shows the average life-cycle profile of housing and financial asset holding, as well as the total wealth in the baseline model economy. The most striking thing is that the portfolio allocation between housing and financial assets vary greatly with age. At the beginning of their life, agents save to prepare for the down payment of their first house. They rent while doing so. Then agents borrow using mortgage loans and accumulate housing assets. Since around age 30, average agents finish replaying mortgage loans and start accumulating savings in the form financial assets, after accumulating sufficient housing asset to support sufficient amount of consumption of housing services. After retirement, agents reduce financial asset holding more quickly compared with housing assets, because agents need housing assets for consumption of housing services. Towards the end of the life-cycle, agents reduce holding of both types of assets. The hump-shape of durable goods, whose main component is housing, is well documented by Fernández-Villaverde and Krueger (2005). In terms of the ratio of housing asset over the total wealth, the ratio is much higher for young agents because they use leverage when they own housing assets whose value is larger than the value of their total wealth. The ratio keeps going down as agents accumulate the financial asset relative to the housing asset. Silos (2007) documents the pattern of the housing-to-wealth ratio in the U.S. data.

Figure 3(b) shows the average life-cycle profile of before-tax and after-tax income, labor income, tax payment, and consumption in the baseline model economy. The after-tax total income in the figure includes the social security benefit, and excludes tax payments and social security contribution. The profile of the after-tax total income is flatter than that of the before-tax total income not only because of the intergenerational transfer through the social security program, and also because workers are taxed more heavily than retirees. As you can see from Figure 3(c), which shows the average life-cycle profile of tax payments, majority of taxes paid by workers is labor income tax. On the other hand, retirees, especially the recent ones, pay only the capital income tax. Consumption of non-housing goods is hump-shaped, as in the U.S. data. The hump-shape of non-housing consumption is carefully documented by Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2005).

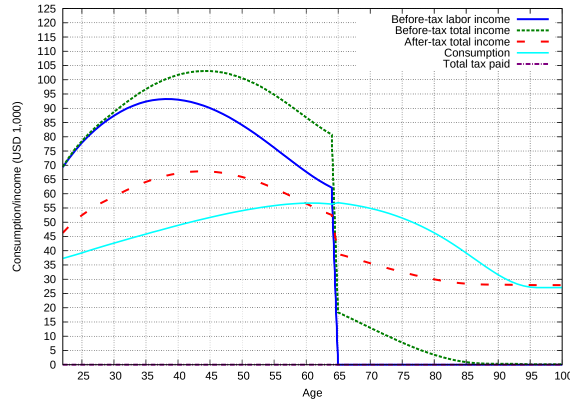
Figure 3(d) shows the average life-cycle profile of hours worked. It is decreasing over the life-cycle up to the mandatory retirement age of 65. After 65, there is no hour for work.

Figure 3(e) shows the homeownership rate for each age group in the baseline model economy. The overall average homeownership rate is 64% in the model. It exhibits a hump-shape, like in the U.S. data. The ratio is low for young agents, peaks around age 55, and goes down after retirement age. Table 2 compares the homeownership rate of different cohorts in the baseline

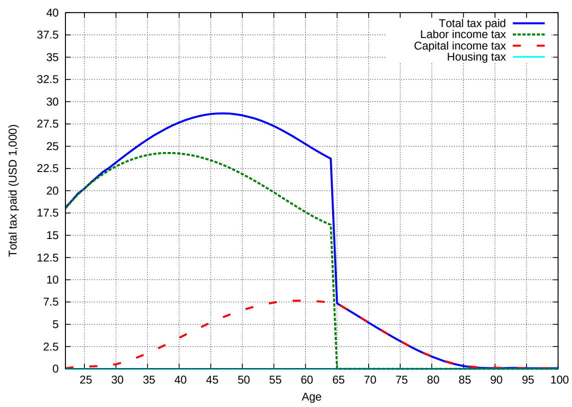
larger than the value of total wealth (x).



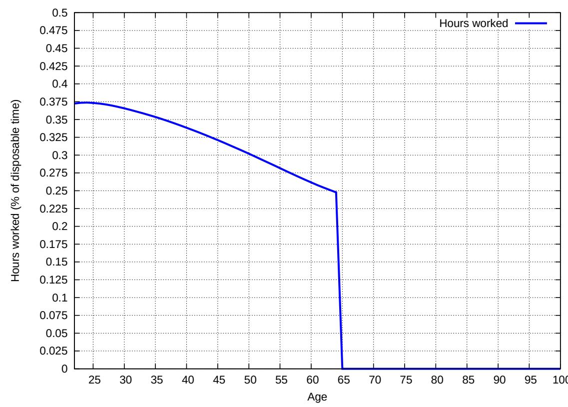
(a) Asset allocation



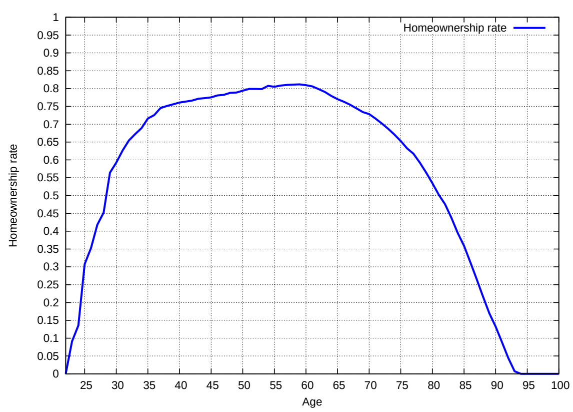
(b) Consumption and income



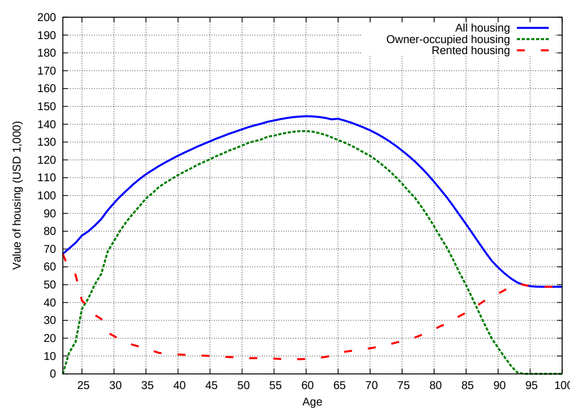
(c) Taxes



(d) Hours worked



(e) Homeownership rate



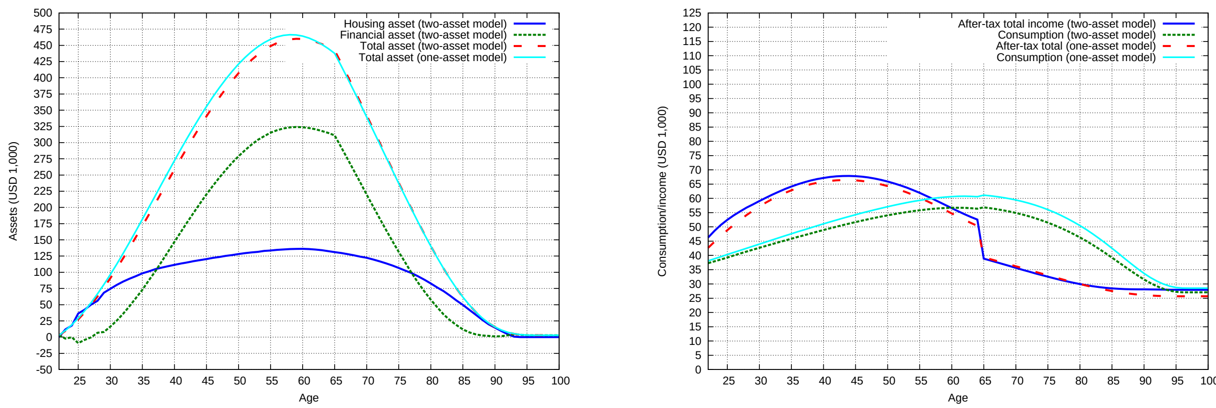
(f) Housing assets: owned and rented

Figure 3: Average life-cycle profiles in the baseline model economy

Table 2: Homeownership rate: U.S. and Baseline Model

Age	Total	–29	30-34	35-44	45-54	55-64	65-74	75–
U.S.[1]	0.658	0.279	0.529	0.665	0.758	0.801	0.818	0.757
Baseline model	0.640	0.324	0.634	0.733	0.781	0.783	0.727	0.512

[1] Source: American Housing Survey for the United States in 1997.



(a) Asset allocation

(b) Consumption and income

Figure 4: Comparison of two-asset and one-asset model

model economies and the U.S. economy in 1997. The homeownership rate declines slightly faster in the late stage of life in the model, but the model captures the general shape of the data counterpart, including the location of the peak.

Figure 3(f) shows the average life-cycle profile of all housing assets, including owner-occupied as well as rented housings. The profile for the owner-occupied housing is the same as the one shown in Figure 3(a). For young and retired agents, more housing assets are rented rather than owned. Therefore, the average life-cycle profile of total housing assets is much higher than the owned housing assets for the young and the retirees.

Figure 4(a) and 4(b) compare average life-cycle profiles of assets and consumption between the two-asset baseline model and the one-asset model. The striking feature is that the life-cycle profiles of total wealth and consumption are very close to each other, although there is an interesting life-cycle for the asset portfolio between the housing and financial assets in the two-asset model.

7 Design of Experiments

7.1 Design of Alternative Tax Schemes

I will investigate the optimal capital income tax rate in the model with housing and financial assets. Once the optimal capital income tax rates in the two-asset environment are obtained, the obtained optimal tax rates are compared with the optimal capital income tax rate in the one-asset model. Since the capital income taxation in the standard one-asset environment corresponds to the taxation on the returns of the financial assets in the current two-asset environment, I will focus on the changes in τ_K.

In order to investigate the optimal capital income taxation, it is necessary to determine the housing tax policy, which is not present in the standard one-asset environment. I investigate two cases. In the first case, I change the capital income tax rate τ_K, while maintaining the preferential tax treatment for owner-occupied housing. In particular, I keep $\tau_H = 0$ and $\tau_M = 0.23$. In the second case, I eliminate the preferential tax treatment for owner-occupied housing. More specifically, I impose that the housing and financial assets be taxed at the same rate, i.e., $\tau_K = \tau_H$, and I eliminate the mortgage interest payment deduction ($\tau_M = 0$). Then I change $\tau_K = \tau_H$ to various rates. Under this polity, the two types of assets not distinguished in terms of taxation.

All experiments are implemented in the revenue neutral manner. In other words, total government expenditure, or the total amount of taxes, G, is fixed at the level of the baseline model economy. The size of G in the baseline two-asset model economy is obtained ex-post from the baseline line tax rates, and is 23.6% of the total output. In order to achieve revenue neutrality, the proportional labor income tax rate τ_L is adjusted such that the total amount of taxes is the same across different experiments with different tax regimes.

7.2 Welfare Measures

In comparing the welfare of agents in economies with different tax regimes, I use the ex-ante expected utility of newborns in the stationary equilibrium. This criterion is used by Conesa et al. (2007). Average welfare is computed by integrating the value of the newborns in the stationary equilibrium with respect to the initial shock to individual labor productivity. The welfare criterion is useful in taking into account both the efficiency effect due to tax reforms and the redistribution or insurance aspect of tax reforms. The consideration of the latter is crucially important in experiments where markets are incomplete, and therefore, agents are ex-post heterogeneous.

In measuring the magnitude of the welfare gain or loss, I use the percentage changes in the flow consumption of non-housing goods. This is a standard measure for welfare analysis in the literature. Using this measure, the welfare gain by moving from one tax regime to another is defined as the percentage increment ϵ to the consumption of non-housing goods in every period and under every contingency in the economy with the original tax regime, which equates average welfare in the economy with the original tax regime to that of the economy with the alternative tax regime. A positive ϵ implies that agents are better off by being born in the alternative environment, in the expected ex-ante sense. Notice that, in the current economy, there are three sources of utility, namely, consumption of non-housing goods and consumption of housing services, and leisure. On the other hand, in computing the welfare gain, percentage ϵ is added only to the consumption of non-housing goods. In other words, welfare changes associated with changes in the consumption of housing services as well as leisure are converted and merged into the welfare changes in the consumption of non-housing goods in computing the welfare gain.

Moreover, for analytical purposes, I decompose ϵ as follows:

$$\epsilon = \epsilon_g + \epsilon_i + \epsilon_d \tag{20}$$

where ϵ_g is the welfare gain associated with the age-independent uniform increase in consumption of non-housing goods, housing services, and leisure. I call it the *average* effect. ϵ_i is the welfare gain associated with the redistribution of consumption across the life-cycle. Therefore, I call it the *life-cycle* effect. Finally, ϵ_d represents the welfare gain associated with the within-cohort redistribution. I call the effect as the *redistribution* effect. The formal definitions and computation of the welfare measures are provided in Appendix A.3.

8 Optimal Capital Income Taxation

8.1 One-Asset Model

I will start describing the optimal capital income tax rates in the model without housing (one-asset model), in order to facilitate the comparison between the one-asset and the two-asset model in the next section.

Table 3 compares the one-asset model economy with the baseline tax rates with the same economy with the optimal capital income tax rate τ_K. The optimal τ_K is 37%, which is not only zero, but far from zero, as Conesa et al. (2007) find. It is indeed close to the average capital income tax rate of the U.S. economy, which is the baseline tax rate of 40%. Conesa et al. (2007) find that the optimal capital income tax rate in a model with utility function that is separable between consumption goods and leisure to be 21%, but with a large amount of deduction, as large as 26% of the average income.[13] The difference between their result and the result here can be understood by the existence of the deduction; with a large amount of deduction, a higher labor income tax can be imposed without making young or low-productivity agents suffer too much. In the current setting where only proportional taxes are available, a high labor income tax rate which is associated with a low capital income tax rate has a strong negative redistribution effect by taxing too much the young and the low-productivity agents.

[13] Conesa et al. (2007) find that the optimal capital income tax rate with non-separable utility function between consumption goods and leisure to be 36%, with a deduction for labor income tax as large as 17% of the average total income.

Table 3: Optimal Capital Income Taxation in the One-Asset Model

Economy	Baseline	Optimal	Zero τ_K
Tax rates			
τ_H	–	–	–
τ_K	0.4000	0.3700	0.0000
τ_L	0.2700	0.2765	0.3466
% change from the baseline[1]			
Output	0.5070	+0.37	+3.46
Total capital	1.3994	+1.48	+15.38
Housing capital	–	–	–
Non-housing capital	1.3994	+1.48	+15.38
Average hours worked	0.3300	−0.16	−1.79
Labor supply	0.3103	−0.17	−1.85
Consumption	0.2799	+0.09	+0.27
Welfare[2]			
Overall change in welfare ϵ	–	+0.01	−1.17
Average effect ϵ_g	–	+0.22	+1.73
Life-cycle effect ϵ_i	–	−0.24	−3.25
Redistribution effect ϵ_d	–	+0.03	+0.35

[1] Level is shown for the baseline economy.

[2] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.

As Conesa et al. (2007) argue, it is optimal to tax capital income heavily in a model where there is a strong life-cycle savings motive and thus savings decision of agents is not strongly elastic against changes in capital income tax rate. They argue that the inelasticity of saving relative to labor supply makes a high capital income tax rate optimal in the economy with life-cycle, while it is optimal *not* to tax capital in an economy without life-cycle.

In response to a decline in the capital income tax rate and an increase in the labor income tax rate, capital stock increases by 1.48%, while labor supply declines by 0.17%. As a result, total output increases by a small 0.37%. Since the difference between the baseline tax rates (40%) and the optimal tax rate (37%) is small, the welfare gain of moving to the optimal tax regime is small as well. The welfare gain is a mere 0.01% of flow consumption.

Suppose capital income tax rate is brought down to zero, which is the optimal rate in the growth model with the infinitely-lived representative agent. There will be a substantial increase in capital stock accompanied by a substantial drop in labor supply. At the end, output increases by 3.5% but consumption does not increase as much as output. Regarding the welfare effect, zero capital income tax rate is predicted to generate a welfare *loss* as large as 1.2% of flow consumption.

Table 4: Optimal Capital Income Taxation in the Two-Asset Model

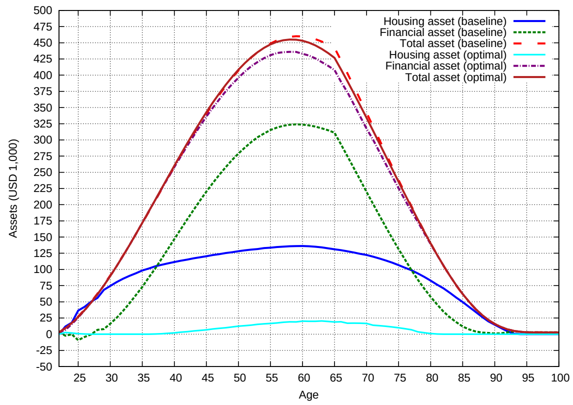
Economy	Baseline	Optimal	Optimal	Zero τ_K
Preferential tax for housing	Yes	Yes	No	Yes
Tax rates				
τ_H	0.0000	0.0000	0.4300	0.0000
τ_K	0.4000	0.1300	0.4300	0.0000
τ_L^1	0.2700	0.2985	0.2398	0.3194
τ_M	0.2300	0.2300	0.0000	0.2300
% change from the baseline[2]				
Output	0.3488	+1.54	+0.34	+1.81
Total capital	0.9627	+3.84	−7.73	+7.41
Housing capital	0.4500	−4.69	−19.04	−0.30
Non-housing capital	0.5128	+11.32	+2.19	+14.19
Average hours worked	0.3300	−1.55	−0.37	−1.99
Labor supply	0.3074	−1.47	−0.26	−1.95
Consumption	0.1876	+1.41	+3.27	+0.73
homeownership rate[3]	0.6400	0.0243	0.0000	0.0000
Welfare[4]				
Overall change in welfare ϵ	–	+2.29	+3.28	+1.63
Average effect ϵ_g	–	+2.23	+0.67	+2.59
Life-cycle effect ϵ_i	–	−0.85	+2.04	−2.01
Redistribution effect ϵ_d	–	+0.91	+0.56	+1.05

[1] Adjusted to guarantee revenue neutrality.
[2] Level is shown for the baseline economy.
[3] Level is shown for all economies.
[4] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.

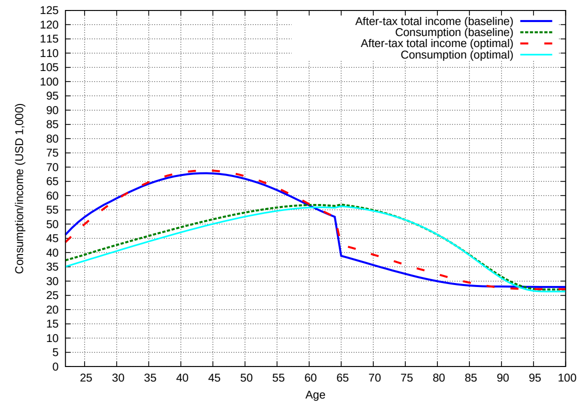
8.2 Two-Asset Model: With Preferential Tax for Housing

Table 4 compares the two-asset model economy with the baseline tax rates and the same model with alternative tax rates. The second column shows the properties of the model with the baseline tax rates. The next three columns show the properties of the model with the optimal capital income tax rate and the preferential tax treatment for housing, the model with the optimal capital income tax rate *without* the preferential tax treatment for housing, and the model with zero capital income tax rate and preferential tax treatment for housing, respectively.

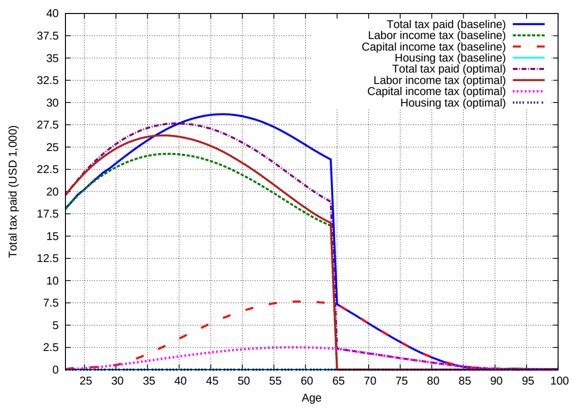
I find that, when the preferential tax treatment for owner-occupied housing is maintained, the optimal capital income tax rate is 13%. This is very low compared the baseline tax rate of 40% and the optimal capital income tax rate in the one-asset model (37%). Labor income tax rate is increased from the baseline rate of 27% to 30% in order to keep the revenue neutrality. As in the one-asset model, labor supply declines while capital stock increases. More interestingly,



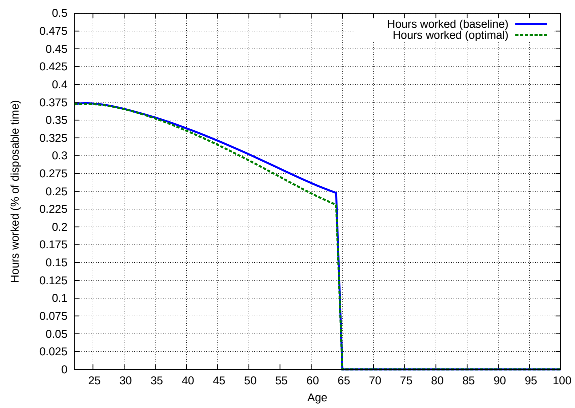
(a) Asset allocation



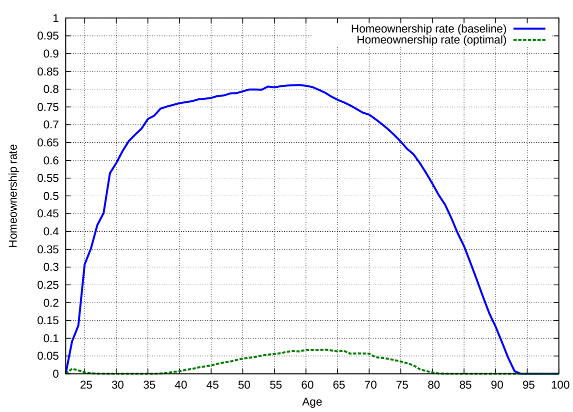
(b) Consumption and income



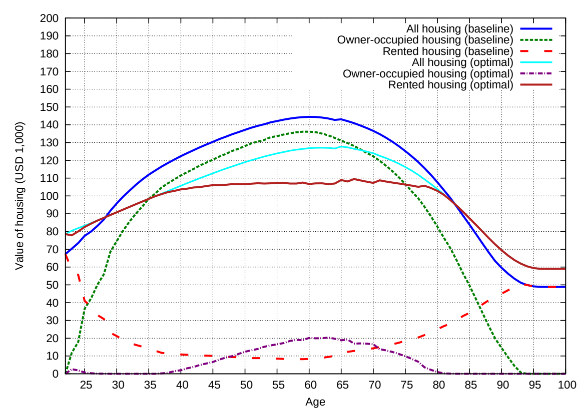
(c) Taxes



(d) Hours worked



(e) Homeownership rate



(f) Housing assets: owned and rented

Figure 5: Average life-cycle profiles of two-asset model with optimal capital income tax rate (with preferential tax treatment for owner-occupied housing)

there is a substantial portfolio reallocation between housing and non-housing capital; housing capital decreases by 5%, while non-housing capital increases by more than 11%. Because of the large increase in non-housing capital stock, output also increases, by 1.5%. Average consumption increases similarly (1.4%).

Homeownership rate drops dramatically from the baseline rate of 64% to a mere 2%. This is because the preferential tax treatment for owner-occupied housing is almost completely nullified when the capital income tax is reduced to 13%. The only agents who still own housing are the ones who have a large amount of assets, and the small tax advantage for owner-occupied housing still makes owning more attractive even with the additional fixed cost associated with ownership. For a large majority of agents, the tax advantage of ownership is not large enough to induce them to own and pay the additional fixed cost associated with ownership.

Figure 5 compares the average life-cycle profiles of model economies with the baseline tax rates and the optimal tax rates together with the preferential tax treatment for owner-occupied housing. Figure 5(a) shows that, even though the change in the total wealth is small, there is a substantial portfolio reallocation from housing to financial assets. The figure shows that agents own very few housing assets with the optimal tax scheme. Instead, most agents rent from the real estate sector. Figure 5(f) shows how the housing capital is reallocated from owner-occupied housings to rental properties. With the optimal tax regime, not only the total housing capital declines, the ownership of the majority of housing capital shifts from agents to real estate firms. Figure 5(e) shows how the homeownership rate drops in response to the change to the optimal tax regime. Figure 5(b) and Figure 5(d) show that the changes in consumption and labor supply are small. But Figure 5(c) show that there is a noticeable change in the life-cycle profile of tax payments; the young pay more taxes in the alternative tax regime. Since the main source of income for the young is labor income rather than capital income, the result implies a possibility of further welfare gain by introducing progressive labor income taxation (including the flat-tax with an deduction).

The bottom part of Table 4 shows the welfare gain associated with the optimal capital income tax of 13%. The welfare gain is as large as 2.3% of flow consumption. The main source of welfare gain is the *average increase* in non-housing consumption goods and leisure. Housing capital drops (-4.7%) but increases in non-housing consumption goods ($+1.4\%$) and leisure (hours worked drop by 1.6%) more than offset the drop in consumption of housing services. Life-cycle effect is negative (-0.85%), which is consistent with a larger tax burden for the young. But the magnitude of the negative life-cycle welfare effect is smaller than the positive average effect. There is also a positive redistribution effect ($+0.91\%$).

The last column implies that the average effect is even bigger if the capital income tax rate is further lowered to zero, but the welfare loss due to the life-cycle effect becomes dominating. This also implies that, if the deduction or progressivity of labor income tax is introduced, the optimal capital income tax rate might be further lowered than the current optimal level of 13%. The optimal capital income tax rate cannot go below 13% because the labor income tax rate becomes too high so that the negative life-cycle effect of high tax payments by the young becomes dominating.

8.3 Two-Asset Model: No Preferential Tax for Housing

The fourth column of Table 4 shows the effect of moving from the baseline tax rates to the optimal tax rates when the preferential tax treatment for owner-occupied housing is eliminated. In particular, *(i)* imputed rents of owner-occupied housing are taxed at the same rate as the capital income ($\tau_K = \tau_H$), and *(ii)* there is no mortgage interest payment deduction($\tau_M = 0$). Figure 6 summarizes the changes in the average life-cycle profile of asset allocations, consumption and income, tax payments, hours worked, homeownership rate, and allocation of housing capital. The optimal capital income tax rate when the tax rate applied to the imputed rents of owner-occupied housing is equalized to capital income tax rate is 43%. This is not only different from the previous case ($\tau_K = 13\%$) but also higher than the baseline tax rate of 40% and the optimal capital income tax rate in the one-asset model (37%).

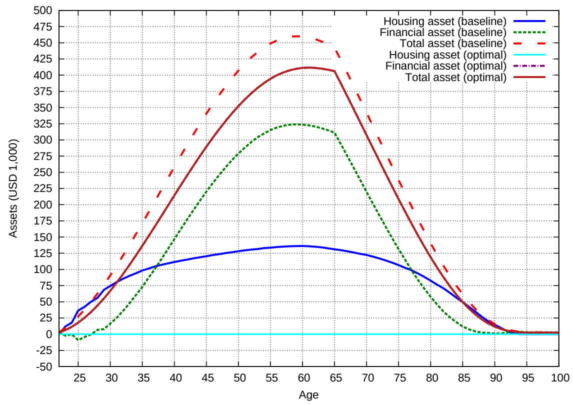
As for the asset level and allocation there are two effects. First, since tax rates applied to both assets are increased, the total wealth drops by as large as 7.7%. Second, since the tax rate applied to owner-occupied housing assets are increased substantially compared with the change in the capital income tax rate, there is a strong reallocation of assets from housing to non-housing capital. As a result, total housing capital declines by 19%, while non-housing capital actually increases by 2.2%. Since the labor income tax rate can be brought down to 24%, labor supply increases. As a result, output increases by 0.3%, but non-housing consumption increases substantially, by 3.3%. The increase in non-housing consumption is due to the shift away from housing capital, more so than the increase in output.

The changes in the average life-cycle profiles exhibited in Figure 6 are similar to the case where $\tau_H = 0$, shown in Figure 5, with one important difference; the average life-cycle profile of tax payments is shifted towards the old. A higher $\tau_K = \tau_H$ enables a lower labor income tax rate and thus a lower tax payment of the young, while keeping the tax wedge between capital income tax and housing tax rate to be zero.

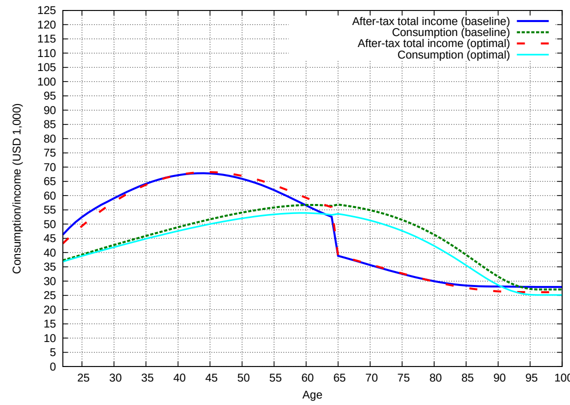
The total welfare gain is large, even larger than the previous case with preferential tax treatment for owner-occupied housing; the welfare gain is equivalent to 3.3% increase in the flow consumption. The average effect (+0.67%) is smaller than in the case of $\tau_H = 0$ because a large increase in non-housing consumption (+3.3%) is offset by a large drop in housing capital (−19%). However, there is a large positive life-cycle effect (+2%). This is associated with the shift of tax payments to older agents. Redistribution effect (+0.56%) is not large compared with other effects.

8.4 Decomposition of Preferential Tax Treatment for Owner-Occupied Housing

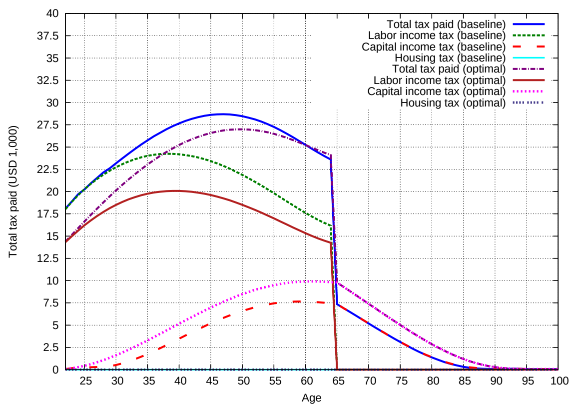
There are two differences between the first and the second experiments; in the second experiment, *(i)* imputed rents of owner-occupied housing are taxed, and *(ii)* there is no mortgage interest payment deduction. What are the effect of each of the two on the optimal capital income tax rate? Table 5 shows the optimal capital income tax rates when either one of the two components



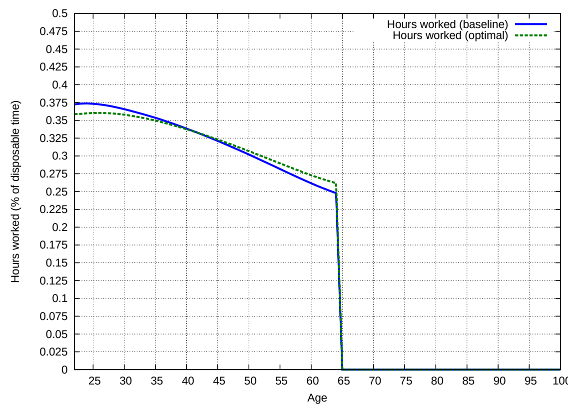
(a) Asset allocation



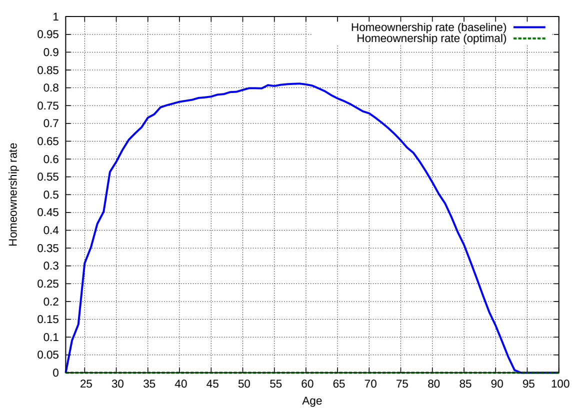
(b) Consumption and income



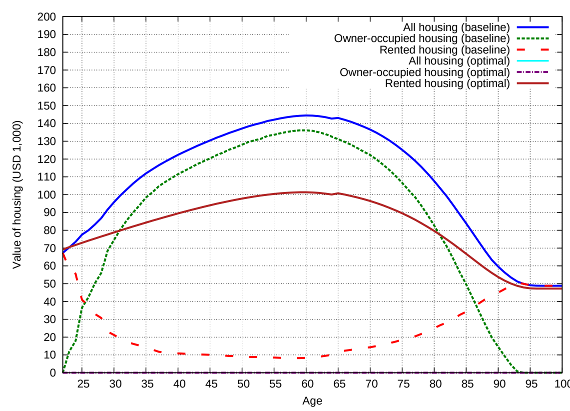
(c) Taxes



(d) Hours worked



(e) Homeownership rate



(f) Housing assets: owned and rented

Figure 6: Average life-cycle profiles of two-asset model with optimal capital income tax rate (No preferential tax treatment for owner-occupied housing)

Table 5: Optimal Capital Income Taxation in the Two-Asset Model: Decomposition

Economy	Baseline	Optimal	Optimal	Optimal	Optimal
Taxing imputed rents	No	No	No	Yes	Yes
Mortgage interest payment deduction	Yes	Yes	No	Yes	No
Tax rates					
τ_H	0.0000	0.0000	0.0000	0.4300	0.4300
τ_K	0.4000	0.1300	0.1300	0.4300	0.4300
τ_L^1	0.2700	0.2985	0.2985	0.2398	0.2398
τ_M	0.2300	0.2300	0.0000	0.2300	0.0000
% change from the baseline[2]					
Output	0.3488	+1.54	+1.54	+0.34	+0.34
Total capital	0.9627	+3.84	+3.83	−7.73	−7.73
Housing capital	0.4500	−4.69	−4.70	−19.04	−19.04
Non-housing capital	0.5128	+11.32	+11.32	+2.19	+2.19
Average hours worked	0.3300	−1.55	−1.55	−0.37	−0.37
Labor supply	0.3074	−1.47	−1.47	−0.26	−0.26
Consumption	0.1876	+1.41	+1.41	+3.27	+3.27
homeownership rate[3]	0.6400	0.0243	0.0235	0.0000	0.0000
Welfare[4]					
Overall change in welfare ϵ	–	+2.29	+2.29	+3.28	+3.28
Average effect ϵ_g	–	+2.23	+2.23	+0.67	+0.67
Life-cycle effect ϵ_i	–	−0.85	−0.85	+2.04	+2.04
Redistribution effect ϵ_d	–	+0.91	+0.92	+0.56	+0.56

[1] Adjusted to guarantee revenue neutrality.
[2] Level is shown for the baseline economy.
[3] Level is shown for all economies.
[4] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.

of the preferential tax treatment for owner-occupied housing is turned off. The basic conclusion is that almost all the results are driven by *(i)* (taxing imputed rents of owner-occupied housing). The role of mortgage interest payment deduction is negligible.

First, compare the third and the fourth columns of Table 5. The only difference between the two is that, in the fourth column, only the mortgage interest payment deduction is turned off. The result is basically identical, except for a small difference in the homeownership rate (it is 2.43% with mortgage interest payment deduction and 2.35% without). Next, compare the last two columns. The only difference is again mortgage interest payment deduction is turned off in the last column. All the results are identical. It is not surprising because homeownership rate is zero when imputed rents of owner-occupied housing is taxed at the rate as high as 43%. It means that the tax rate for mortgage loans does not matter.

8.5 Optimal Capital Income and Housing Tax Rates: Intuition

What can we learn from the experiments? First of all, in the one-asset model, the optimal capital income tax rate is determined to balance the following two opposing forces: *(i)* higher capital income tax is welfare improving as efficiency loss associated with taxing capital rather than labor is smaller due to the inelasticity of capital, *(ii)* higher capital income tax is welfare reducing as a higher tax on labor income puts disproportionately large tax burden on the young and the low-productivity agents. The 37% optimal capital income tax rate is mainly the result of the balance between the two forces.

If the difference between hosing and non-housing capital is explicitly taken into account, there are following three additional forces which affect the optimal capital income tax rate: *(iii)* a smaller tax wedge between housing and non-housing capital relaxes the over-accumulation of housing capital induced by the preferential tax treatment for owner-occupied housing, *(iv)* a smaller wedge between housing and non-housing capital makes the relative cost of renting lower compared with owning and thus inducing agents from owning and paying the additional fixed cost of owning, *(v)* efficiency loss of taxing capital is higher when agents can *evade* from capital income taxation by shifting their portfolio to housing, if owner-occupied housing is taxed at a lower rate.

The point *(iv)* requires further explanation. The budget constrain for a renter, with the equilibrium rent $q = r + \delta_H$, is:

$$(1 + r(1 - \tau_K))x + y = c + x' + (r + \delta_H)h^r \tag{21}$$

where $y = we\bar{e}_i\ell\left(1 - \tau_S - \tau_L\left(1 - \frac{\tau_S}{2}\right)\right) + b_i + tr$ is a short-hand notation of income other than those related to assets. The budget constraint for an owner is:

$$(1 + \widetilde{r})a + (1 - \delta_H - r\tau_H)h^o + y = c + \chi + x' \tag{22}$$

First, assume that the owner is not borrowing ($a \geq 0$), then $\widetilde{r} = r(1 - \tau_K)$. Plugging $x = a + h^o$ into the owner's budget constraint and rearrange terms, we can obtain:

$$(1 + r(1 - \tau_K))x + y = c + x' + \chi + (r + \delta_H + r(\tau_H - \tau_K))h^o \tag{23}$$

By comparing (21) and (23), it is easy to see that the additional benefit of owning rather than renting is $r(\tau_K - \tau_H)h^o$, which is bigger when $(\tau_K - \tau_H)$ is larger, while the additional cost of owning is χ. The additional benefit of owning becomes lower when either τ_H is increased or τ_K is lowered. When $\tau_H = 0$ is imposed, lowering τ_K reduces the additional benefit of owning, and induces agents away from owning, exactly in the same way as the case when τ_H is increased. Therefore, there is a close relationship between eliminating the preferential tax treatment of owner-occupied housing and lowering capital income tax rate.

Second, in case the owner is borrowing ($a < 0$), then $\widetilde{r} = r(1 - \tau_M)$. The budget constraint for the owner can be transformed into:

$$(1 + r(1 - \tau_K))x + y = c + x' + \chi + (1 + \delta_H + r(\tau_H - \tau_M))h^o + rx(\tau_M - \tau_K) \tag{24}$$

By comparing (21) and (24), one can see that the additional benefits of owning with mortgage loans are $r(\tau_M - \tau_H)h^o$ (which is positive with the baseline tax rates) and $rx(\tau_K - \tau_M)$ (which is also positive with the baseline tax rates). The additional cost of owning relative to renting is same as for the savers, which is χ. When τ_K is lowered, the relative benefit of owning with mortgage loans also goes down. Moreover, the benefits of owning with mortgage loans relative to renting disappear when $\tau_K = \tau_M = \tau_H$.

In case the preferential tax treatment for owner-occupied housing is preserved ($\tau_H = 0$ and $\tau_M = 0.23$), the additional considerations associated with two-asset model works to lower the optimal capital income tax rate. The 13% optimal capital income tax rate is the result of balancing the five forces.

In case the preferential tax treatment for owner-occupied housing is eliminated ($\tau_K = \tau_H$ and $\tau_M = 0$), tax benefit of owning disappears for savers, and decreases for mortgage-loan borrowers. The tax wedge between financial assets and housing assets disappears, and the benefit of owning rather than renting declines. In this case, there is no need to lower τ_K to achieve the reduced benefits of owning, as it is already built-in with $\tau_K = \tau_H$. That is why the optimal capital income tax rate is back to as high as 43%. The reason why the optimal capital income tax rate is even higher than in the one-asset model (37%) might be the extra inelasticity of saving, as housing asset accumulation is also inelastic to changes in tax rates. In other words, because agents need housing for enjoying services, and there is no way to *evade* away from housing taxation, agents are less even elastic in response to taxing housing and capital, more so than in the one-asset model.

The discussion so far implies that a part of the welfare gain of implementing optimal capital income tax rate is related to the inefficiency of owned housing. How important is it? In order to answer the question, I turn off the tenure choice from the baseline model, recalibrate the model, and investigate the optimal capital income tax rate. The new model still has the portfolio choice between housing and financial assets, but housing can only be owned. And there is no additional cost associated with ownership ($\chi = 0$).

Table 6 summarizes the macroeconomic and welfare effects of implementing optimal capital income tax rate in the two-asset model without tenure decision. The optimal capital income tax when keeping the preferential tax treatment for owner-occupied housing is 29%, which is lower than the optimal capital income tax rate in the one-asset model (37%) but not as low as in the baseline case (13%). When the preferential tax treatment is eliminated, the optimal capital income tax rate is 39%, which is higher than the other case, like in the baseline experiment, but again lower than the baseline case (43%). As expected, when the inefficiency associated with ownership is turned off and there is no need to consider *(iv)* , there is no efficiency gain from shifting the tenure decision from owning to renting, and thus the optimal capital income tax rate is drawn to a higher level than in the baseline model. However, the key properties of the baseline result, namely a lower optimal capital income tax rate when the preferential tax treatment for owner-occupied housing is preserved, and higher optimal capital income tax rate when imputed rents are taxed at the same rate as capital in come, are still obtained. A lower capital income tax still induces a welfare gain by reducing the tax wedge between housing and financial assets.

Table 6: Optimal Capital Income Taxation in the Two-Asset Model without Tenure Choice

Economy	Baseline	Optimal	Optimal	Zero τ_K
Preferential tax for housing	Yes	Yes	No	Yes
Tax rates				
τ_H	0.0000	0.0000	0.3900	0.0000
τ_K	0.4000	0.2900	0.3900	0.0000
τ_L^1	0.2700	0.2837	0.2410	0.3140
τ_M	0.2300	0.2300	0.0000	0.2300
% change from the baseline[2]				
Output	0.3492	+0.98	+2.31	+2.73
Total capital	0.9638	+2.16	−5.54	+6.52
Housing capital	0.4505	−0.97	−20.61	−2.49
Non-housing capital	0.5133	+4.91	+7.69	+14.43
Average hours worked	0.3300	−0.27	+0.63	−0.76
Labor supply	0.3077	−0.28	+0.61	−0.84
Consumption	0.1931	+0.26	+3.10	+0.46
homeownership rate[3]	1.0000	1.0000	1.0000	1.0000
Welfare[4]				
Overall change in welfare ϵ	–	+0.10	+0.90	−0.30
Average effect ϵ_g	–	+0.37	−0.54	+0.83
Life-cycle effect ϵ_i	–	−0.35	+1.68	−1.29
Redistribution effect ϵ_d	–	+0.08	−0.24	+0.16

[1] Adjusted to guarantee revenue neutrality.
[2] Level is shown for the baseline economy.
[3] Level is shown for all economies.
[4] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.

As is also expected, the size of the welfare gain by implementing the optimal capital income tax is lower, partly because the optimal capital income tax rate is much closer to the baseline rate, and partly because there is no welfare gain associated with eliminating efficiencies associated with ownership. The welfare gain is 0.1% of flow consumption when the preferential tax treatment is maintained, and 1.07% when it is eliminated.

In sum, *(iv)* plays an important role in shaping the optimal capital income tax rate in the two-asset baseline model, but not all of the results are solely based on *(iv)* , and thus the main results obtained in the baseline model are still valid.

Table 7: Optimal Capital Income Taxation with Inelastic Labor Supply

Economy	One-asset model		Two-asset model		
	Baseline	Optimal	Baseline	Optimal	Optimal
Preferential tax for housing			Yes	Yes	No
Tax rates					
τ_H	–	–	0.0000	0.0000	0.3300
τ_K	0.4000	0.2200	0.4000	0.0800	0.3300
τ_L^1	0.2700	0.3053	0.2700	0.3009	0.2578
τ_M	–	–	0.2300	0.2300	0.0000
% change from the baseline[2]					
Output	0.5515	+2.09	0.3830	+3.14	+1.84
Total capital	1.5222	+6.55	1.0570	+5.52	−1.63
Housing capital	–	–	0.4940	−3.39	−12.22
Non-housing capital	1.5222	+6.55	0.5630	+13.34	+7.69
Average hours worked	0.3300	–	0.3300	–	–
Labor supply	0.3375	–	0.3375	–	–
Consumption	0.3044	+1.23	0.2068	+3.40	+3.50
homeownership rate[3]	–	–	0.6400	0.0103	0.0000
Welfare[4]					
Overall change in welfare ϵ	–	+0.25	–	+3.31	+3.76
Average effect ϵ_g	–	+1.23	–	+2.92	+1.69
Life-cycle effect ϵ_i	–	−1.31	–	−0.97	+1.24
Redistribution effect ϵ_d	–	+0.34	–	+1.37	+0.83

[1] Adjusted to guarantee revenue neutrality.
[2] Level is shown for the baseline economy.
[3] Level is shown for all economies.
[4] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.

9 Sensitivity Analysis

Table 7 summarizes the aggregate and welfare effects of moving to the optimal tax regime in two-asset and one-asset model economies, with *inelastic* labor supply. The results in the table confirm that the results obtained with the baseline model do not depend on the elastic labor supply, although the optimal tax rates are all lower than in the models with inelastic labor supply. The model with housing and non-housing capital has a much lower optimal capital income tax rate (8%) if the preferential tax treatment for owner-occupied housing is preserved, and a higher optimal capital income tax rate (34%) when the preferential tax treatment is eliminated, compared with the optimal capital income tax rate in the one-asset model (22%). It is intuitive since there is no distortionary effect of labor income tax in the models with inelastic labor supply, which has a positive (negative) effect to the optimal labor income tax rates (other

Table 8: Optimal Capital Income Taxation without Idiosyncratic Shocks to Individual Productivity

Economy	One-asset model		Two-asset model		
	Baseline	Optimal	Baseline	Optimal	Optimal
Preferential tax for housing			Yes	Yes	No
Tax rates					
τ_H	–	–	0.0000	0.0000	0.2300
τ_K	0.4000	0.1600	0.4000	0.2300	0.2300
τ_L^1	0.2700	0.3166	0.2700	0.2838	0.2838
τ_M	–	–	0.2300	0.2300	0.0000
% change from the baseline[2]					
Output	0.5466	+3.06	0.3801	+0.73	+0.73
Total capital	1.5086	+12.31	1.0492	+0.83	+0.83
Housing capital	–	–	0.4904	−7.81	−7.81
Non-housing capital	1.5086	+12.31	0.5588	+8.41	+8.41
Average hours worked	0.3300	−0.75	0.3300	−1.59	−1.59
Labor supply	0.3345	−1.13	0.3350	−1.67	−1.67
Consumption	0.3017	+0.75	0.2017	+2.25	+2.25
homeownership rate[3]	–	–	0.6510	0.0000	0.0000
Welfare[4]					
Overall change in welfare ϵ	–	+0.61	–	+2.90	+2.90
Average effect ϵ_g	–	+1.47	–	+2.85	+2.85
Life-cycle effect ϵ_i	–	−0.86	–	+0.06	+0.06
Redistribution effect ϵ_d	–	–	–	–	–

[1] Adjusted to guarantee revenue neutrality.
[2] Level is shown for the baseline economy.
[3] Level is shown for all economies.
[4] Measured by the uniform percentage increase in flow consumption of non-housing goods, against the welfare in the baseline model economy.

tax rates). Without labor supply distortion, it is possible to lower capital income tax rate and raise labor income tax rate further. The size of the welfare gain measured by the uniform increase in the flow consumption is higher than the counterpart with the elastic labor supply, because of there is no inefficiency associated with increasing the labor income tax rate. The size of the welfare gain is 3.3% in case the preferential tax treatment for owner-occupied housing is preserved, and 3.8% when the preferential tax treatment is eliminated.

Table 8 summarizes the aggregate and welfare effects of moving to the optimal tax regime in two-asset and one-asset model economies, *without* idiosyncratic shocks to individual labor productivity. The optimal capital income rate in the one-asset model is now at a relatively low level at 16%. One potential explanation for the difference from the baseline result (37% optimal capital income tax rate) is that the single agent in the model economy is not borrowing constrained. Therefore, labor income tax can be increased without causing a welfare loss for the borrowing-constrained.

As for the two-asset model, surprisingly, the optimal capital income tax rate is the same between the two environments with different tax treatment of owner-occupied housing; the optimal rate is higher than in the one-asset model, at 23%. Even with the preferential tax treatment for owner-occupied housing, the single representative agent does not own housing at a relatively high capital income tax rate of 23%. Since the agent already does not own any housing, eliminating the preferential tax treatment for owner-occupied housing does not make any difference. Intuitively, the optimal capital income tax rate when the preferential tax treatment for owner-occupied housing is maintained is relatively high because even with the 23% capital income tax rate, all agents are induced away from owning housing assets, and there is no more welfare gain from this margin.

In the model with two-assets but without tenure decision, the optimal capital income tax rate is 14% with the preferential tax treatment for owner-occupied housing and 20% without. In this model, the properties of the optimal capital income taxation obtained in the baseline experiment are preserved; the optimal capital income tax rate is very different depending on how housing is taxed, and the optimal capital income tax rate is low with the preferential tax for housing and high without.

Finally, eliminating the social security, or using non-separable utility function between consumption and leisure instead of separable utility function, does not change the main results of the optimal capital income tax rate obtained in the baseline experiment, either. In both cases, the optimal capital income tax rates are higher than in the baseline experiment with social security and separable utility function.

10 Conclusion

In this paper, I quantitatively study the optimal capital income taxation in the general equilibrium overlapping generations model with uninsurable idiosyncratic income shocks and with housing and financial assets. Key characteristics of housing are explicitly modeled: *(i)* housing is held for the dual purpose of consumption and savings, *(ii)* housing can be either owned or rented, *(iii)* if owned, housing can be used as a collateral for mortgage loans, and *(iv)* there is a preferential tax treatment for owner-occupied housing through tax-exemption of imputed rents and mortgage interest payment deduction. Using the calibrated version of the model, I investigate whether and how the optimal capital income tax rate differs between the model with both housing and financial assets and the standard model without housing.

I find that the optimal capital income tax rate changes significantly depending on how housing is taxed. This is because capital income tax affects both the portfolio choice decision between housing and financial assets, and the tenure decision between renting and owning. In particular, I find that, when the preferential tax treatment for owner-occupied housing is maintained, the optimal capital income tax rate is low at 13%, while the optimal tax rate is 37% in the standard one-asset model. The difference occurs because of the additional effect of lowering capital income tax; lower capital income tax implies a smaller tax wedge between financial and housing assets, as well as a smaller wedge in the after-tax cost of renting and owning. When these wedges are reduced with a lower capital income tax rate, over-accumulation of housing asset as well as the distortion induced by the preferential tax treatment in favor of owner-occupied housing are relaxed. I find that the welfare gain by moving to the optimal tax regime is as large as 2.3% of average flow consumption.

On the other hand, if the preferential tax treatment for owner-occupied housing is eliminated, it becomes optimal to tax both assets heavily, like in the one-asset life-cycle model. Indeed, the optimal capital income tax rate becomes higher than in the one-asset model. The optimal capital income tax rate turns out to be 43%, which is higher than the 37% optimal capital income tax rate obtained from the one-asset model. The comparison of the optimal capital income tax rate under the two settings confirm that, while both reducing the wage distortion and reducing the tax wedge between owning and renting and the tax wedge between the financial and housing assets are both important source of welfare gain, the latter is more important if the two cannot be obtained simultaneously.

In general, all the results obtained in this paper suggests the importance of considering the capital income taxation and housing taxation simultaneously, because there is a nontrivial interaction among saving decision, portfolio choice decision, and tenure decision. Tax reforms involving capital income taxation should take into account the interaction. If the optimal tax scheme is computed using the one-asset model and ignoring the existence of housing asset, one might end up having a capital income tax rate which is very different from optimal under the environment where housing is considered explicitly.

Given the large welfare gain from nullifying the preferential treatment for owner-occupied housing, a natural question is why we do not observe a tax reform against the preferential treatment of owner-occupied housing. There are two possible explanations. One is that it is not easy to adjust housing asset holding and thus the cost on the transition is significant. Another explanation is the potential loss of homeowners makes the reforms difficult to implement politically. In order to investigate the hypotheses, it is necessary to extend the current framework by introducing transition between steady states and various costs associated with adjustments of housing assets, together with the tenure choice between owning and renting. I leave these to the future research.

Appendix A

A.1 Computing Cross-Sectional Variances of Hourly Wage from PSID

I use the Panel Study on Income Dynamics (PSID), 1967-1996.[14] Since each wave of PSID covers income and hours worked in the previous year, the data set covers the years 1966-1995. Following Storesletten et al. (2004), each *year* is an overlapping panel of three years. For example, *year* 1968 consists of actual years of 1967, 1968, and 1969. This overlapping panel structure helps maintaining a broad cross-section and a stable age distribution, but still enables identification of time series parameters.

I use the households *(i)* whose head is between 22 and 65 years of age, *(ii)* have a positive core weight, *(iii)* labor income of the head is not top-coded, *(iv)* hourly wage of the head (computed by dividing the annual labor income of the head by the total hours worked by the head in the same year) is above half of the minimum wage of the respective year, *(v)* hours worked by the head is between 520 and 5096 hours, and *(vi)* all the conditions are satisfied in two consecutive years. The nominal hourly wage is deflated using the Consumer Price Index (CPI) for respective years.

I compute the cross sectional variances of the logarithm of real hourly wage of the heads of households, for each age between 22 and 65. The variances are net of cohort effect, i.e., the variance for each age captures the one to a group of households with heads born in the same year. This is accomplished by a cohort and age dummy-variable regression as proposed by Deaton and Paxon (1994) and also used in Storesletten et al. (2004).

Figure 7 shows the age effect to the cross-sectional variances of logarithm of the real hourly wage of heads of households. It takes the value of 0.197 and 0.691 for age 22 and 65, respectively, and almost linearly increases between age 22 and 65.

A.2 Computation

This appendix gives details about the solution algorithm of the stationary recursive competitive equilibrium.

1. Guess prices r, w, amount of lump-sum transfer tr, amount of social security benefit $\bar{b}$, and tax rates: τ_K, τ_L, τ_H, and τ_M. The equilibrium price of rental properties is easily computed using $q = r + \delta_H$. When running the baseline model, tax rates for labor and capital income, and housing and rate of mortgage interest payment deduction are fixed at $\tau_L = 27\%$, $\tau_K = 40\%$, $\tau_H = 0\%$, $\tau_M = 23\%$, respectively. In other cases one of the tax rates will be adjusted such that the total amount of taxes is equal to the target total tax payments, which guarantees the fiscal revenue neutrality across experiments.

[14] I do not use waves after 1996, when PSID is no longer annual, but bi-annual.

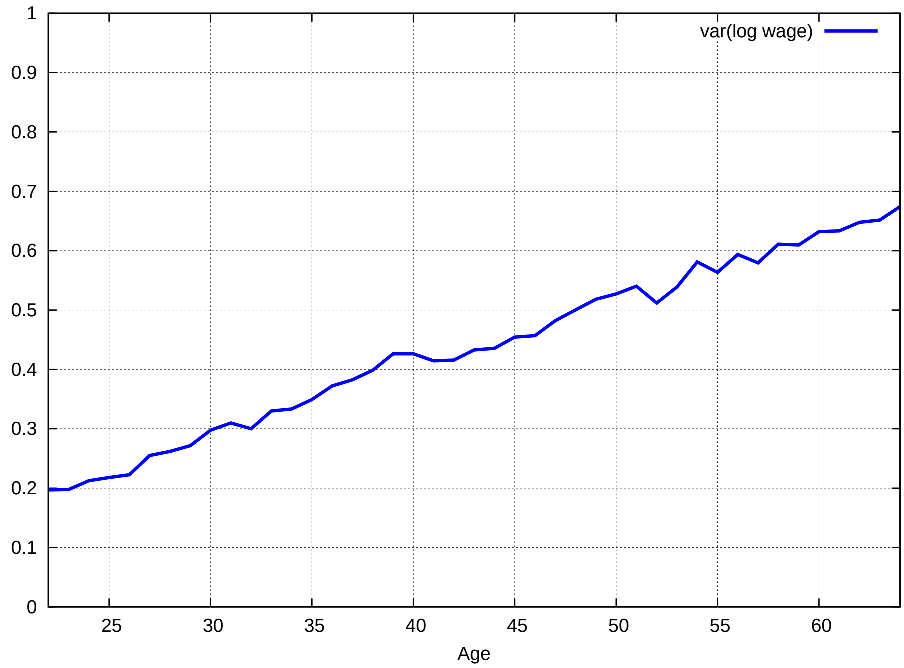


Figure 7: Cross-sectional variances of log-hourly wages

2. Given the guess, solve the problem of agents. Specifically, follow the steps below and find the optimal decision rules $c = g_c(i,e,x)$, $h^o = g_o(i,e,x)$, $h^r = g_r(i,e,x)$, $a = g_a(i,e,x)$, $x' = g_x(i,e,x)$, and $\ell = g_\ell(i,e,x)$.

   (a) Find the optimal decisions in the last period of life I. It is easier because age$-I$ is the last period of life and thus the maximand contains only the current utility for age I. I use the first order conditions that characterize the optimal choice to find the optimal decision rules.

   (b) The obtained optimal decisions can be used to compute the value for age I.

   (c) Given the value function for age I which is obtained in the last step, go back one step and find the optimal decision rules for age $I-1$.

   (d) Keep going back up to age 1.

3. Having obtained the optimal decision rules $c = g_c(i,e,x)$, $h^o = g_o(i,e,x)$, $h^r = g_r(i,e,x)$, $a = g_a(i,e,x)$, $x' = g_x(i,e,x)$, and $\ell = g_\ell(i,e,x)$, run a simulation with a large number of agents (I use $N = 1,000,000$ agents). Specifically, follow the steps below:

   (a) For each of N agents, draw the initial e from $\{p_e^0\}$, using a random number generator. Initial x is zero. Initial i is 1.

   (b) For each of N agents, compute the optimal decisions $(c, h^o, h^r, a, x', \ell)$ and update the state variables using the optimal decisions. e is updated using the assumed first order

Markov process together with a draw from a random number generator. x is updated using the optimal decision rule $x' = g_x(i, e, x)$.

(c) Keep updating the individual state variables up to age I.

4. Using the simulation results, compute the aggregate variables. When aggregating individual variables, normalize the measure of a single newborn as 1. Because of the population growth and the mortality risk, the measure of a single age-2 agent is $\frac{\pi_1}{1+\gamma}$. Similarly, the measure of a single age-3 agent is $\frac{\pi_1 \pi_2}{(1+\gamma)^2}$, and so on.

5. Use the aggregate variables to construct new guess for prices, lump-sum transfer, social security benefit and tax rates.

   (a) The new prices, $\hat{r}$ and $\hat{w}$, can be constructed using the profit maximizing conditions for the firm, and the aggregate capital stock labor supply that are obtained by aggregating individual agents' decision.

   (b) The new amount of transfer $\hat{tr}$ can be constructed by computing the total amount of accidental bequests (total amount of assets, including interests and depreciation, held by the agents which are not surviving), and dividing the total accidental bequests by the number of living agents in the next period.

   (c) The new social security benefit $\hat{b}$ can be constructed by computing the total social security contribution and divide the total by the number of retirees.

   (d) In case τ_L is used to guarantee the revenue neutrality, the new labor income tax rate $\hat{\tau}_L$ can be obtained from the government budget balance constraint. $\hat{\tau}_L$ is chosen such that the budget balance is achieved. In case of baseline model, tax rates are all fixed; τ_L is always set at 27% and there is no need for updating τ_L.

6. Compare the old and the new guess. If the distance of the two is smaller than a predetermined criteria, it's done. Otherwise, update the guess and go back to step 2.

7. When calibrating the model, change the parameters, and solve the equilibrium. If, in the equilibrium, all the targets are satisfied up to a predetermined accuracy, done. Otherwise, change the parameters and solve the equilibrium again.

A.3 Definition of Welfare Measures

Suppose we are comparing two economies $j = 0, 1$. Economy 0 is the baseline economy and economy 1 is the counterfactual one. The optimal combination of consumption of non-housing goods, housing services, and leisure in economy j, conditional on the initial $e = e_0$ and the history of realization of labor productivity shocks $\widetilde{e}$ are denoted by $(c_i^j(e_0, \widetilde{e}), d_i^j(e_0, \widetilde{e}), m_i^j(e_0, \widetilde{e}))$. Moreover, let $\widetilde{s}$ denotes the history of realizations of mortality shocks. In particular, $\widetilde{s}_i = 1$ when the agent is alive in age-i, and $\widetilde{s}_i = 0$ when the agent is dead in age-i.

The ex-ante expected welfare of a newborn in economy j in the stationary equilibrium can be represented as follows:

$$\omega^j = \sum_{e_0} p_e^0 \sum_{\widetilde{e}} \sum_{\widetilde{s}} \; q_{\widetilde{e}|e_0} \; q_{\widetilde{s}} \; \sum_{i=1}^{I} \mathcal{I}_{\widetilde{s}_i=1} \beta^{i-1} u(c_i^j(e_0,\widetilde{e}), d_i^j(e_0,\widetilde{e}), m_i^j(e_0,\widetilde{e})) \tag{25}$$

where p_e^0 is the probability with which e_0 is drawn, $q_{\widetilde{e}|e_0}$ is the probability of a history $\widetilde{e}$ conditional on e_0, $q_{\widetilde{s}}$ is the unconditional probability of a history $\widetilde{s}$, $\mathcal{I}$ is the indicator function which takes the value of 1 is the statement attached to it is true, and 0 otherwise.

The welfare gain by moving from the economy 0 to the economy 1, measured by the uniform percentage increase in non-housing consumption goods, ϵ, can be defined implicitly as follows:

$$\omega^1 = \sum_{e_0} p_e^0 \sum_{\widetilde{e}} \sum_{\widetilde{s}} \; q_{\widetilde{e}|e_0} \; q_{\widetilde{s}} \; \sum_{i=1}^{I} \mathcal{I}_{\widetilde{s}_i=1} \beta^{i-1} u(c_i^0(e_0,\widetilde{e})(1+\epsilon), d_i^0(e_0,\widetilde{e}), m_i^0(e_0,\widetilde{e})) \tag{26}$$

Suppose consumption of non-housing goods, and housing services, and leisure increased by the proportion g^c, g^d and g^m, respectively, by moving from the economy 0 to the economy 1. The welfare gain measured by uniform percentage increase in non-housing consumption goods, associated with the uniform increase in consumption of non-housing goods, and housing services and leisure, ϵ_g, can be defined implicitly as follows:

$$\begin{aligned} &\sum_{e_0} p_e^0 \sum_{\widetilde{e}} \sum_{\widetilde{s}} q_{\widetilde{e}|e_0} \; q_{\widetilde{s}} \; \sum_{i=1}^{I} \mathcal{I}_{\widetilde{s}_i=1} \beta^{i-1} u(c_i^0(e_0,\widetilde{e})(1+\epsilon_g), d_i^0(e_0,\widetilde{e}), m_i^0(e_0,\widetilde{e})) \\ &= \sum_{e_0} p_e^0 \sum_{\widetilde{e}} \sum_{\widetilde{s}} q_{\widetilde{e}|e_0} \; q_{\widetilde{s}} \; \sum_{i=1}^{I} \mathcal{I}_{\widetilde{s}_i=1} \beta^{i-1} u(c_i^0(e_0,\widetilde{e})(1+g^c), d_i^0(e_0,\widetilde{e})(1+g^d), m_i^0(e_0,\widetilde{e})(1+g^m)) \end{aligned} \tag{27}$$

Next, suppose consumption of non-housing goods, and housing services, and leisure for age i increased by the proportion g_i^c, g_i^d and g_i^m, respectively, by moving from the economy 0 to the economy 1. The welfare gain measured by uniform percentage increase in non-housing consumption goods, associated with the age-dependent increase in consumption of non-housing goods, and housing services and leisure, ϵ_e, can be defined implicitly as follows:

$$\begin{aligned} &\sum_{e_0} p_e^0 \sum_{\widetilde{e}} \sum_{\widetilde{s}} q_{\widetilde{e}|e_0} \; q_{\widetilde{s}} \; \sum_{i=1}^{I} \mathcal{I}_{\widetilde{s}_i=1} \beta^{i-1} u(c_i^0(e_0,\widetilde{e})(1+\epsilon_e), d_i^0(e_0,\widetilde{e}), m_i^0(e_0,\widetilde{e})) \\ &= \sum_{e_0} p_e^0 \sum_{\widetilde{e}} \sum_{\widetilde{s}} q_{\widetilde{e}|e_0} \; q_{\widetilde{s}} \; \sum_{i=1}^{I} \mathcal{I}_{\widetilde{s}_i=1} \beta^{i-1} u(c_i^0(e_0,\widetilde{e})(1+g_i^c), d_i^0(e_0,\widetilde{e})(1+g_i^d), m_i^0(e_0,\widetilde{e})(1+g_i^m)) \end{aligned} \tag{28}$$

Notice that the welfare measure includes both the welfare gain associated with the *age-independent uniform increase* in the consumption of non-housing goods and housing services, and leisure, ϵ_g, and the welfare gain associated with the redistribution across the life-cycle, ϵ_i. Therefore, ϵ_i can be obtained as residual, as follows:

$$\epsilon_i = \epsilon_e - \epsilon_g \tag{29}$$

The welfare gain measured by uniform percentage increase in non-housing consumption goods, associated with the *intra-cohort redistribution* of consumption of non-housing goods, and housing services and leisure, ϵ_d, can be defined as the residual, as follows:

$$\epsilon_d = \epsilon - \epsilon_i - \epsilon_g \tag{30}$$

References

Aiyagari, S. Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 1994, *109* (3), 659–684.

_ , "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 1995, *103* (6), 1158–1175.

Atkeson, Andrew, V. V. Chari, and Patrick Kehoe, "Taxing Capital Income: A Bad Idea," *Federal Reserve Bank of Minneapolis Quarterly Review*, 1999, *23*, 3–17.

Chambers, Matthew, Carlos Garriga, and Don E. Schlagenhauf, "The Tax Treatment of Homeowners and Landlords and the Progressivity of Income Taxation," 2007. Unpublished Manuacript.

_ , _ , and _ , "Accounting for Changes in the Homeownership Rate," 2008. forthcoming, International Economic Review.

Chamley, Christophe, "Optimal Taxation of Capital in General Equilibrium with Infinite Lives," *Econometrica*, 1986, *54* (3), 607–622.

Chari, V. V. and Patrick Kehoe, "Optimal Fiscal and Monetary Policy," in John Taylor and Mike Woodford, eds., *Handbook of Macroeconomics*, Vol. 1-3, Amsterdam: North Holland, 1999, chapter 26.

Conesa, Juan C. and Dirk Krueger, "Social Security Reform with Heterogeneous Agents," *Review of Economic Dynamics*, 1999, *2*, 757–795.

Conesa, Juan Carlos, Sagiri Kitao, and Dirk Krueger, "Taxing Capital? Not a Bad Idea After All!," 2007. Unpublished Manuscript.

Deaton, Angus and Christina Paxon, "Intertemporal Choice and Inequality," *Journal of Political Economy*, 1994, *102*, 437–467.

Díaz, Antonia and María J. Luengo-Prado, "The Wealth Distribution with Durable Goods," 2006. Unpublished Manuscript.

Domeij, David and Jonathan Heathcote, "On the Distributional Effects of Reducing Capital Taxes," *International Economic Review*, 2004, *45*, 523–554.

Erosa, Andre's and Martin Gervais, "Optimal Taxation in Life-Cycle Economies," *Journal of Economic Theory*, 2002, *105* (2), 338–369.

Feenberg, Daniel R. and James M. Poterba, "The Alternative Minimum Tax and Effective Marginal Tax Rates," *National Tax Journal*, 2004, *57*, 407–427.

Fernández-Villaverde, Jesús and Dirk Krueger, "Consumption and Saving over the Life Cycle: How Important Are Consumer Durables?," 2005. Unpublished Manuscript.

Garriga, Carlos, "Optimal Fiscal Policy in Overlapping Generations Models," 2003. Unpublished Manuscript.

Gervais, Martin, "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics*, 2002, *49*, 1461–1489.

Gourinchas, Pierre-Olivier and Jonathan A. Parker, "Consumption over the Life-Cycle," *Econometrica*, 2002, *70* (1), 47–89.

Hansen, Gary D., "The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, 1993, *8*, 71–80.

Huggett, Mark, "Wealth Distribution in Life-Cycle Economies," *Journal of Monetary Economics*, 1996, *38*, 469–494.

Joines, Douglas H., "Estimates of Effective Marginal Tax Rates on Factor Incomes," *Journal of Business*, 1981, *54* (2), 191–226.

Judd, Kenneth L., "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 1985, *28* (1), 59–83.

McGrattan, Ellen, "The Macroeconomic Effects of Distortionary Taxation," *Journal of Monetary Economics*, 1994, *33*, 573–601.

Mendoza, Enrique G., Assaf Razin, and Linda L. Tesar, "Effective Tax Rates in Macroeconomics. Cross-Country Estimates of Tax Rates on Facto Incomes and Consumption," *Journal of Monetary Economics*, 1994, *34*, 297–323.

Nakajima, Makoto, "Rsing Earnings Instability, Portfolio Choice, and Housing Prices," 2005. Unpublished Manuscript.

Ríos-Rull, José-Víctor, "Computation of Equilibria in Heterogeneous-Agent Models," in Ramon Marimon and Andrew Scott, eds., *Computational Methods for the Study of Economic Dynamics*, Oxford: Oxford University Press, 1999, chapter 11.

Rosen, Harvey S., "Housing Subsidies: Effects on Housing Decisions, Efficiency and Equity," in Alan Auerbach and Martin S. Feldstein, eds., *Handbook of Public Economics*, Vol. 1, Amsterdam: North Holland, 1985, chapter 7.

Silos, Pedro, "Housing, Portfolio Choice, and the Macroeconomy," *Journal of Economic Dynamics and Control*, 2007, *31* (8), 2774–2801.

Social Security Administration, *Annual Statistical Supplement, 2004, to the Social Security Bulletin*, Washington, DC: Social Security Administration, 2005.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron, "Consumption and Risk Sharing over the Life Cycle," *Journal of Monetary Economics*, 2004, *51*, 609–633.

Tauchen, George, "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 1986, *20*, 177–181.