

# Equilibrium Sticky Prices \*

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## Abstract

I offer a macroeconomic view of product markets that combines the desirable properties of the market-clearing models descended from the real business cycle model, on the one hand, and the sticky-price disequilibrium model of modern inflation analysis, on the other hand. I adhere strictly to the basic concept of equilibrium, in that buyers and sellers never transact in my model under conditions that could be improved through bilateral action. No transactors are prisoners of a sticky price that results in bilateral inefficiency. But I adopt the sticky-price view that transactions do occur at stale prices. I show that the equilibrium set for prices (and wages) is big enough under a reasonable calibration to include a wide range of stale prices. I show that many of the propositions about central-bank policy for controlling product prices from the established sticky-price models carry over to this view. A innovation in this model is an analysis of the effects of alternative policy regimes in amplifying disturbances in the product market as they reduce the effects in the labor market. When policy narrows the profit margin in retailing in order to sustain incentives for job creation when productivity is weak, the product market moves toward shortages, in a way made coherent in the model.

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# 1 Introduction

I propose a new view of the role of product markets in aggregate fluctuations and in stabilization policy. In this view, product markets are in equilibrium in the sense that the bilateral relationship between customer and store offers no private opportunity for joint improvement. The view shares the equilibrium character of the real business cycle model and its progeny. On the other hand, product prices are sticky—transactions occur at stale prices based on past information and do not respond to the most recent shocks. This aspect of the view accords with a branch of macroeconomics founded on sticky prices generally associated with Keynes.

I join other authors in trying to find the realistic middle ground between models with competitive market-clearing prices and those with sticky prices. Blanchard and Gali (2006) is an important recent effort of this type. But their model retains the traditional Keynesian view of the product market, in which sellers stand ready to serve all customers at the predetermined price. The model does not describe any differences in the product market when the predetermined price squeezes the margins of retailers. Arseneau and Chugh (2007) incorporate a product market similar to the one in this paper, but explore rather a different range of implications, with a focus on explaining literally fixed prices.

The product market in my model tracks the Mortensen-Pissarides search and matching model of the labor market closely, even slavishly. I do so to reduce the expository burden, not because I believe that the labor market and product market work in the same way. But I believe that many of the features of the MP model apply to product markets. The model has a coherent view of market tightness, surely an important element of any model where transactions occur at stale prices. The model also recognizes that shopping takes time—the average American adult spends more than an hour shopping for each five hours of work. I anticipate that modifications of the MP model to make it fit the product market more closely will retain many of the properties I discuss here.

Although this paper is about sticky product prices, I adopt the equilibrium sticky-wage view of the labor market, following Blanchard and Gali (2006) and Hall (2005). I study a simple economy with one product and one type of labor.

The primary approach I take to studying the sticky product price is to find the equilibrium set for the price and the wage. A price-wage pair is in the equilibrium set if there is a configuration of the economy where price is in the bargaining set of shoppers and merchants

and the wage is in the bargaining set of workers and producers. I start by describing the equilibrium set for a static version of the economy. It is a triangle where one leg is the minimum price where product trade can occur, a second is the maximum wage where employment can occur, and a third is the minimum real wage where households will participate in the two markets. In a reasonable calibration, the typical values of prices and wages are close to the first two bounds and far from the household participation bound. The model provides a coherent account of the limited role that labor supply plays in sticky-price macro models.

The static equilibrium set is a good guide to the dynamic model. With constant productivity, every point in the static equilibrium set is a constant equilibrium of the dynamic model, not surprisingly. In addition, wage-price combinations outside the static equilibrium set may be equilibria in one state of the dynamic model.

With state-dependent productivity, I focus on dynamic equilibria where the price and wage are sticky in the sense that they reflect conditions not of the current state, but of the preceding one. Specifically, the price and wage in the current state are those that would have prevailed in the previous state under price and wage flexibility. I define flexibility as involving a price that divides the transaction surplus equally between buyer and seller and a wage that divides the surplus equally between worker and employer.

In the dynamic setting, I introduce a central bank that provides a nominal anchor for the economy. In keeping with modern price-level analysis, I presume the bank's ability to set a nominal quantity and do not delve into the way the bank accomplishes this peg. To explain monetary non-neutrality—the central bank's ability to influence allocations and not just prices—I presume that the stale price and wage from the previous state is set in the unit of value that the central bank provides. The resulting nominal stickiness will result in equilibrium stale price and wage provided the shock that hits the economy between the time when the stale price and wage are determined and when they take effect is not too big.

I derive the policy frontier for central-bank management of the economy. As usual, if the bank chooses to stabilize the product price (and thus stabilize conditions in the product market), the policy creates volatility in the labor market in the form of variations in the unemployment rate and, to a remarkably modest extent, in the wage. A more novel finding is that a policy to stabilize the labor market, partially or totally, results in substantial volatility of conditions in the product market. When the bank leans against a negative shock that

would otherwise cause the labor market to slacken, the result is a sharp slackening of the product market. The bank's action squeezes retail margins and makes it hard for customers to find willing sellers. This aspect of stabilization policy has escaped formal modeling, I believe.

## 2 Model

The model embodies a product market with search frictions. Because shoppers and merchants invest in search effort before locating each other, they enjoy a surplus from their relationship once it is formed. Any price that gives both the shopper and the merchant a non-negative share of the surplus is an equilibrium. Hence if they choose to transact at a stale price, their relationship will be efficient as long as the price is not outside their bargaining set.

Although bargaining is common in retail transactions for items costing more than \$100 or so, the relevance of the analysis of the retail market in terms of a bargaining set is not restricted to circumstances where bargaining actually occurs. In situations where bargaining would be practical but does not occur, the market protocol may be alternating-offer bargaining with the seller making the first offer. The unique equilibrium of this bargaining game is immediate acceptance of an offer that is attractive enough to deter the buyer from incurring the cost of a counter-offer—see Binmore, Rubinstein and Wolinsky (1986).

Even for products where the non-negotiable posted price is the rule—notably on the shelves of a supermarket—transactions occur within fairly wide bargaining sets. The supermarket does not post a price at the upper end of the bargaining set, the shopper's reservation price. Once in a store, the shopper would be willing to pay quite a high price for any one item, because the alternative would be to visit another store for just that one item. But if all the prices were that high, the shopper would go to another store.

I assume that transactions occur at stale prices as long as they are within the bargaining set. Moreover, the stale prices are in the economy's unit of value, without indexation. Neither of these assumptions offends equilibrium. No principle of economics is violated when a buyer purchases at a stale nominal price that is within the bargaining set. There is no need to invoke a cost of changing prices to rationalize these transactions.

The modern literature on sticky prices has been transfixed by the idea that sellers—when they are free to set prices—set them at levels that are best for the future. The paradoxes

of this forward-looking behavior are well known. Although forward-looking prices are not ruled out in the model of this paper, they are not compelled either. No actor is irrational in the purely backward-looking equilibrium that I consider here.

The claim that stale prices result in efficient transactions is strictly true only when the size of each purchase is exogenous. If the shopper chooses the number of jars of peanut butter, only marginal-cost pricing will be bilaterally efficient. The situation corresponding to my assumptions when buyers choose quantity is that the seller receives a lump sum independent of quantity plus marginal cost multiplied by the quantity purchased. In that setting, the lump sum lies within a bargaining set that may be wide. Stale pricing would apply to the lump sum part only. Two-part pricing is rarely seen in direct form. But stores offer a wide variety of package sizes, which may get close to the case where customers buy one unit in each visit to the store, the case assumed in this paper.

The model adopts the Mortensen-Pissarides setup for the product market. Merchants spend resources attracting customers, a realistic aspect of the model. What strains realism is the vocabulary needed to use the MP setup. I refer to *customer openings* as the analog of vacancies. The way that merchants go about attracting customers is to create an opening and then use resources—advertising and the like—until the opening is filled. While a vacancy is plainly a tool that employers use in the process of recruiting workers, the customer opening is purely artificial. The only reason I use it here is that the mechanics of the MP model are so well known. It would be straightforward to alter the model to remove customer openings and relate the cost of attracting a customer to conditions in the market, but that would lose the close analogy to MP.

The model speaks coherently about tight and slack product markets, a property of the real world that has not previously been incorporated in macro models. The MP model transformed macroeconomic thinking about the labor market by providing a rigorous and sensible model of market tightness. I use the same model for the product market. Earlier sticky-price models lacked any differentiation between the operation of the product market when sellers are called upon to provide high volumes of output when prices and margins are low and the reverse.

The product market is tight when a shopper finds a suitable store rapidly, thanks to enthusiastic customer-attracting efforts by stores. In a tight market, it takes more resources than usual for a merchant to land a new customer. Markets are tight when the retail price

is high relative to the wholesale price. If the price adapts to current conditions, it will fall in the future and restore normal conditions. A tight product market is analogous in every way to a tight labor market, where a job seeker finds an opening rapidly, thanks to enthusiastic recruiting by employers. The labor market is tight when the wage is low relative to the wholesale value of the worker's marginal product.

In addition to the product price and wage, both of which may govern transactions at stale values, the model contains two fully flexible prices. One is the wholesale price of the product—the price at which producers sell the product to retailers. I model this as a market price, but it could be an internal shadow price of a firm vertically integrated from production to retailing. The second is the shadow price of the product after it arrives in the household. This price, about 6 percent above the retail price, reflects the value added by the shopper.

## 2.1 Households

A household has a continuum of members. Each member takes on one of four roles:

- *S*: Shopping, that is, searching for an appropriate goods supplier
- *R*: Buying under an established relationship with a supplier
- *U*: Seeking work
- *J*: Working at a job

In the output market, the ratio of customer openings to shoppers is  $\theta_R$  and in the labor market, the ratio of vacancies to job-seekers is  $\theta_J$ . I assume that a shopper has a probability  $\frac{\theta_R}{1+\theta_R}$  of finding a supply relationship each period and a job-seeker has a probability  $\frac{\theta_J}{1+\theta_J}$  of finding a job each period. The probability that a seller will attract a customer during a period is  $\frac{1}{1+\theta_R}$  and the probability that a producer will hire a worker is  $\frac{1}{1+\theta_J}$ .

Households have linear preferences. They value the time spent shopping, buying, and seeking work at zero and the time spent working at  $-z$  units of consumption. See Hall (2006) for a derivation of  $z$  from underlying preferences. Thus household utility in consumption units is  $c - zf$ , where  $c$  is consumption per member and  $f$  is the fraction of household members holding jobs.

A buyer has an exogenous probability  $s_R$  of terminating the relationship with a seller and a worker has an exogenous probability  $s_J$  of the termination of the current job. A buyer

purchases  $\alpha$  units of consumption goods each period from the chosen supplier. The household has a discount ratio of  $\beta$ . Buyers pay  $p$  for each unit at the store and the household places a shadow value  $\tilde{p}$  on consumption goods at home. Workers receive a wage of  $w$  per period. These and other value-related variables are measured in the economy's unit of value.

A household assigns Bellman values  $H_S$ ,  $H_R$ ,  $H_U$ , and  $H_J$  to the four roles. The household chooses the fraction of its members in roles  $S$  and  $U$  so as to equalize their values:

$$H_S = H_U. \quad (1)$$

The Bellman values satisfy the following static conditions:

$$H_S = \beta \left( \frac{\theta_R}{1 + \theta_R} H_R + \frac{1}{1 + \theta_R} H_S \right) \quad (2)$$

$$H_R = \alpha(\tilde{p} - p) + \beta [s_R H_S + (1 - s_R) H_R] \quad (3)$$

$$H_U = \beta \left( \frac{\theta_J}{1 + \theta_J} H_J + \frac{1}{1 + \theta_J} H_U \right) \quad (4)$$

$$H_J = w - z\tilde{p} + \beta [s_J H_U + (1 - s_J) H_J] \quad (5)$$

Given values of the parameters and of the endogenous variables  $\theta_R$ ,  $\theta_J$ ,  $p$ , and  $w$ , the five equations above can be solved for the four Bellman values and the home consumption goods price  $\tilde{p}$ .

## 2.2 Firms

The economy has two types of firms: Producers, who hire workers and make goods, and sellers, who buy goods from the producers and sell them to households. To preserve the analogy of the customer side of the model to the MP model, I view the seller as creating a customer opening, denoted  $O$ . An opening is analogous to a vacancy. When a seller and a shopper meet, they form a customer relationship, denoted  $R$ . I denote a vacancy as  $V$ . It costs a seller  $k_R$  units of output per period to maintain a customer opening and a producer  $k_J$  to maintain a vacancy.

One worker produces  $x$  units of output. Output sells for  $y$  in the intermediate market. The margin earned from production is  $yx - w$  and the margin earned from sales is  $p - y$ .

The firm assigns values  $F_O$  to customer openings,  $F_R$  to customer relationships,  $F_V$  to vacancies, and  $F_J$  to filled jobs. These satisfy the following static conditions:

$$F_O = -k_R y + \beta \left( \frac{1}{1 + \theta_R} F_R + \frac{\theta_R}{1 + \theta_R} F_O \right) \quad (6)$$

$$F_R = \alpha(p - y) + \beta (1 - s_R)F_R \quad (7)$$

$$F_V = -k_J y + \beta \left( \frac{1}{1 + \theta_J} F_J + \frac{\theta_J}{1 + \theta_J} F_V \right) \quad (8)$$

$$F_J = yx - w + \beta (1 - s_J)F_J \quad (9)$$

I make the standard assumption of free entry to the creation of vacancies and similarly assume free entry to the creation of consumer openings. Under free entry, competition holds the values of openings and vacancies at zero:

$$F_O = 0 \quad (10)$$

and

$$F_V = 0. \quad (11)$$

Under these assumptions, the intermediate product market is perfectly competitive—producers have perfectly elastic supply at the price  $y$  because they would expand infinitely at any higher price. Similarly, sellers have perfectly elastic demand.

Given values for the three prices,  $p$ ,  $w$ , and  $y$ , the six equations above imply unique values for the two market tightness variables,  $\theta_R$  and  $\theta_J$ .

### 3 Calibration

I take the separation rates  $s_R$  and  $s_J$  to have the known value for the labor market of 3 percent per month. I calibrate to  $\theta_R = \theta_J = 1$ , which is high by a factor of two for the U.S. labor market, but generates realistic supplier-finding and job-finding rates of 0.5, implying an unemployment rate of 5.7 percent in both roles.

For  $\alpha$ , the number of units of output purchased each period by a buyer, I use the ratio of work hours to shopping hours reported in the Bureau of Labor Statistics' American Time Use Survey for 2005, which is 4.6.<sup>1</sup>

I use a value of  $z$ , the disamenity of work, of  $z = 0.6$ , following Hall (2006). I take the monthly discount to be  $\beta = 0.95^{1/12}$ . I normalize productivity at  $x = 1$  and the intermediate product price at  $y = 1$ .

I calibrate at the point where households and sellers have equal shares of the transaction surplus and where workers and producers have equal shares of the employment surplus. At

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<sup>1</sup>[www.bls.gov/news.release/atus.t01.htm](http://www.bls.gov/news.release/atus.t01.htm)



this point, the value of a shopper or a job-seeker is  $H_S = H_U = 73.0$ , the value of a buyer or a worker is  $H_R = H_J = 73.6$ , and the value of a customer relationship or employment relationship to the firm is  $F_R = 0.625$ . The wage is  $w = 0.979$  and the product price is  $p = 1.005$ . The shadow price of the product delivered to the home is  $\tilde{p} = 1.077$ . The cost of maintaining a consumer opening is  $k_R = 0.063$  and the cost of maintaining a vacancy is  $k_J = 0.313$ .

The calibration implies that households enjoy most of the fruits of production as rents. Producers do not use a large fraction of output for recruiting and sellers do not use a large fraction for attracting customers. The real wage  $w/p = 0.974$  is close to its maximum feasible value of one. Evidence in Hagedorn and Manovskii (2006) supports this aspect of the calibration, though they reconcile it with evidence on job-finding rates by assigning workers a tiny share of the surplus and attributing a high disamenity to work.

## 4 Equilibrium Price and Wage

### 4.1 The static equilibrium set

Here I derive the set of static equilibrium prices of the economy. I consider the three prices  $p$  (amount paid to merchants),  $y$  (wholesale price of output), and  $w$  (compensation to a worker). As will become apparent shortly, the fourth price,  $\tilde{p}$ , is a function of the other three, so I do not include it explicitly. The prices have the usual homogeneity property that if  $(p, y, w)$  is an equilibrium, so is  $(\lambda p, \lambda y, \lambda w)$  for any positive  $\lambda$ . Accordingly, I normalize the members of  $Q$  in the form  $(p/y, w/y)$ .

Figure 1 shows the static equilibrium set for the economy. The left side shows the entire set and the right side magnifies the relevant portion, containing the lowest prices and the highest wages. Household equilibrium requires  $w \geq zp$ —the wage has to be high enough to clear the value of time,  $zp$ . If this condition holds, the shadow value  $\tilde{p}$  can take on a value such that  $\tilde{p} \geq p$  and  $w \geq z\tilde{p}$ , in which case equations (2) through (5) imply that  $H_R \geq H_O$  and  $H_J \geq H_U$ . The household is engaging in voluntary trade in the sense that having a relationship with a supplier or having a job is at least as valuable as not having the relationship or job. Below the slanting line, the real wage  $w/p$  is too low to support voluntary trade. The left edge of the equilibrium set is determined by the seller's willingness to engage in sales to customers. Further to the left, the price is too low to induce supply. At the critical price, the customer-finding rate,  $\frac{1}{1+\theta_R}$ , is one. Solving equations (6), (7), and

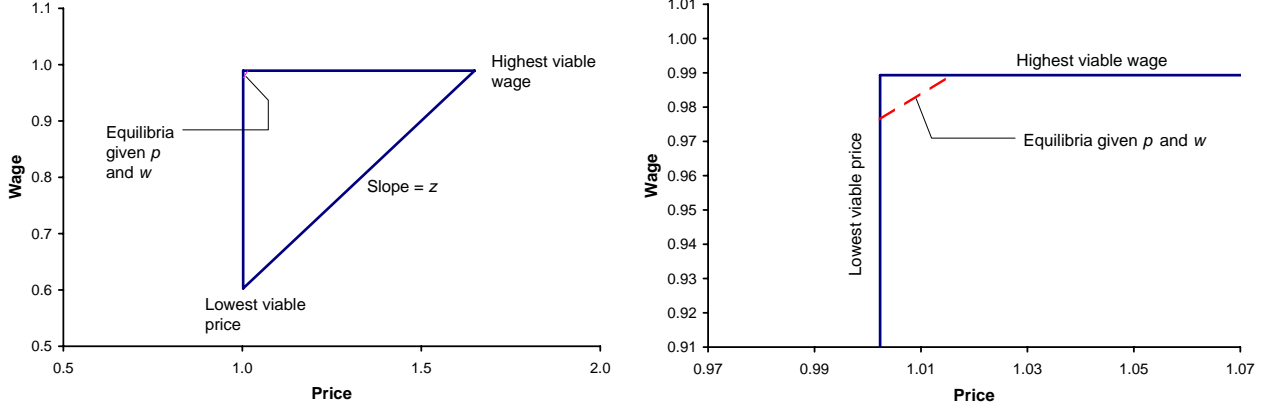


Figure 1. Static Equilibria

(10), I find

$$\frac{p}{y} = 1 + \frac{1 - \beta(1 - s_R)}{\alpha\beta} k_R. \quad (12)$$

The price needs to be at least the cost of the product, 1 when the price is stated as a ratio to the intermediate product cost,  $y$ , plus the annuity cost of forming the sales relationship.

The top edge of the set is determined by the producer's willingness to hire workers. Above the horizontal line in Figure 1, the wage is too high to permit hiring. The critical wage, where the recruiting rate is one, is

$$\frac{w}{y} = x - \frac{1 - \beta(1 - s_J)}{\beta} k_J. \quad (13)$$

This is the worker's productivity  $x$  less the annuity cost of hiring.

This establishes

**Proposition 1** *The static equilibrium set is the triangle,*

$$z \leq \frac{w}{y} \leq x - \frac{1 - \beta(1 - s_J)}{\beta} k_J \quad (14)$$

$$\frac{1 - \beta(1 - s_R)}{\alpha\beta} k_R \leq \frac{p}{y} \leq \frac{w}{yz}. \quad (15)$$

## 4.2 Equilibrium set for the real wage

Figure 1 and Proposition 1 places bounds on the real wage:

**Proposition 2** *The equilibrium set for the real wage is:*

$$z \leq \frac{w}{p} \leq \frac{x - \frac{1-\beta(1-s_J)}{\beta} k_J}{\frac{1-\beta(1-s_R)}{\alpha\beta} k_R - 1} \quad (16)$$

If the real wage is too high, there is no value of the intermediate product price  $y$  under which both producers and sellers are viable—the household is receiving an untenably high share of the benefits of production and distribution. If the real wage is too low, households are unwilling to participate in the market. As I noted in the section on calibration, the relevant bound is the upper one, because households capture most of the value of production as rents.

### 4.3 The equilibrium set for given price and wage

If the values of  $p$  and  $w$  are sticky—fixed in advance—but  $y$  is flexible, the equilibria of the economy lie along the part of the line  $\{(p/y, w/y) | y \geq 0\}$  that is inside the equilibrium set of the proposition. This line is shown as the slanting dashed line in the figure. To understand the equilibria along this line, it is useful to examine the allocation of household members among the four roles and the resulting level of output.

In principle, the numbers of shoppers and job-seekers are state variables of the model. However the speed with which the two variables reach their stochastic equilibrium values is so much higher than the speed of other changes in the model that nothing is lost by assuming that the search-related allocations are always at their stochastic equilibrium values rather than following their laws of motion. The result is a huge simplification of the discussion of allocations.

With the measure of household members taken to be 1, output  $q$  is the solution to the household time-allocation condition,

$$\left(1 + \frac{s_R(1 + \theta_R)}{\theta_R}\right) \frac{q}{\alpha} + \left(1 + \frac{s_J(1 + \theta_J)}{\theta_J}\right) \frac{q}{x} = 1. \quad (17)$$

The first term comprises the number of buyers needed to buy the output together with the number of searchers needed to generate that many buyers in stochastic equilibrium. The second term embodies the same calculation for the number of workers and job-seekers. They add to the total number of people available for the four roles, 1.

At the lower left end of the line, where it touches the vertical line, the product market is as slack as it can be. The rate at which searchers find customer relationships is zero. If

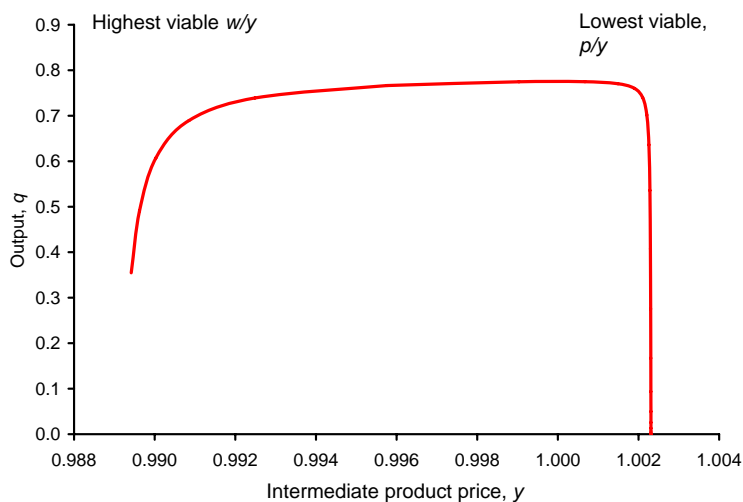


Figure 2. Equilibrium Set for Output and Intermediate Product Price, for Given Price and Wage

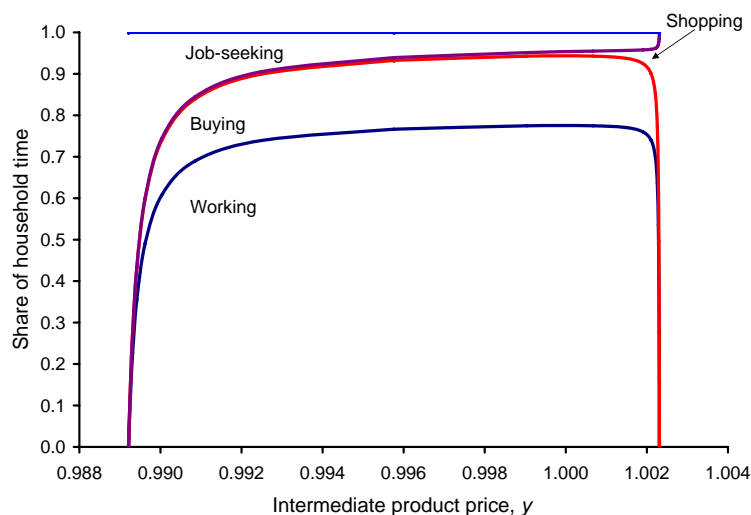


Figure 3. Allocations of Time

any output were produced, all household members would need to be searchers, so output is zero. At the upper right, output is also zero, because all household members would need to be job-seekers if any output were produced. Figure 2 shows output as a function of  $y$  along a representative sticky-price-wage equilibrium line and Figure 3 shows the corresponding allocations of household time.

The left sides of the figures correspond to the upper-right end of the dashed line in Figure 1—it is the point where the wage in relation to the intermediate price,  $w/y$ , is at its maximum feasible level. The labor market is slack, with a low job-finding rate. A large fraction of household time is devoted to job-seeking. As  $y$  rises and  $w/y$  falls in proportion, the fraction working rises rapidly and output rises by the same amount. Buying also rises

in proportion. This process continues as  $y$  rises further until slightly below the maximum feasible value, where the economy suddenly collapses because sellers lose the incentive to find customers. The fraction of time that households spend shopping—searching for available sellers—rises dramatically and eventually consumes all available time, as the product market ceases functioning. Output collapses.

The pronounced asymmetry in Figures 2 and 3 arises from an asymmetry in the model. People place a substantial disamenity  $z$  on working relative to buying, shopping, and job-seeking. The figures would be symmetric if the disamenity also applied to buying and if the model were calibrated to the same tightness in the product and labor markets.

#### 4.4 Equilibrium with given price and wage and a nominal anchor

The economy's nominal anchor picks out a point along the dashed line in Figure 1, corresponding to a point on the horizontal axes of Figures 2 and 3. Although an economy can choose among many methods for setting a nominal anchor, I will assume, for the sake of specificity, that the central bank pegs the flexible wholesale price  $y$ . Following the custom of modern price-level analysis, I do not specify the nature of the intervention needed to accomplish the peg.

The dashed line in Figure 1 together with Figures 2 and 3 describe the central bank's policy choice set with respect to the instrument  $y$ . With  $p$  fixed, different values of  $y$  correspond to different values of output,  $q$ , in the same proportion.

Under this assumption, the choice facing the central bank is how tight to make the labor market and how tight to make the product market, given the negative tradeoff between the two. Although the model lacks any dynamics at this point, one might reason casually that a tighter labor market puts upward pressure on the nominal wage  $w$  and a tighter product market puts downward pressure on the nominal product price,  $p$  (in a tight product market, potential customers find sellers easily and sellers find customers at low rates, so sellers might lower prices to attract more customers). Thus a choice of a lower level of output is anti-inflationary relative to a higher level, the standard view about central-bank policy. But the model as stated provides no basis for that view.

## 4.5 Monetary non-neutrality

The model generates monetary non-neutrality in a standard way—nominal prices and wages are determined before the central bank sets the nominal anchor. The model would satisfy monetary neutrality if prices and wages were set after the central bank acted. For example, if the product price were set in a bargain struck after the central bank acted (or indexed to the bank’s action) and the wage was set similarly, the nominal anchor would affect only the product price, the intermediate price, and the wage, and not any allocation. This point would hold for any bargaining equilibrium, such as Nash or alternating-offer bargaining. The bargain would pick a point along the dashed line in Figure 1, corresponding to levels of output and other allocations as in Figures 2 and 3.

## 5 Dynamic Model

The dynamic economy moves among  $N$  fundamental states, labeled  $\iota$ . Productivity  $x_\iota$  depends on the fundamental state. Endogenous variables may depend on the most recent previous fundamental state, because of stale prices and wages determined in that state. Note that the most recent previous state is not the fundamental state in the previous period, but rather the one before the most recent transition between fundamental states. To accommodate this dependence, I define  $M$  compound states, labeled  $i$ . This variable encodes both the current fundamental state,  $\iota = C(i)$ , and the most recent earlier fundamental state,  $L(i)$ . I denote the compound state in the next period as  $i'$ . The driving force  $x_{C(i)}$  is a function only of the current fundamental state while the price  $p_{L(i)}$  and the wage  $w_{L(i)}$  are restricted to depend only on the past state—this makes them stale and sticky. The central-bank policy variable  $y_i$  can depend on the current state, so the bank can pursue an active policy.

The equations governing the dynamic equilibrium are:

$$H_{S,i} = H_{U,i}. \quad (18)$$

$$H_{S,i} = \beta E \left( \frac{\theta_{R,i}}{1 + \theta_{R,i}} H_{R,i'} + \frac{1}{1 + \theta_{R,i}} H_{S,i'} \right) \quad (19)$$

$$H_{R,i} = \alpha(\tilde{p}_i - p_{L(i)}) + \beta E [s_R H_{S,i'} + (1 - s_R) H_{R,i'}] \quad (20)$$

$$H_{U,i} = \beta E \left( \frac{\theta_{J,i}}{1 + \theta_{J,i}} H_{J,i'} + \frac{1}{1 + \theta_{J,i}} H_{U,i'} \right) \quad (21)$$

$$H_{J,i} = w_{L(i)} - z\tilde{p} + \beta E [s_J H_{U,i'} + (1 - s_J) H_{J,i'}] \quad (22)$$

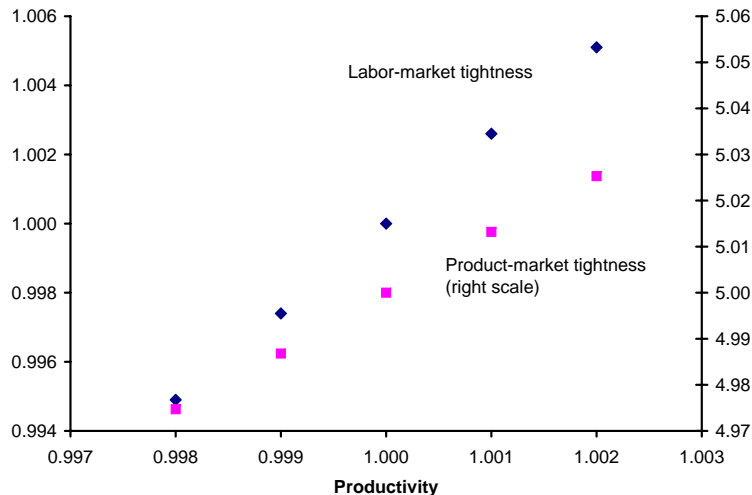


Figure 4. Product- and Labor-Market Tightness in Response to Productivity with Flexible Price and Wage

$$0 = -k_R y_i + \beta E \left( \frac{1}{1 + \theta_{R,i}} F_{R,i'} \right) \quad (23)$$

$$F_{R,i} = \alpha (p_{L(i)} - y_i) + \beta E [(1 - s_R) F_{R,i'}] \quad (24)$$

$$0 = -k_J y_i + \beta E \left( \frac{1}{1 + \theta_{J,i}} F_{J,i'} \right) \quad (25)$$

$$F_{J,i} = y_i x_{C(i)} - w_L(i) + \beta E [(1 - s_J) F_{J,i'}] \quad (26)$$

The exogenous driving force of the economy is productivity,  $x$ , which follows a Markoff process with transition matrix  $\pi_{l,l'}$ . The process has five states and transits only among adjacent states. The probability of transiting to the next higher or next lower state is 0.025, except that the probability of transiting from states 1 to 2 and 5 to 4 are 0.05. The stationary probabilities of the five states are equal. Productivity ranges in equal increments from 0.998 in state 1 to 1.002 in state 5.

First I consider the dynamic model with flexible, timely price and wage. The price and the wage in each state divides the transaction surplus equally. Figure 4 shows the relation between productivity and product- and labor-market tightness in the flexible case. Both markets are tighter when productivity is higher, but the response is tiny. Not surprisingly, the model generates the same behavior in the product market that Shimer (2005) found in the labor market. The model has monetary neutrality, so the policy  $y_i$  has no effect on the allocations shown in the figure.

Next I consider the case where the price  $p$  and the wage  $w$  are predetermined. I investigate the set of prices and wages that can be equilibria of the dynamic model. To this end, retrace

	<i>Productivity</i>				
	<i>Lowest</i>	<i>Low</i>	<i>Medium</i>	<i>High</i>	<i>Highest</i>
Productivity	0.998	0.999	1.000	1.001	1.002
<b>Stabilize labor market</b>					
Labor market tightness	1.00	1.00	1.00	1.00	1.00
Product market tightness	2.2	3.3	5.0	6.7	7.8
Intermediate price	0.9993	0.9982	0.9972	0.9962	0.9951
<b>Stabilize product market</b>					
Labor market tightness	0.88	0.93	1.00	1.07	1.12
Product market tightness	5.0	5.0	5.0	5.0	5.0
Intermediate price	0.9972	0.9972	0.9972	0.9972	0.9972

Table 1. Alternative Outcomes with Sticky Price and Wage

the logic of Proposition 1 for the three boundary cases considered there. Suppose that a given boundary condition holds for every state. The same conclusions follow about any one of those states as followed for the single state in that proposition. Hence

**Proposition 3** *The dynamic equilibrium set for state  $i$  includes the static equilibrium set with  $x = x_i$ .*

The dynamic equilibrium set for a given state also includes other points in the  $(p, w)$  space, outside the triangle of Figure 1, because, for example, it may pay in a particular low-productivity state to look for a job anyway because the state is transitory. Proposition 3 establishes that the dynamic model will have an equilibrium for sure if all the states satisfy the real-wage bound in Proposition 2.

Table 1 illustrates a simple dynamic equilibrium. Here the price and wage are strictly predetermined at constant levels that do not depend even on the previous state—they are not linked in any way to central-bank policy as embodied in  $y$ . As discussed earlier in connection with the static equilibrium, with a predetermined price and wage, the central bank can pick any point along a line in the equilibrium set, with the fixed real wage determining the slope of the line.

The table shows two different choices the bank might make. One is to stabilize the labor market, so that the labor tightness  $\theta_J$  is held constant at 1. In this equilibrium, the central bank moves  $y$  enough upward when productivity is low and downward when productivity is high to keep  $yx - w$  constant. In conventional terms, the bank inflates enough in bad times and deflates enough in good times to insulate the level of unemployment from the



driving force. The new feature of the model is the analysis of the corresponding effects in the product market. When productivity is low and the bank raises the wholesale price to stabilize unemployment, retailers are squeezed because they sell at the fixed price  $p$  but buy at the higher price  $y$ . The product market slackens dramatically— $\theta_R$  falls by more than half. Customers find it much more difficult to find a willing seller. Merchants find it correspondingly easy to attract customers. The bank’s policy creates something like a shortage of goods in the product market by stabilizing unemployment. The reverse happens when the bank prevents unemployment from falling when productivity is strong, as shown in the right column of the upper panel of the table.

Table 1 shows that a constant price and wage are in the dynamic equilibrium set only for relatively small movements of productivity. If productivity in the lowest state is 0.997 instead of 0.998, the model no longer has an equilibrium in that state.

The opposite policy, shown in the bottom panel of Table 1, insulates the product market from fluctuations at the expense of movements of labor-market tightness  $\theta_J$ . This policy is one of strict insulation of the flexible wholesale price  $y$  from changes in productivity, so that the retail margin  $p - y$  remains constant in the face of productivity fluctuations. Because of the two asymmetries in the model—higher calibrated  $\theta_R$  than  $\theta_J$  and zero disamenity for time spend buying goods—the effects in the labor market are not as large.

Table 1 reveals a tradeoff of a conventional sort between a dovish policy of stabilizing unemployment at the cost of variations in the price level and high volatility of product-market conditions, on the one hand, and a hawkish policy of stabilizing the product market with a constant price level at the cost of high variability in the labor market, on the other hand. Although the model reveals new features of the tradeoff in the product market, it does not do justice to the challenge of central banking, because there is no feedback from central-bank policy in one period to prices and wages in subsequent periods. The principle of equilibrium does not require any feedback, but that only shows that the principle is not by itself able to limit outcomes adequately. In the model as developed so far, all kinds of unreasonable behavior are equilibria.

## 6 Stale Price and Wage

Within the infinitely rich set of equilibria of the model, I now explore those where the price and wage are stale, but otherwise have a market-clearing character. The price and the wage

in the current state are those that would have cleared the market (split the transactions surpluses) in the previous state. This setup makes the price and wage adaptive—they are reasonably close to clearing the markets in an economy where the most likely transitions are to nearby levels of productivity.

In this economy, the central bank adopts a monetary policy  $y_i$  that varies over the compound state  $i$ —the bank sets a different policy based on the history of the economy encoded in  $i$  as well as the current level of productivity. Households and firms know  $y_i$ .

The price  $p_{L(i)}$  and the wage  $w_{L(i)}$  clear a hypothetical market with conditions set in the previous fundamental state, except for the central-bank policy that actually prevailed then. I assume that the hypothetical market ignores the part of that policy that was responding to the even earlier state. In the market, bank policy is taken to be

$$\bar{y}_\iota = E(y_i | C(i) = \iota) \quad (27)$$

where the expectation is taken over the stationary distribution of  $i$ . Though this assumption is needed to keep the state space finite, I believe it is reasonable on its own grounds.

Given central-bank policy  $y_i$ , I find the equilibrium of a model comprising equations (18) through (26), written with the fundamental state variable  $\iota$  in place of the compound  $i$ , the value-splitting conditions,

$$H_{R,\iota} - H_{S,iota} = F_{R,\iota}, \quad (28)$$

$$H_{J,\iota} - H_{U,iota} = F_{J,\iota}, \quad (29)$$

and central-bank policy,

$$y_\iota = \bar{y}_\iota. \quad (30)$$

Let the price and wage in this hypothetical economy be  $\hat{p}_\iota$  and  $\hat{w}_\iota$ .

Then I find the equilibrium of the economy with central-bank policy  $y_i$  and predetermined price  $p_i = \hat{p}_{L(i)}$  and wage  $w_i = \hat{w}_{L(i)}$ . This economy has stale prices that reflect the market-clearing equilibrium of a virtually identical economy one state earlier.

It remains to specify the policy. In the modern literature on sticky-price models, the central bank chooses its policy to minimize its loss function, typically depending on the deviations of inflation and unemployment from fixed values. But assigning a cost to inflation volatility generally takes one outside the model at hand, which is attributes no cost to the volatility.

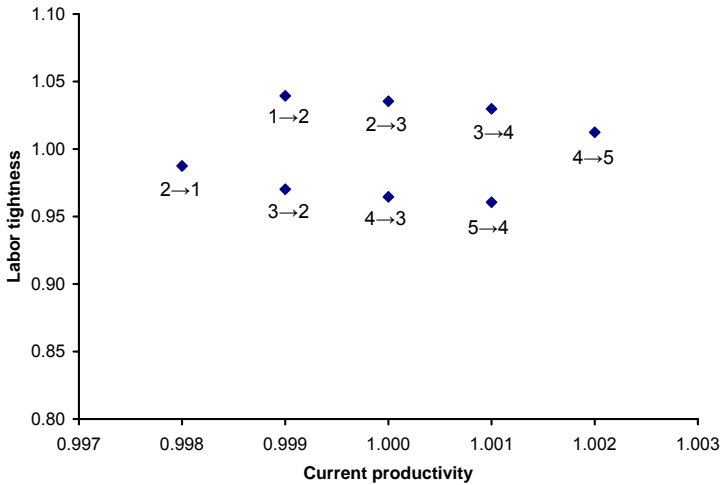


Figure 5. Labor-Market Tightness in Relation to Current and Previous Productivity State when Policy Stabilizes the Product Market

Instead, I will consider a class of central-bank policies that range from hawk to dove, that is, from price-level stabilization to unemployment stabilization. For the hawk policy, I solve the hypothetical model for the policy  $y_{H,i}$  that keeps the price level  $p_i$  constant across the fundamental states. For the dove policy, I solve the two models described above jointly for the policy  $y_{D,i}$  that keeps unemployment constant—that is, it keeps  $\theta_J$  constant.

## 6.1 Implications of hawk and dove policies

Figure 5 shows the implications of the hawk policy in the same format as Figure 4, for the labor market (the product market remains at the same level of tightness for all levels of productivity). Each point is marked with the prior and current productivity state. In the three intermediate states, which can be reached from below and above, small differences in earlier productivity induce large differences in labor-market tightness. A decline in productivity—current state below previous state—saddles the current economy with a high sticky wage and thus a softer labor market with higher unemployment.

Figure 6 is the counterpart of Figure 5 for the product market under the dove policy that stabilizes the labor market. Under the dove policy, keeping unemployment constant results in high volatility of the product market. This aspect of central-bank policies is novel to the product-market setup in this paper—it has escaped discussion in the New Keynesian literature, which presumes that firms sell products to customers with equal enthusiasm whether margins are high or low. When the product price inherited from the previous state is low

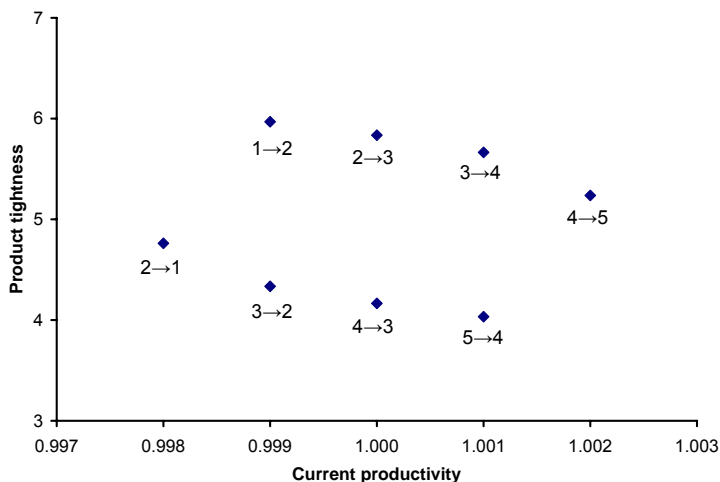


Figure 6. Product-Market Tightness in Relation to Current and Previous Productivity State when Policy Stabilizes the Labor Market

because productivity was higher in that state than in the current one, the product market slackens substantially. Retailers have low margins and make less effort to attract customers. Shoppers need to spend rather more time finding willing merchants. More resources are devoted to shopping in times of low productivity, the counterpart of the increase in resources devoted to job-seeking in those times if policy stabilizes the product market instead.

## 6.2 A policy frontier

I define a class of policies of the form

$$y_i = h y_{H,i} + (1 - h) y_{D,i}. \quad (31)$$

Here  $h$  is an index of hawkishness—that is, tendency to stabilize the product price—ranging from 0 (dove policy to stabilize the labor market) to 1 (hawk policy to stabilize the product market).

Figure 7 shows the policy frontier with respect to the volatilities of conditions in the product and labor markets, measured by the coefficient of variation (ratio of standard deviation to mean). The policies have a pronounced asymmetry derived from the two asymmetries in the calibration—higher equilibrium tightness in the product market and no disamenity to shopping comparable to the disamenity of work. The policy of stabilizing the labor market induces high volatility of product-market tightness relative to the modest volatility of labor-market conditions resulting from stabilization of the product market.

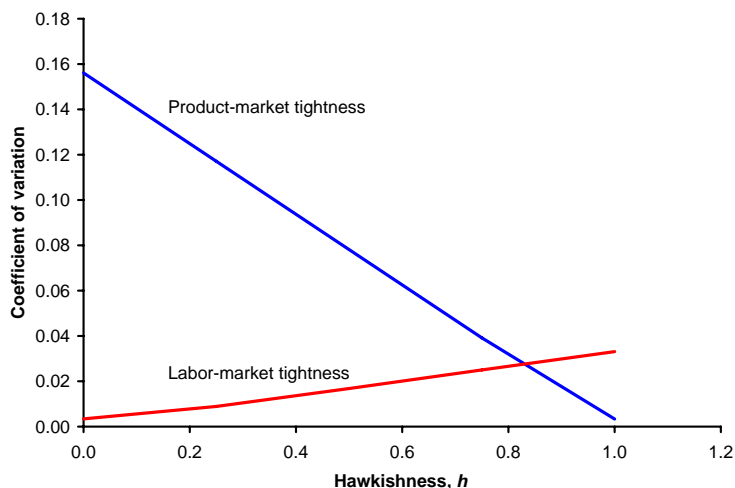


Figure 7. Relation between Hawkishness of Policy and Volatilities of Product- and Labor-Market Tightness

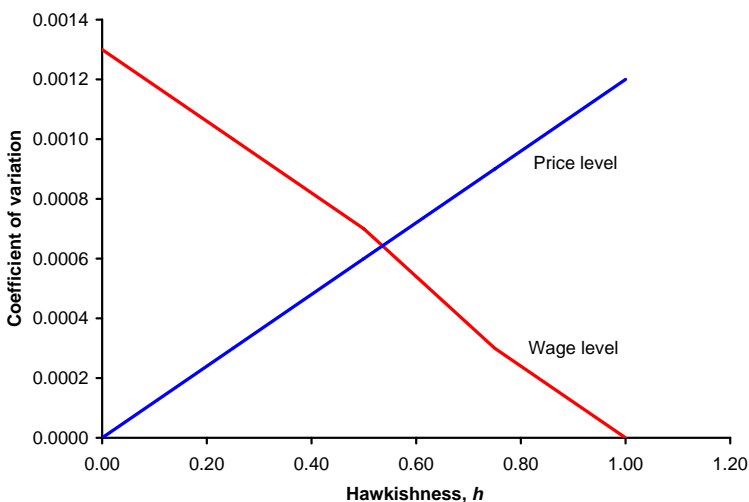


Figure 8. Relation between Hawkishness of Policy and Volatilities of Price and Wage

Figure 8 shows that the asymmetry in tightness does not apply to the product price and the wage. These have similar volatilities at  $h = 0$  for the price and  $h = 1$  for the wage. Both volatilities are low. Any choice of policies in the  $h$ -class delivers a stable price and a stable wage. Other policies that assigned substantially different values of  $y_i$  to different states could bring about high volatilities of the price and the wage, but without any purpose.

## 7 The New Keynesian View of the Product Market

The New Keynesian model recognizes that a competitive product market is inconsistent with a sticky product price. The seller with the lowest sticky price will attract all of the customers

and will incur unlimited losses if that price is below marginal cost. This consideration has led New Keynesian modelers to impute market power to sellers. The assumption of market power solves the problem of explaining sellers' willingness to serve customers even when the sticky price is below the profit-maximizing price. But the model fails to consider the changes in the relation between customers and merchants that occurs when the gap between the profit-maximizing and sticky price rises and the payoff to serving a customer falls.

## 8 Concluding Remarks

In a model where customer and merchant enjoy a small surpluses from their relationship, they may transact at any price in their bargaining set and still be in equilibrium, in the sense that no bilateral alteration can make both better off. This perspective differs from existing sticky-price models. A wide variety of state-dependent prices can be equilibria, but the interesting ones may be those involving stale prices. These are prices that reflect conditions in the recent past.

A model with equilibrium stale prices reproduces some of the properties of modern models of monetary non-neutrality, where the price today is derived from conditions in an earlier state and is not indexed to changes in central-bank policy in the current state. In particular, the nominal wage may be higher than usual in the bargaining set, resulting in unusually low incentives to create jobs and thus in higher unemployment. The model provides an equilibrium account of the influence of sticky—that is, stale—nominal wages on real activity that is an alternative to the New Keynesian model.

The primary new insight that this model delivers is the potential importance of fluctuations in product-market conditions. These fluctuations will be substantial if the central bank uses its policy lever to stabilize the labor market.

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