

Carry Trades and Speculative Dynamics*

Guillaume Plantin
London Business School
gplantin@london.edu

Hyun Song Shin
Princeton University
hsshin@princeton.edu

Revised, June 2007

Abstract

We develop a model of foreign exchange trading with imperfect liquidity. Speculators have a collective impact on market liquidity. Moreover, their margin requirements decrease with market liquidity. Such circumstances can turn carry trades into self-enforcing arbitrage opportunities: Carry trades generate all the more value because many speculators enter them. As a result, rational speculation destabilizes the exchange rate. Applying recent advances in dynamic coordination games, we obtain a unique equilibrium exchange rate with high conditional skewness. Namely, extended periods of slow depreciation of the low rate currency are followed by abrupt reversals. Reversals are stochastic, but their distribution is uniquely determined by the distribution of the fundamentals.

*We thank participants in various seminars and conferences, and in particular Patrick Bolton, Doug Diamond, Gur Huberman, Pete Kyle, Ady Pauzner, Lasse Pedersen, Tano Santos, and Dimitri Vayanos for their comments.

Introduction

Currency carry trades consist in selling currencies forward that are at a significant forward premium - that is, selling a low interest rate currency to fund the purchase of a high interest rate currency. In particular, the yen carry trade has been a topical subject of debate over the last decade given the extended period of low interest rates in Japan. Carry trades aim in practice at exploiting the well-documented "forward discount bias" - the fact that currencies that trade at a forward premium tend to depreciate. Profits from carry trades stem partly from the interest rate differential, and mostly from this subsequent appreciation of the high rate currencies. In a recent study, Burnside et al. (2006) find that currency carry trades generate high Sharpe ratios that do not seem to correspond to a compensation for a variety of risk factors such as consumption growth. More generally, from the perspective of asset pricing theory, the forward discount bias is by and large an anomaly. Deriving it from pure risk to consumption arguments has proven difficult for a whole range of "plausible" preferences (see Backus, Foresi, and Telmer, (2001) for a discussion).

A popular view is that the forward discount bias is not only the *pre-condition* for carry trades, but is also a *consequence* of carry trades.¹ The rationale behind this view starts with the observation that most central banks set official overnight interest rates mainly with domestic monetary policy considerations in mind, rather than the external exchange rate environment. When official interest rates are held fixed by central banks, carry trades can become self-reinforcing. As more and more speculators pile into the carry trade, they sustain the appreciation of the high interest rate currency relative to the low interest rate currency. This notion that an arbitrage opportunity

¹see, e.g., *Carry on Speculating*, The Economist, February 22nd 2007: "One obvious possibility is that the actions of carry traders are self-fulfilling; when they borrow the yen and buy the dollar, they drive the former down and the latter up."

can be magnified by rational speculation is at odds with the usual view that arbitrage opportunities should become less, not more profitable as more speculators exploit them.

This paper develops a theoretical model that identifies plausible conditions under which carry trades lead to *self-enforcing arbitrage*². We also derive carry trades' impact on speculative dynamics in FX markets. We consider speculators who have a collective ability to move an exchange rate because they face dealers who do not supply perfect liquidity.³ By contrast, at least over some random period, short-term funds are in perfectly elastic domestic supply and demand at the rates prevailing in each currency. In other words, the speculators do not believe that their impact on exchange rates is factored in by monetary policies over this random period.

With these two ingredients only, we obtain the non-surprising standard result that speculation is stabilizing. Anticipating future corrections, speculators bring the market exchange rate in line with their view of the fundamental parity between the two currencies. This result has a particularly strong form in our setup, however. Stabilizing speculation is not only a Nash equilibrium of the trading game, it is the only possible outcome when it is common knowledge that all speculators are rational.

Imposing additional - possibly small - funding constraints on speculators can change everything in this framework. Following Brunnermeier and Pedersen (2007), we assume that speculators can structure a carry trade at a lower cost whenever there is a lot of liquidity in the market, for instance because in this case, their positions have a higher collateral value in the eyes of their financiers. This may destabilize the exchange rate. Carry trades become self-justified arbitrages: A speculator is willing to enter the carry

²We are grateful to the editor for suggesting this term.

³In line with the important role played by illiquidity in our model, Burnside et al. (2006) argue that the main limitation of the arbitrage profits generated by carry trades are liquidity issues in FX markets.

trade only if she believes that other speculators will do so. In our dynamic trading game, such strategic complementarities do not give rise to multiple equilibria, but rather to a unique equilibrium exchange rate process with stochastic bifurcations. Extended periods of slow appreciations of the high rate currency are stochastically punctuated by endogenous crashes. Currency traders refer to such patterns as “going up by the stairs and coming down in the elevator” (see Breedon, 2001).

While focussed on this endogenously generated speculative dynamics, our paper closely relates to three contributions. Our justification of the forward discount bias is close to the explanation based on "positive feedback trading" developed in Froot and Thaler (1990). Our modelling of speculation as a dynamic coordination game relates to Abreu and Brunnermeier (2003), although both "bubbles" and "crashes" are endogenous in our framework. Finally, our refinement of multiplicity of equilibria closely follows from Frankel and Pauzner's (2000).

1 Baseline Model

Time is continuous and is indexed by $t \in [0, +\infty)$. There are two assets. One asset is denominated in Japanese yen and serves as *numéraire*, and the other asset is U.S. dollar denominated. The relative price of the dollar-denominated asset at date t is denoted p_t . Each asset may be interpreted as a deposit and we will interpret p_t as the dollar/yen exchange rate. These two assets are exchanged between two types of agents, speculators and dealers. Speculators bet on the evolution of p_t and dealers supply liquidity. Each type comprises a continuum of agents with unit mass.

The speculators (also called "traders" henceforth) are risk neutral and do not discount the future. Their date- t portfolio choice consists of holding either one dollar denominated asset or p_t yen assets. It is common knowledge

among the speculators that the *fundamental value* of p_t is $v \in (0, 1)$. More precisely, they know that there is a stopping time at which the market price p_t will snap back to v for exogenous reasons, and then remain there forever. The stopping time has Poisson arrival intensity ρ . The idea here is similar to the notion of a “day of reckoning” in Duffie, Gârleanu, and Pedersen (2002) on which there is an exogenous public announcement that reveals the relative value of the future consumption generated by the dollar asset to all market participants. The assumption that the price remains at v forever once it has snapped back to v is offered as a simplification. Our focus is on how traders behave *in anticipation* of this anchor to the fundamental. In this section, for simplicity, assets generate no consumption until the day of reckoning.

The situation that we aim to capture with this stylized setup is one in which the speculators expect that the Bank of Japan and the Fed will maintain short-term rates unchanged until the "day of reckoning". In particular, traders believe that neither country will attempt to respond to exchange rate fluctuations before this random date. We study whether speculation stabilizes or destabilizes the exchange rate under such circumstances.

Speculators aim at maximizing their expected trading profits before the day of reckoning. They face a small friction in how often they can trade. A speculator can only trade at discrete designated trading dates that are generated by a Poisson process with intensity λ . The processes are independent across traders, so that a fraction λdt of the traders gets a chance to trade between t and $t + dt$. The ratio given by

$$\lambda/\rho \tag{1}$$

indicates the number of times a trader may be expected to get an opportunity to trade before p_t snaps back to fundamentals. In a very active market such as the FX market, we would expect the traders to have a free hand in trading, and so for this reason our main focus will be on the limiting case where the

ratio λ/ρ is large.

This small friction may be interpreted as the time it takes to a hedge fund to structure a large deal with prime brokers, or to a proprietary trader to clear internal risk controls before a large trade. Let x_t denote the fraction of traders who are invested in dollars at date t . This fraction has the following dynamics:

$$\begin{cases} \dot{x}_t = -\lambda x_t & \text{when traders sell the dollar} \\ \dot{x}_t = \lambda(1 - x_t) & \text{when traders buy the dollar} \end{cases}$$

This departure from continuous trading strategies is the key feature of the model that warrants equilibrium uniqueness in Section 3.

When she has a chance to trade at date t , a trader meets the market-making sector. This sector is comprised of a continuum of dealers who have heterogeneous valuations of the dollar asset with c.d.f. $F(\cdot)$ until the day of reckoning. This may stem from heterogeneity in their inventories, or from heterogeneous beliefs about the fundamentals v . Like the traders, each dealer can be long up to one dollar asset. At each trading date t , the trader submits a supply or demand schedule to the dealers, and the non-filled part is cancelled. As a result, the price of the risky asset p_t solves:

$$c_t = F(p_t) \tag{2}$$

where c_t is the cash that traders have invested in the market up to date t . Equation (2) formalizes that the date t trader buys the dollar asset from the dealer who owns it and values it the least at date t , or sells it to the dealer who does not own it and values it the most at date t . This corresponds to a trade with a dealer with a valuation of $F^{-1}(c_t)$ in both cases. For expositional simplicity only, we will assume in this paper that the valuations of the dealers are uniformly distributed over $(0, 1)$.

Since we have assumed that traders are long up to one dollar, any capital gains or losses realized by a trader between two trading dates are accumulated in yen: There is no compounding of gains or losses. Thus, the date t price before the day of reckoning satisfies

$$p_t = x_t \tag{3}$$

where x_t is the proportion of the traders who hold one dollar asset. Note that there is ample evidence that prices respond to flows in FX markets (see, e.g., Cao, Evans, and Lyons (2006)).

At trading date t , a trader who holds the dollar asset faces a binary decision - to keep it or to sell it for p_t yen assets. For a trader who does not already hold the dollar asset, the binary decision is either to buy it at price p_t , or to maintain her yen holdings. At the time of making a decision, the trader can condition on the realized price path as well as the calendar date t . Thus, the trading strategy of a trader is a mapping:

$$(t, (p_u)_{u < t}) \mapsto \{\text{dollar asset, yen asset}\} \tag{4}$$

that specifies whether a trader will hold dollars or yens for all pairs of dates and price histories.

Dominance Solvable Outcome

Our baseline model allows us to draw a very strong conclusion - starting from any price p_0 , the price until the day of reckoning returns to the fundamental value v at the fastest possible rate. Any other outcome can be ruled out by the iterated deletion of strictly dominated strategies. Iterated dominance is a weaker solution concept than Nash equilibrium in the sense that Nash equilibria survive iterative elimination of dominated strategies.

Suppose that the price is p_t . The most pessimistic scenario for the holder of the dollar asset is that all future traders either switch out of it, or refrain

from buying it so that the price path is declining over time. Under this most pessimistic scenario, the price path is given by $\{p_{t+u}\}_{u \geq 0}$, where

$$p_{t+u} = p_t e^{-\lambda u}. \quad (5)$$

In other words, the price converges to 0 at the rate λ , as each trader whose trading date arrives switches out of the dollar asset.

Even under this most pessimistic scenario, there is a price at which a trader is better off holding the dollar asset than the yen asset. Consider a speculator who has a chance to trade at date t . If the price path from date t onward is given by $\{p_{t+u}\}_{u \geq 0}$ then the expected excess rate of return on the dollar is:

$$\int_0^\infty \frac{\lambda p_{t+u} + \rho v}{p_t} e^{-(\lambda + \rho)u} du - 1. \quad (6)$$

Thus, if the future price path is given by $\{p_{t+u}\}_{u \geq 0}$, the trader buys the dollar asset or holds on to it whenever (6) is greater than 0.

By substituting (5) into the expression for expected return given by (6) we can obtain the price \underline{p}^0 at which a trader is indifferent between holding dollar and yen under this most pessimistic scenario. This threshold price \underline{p}^0 is given by

$$\underline{p}^0 \equiv \frac{(1 + 2\theta)v}{(1 + \theta)^2} \quad (7)$$

where θ is defined as the ratio λ/ρ . If the price falls below \underline{p}^0 , then holding yen is dominated. Note that \underline{p}^0 tends to 0 as $\theta \rightarrow \infty$.

But then, the most pessimistic price path given by (5) is *too pessimistic* in that it assumes that some future traders may choose dominated actions. By ruling out trading strategies that are dominated the most pessimistic price path now becomes:

$$\{\max(\underline{p}^0, p_t e^{-\lambda u})\}_{u \geq 0} \quad (8)$$

Since (8) implies strictly higher prices than (5) beyond some date in the future, we can define a new threshold price given by \underline{p}^1 below which holding yen is dominated. Clearly, $\underline{p}^0 \leq \underline{p}^1$. If the price is below \underline{p}^1 , the trader will not hold yen. Thus, any trading strategy in which a trader chooses the yen asset at a price below \underline{p}^1 is ruled out after *two* rounds of deletion of dominated strategies.

We can iterate this argument. After $n+1$ rounds of deletion of dominated strategies, the most pessimistic price path starting from p_t is given by:

$$\{\max(\underline{p}^n, p_t e^{-\lambda u})\}_{u \geq 0}$$

This sets a new threshold \underline{p}^{n+1} for the trading strategy, in which choosing yen for any price below \underline{p}^{n+1} is ruled out by $n+2$ rounds of deletion of dominated strategies. We thus obtain the increasing sequence:

$$\underline{p}^0 \leq \underline{p}^1 \leq \underline{p}^2 \leq \dots \leq \underline{p}^n \leq \dots$$

Since price is bounded above, this sequence converges to some limit, denoted by \underline{p} . No trader will choose yen below \underline{p} in any rationalizable outcome, since such an action is ruled out by iterated dominance. Thus, \underline{p} constitutes a floor for the price of the dollar asset in any price path $\{p_{t+u}\}_{u \geq 0}$.

Analogously, we can define a *decreasing* sequence of thresholds that corresponds to the most *optimistic* price paths that are consistent with n rounds of deletion of dominated strategies. If the price is sufficiently close to the upper bound 1, then yen is strictly preferred since the price will never rise sufficiently to compensate for the risk that it could possibly fall to its fundamental value v . Let \bar{p}^0 be the price above which selling is dominant. Thus, the price path will never rise above this level. We can then iterate the argument to derive the decreasing sequence:

$$\bar{p}^0 \geq \bar{p}^1 \geq \bar{p}^2 \geq \dots$$

Denote by \bar{p} the limit of this sequence. This limit would constitute a ceiling for any price path. Clearly,

$$\underline{p} \leq \bar{p}. \quad (9)$$

We will now show that the reverse inequality must hold, too. Consider the floor price \underline{p} . We must have $\underline{p} \geq v$. To see this, suppose (for the sake of argument) that $\underline{p} < v$. Since no trader sells dollars below \underline{p} , the future path $\{p_{t+u}\}_{u \geq 0}$ lies on or above \underline{p} . Thus, conditional on a price \underline{p} , the expected return on the dollar asset is *strictly* greater than one since all possible future values of the asset are larger than \underline{p} . But this contradicts the fact that \underline{p} is the upper limit of the sequence of indifference thresholds. Hence, we must have

$$\underline{p} \geq v. \quad (10)$$

From an exactly analogous argument, we conclude that $v \geq \bar{p}$. Thus, we have

$$\underline{p} \geq v \geq \bar{p} \quad (11)$$

From (11) and (9), we conclude that $\underline{p} = \bar{p} = v$. We have thus proved the following.

Proposition 1

In any subgame, the only trading strategy that survives the iterated deletion of dominated strategies is to hold the dollar asset when $p_t \leq v$ and hold the yen asset when $p_t > v$.

Corollary 2 *In the unique equilibrium price path in the subgame that starts with price p_t , the price converges to the fundamental value at the maximum speed that trading constraints allow for.*

Our baseline model shows the power of the stabilizing role of speculation, as argued by Friedman (1953). No matter how loose the anchor is to the

fundamentals, the speculative behavior of traders push the price to coincide with the fundamentals. This result does not rely upon any particular equilibrium concept, but on the mere assumption that traders are rational and that this is common knowledge.

Our result can be understood as the resolution of two competing externalities generated by the predecessors of the date t trader. As the predecessors throw more “weight of money” into the dollar asset, there are two effects. First, the positive externality is that the future resale values $(p_{t+u})_{u \geq 0}$ will be high, other things being equal. But the negative externality is of course that the dollar asset is currently expensive. Because of the risk that the dollar asset reverts to its fundamental value, the negative externality ultimately wins out. Thus, a trader has no incentive to join in pushing the price away from its fundamental value. Instead, the trader will seek to trade against her predecessors to bring the price back into line with fundamentals. When θ is large, fundamental risk is small compared to the risk that other speculators create an adverse price move. In this case, the competition between positive and negative externalities is more even, in the sense that very small additional positive externalities tip the balance toward conditions that are more fertile to the emergence of destabilizing speculation, as we see now.

2 Funding Externalities

We now add to this baseline model two features that capture important practical aspects of yen carry trades.

First, we introduce a positive carry. The initial motive for funding carry trades in yen is the persistence of very low Japanese official rates. Accordingly, we assume that the dollar asset generates a higher (real) rate of return than the yen asset. Formally, we posit that the excess rate of return from holding dollar assets over yen assets (6) features a carry equal to $\delta > 0$ per

unit of time.

Second, we take into account that speculation requires capital. When she enters a carry trade at date t - sells yen assets to obtain a dollar asset in our setup - an investor needs to tie up some capital. More precisely, only a fraction of the dollar asset equal to

$$1 - h(p_t)$$

where $h(p_t) \in (0, 1)$ can be financed by the sale of yen assets. The remaining fraction $h(p_t)$ has to be financed by the trader's own capital. This captures the haircut that a broker would require as collateral from the speculator. The trader's own capital has an opportunity cost of $\Delta > 0$ per unit of time.

Our key assumption is that the collateral requirements to enter a yen carry trade are lower when there is already a lot of liquidity invested in the trade. *Namely, we assume that $h(p)$ decreases with respect to p .* This assumption follows Brunnermeier and Pedersen (2007). Their paper describes a variety of practical situations in which funding liquidity increases as market liquidity increases. A plausible explanation for this feature of the haircut $h(\cdot)$ is that the lenders are less informed than the speculators, and thus do not know if an increase in p_t is due to a speculative flow or reflects some fundamental news. In the latter case, the collateral value of the dollar asset is enhanced in their eyes.⁴ If more cash in the market implies a possible higher collateral value of the trade in the eyes of the financiers, then speculators' leverage should increase with respect to p_t . An explicit modelling of such funding frictions is beyond the scope of this paper. Rather, we take this feature as given and study its impact on exchange rate dynamics. We assume that the speculators are protected by limited liability, and that there exists $p^* \in (0, 1)$ such that

$$h(p^*) = \frac{\delta}{\Delta}. \tag{12}$$

⁴Brunnermeier and Pedersen (2007) formalize this argument.

Note that assumption (12) does not impose that the haircut takes large values since the opportunity cost of capital of a highly leveraged speculator would easily be much larger than the carry (possibly up to five or six times). Finally, we assume that h' is bounded away from 0 over $(0, 1)$.

With these two additional features, the expected excess rate of return on the carry trade now becomes:

$$\begin{aligned} & \underbrace{\frac{1}{p_t} \int_0^{+\infty} [\lambda \max(p_{t+u}, (1-h(p_t)) p_t) + \rho \max(v, (1-h(p_t)) p_t) e^{-(\lambda+\rho)u}] du - 1}_{\text{Profit or loss due to exchange rate fluctuations}} \\ & - \underbrace{\left(\int_0^{+\infty} (\lambda + \rho) (\Delta h(p_t) - \delta) u e^{-(\lambda+\rho)u} du \right)}_{\text{Cost of capital minus carry}} \end{aligned} \quad (13)$$

That the cost of funding a carry trade decreases w.r.t. p_t implies that speculators create additional positive externalities for each other by entering carry trades. We now show that these externalities may suffice to dramatically change the situation of the baseline model.

Proposition 3

Starting from any price p_t , if λ is sufficiently large and ρ sufficiently small, there are multiple equilibria. In particular, there is both an equilibrium in which all traders keep entering the carry trade after t , and also an equilibrium in which all traders unwind their carry trade after t .

Proof. If all traders enter the carry trade after t' , then

$$p_{t'+u} = p_{t'} e^{-\lambda u} + (1 - e^{-\lambda u})$$

and the yen profit of a carry trade Π is

$$\Pi = \underbrace{\frac{\lambda^2}{(2\lambda + \rho)(\lambda + \rho)} (1 - p_{t'})}_{(1)} + \underbrace{\frac{1}{\lambda + \rho} (\rho (\max(v - p_{t'}, -h(p_{t'}) p_{t'}) - (\Delta h(p_{t'}) - \delta) p_{t'}))}_{(2)}.$$

For λ sufficiently large and ρ bounded above, Π is positive for all $p_t \leq \frac{p^*+1}{2}$ because term (1) is positive and dominates term (2) for all $p_t \leq \frac{p^*+1}{2}$. Fix such a λ . For ρ sufficiently small, Π is positive for all $p_t > \frac{p^*+1}{2}$ because (2) is positive as well for such p_t since $h(\cdot)$ is decreasing. Thus, we have shown that starting from any price, for λ sufficiently large and ρ sufficiently small, all speculators find it profitable to enter the carry trade if they believe that other speculators will do so. Entering the carry trade is therefore a self-enforced arbitrage.

The proof that exiting the carry trade is also a self-enforced arbitrage is symmetric. ■

The contrast between the stabilizing role of speculation in the benchmark case and the de-stabilizing role of speculation in Proposition 3 is very striking. We can give an alternative interpretation of why we have multiple trading equilibria. When λ becomes large, we get closer to a single-shot game between the traders since they can trade very frequently. The two extreme steady states (all sell, all hold) resemble the Nash equilibria of a binary action game between the traders.

The fact that the two extreme steady states resemble Nash equilibria in the single-shot game suggests that trading decisions are strategic complements - that is, the more other traders buy, the greater my incentive is to buy (and conversely, the greater the other traders sell, the more I want to sell). Thus, the strategic incentives become inverted, as compared to the benchmark case. We commented after our benchmark Proposition in the previous section that the reason why speculation is stabilizing comes from the fact that the *negative* externalities created by previous buyers outweigh the *positive* externalities. In Proposition 3, the roles are reversed. If λ is sufficiently large and ρ sufficiently small, the positive externality of raising the price higher is larger than the negative externality even if the sensitivity of the haircut to the price h' is arbitrarily close to 0. This is because in

this case, the probability that the asset price will snap back to v during the current trade is so small that the positive funding externalities always offset the risk of holding an overvalued asset.

When funding constraints create such strategic complementarities, the price path itself will influence expected payoffs, and we cannot come to any firm conclusions regarding predictable outcomes without additional argument. In general, we can envisage very complicated dynamic strategies that try to balance the negative and positive externalities between traders, and we cannot say much more without additional structure on the problem. Rather than going further in investigating complex dynamics, we will now go in a different direction. We will now examine what happens when the carry itself is stochastic.

3 Stochastic Fundamentals

It turns out that the multiplicity of equilibria in Proposition 3 is not robust to the addition of some variation in the carry δ . Adding (possibly arbitrarily small) shocks on δ , we obtain a unique dominance-solvable equilibrium. Such shocks may be interpreted as liquidity trading in domestic markets or noise in monetary policies. We draw on the work of Burdzy, Frankel and Pauzner (2001) and Frankel and Pauzner (2000), who showed that in binary action coordination games with strategic complementarities, the addition of small stochastic shocks to the fundamentals of the payoffs generates a unique, dominance solvable outcome. The arguments in these papers are similar to the “global game” arguments of Carlsson and van Damme (1993) and Morris and Shin (1998). We return to an interpretation of the results later in the paper.

Formally, we assume in this section that the carry obeys the process:

$$\delta_t = \delta + \mu t + \sigma W_t,$$

where W_t is a standard Brownian motion, $\mu \in \mathbb{R}$, and $\sigma > 0$.

In addition, we also assume that the anchor to the fundamental v is weaker than in the previous sections, in that the exogenous correction at the day of reckoning τ cannot move the price at an arbitrarily large rate. Formally, v is a function of the market price at date τ , $v(p_\tau)$, that satisfies:

Condition 4 *There exists $K > 0$ such that the rate of appreciation/depreciation of the dollar at the day of reckoning τ*

$$r(p_\tau) = \frac{v(p_\tau) - p_\tau}{p_\tau}$$

is Lipschitz-continuous with constant K . In words, the impact of the exogenous intervention on return is limited.

Note that for K arbitrarily large, condition 4 does not prevent the exchange rate from snapping back to a fixed level v unless p_τ is arbitrarily close to 0. For sufficiently small p_τ , the exogenous correction of the exchange rate cannot bring the dollar back to an arbitrarily large level. Intuitively, this corresponds to a free-floating regime in which interventions might only moderate market fluctuations, but do not target a specific parity. The exact role of this condition will be explained shortly. Note that this condition would create even more instability in Section 2, in that it makes self-fulfilling "runs" on the dollar easier to sustain. Despite this additional source of instability, we have the following result:

Proposition 5

If $\theta = \frac{\lambda}{\rho}$ is sufficiently large, there is a Lipschitz downward-sloping function $Z(\cdot)$ such that in any subgame starting at date t with a carry δ_t and an exchange rate p_t , there is a unique, dominance solvable solution to the trading game. In this solution, a trader who trades at date t engages in the carry trade if and only if $\delta_t \geq Z(p_t)$.

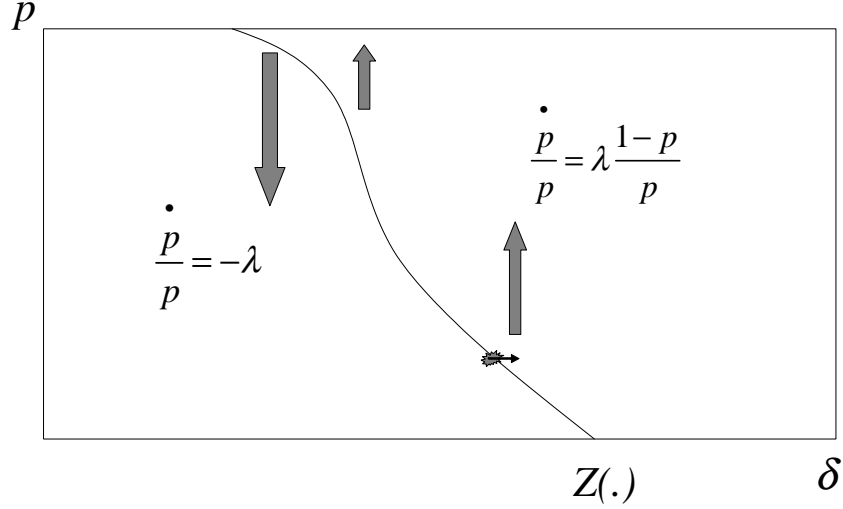


Figure 1: Unique equilibrium with stochastic δ_t

Proposition 5 does not impose any restriction on μ, σ or $h(\cdot)$, but only on the expected number of trades $\frac{\lambda}{\rho}$. Proposition 5 states that the multiplicity of equilibria that we saw in the previous section disappears when the carry moves around stochastically. Not only is the equilibrium unique, but it is dominance solvable. This proposition can be illustrated in figure 1. The curve $Z(p_t)$ divides the square into two regions. Proposition 5 states that in the unique equilibrium, any trader decides to hold dollar to the right of the $Z(\cdot)$ curve, and holds yen to the left of the $Z(\cdot)$ curve. Thus, the price will tend to rise in the right hand region, and tend to fall in the left hand region, as indicated by the arrows in figure 1.

The price dynamics implied by the unique equilibrium is given by:

$$dp_t = 1_{\{\delta_t > Z(p_t)\}} \lambda (1 - p_t) dt - 1_{\{\delta_t < Z(p_t)\}} \lambda p_t dt. \quad (14)$$

where $1_{\{\cdot\}}$ denotes the indicator function that takes the value 1 when the

condition inside the curly brackets is satisfied. These processes are known as *stochastic bifurcations*, and are studied in Bass and Burdzy (1999) and Burdzy et al. (1998). From Theorem 1 in Burdzy et al. (1998), for a given initial price p_0 , and for almost every sample path of δ , there exists a unique Lipschitz solution $(p_t)_{t \geq 0}$ to the differential equation (14) defining the price dynamics for Z Lipschitz decreasing.

Some suggestive features of the price dynamics can be seen from figure 1. When the price of the risky asset is near its upper bound (that is when p_t is close to 1) the rate of return when the currency appreciates is given by

$$\frac{\dot{p}}{p} = \lambda \frac{1-p}{p} \simeq 0$$

However, when the price crosses the Z boundary, the rate of depreciation is

$$\frac{\dot{p}}{p} = -\lambda$$

In other words, when p is high and the currency crosses the Z boundary from above, there is a sharp depreciation that was preceded by a slow appreciation. Such dynamics are suggestive of the price paths of high-yielding currencies in carry trades that “go up by the stairs and come down in the elevator”.

We provide a sketch of the proof of Proposition 5 that follows closely the argument given by Frankel and Pauzner (2000) for their discussion of binary coordination games. The difference between our setup and the game studied in Frankel and Pauzner (2000) is that viewed from date t , the future instantaneous profits at date $t + u$ depend on p_{t+u} , but also on p_t (see expression (13)). It is easy to see that their proofs apply almost identically, however. This is because condition 4 ensures that for θ sufficiently large, the yen profit from the carry trade will always be increasing in p_t . To see this, note that

the yen profit from a date t carry trade, $\pi(p_t)$, can be written:

$$\pi(p_t) = \int_0^{+\infty} \left[\begin{array}{l} \lambda \max(p_{t+u} - (1 - h(p_t))p_t, 0) \\ + \rho \max(v(p_t) - (1 - h(p_t))p_t, 0) \end{array} \right] e^{-(\lambda+\rho)u} du \quad (15)$$

$$- \frac{p_t}{\lambda + \rho} ((\lambda + \rho + \Delta)h(p_t) - \delta_t).$$

Denote by $Z_0(p_t)$ the boundary of the dominance region to the right of which it is dominant to hold the dollar asset. Namely, $Z_0(p_t)$ is the smallest number such that when

$$\delta_t \geq Z_0(p_t),$$

then $\pi(p_t) \geq 0$ even when for all $u \geq 0$,

$$p_{t+u} = p_t e^{-\lambda u} \quad (16)$$

We start with the following lemma.

Lemma 6

$Z_0(\cdot)$ is Lipschitz and nonincreasing for θ sufficiently large.

Proof. Condition 4 implies that $\frac{v(p)}{p}$ is Lipschitz-continuous. Plugging (16) in (15) yields that

$$\lim_{\theta \rightarrow \infty} Z_0(p_t) = \lambda h(p_t) \left(1 - \frac{h(p_t)}{2} \right) + \Delta h(p_t),$$

and that $Z_0(\cdot)$ is Lipschitz and decreasing for θ sufficiently large. ■

Note that absent condition 4, $Z_0(\cdot)$ would be Lipschitz decreasing for all values of p_t except in a neighborhood of 0. It is unclear to us whether the stochastic bifurcation equation (14) would still admit uniquely defined Lipschitz paths in this case. Condition 4 essentially allows us to circumvent this open mathematical question. Under condition 4, speculators worry less about fundamental mispricings, and thus worry more about interest rate fluctuations when making investment decisions.

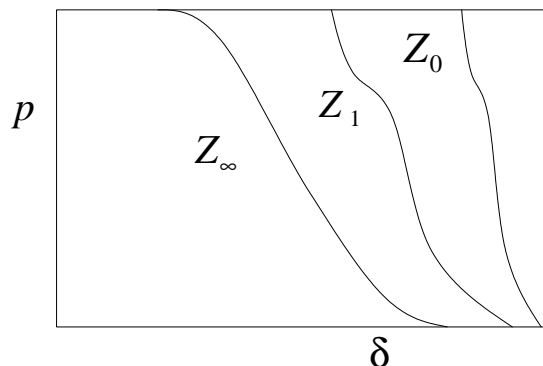


Figure 2: Iterative dominance from right. The curves Z_i are recursively defined as follows. Z_i is such that, if $\delta \geq Z_i(p)$, then a trader is willing to hold dollar if she believes that other traders hold dollar when they are on the right of Z_{i-1} .

Refer now to figure 2. Ruling out any strategy in which the trader holds yen to the right of Z_0 , we can derive a boundary Z_1 for the second-round dominance region which indicates the region where it is dominant to hold dollar in the absence of any first-round dominated trading strategies. In other words, if she knows that other traders hold dollars at least when they are on the right of Z_0 at their trading dates, a trader will be willing to hold dollar at least when she is on the right of Z_1 . We skip the proof that $Z_1(\cdot)$ is Lipschitz with at most the same constant as $Z_0(\cdot)$ (identical to Frankel Pauzner 2000). Condition 4 ensures that Z_1 is decreasing. To see this, note that we know from Lemma 6 that Z_1 would be decreasing if traders were selling dollar all the time. In the case in which the other traders use $Z_0(\cdot)$ as a buy/sell dollar frontier, all else equal, a higher p increases the probability of future dollar buys because $Z_0(\cdot)$ is nonincreasing. In sum, if $p' \geq p$, then $Z_1(p') \leq Z_1(p)$ because i) even absent any future dollar purchases, the yen profit would be higher in p' from Lemma 6, ii) in addition there will be more future dollar purchases starting from (v, p') than from (δ, p) since Z_0 is

nonincreasing.

By iterating this process, we can obtain the boundary Z_∞ for the region where a trader holding yen can be eliminated by iterated dominance. Z_∞ is decreasing Lipschitz as a limit of decreasing Lipschitz functions with decreasing Lipschitz constants. The boundary Z_∞ defines an equilibrium strategy since, if all traders hold yen to the left and hold dollar to the right, the indifference point between dollar and yen for the trader also lies on Z_∞ .

Consider now a translation to the left of Z_∞ so that the whole of the curve lies to the left of the yen-dominance region. Call this translation Z'_0 . To the left of Z'_0 , holding cash is dominant. Then construct Z'_1 as the *rightmost translation* of Z'_0 such that a trader must choose cash to the left of Z'_1 if she believes that other traders will play according to Z'_0 . By iterating this process, we obtain a sequence of translations to the right of Z'_0 . Denote by Z'_∞ the limit of the sequence. Refer to figure 3. The boundary Z'_∞ does not necessarily define an equilibrium strategy, since it was constructed as a translation of Z'_0 . However, we know that if all others were to play according to the boundary Z'_∞ , then there is at least one point A on Z'_∞ where the trader is indifferent between holding cash and holding the risky asset. If there were no such point as A , this suggests that Z'_∞ is not the *rightmost translation*, as required in the definition.

We claim that Z'_∞ and Z_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between Z'_∞ and Z_∞ . Then, choose point B on Z_∞ such that A and B have the same height - i.e. have the same second component. But then, since the shape of the boundaries of Z'_∞ and Z_∞ are identical, the stochastic bifurcation process starting from A must have the same distribution over payoffs as the process starting from B . Thus, the uncertainty governing the expected payoffs are identical at points A and B , except for the fact that B has a higher current value δ_t . This contradicts the hypothesis that a trader is indifferent between the two actions both at

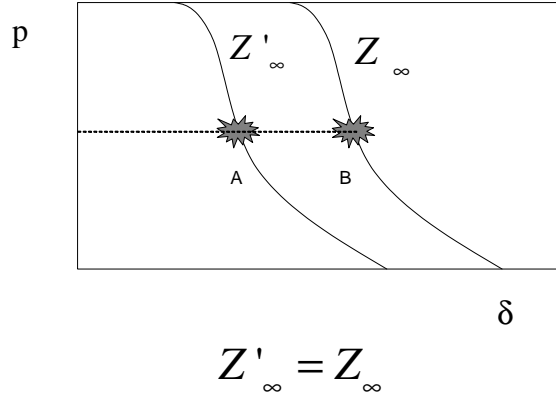


Figure 3: If a trader is in A and thinks that other traders buy dollar if and only if they are at the right of Z'_∞ , then future price trajectories will just be horizontal translations of the trajectories realized when a trader is in B and thinks that other traders buy dollar if and only if they are at the right of Z_∞ . Thus a trader can be indifferent between both situations only if A and B correspond to the same δ and thus $Z_\infty = Z'_\infty$.

A and at B . If she were indifferent at A , she would strictly prefer to hold dollar at B , and if she is indifferent at B , she would strictly prefer to hold yen at A . But we constructed A and B so that traders are indifferent. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no “gap”, and we must have $Z'_\infty = Z_\infty$. In other words, we have the situation depicted in figure 1 as claimed.

Interpreting the Results

Proposition 5 demonstrates the impact of adding some uncertainty to the carry δ_t . The multiplicity of equilibria reported in the previous section resulted from the feature that, if the fundamentals were fixed and known, then one cannot rule out all other players trading in one direction, provided that the fundamentals were consistent with such a strategy. However, the intro-

duction of shocks changes the picture radically. Since δ_t follows a Brownian motion, while traders must wait for their trading opportunities, the traders are far less nimble than the shifts in the fundamental value itself. Thus, choosing to hold dollar *versus* yen entails a substantial degree of commitment over time to fix one's trading strategy.

Suppose that the (δ, p) pair is close to a dominance region, but just outside it. If δ is fixed, it may be possible to construct an equilibrium for both actions, but when δ moves around stochastically, it may wander into the dominance region between now and the next opportunity that the trader gets to trade. This gives the trader some reason to hedge her bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on. Essentially, adding Brownian shocks to the carry enables us to extend to the two dimensional space of (δ, p) pairs the dominance argument we showed in our benchmark result without funding externalities.

That $Z(\cdot)$ is nonincreasing implies that price paths exhibit hysteresis. If the dynamic system (δ_t, p_t) is in the area where buying is dominant ($\delta_t > Z(p_t)$), then the buy pressure takes the system away from $Z(\cdot)$, making the continuation of a bullish market even more likely, all else equal. The reader may wonder whether Brownian excursions completely swamp this effect at the proximity of $Z(\cdot)$, so that runs never develop and the system is "trapped" in the vicinity of $Z(\cdot)$. The next proposition shows that it is not the case provided μ and σ are sufficiently small.

Proposition 7

Assume that the system is in the state (p_t, δ_t) such that

$$\delta_t = Z(p_t).$$

For any $\varepsilon > 0$, as $\mu, \sigma \rightarrow 0$, the last time at which the system hits $Z(\cdot)$ before

p_{t+u} becomes larger than $1 - \varepsilon$ or smaller than ε tends to t in distribution.
The probability that the price will go up tends to $1 - p_t$.

Proof Theorem 2 in Burdzy, Frankel, and Pauzner (1998).■

The broad intuition for this result is that when μ and σ are small, the price path around $Z(\cdot)$ is mostly driven by changes in p : Liquidity flows are more important than changes in the carry. The speed at which the price goes up is $\lambda(1 - p_t)$, while it decreases with speed $-\lambda p_t$. The price path does not revert to $Z(\cdot)$ once it has headed off towards one direction, and the ratio of the probabilities to go up or down is the ratio of the speeds at which the price goes in each direction. If the system hits $Z(\cdot)$ when p_t is very high (low), then it is most likely to bifurcate downwards (upwards). Thus, for μ, σ sufficiently small, the price paths will exhibit “runs”, or long series of identically signed returns, with sudden and large reversals. Very small variations in traders’ opinions may some times trigger very large fluctuations, depending on whether the system is close to Z or not.

Such trajectories, in which a currency appreciates at a decreasing rate for a long time after an interest rate hike, and then eventually crashes, are reminiscent of the “delayed overshooting” in FX markets documented in Bacchetta and van Wincoop (2007). They find a persistence of the forward discount bias: A current positive shock on δ_t predicts excess dollar returns at future dates, but the slope in the regression decreases to 0 or even becomes positive over longer horizons. Interestingly, the “positive feedback trading” that we need to generate this phenomenon can be small in the sense that h' can be arbitrarily small.

More generally, these price paths share features with the rational bubbles that burst stochastically in Blanchard and Watson (1982). The equilibrium is unique, however, and these endogenous “slow booms and sudden crashes” are generated by purely static externalities in an economy with finite wealth,

and a finite horizon. The probabilities of reversals in our model are intrinsic, and depend on the magnitude of the deviation of the price from the “fundamental” value.

4 Discussion and Conclusion

*Deterministic day of correction*⁵

Assuming a constant ρ is meant to preserve time-homogeneity. Our results do not depend on this restriction. In fact, our results would hold even with a deterministic "day of reckoning". Since the positive funding externalities that traders create for each other are static, and not intertemporal, then for a sufficiently large λ the type of equilibrium described in Proposition 5 would still prevail at dates bounded away from the day of reckoning.

Binary portfolio choice

An important restriction in our setup is that traders choose only between being long one dollar asset or not. Our model would quickly become intractable with a larger number of options since one would have to study each pair of options and keep track of the fraction of the traders in each position. But it is worthwhile emphasizing that the absence of short sales of dollar plays no role here. We might as well have assumed that one of the options was to short dollar: only the differential return between the two options matters.

Carry trades in other markets

The main intuition that our model illustrates may be described in general terms as follows. An asset whose price is sufficiently sensitive to the flow of funds from a group of speculators would give rise to carry trades and speculative dynamics if i) short-term funds are in sufficiently elastic supply, and ii) the speculators are sufficiently leveraged that they create positive

⁵We are indebted to our referee for this remark.

funding externalities for each other. While we view this set of assumptions as particularly plausible in the FX market, we also believe that our model can describe the destabilizing impact of carry trades in other markets such as the bond market.

We have developed a dynamic asset pricing model in which speculators face a coordination problem because of destabilizing margins. Using recent methodological advances in game theory, we obtain a unique equilibrium price that has appealing qualitative features: It implies a risk premium that is time-varying and countercyclical. The required return decreases in a highly non-linear fashion with respect to the value of the fundamentals. A natural route for future research is to improve the tractability of this baseline model in order to enrich it, and check its ability to generate quantitative features of empirical risk premia.

References

- [1] Dilip Abreu; Markus K. Brunnermeier, "Bubbles and Crashes", *Econometrica*, Vol. 71, No. 1 (2003), pp. 173-204
- [2] Philippe Bacchetta; Eric van Wincoop, "Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle, working paper, (2007)
- [3] David K. Backus; Silverio Foresi; Chris I. Telmer, "Affine Term Structure Models and the Forward Premium Anomaly", *The Journal of Finance*, Vol. 56, No. 1 (2001), pp. 279-304.
- [4] Richard Bass; Krzysztof Burdzy, "Stochastic bifurcation models," *Annals of Probability*, Vol. 27 (1999), pp. 50-108.
- [5] Olivier J. Blanchard; Mark W. Watson, "Bubbles, Rational Expectations and Financial Markets," NBER Working Paper No. W0945, July 1982.
- [6] Francis Breedon, "Market Liquidity under Stress: Observations in the FX Market" Lehman Brothers working paper, (2001) presented at the BIS workshop on market liquidity <http://www.bis.org/publ/bispap02g.pdf>
- [7] Markus Brunnermeier; Lasse Pedersen, "Market Liquidity and Funding Liquidity," working paper (2007).
- [8] Krzysztof Burdzy; David Frankel; Ady Pauzner, "Fast Equilibrium Selection by Rational Players Living in a Changing World.," *Econometrica*, Vol. 68 (2001), pp. 163-190.
- [9] Krzysztof Burdzy; David Frankel; Ady Pauzner, "On the Time and Direction of Stochastic Bifurcation." In B. Szyszkowicz, ed. *Asymptotic Methods in Probability and Statistics: A Volume in Honour of Miklos Csorgo*. Holland: Elsevier (1998).

- [10] Craig Burnside; Martin Eichenbaum; Isaac Kleshchelski; Sergio Rebelo, "The Returns to Currency Speculation", (2007), NBER Working Paper No. 12489
- [11] H. Henry Cao; Martin D. Evans; Richard K. Lyons, "Inventory Information", *The Journal of Business*, Vol. 79 (2006), pp. 325-363.
- [12] Hans Carlsson; Eric van Damme, "Global Games and Equilibrium Selection", *Econometrica*, Vol. 61, No. 5 (1993), pp. 989-1018
- [13] Darrell Duffie; Nicolae Gârleanu; Lasse Pedersen, "Securities Lending, Shorting, and Pricing," *Journal of Financial Economics*, Vol. 66 (2002), pp. 307-339.
- [14] David Frankel; Ady Pauzner, "Resolving Indeterminacy in Dynamic Settings: The Role of Shocks," *Quarterly Journal of Economics*, Vol. 115 (2000), pp. 283-304.
- [15] Milton Friedman *Essays in Positive Economics*, (1953) University of Chicago Press.
- [16] Kenneth A. Froot; Richard H. Thaler, "Anomalies: Foreign Exchange," *The Journal of Economic Perspectives*, Vol. 4, No. 3 (1990), pp. 179-192
- [17] Stephen Morris; Hyun Song Shin, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks", *The American Economic Review*, Vol. 88, No. 3 (1998), pp. 587-597